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OF THE

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OF

LONDON.

FOR THE YEAR MDCCCLXIV.

VOL. 154.

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MDCCCLXV.

CONTENTS

OF VOL. 154.

APPENDIX.

LIST OF ILLUSTRATIONS.

ADJUDICATION of the MEDALS of the ROYAL SOCIETY for the year 1864 by the PRESIDENT and COUNCIL.

The COPLEY MEDAL to CHARLES DARWIN, Esq., F.R.S., for his important Researches in Geology, Zoology, and Botanical Physiology.

The RUMFORD MEDAL to Professor JOHN TYNDALL, F.R.S., for his Researches on the Absorption and Radiation of Heat by Gases and Vapours.

A ROYAL MEDAL to JACOB LOCKHART CLARKE, Esq., F.R.S., for his Researches on the Intimate Structure of the Spinal Cord and Brain, and on the Development of the Spinal Cord, published in five Memoirs in the Philosophical Transactions and in other writings.

A ROYAL MEDAL to WARREN DE LA RUE, Esq., F.R.S., for his Observations on the Total Eclipse of the Sun of 1860, and for his improvements in Astronomical Photography.

Professor J. TYNDALL'S Paper, entitled "Researches on Radiant Heat,—Fifth Memoir," was appointed as the BAKERIAN LECTURE.

The CROONIAN LECTURE was delivered by Professor HERMANN HELMHOLTZ, For. Mem. R.S.: it was entitled "On the Normal Motions of the Human Eye in relation to Binocular Vision."

ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the

thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

The Meteorological Journal hitherto kept by the Assistant Secretary at the Apartments of the Royal Society, by order of the President and Council, and published in the Philosophical Transactions, has been discontinued. The Government, on the recommendation of the President and Council, has established at the Royal Observatory at Greenwich, under the superintendence of the Astronomer Royal, a Magnetical and Meteorological Observatory, where observations are made on an extended scale, which are regularly published. These, which correspond with the grand scheme of observations now carrying out in different parts of the globe, supersede the necessity of a continuance of the observations made at the Apartments of the Royal Society, which could not be rendered so perfect as was desirable, on account of the imperfections of the locality and the multiplied duties of the observer.

A List of Public Institutions and Individuals, entitled to receive a Copy of the Philosophical Transactions of each year, on making application for the same directly or through their respective agents, within five years of the date of publication.

Observatories.

Armagh.
Cape of Good Hope.
Dublin.
Edinburgh.
Greenwich.
Kew.
Liverpool.
Madras.
Oxford (Radcliffe).

Institutions.

Barbadoes.... Library and Museum.
Calcutta ..... Asiatic Society.
Geological Museum.
Cambridge ... Philosophical Society.
Cape Town .. South African Library.
Dublin ...... Royal Dublin Society.
Royal Irish Academy.
Edinburgh ... Royal Society.
London ...... Admiralty Library.
Chemical Society.
College of Surgeons.
Entomological Society.
Geological Society.
Geological Survey of Great Britain.
Horticultural Society.
Institute of British Architects.
Institution of Civil Engineers.
Linnean Society.
London Institution.
Royal Asiatic Society.
Royal Astronomical Society.
Royal College of Physicians.
Royal Geographical Society.
Royal Institution of Great Britain.
Royal Medical and Chirurgical Society.
Royal Society of Literature.
Society of Antiquaries.
Society of Arts.
The Queen's Library.
The Treasury Library.
United Service Museum.
Zoological Society.
Malta .... Public Library.
Manchester .. Literary and Philosophical Society.
Melbourne .. University Library.
Montreal .... McGill College.
Oxford ...... Ashmolean Society.
Radcliffe Library.
Swansea ..... Royal Institution.
Sydney ...... University Library.
Woolwich .... Royal Artillery Library.

Belgium.

Brussels ..... Académie Royale de Médecine.
Royal Academy of Sciences.

Denmark.

Altona ...... Royal Observatory.
Copenhagen .. Royal Society of Sciences.

France.

Montpellier .. Academy of Sciences.
Faculté de Médecine.
Paris .... Academy of Sciences.
Dépôt de la Marine.
Ecole des Mines.
Geographical Society.
Geological Society.
Jardin des Plantes.
Société d'Encouragement pour l'Industrie Nationale.

Germany.

Berlin .... Royal Academy of Sciences.
Society of Experimental Philosophy.
Brünn .... Naturforschender Verein.
Dresden .... Cæsarean Acad. of Naturalists.
Frankfort .... Natural History Society.
Giessen .... University.
Göttingen .... University.
Hamburg .... Naturwissenschaftlicher-Verein.
Königsberg ...... Königlichen Physikalisch Ökonomischen Gesellschaft.
Leipzig .... Royal Saxon Society of Sciences.
Mannheim ...... Observatory.
Munich .... Royal Academy of Sciences.
Prague .... Bohemian Society of Sciences.
Vienna .... Imperial Academy of Sciences.
Geologische Reichsanstalt.
Würzburg .... Physico-Medical Society.

Hungary.

Pesth .... Hungarian Academy of Sciences.

Italy.

Bologna .... Academy of Sciences.
Catanea .... Accademia Gioenia di Scienze Naturali.
Milan .... Institute of Sciences, Letters and Arts.
Modena.......... Italian Society of Sciences.
Naples .... Institute of Sciences.
Palermo .... Academy of Sciences and Letters.
Rome .... Academy de' Nuovi Lincei.
Collegio Romano.
Turin .... Royal Academy of Sciences.
Venice .... Institute of Sciences, Letters, and Arts.

Java.

Batavia .... Batavian Society of Sciences.

Netherlands.

Amsterdam ...... Royal Institute.
Haarlem .... Dutch Society of Sciences.
Rotterdam ...... Batavian Society of Experimental Philosophy.

Portugal.

Lisbon .... Royal Academy of Sciences.

Russia.

Kazan .... Imperial University.
Moscow.......... Imperial Society of Naturalists.
Public Museum.
Pulkowa .... Observatory.
St. Petersburg .... Imperial Academy of Sciences.

Spain.

Cadiz............ Observatory.
Madrid .... Royal Academy of Sciences.

Sweden and Norway.

Christiania ...... Royal University.
Drontheim ...... Royal Society of Sciences.
Gottenburg ...... Kongl. Vetenskaps och Vitterhets Samhälle.
Stockholm........ Royal Academy of Sciences.

Switzerland.

Bern .... Allg. Schweizerischen Gesellschaft.
Geneva .... Société de Phys. et d'Hist. Naturelle.

Transylvania.

Klausenburg...... Society of the Transylvanian Museum.

United States.

Albany .... New York State Library.
Boston .... American Academy of Sciences.
Newhaven (Conn.). The Editors of the American Journal.
Cambridge ...... Harvard University.
Philadelphia...... Academy of Natural Sciences.
American Philosophical Society.
Washington ...... Smithsonian Institution.
Observatory.

A List of Public Institutions and Individuals, entitled to receive a Copy of the Astronomical Observations (including Magnetism and Meteorology) made at the Royal Observatory at Greenwich, on making application for the same directly or through their respective agents, within two years of the date of publication.

Observatories.

Altona.
Armagh.
Berlin.
Breslau.
Brussels.
Cadiz.
Cambridge.
Cape of Good Hope.
Coïmbra.
Copenhagen.
Dorpat.
Dublin.
Edinburgh.
Helsingfors.
Königsberg.
Madras.
Mannheim.
Marseille.
Milan.
Munich.
Oxford.
Palermo.
Paris.
Seeberg.
Tübingen.
Turin.
Vienna.
Wilna.

Institutions.

Aberdeen	University.
Berlin	Academy of Sciences.
Bologna	Academy of Sciences.
Boston	American Academy of Sciences.
Brunswick, U.S.	Bowdoin College.
Cambridge	Trinity College Library.
Cambridge, U.S.	Harvard University.
Dublin	University.
Edinburgh	University.
	Royal Society.
Glasgow	University.
Göttingen	University.
Leyden	University.
London	Board of Ordnance.
	Royal Institution.
	Royal Society.
	The Queen's Library.
Oxford	Savilian Library.
Paris	Academy of Sciences.
	Board of Longitude.
	Dépôt de la Marine.
Pesth	Hungarian Academy of Sciences.
Philadelphia	American Philosophical Society.
St. Andrews	University.
St. Petersburg	Imperial Academy.
Stockholm	Royal Academy of Sciences.
Upsal	Royal Society.
Waterville, Maine (U.S.)	College.

Individuals.

Christie, S. H., Esq.	Twickenham.
Lubbock, Sir John William, Bart.	London.
Lowndes' Professor of Astronomy	Cambridge.
Plumian Professor of Astronomy	Cambridge.
President of the Royal Society	London.
Smyth, Admiral W. H.	Aylesbury.
South, Sir James	Kensington.
The Earl of Rosse	Parsonstown.

A List of Observatories, Institutions and Individuals, entitled to receive a Copy of the Magnetical and Meteorological Observations made at the Royal Observatory, Greenwich.

Observatories.

Bombay	Lieut. P. W. Mitcheson.
Cambridge, United States	Prof. J. Lovering.
Christiania	C. Hansteen.
Gotha	P. A. Hansen.
Heidelberg	M. Tiedemann.
Kew	B. Stewart.
Kremsmünster	P. A. Reslhuber.
Leipzig	Professor Möbius.
Lisbon	Senhor da Silveira.
Marburg	Professor Gerling.
Prague	K. Jelinek.
Stockholm	Professor H. Selander.
Toronto	Professor Kingston.
Upsal	Professor Svanberg.
Washington	Capt. Gilliss.

Institutions.

Bombay	Geographical Society.
Bonn	University.
Boston, U.S.	The Public Library (late Bowditch).
Cambridge	Philosophical Society.
Cherkow	University.
Falmouth	Royal Cornwall Polytechnic Society.
London	House of Lords, Library.
	House of Commons, Library.
	King's College.
	Royal Society.
	University College, Library.
Oxford	Radcliffe Observatory.
Paris	Meteorological Society.
St. Bernard	Convent.
Washington	Smithsonian Institution.
Woolwich	Office of Mag. and Met. Publication.

Individuals.

Bache, Dr. A. D.	Washington.
Buys Ballot, Dr.	Utrecht.
Dove, Prof. H. W.	Berlin.
Erman, Dr. Adolph	Berlin.
Fox, R. W., Esq.	Falmouth.
Harris, Sir W. Snow	Plymouth.
Hoskins, Dr. S. E.	Guernsey.
Kaemtz, Prof. L. F.	Dorpat.
Kreil. Prof. K.	Vienna.
Kupffer, A. T.	St. Petersburg.
(Twelve copies for distribution to the Russian Mag. and Met. Obs.)	
Lloyd, Rev. Dr.	Dublin.
Loomis, Prof. E.	Yale College, Newhaven (Conn.).
Phillips, Prof. John	Oxford.
Quetelet, A.	Brussels.
Sabine, Major-General, R.A.	London.
Senhor de Souza	Coimbra.
Vernon, G. V., Esq.	Manchester.
Wartmann, Prof. Elie	Geneva.
Younghusband, Col., R.A.	Woolwich.

I. *Catalogue of Nebulæ and Clusters of Stars.*

By Sir John Frederick William Herschel, *Bart.*, *F.R.S.*

Received October 16,—Read November 19, 1863.

Introduction.

The study of the Nebulæ has, within the last quarter of a century, attracted much more of the attention of observers than heretofore—as well on account of the singularity of the phenomena presented by many of these objects, as in consequence of the increased optical power of the telescopes which the skill and industry of modern inventors and artists have placed within their reach. The brighter nebulæ cannot be viewed to any advantage, and the fainter cannot be seen at all, except by the aid of telescopes of large aperture; and, thanks to the exertions of Lord Rosse, Mr. Lassell, Messrs. Nasmyth and De la Rue in England, and Messrs. Steinheil, Foucault, and Porro in Germany and France, as regards reflecting telescopes, and to those of Fraunhofer, Merz, Cauchoix, Clarke, Cook, Secretan, Ross, and Dallmeyer as regards refractors; instruments of abundantly sufficient optical capacity not only to repeat and verify the earlier observations, but to disclose new and more interesting features in many cases, have now come into the hands of many observers, both professional astronomers and amateurs, and may be had by any one who is willing to incur a cost which may be considered moderate when it is remembered that instruments of similar dimensions and goodness could not be obtained fifty years ago at any price. In consequence we find a continually increasing attention directed to this department of astronomy. Not to insist on the observations of the Earl of Rosse and Mr. Lassell with their transcendent reflectors, we find a systematic examination and review of them undertaken by M. D'Arrest in the year 1855, by the aid of a refractor of 6-feet focal length and 4½ inches aperture in the Leipzig Observatory, whose results, consisting in the carefully determined places, by repeated observations, of about 230 nebulæ, were published in 1856, in a work entitled "Resultate aus Beobachtungen der Nebelflecken und Sternhaufen" (Erste Reihe, Leipzig). This review has since been carried on by the same excellent astronomer, with the great refractor by Merz of 11 inches in aperture and 16-feet focus, erected in the year 1861 at the Royal Observa-

tory of Copenhagen. Again, from the Observatory of the Collegio Romano, under the direction of Signor SECCHI, have emanated many valuable observations, and from that at Harvard College, Cambridge, U.S., under the late and present Professors BOND, some of the most striking pictorial representations of particular nebulæ which we possess. Neither ought a short but very valuable memoir by the late E. MASON, printed in the 7th volume of the Memoirs of the American Academy of Arts and Sciences, to be passed in silence; containing as it does a very elaborate and minute examination, and some excellent delineations of several highly interesting nebulæ, particularly those in the great nebulous region of Cygnus. To M. AUWERS also we owe many accurate and valuable observations, besides a Catalogue comprising the whole series of Sir WILLIAM HERSCHEL'S nebulæ arranged in order of right ascension and reduced to a common epoch, of which more hereafter. Should the efforts which are now making to procure for the University of Melbourne in Australia a reflector of the first magnitude prove, as is to be hoped, successful, it is understood that one of the principal uses to which it will be devoted will be the examination and exact delineation of the numerous and wonderful objects of this class which the southern hemisphere presents.

These circumstances, but more especially the last-mentioned, render it extremely desirable to have presented in one work, without the necessity of turning over many volumes, a general catalogue of all the nebulæ and clusters of stars actually known, both northern and southern, arranged in order of right ascension and reduced to a common and sufficiently advanced epoch which may serve as a general index to them, and enable an observer at once to turn his instrument on any one of them, as well as to put it in his power immediately to ascertain whether any object of this nature which he may encounter in his observations is new, or should be set down as one previously observed. For want of such a general catalogue, in fact, a great many nebulæ have been, from time to time, in the 'Astronomische Nachrichten' and elsewhere, introduced to the world as new discoveries, which have since been identified with nebulæ already described and well known. Many a supposed comet, too, would have been recognized at once as a nebula, had such a general catalogue been at hand, and much valuable time been thus saved to their observers in looking out for them again.

Besides these there are other considerations which have weighed with me in undertaking the task of compiling such a general catalogue. Having, in the course of my own observations, received the greatest possible assistance from the possession of a Manuscript Catalogue of all the nebulæ and clusters discovered by my Father, brought to the common epoch 1800·0, and arranged in zones of 1° in breadth in polar distance, by his sister the late Miss CAROLINA L. HERSCHEL, it seemed to me nothing less than a debt of gratitude, not merely to acknowledge that assistance, but to avail myself still further of it to complete the list of his nebulæ by supplying from that catalogue the places of all those nebulæ among them which had escaped my own observation (a very numerous list), and by inserting from it all those places of nebulæ observed by myself which were deficient in either element (of R.A. or P.D.), or in which I had reason to apprehend

greater errors than those which probably affected her results. This I have accordingly done. But to do it effectually, and at the same time to effect a thoroughly correct identification of the objects in my catalogues with those of the older series, involved, as a necessary preliminary step, the reduction to 1830·0 of the whole of her catalogue, an operation in which I received the assistance of my sons; the computations being executed for each nebula in duplicate and checked by myself, and which, taken leisurely, zone by zone, as time and circumstance permitted, proved less onerous and wearisome than might have been expected. The Catalogue thus reduced to the same epoch as my own, afforded the means of detecting and rectifying a great many errors of nomenclature in the latter. And it was in the course of this part of the inquiry, in which many cases of considerable intricacy and difficulty occurred (as will be evident on a perusal of the notes appended to this Catalogue), and in which it became necessary to recur both to the original sweeps and to a series of registered extracts from them (the nature of which will be more distinctly stated hereafter), that I learned fully to appreciate the skill, diligence, and accuracy which that indefatigable lady brought to bear on a task which only the most boundless devotion could have induced her to undertake or enabled her to accomplish.

Arrived at this stage—that is to say, the mean results of all the observations in my own Catalogues taken, and all the deficient or imperfectly observed nebulæ in my Father's list supplied, as above stated, and the whole arranged, not in zones, but in general order of right ascension,—it then became necessary, in order to produce a work available for future observation, to bring the whole up to a still more advanced epoch. The work required for this purpose, calling no longer for any discussion, or collation of the original observations or registers, but being one of simple arithmetical computation from a definite formula—the Royal Society, at my application, very liberally undertook to supply, from the funds at their annual disposal, the amount necessary to procure its execution by an experienced computer (Mr. KERSCHNER, one of the occasional computists for the Royal Observatory of Greenwich). This work the Astronomer Royal most obligingly offered to superintend, affording at the same time his advice as to the general principle on which the computation should be conducted. The plan suggested by him and adopted in effect was this. Each object in the Catalogue was first roughly brought up to the year 1880 by the application of approximate precessions in R.A. and P.D. The places so obtained were then employed to compute the exact precessions in both by the usual formulæ, with coefficients for the year 1880·0, viz.

$$\text{Precession in R.A.} = 3^{s}{\cdot}072 + 1^{s}{\cdot}337 \,.\, \sin \text{R.A.} \cot \text{P.D.}$$

$$\text{Precession in P.D.} = -20''{\cdot}06 \cos \text{R.A.}$$

And the precessions, so calculated, were then used to bring up the places from 1830 to 1860, the epoch of the Catalogue; so that, the places being given for 1860 and the precessions for twenty years in advance, the application of those precessions to those places shall give dependable places for any year up to the year 1930, at which time the small error in excess or defect of the true precession consequent on using the fifty years'

antecedent place of the object will be exactly compensated by the further change of place in the same direction in the *subsequent* fifty. Two cases of excessive proximity to the poles, northern and southern, viz. those of the nebulæ Nos. 2043 and 1652 of the present Catalogue, are excepted, the precessions changing so rapidly, and with so much deviation from uniformity, that a rigorous computation, at least in R.A., will always be necessary. In the case of No. 2043, the effect of precession in the thirty years from 1830 to 1860 has been to change the R.A. from $2^h\ 32^m$ to $10^h\ 8^m$.

This computation was completed, and a fair copy of the resulting places, arranged *de novo* in their order of R.A. for 1860, forwarded to me on the 6th of February last (1863). The nomenclature of the objects having in the interim been settled satisfactorily by myself, and a description of each nebula, from a careful comparison of all the descriptions given, prepared, it remained only to fill in the columns left blank for these and the other necessary particulars, and to complete the Catalogue by the insertion in their proper places of the places and descriptions of all such other nebulæ, non-observed by either my Father or myself, similarly reduced, of which I could gather any accounts. These will be found enumerated further on in the "Explanation and arrangement of the Catalogue."

On the 23rd of February last, while engaged in this work, I received, by the kindness of the Astronomer Royal, a copy of the important work of M. Auwers before alluded to, entitled "William Herschel's Verzeichnissen von Nebelflecken und Sternhaufen, bearbeitet von Arthur Auwers. Königsberg, 1862," of whose existence this was my first notice. It contains a complete and most elaborate reduction to 1830, from the observed differences in R.A. and P.D. with known stars, recorded in the Philosophical Transactions, of all the nebulæ and clusters in my Father's three Catalogues; together with a separate catalogue of all those collected by Messier from his own observations, or those of Mechain and others (101 in number), similarly reduced; another of Lacaille's southern nebulæ, and one of 50 " new nebulæ," comprising nearly all those observed by other astronomers (Lord Rosse excepted) in this hemisphere—all brought up to the same epoch.

It may be readily supposed that I lost no time in comparing my own previous work with this of M. Auwers; the places of which having been obtained by the aid of far better and more dependable catalogues of stars, to give the true positions of the zero-points or determining stars in the differential observations, as well as of more exact precessions, and doubtless, a much more systematic process of treatment, would be entitled, *observation for observation*, to be considered as representing the original sweeps more faithfully than could be expected from my own preparatory catalogue. On the other hand, however, the Zone Catalogue from which that was derived possessed the advantage of having been deduced, not from a single difference of R.A. and P.D. between each nebula and a single determining star, but from *all* the observations of each nebula; often in many different sweeps, and in the same sweep often from more than one star; thus eliminating, no doubt, a great deal of casual error. In that catalogue, too, as in my own catalogues of 1833 and that of the southern nebulæ, the individual results of each observation, or, to speak more exactly, of each differential comparison, is separately

recorded, so that any suspiciously large deviation from the mean of all may be at once noticed and traced to its origin in the sweeping books. My reduction was of course based on the means of all these (rejecting such as were obviously and grossly faulty), and might therefore, *pro tanto*, be regarded as of superior authority. This consideration, joined to that before adduced, decided me to retain those places in the present Catalogue which had been derived from this source, except in a few instances (specified in the notes) when it proved, by careful examination of the causes of discordance, that actual mistakes had been committed. And I must not omit to add that the comparison so instituted with M. AUWERS's results has led me to the detection of several grave errors in my own work which would certainly have otherwise escaped notice (and in some cases have caused the loss of future observations by missetting the telescope), and whose rectification has added materially to its value. On the other hand, as no human work is perfect, I have been led to notice some errors in M. AUWERS's work itself, which are set down in a list of *errata* and *corrigenda* at the end of this Catalogue; and besides, a good many cases in which, owing to mistakes in the printed catalogue in the volumes of the Philosophical Transactions (many of which stood corrected in MS. in the margin of the copy of those Transactions in my possession, and many more have been silently detected and rectified by Miss C. H. in her subsequent computations), his calculations have been founded on erroneous data, and have therefore led him to assign erroneous places to the objects so affected. Thus on every account the result has been what may be considered a complete expurgation of both our catalogues.

It remains for me to say a few words on the way in which the reduction to 1860 and the calculation of the precessions have been performed by Mr. KERSCHNER, the computist employed by the Astronomer Royal for that purpose. The whole work has been executed on printed forms, which being preserved may at any time be referred to. Since error in computation, however practised the computer, and however checked, is always possible, and occasional error of copying, especially when the order of the entries has to be rearranged, is absolutely unavoidable, I considered it incumbent on me to recalculate, *seriatim* from my original MS. Catalogue for 1830, and taking for granted the precessions set down in the fair copy, for 1880, the places both in R.A. and P.D. of every object included in the Catalogue; keeping an eye meanwhile to the precessions themselves, and their signs, to seize the least indication of error in that quarter. It would have been too laborious to recompute these. As for the precessions in P.D., their regular progression of itself ensures their correctness, as far at least as the integer seconds and the first decimal place. A pretty considerable number of errors (most of them of little moment) was thus detected and corrected—not more, however, than might reasonably be expected in the work of the most expert computist in so extensive a work, consisting of between nine and ten thousand computed entries (taking both elements), and traceable moreover in many instances to obvious misreading, and in some to actual misentry on my part, of figures in the original MS., which but for this further examination would also have escaped notice altogether.

The correction of these and the other errors already spoken of necessitated, in a great

many instances, a change in the order of R.A., and a consequent erasure and interlineation in the MS. The introduction, too, of the other nebulæ (those of M. AUWERS's catalogue of "novæ," those communicated to me for insertion by M. D'ARREST, and those noticed by Lord ROSSE in his memoir of 1861, amounting altogether to 433 objects) necessitated many more interlineations, often occurring very inconveniently, two or three together, in a way to disfigure the MS. considerably. Unfortunately, too, in the MS. itself the column headed "No. in the Catalogue," which I had intended to have been left blank till all the rest of the work was completed, had been filled in by the transcriber with a series of numbers in regular progression, from 1 to 4629, the actual number of lines of which it then consisted. This made it necessary to renumber the whole *ab initio* in red ink, striking out the former numbers, and thus producing a still more unsightly appearance. Under these circumstances, I debated whether or not to recopy the whole. But, to say nothing of the sacrifice of time (since I could have entrusted it to no other hand), I believe it impossible to copy so voluminous a mass of figures and abbreviated writing without numerous errors. And being satisfied, from the repeated and careful revision it has undergone, of its present correctness, and equally so that with ordinary care on the part of the compositor (should the Council of the Royal Society decide on printing it) no *mistake* can arise from any of the alterations and interlineations it contains, I have decided in favour of presenting it as it stands, with the exception of two sheets which it was absolutely necessary to recopy owing to the extreme closeness of the interlineations, the smallness of the writing, and the transpositions needed. These have each been twice carefully read with the original.

In presenting to the Royal Society this Catalogue, it will be accompanied by the following series of records and documents which it may become desirable hereafter to refer to in elucidation of any point which may arise respecting the history or reduction of such of the objects as occur in my Father's classes and numbers printed in the Philosophical Transactions, viz.—

1st. A series of "*register sheets*," in which are entered up *all* the observations of *each* nebula or cluster copied verbatim from the sweeps, the nebulæ, &c. being arranged in the order of their dates of discovery. These are the "register sheets" referred to in the notes on this Catalogue, and cited by their *general* (*i. e.* current) *number*, as H, 1; H, 2; ... H, 2508.

2nd. A similar set of register sheets of all the observations of each of MESSIER's nebulæ, arranged according to MESSIER's numbers.

3rd. A general index of the 2508 nebulæ in classes and numbers, to find the "general number" of each to facilitate reference to the register sheets. (This index was drawn up by myself.)

4th. An index list of the same nebulæ, &c. arranged according to the "general number," to find the class and number of each.

5th. A more complete ditto ditto, containing also the rough approximate R.A. and P.D. of each object for 1800, and the determining stars as in the Philosophical Transactions.

6th. A catalogue in zones of P.D. of all the said nebulæ and clusters arranged in each

zone in order of R.A., and reduced to the year 1800 by Miss C. L. Herschel, exhibiting the reduced result of each separate observation of each nebula; together with the determining star or stars in each case, and the differences of R.A. and P.D. from such star, with references to the current number of the sweep in which the observation is contained.

7th. The original sweeps with the 20-feet reflector at Slough in which the nebulæ were observed, contained in three small quarto and four folio volumes of MS.

All these manuscripts, with exception of the index No. 3, are in the original handwritings of my Father and his Sister, in most cases easily distinguishable, in some others not so readily. The Zone Catalogue No. 6 is entirely the autograph of the latter.

Explanation and arrangement of the Catalogue.

The Catalogue is arranged in twelve columns, of which the first contains the general or current number in order from 1 up to 5063, the total number of objects comprised, including six supplementary ones, whose insertion in their proper order in R.A. would have involved altering all the numbering both of the catalogue and the annotations, &c., and would have proved a source of confusion and unavoidable error. Nevertheless, to prevent their being overlooked by any observer who may consult the catalogue for the purpose of a general review of the nebulæ, or for the verification of a new one, their numbers are interpolated into the general series so as to catch the eye, and a reference made to the supplementary catalogue in each case in the column of descriptions.

Column 2 contains the numbers of those nebulæ of which observations are given in my two former catalogues, and those of the two nubeculæ; the numbers from 1 to 2307 inclusive being from that in Philosophical Transactions 1833, and from 2308 to 4021 from my Cape observations. Where a number in this column is enclosed in hooks thus [ ], it is taken from the Catalogue of Objects in the Nubecula minor in pp. 153 to 155 of that work. Where in parentheses thus ( ), from those in the Nubecula major, pp. 156 to 163.

Column 3 contains the classes and numbers of nebulæ as given by my Father in his three Catalogues in the Philosophical Transactions for 1786, 1789, and 1802. One only is omitted, viz. V. 35. It is an immense diffused nebulosity, extending from $5^h\ 27^m$ to $5^h\ 42^m$ in R.A., and from 98° 6′ to 87° 43′ in P.D. A special list of these great diffusions of nebula is given by M. Auwers in p. 42 of the work above cited.

Column 4 contains references to other authorities, and gives either the name of the first discoverer of the nebula, or a reference to the particular list or catalogue of nebulæ which has been taken as the authority for the place set down. The principal of these are —1st. The list of "new nebulæ" (Verzeichniss neuer Nebelflecke), in pp. 73 to 76 of the work of M. Auwers already cited. These are referred to in the following form:—Auw. N. 1, Auw. N. 2, &c. 2ndly. Under the form D'Arr. 1, 2, &c., are given a series of objects contained in a MS. list of 125 nebulæ, kindly communicated to me by their discoverer, M. D'Arrest, Director of the Royal Observatory of Copenhagen, and reduced by him to the epoch (1860·0) of this Catalogue, with their precessions for 1880. 3rdly. A great number of nebulæ cited under the form "R. novæ," whose places have been approxi-

mately obtained from the diagrams accompanied by micrometrical measures of position and distance, or from more loose and general indications contained in Lord ROSSE's paper in the Philosophical Transactions for 1861, the comparisons being in all cases made with those nebulæ in my Catalogue of 1833 whose numbers stand annexed to them in column 2, with an italic letter appended, thus:—

322, *a* R. nova; 319, *a* R. 3 novæ.

In cases of which latter kind it is intended to express merely that nebulæ to the number indicated, not otherwise identifiable, will be found on due search in the immediate neighbourhood of the place approximately set down. Lastly. The names of Professor G. P. BOND, Mr. S. COOLIDGE, and Mr. J. T. SAFFORD in this column of the supplementary list of nebulæ refer to the places of nebulæ and clusters in a list of objects of that description discovered at the Observatory of Harvard College, obligingly communicated to me by Professor BOND, Director of that establishment, too late for their introduction into the body of the Catalogue.

Besides these references, in which the places set down have been adopted from the catalogues above mentioned, column 4 also contains synonyms or identifications of objects observed by myself with those contained in MESSIER's lists communicated to the French Academy, or to the Connoissance des Temps for 1783 and 1784. These are cited by the number they bear in MESSIER's own list, thus, M. 1, M. 2, &c. They have, with very few exceptions, been observed and described by myself or my Father, and their places here set down are given as results from our observations. In the few excepted cases they are taken from M. AUWERS's catalogue already spoken of. The nebulæ also whose identity has been (sometimes satisfactorily, but for the most part very doubtfully) made out with objects in Mr. DUNLOP's Catalogue of Southern Nebulæ, are indicated by the letter Δ, thus, Δ. 169, &c. In a few cases, chiefly those of nebulous stars, planetary nebulæ, or very star-like objects, which have been set down as stars in catalogues of authority; these are also referred to by name and number in column 4.

Many of Mr. DUNLOP's nebulæ are contained in LACAILLE's catalogue, as also some of MESSIER's, but of that catalogue two objects only, not so identifiable, viz. Nos. 38 and 40 of M. AUWERS's catalogue of LACAILLE's nebulæ, have been considered as definitely enough described (nébuleuses sans étoiles) by that astronomer to be inlcuded in the present Catalogue.

Column 5 contains the Right Ascension in time for 1860·0 of each object in the Catalogue. When this is given to decimals of seconds, it is to be understood as having been brought up from the mean of the observations given in my former Catalogues, or from the mean of those (where not observed by myself) in Miss C. HERSCHEL's Zone Catalogue above mentioned *. When the R.A. is given only to the nearest minute or degree, it will of course be understood that the place is too loosely determined to render

* In some cases a careful subsequent revision of the catalogued observations *seriatim* has necessitated altering these R.A.'s by a few decimals of a second (seldom more) *after the process of reduction to* 1860. In all such cases the alteration has been applied *as a correction* to Mr. KERSCHNER's figures, so as not to disturb the amounts of precession allowed—a procedure perfectly legitimate and productive of no error. The same remark applies to col. 8.

further precision of statement other than illusory. This is the case with the greater part of those set down as "R. novæ."

Column 6 contains the precession, in seconds and decimals, in R.A. for 1880·0.

Column 7 contains the number of observations in R.A. which have been actually used in concluding the R.A. for 1830, from which that for 1860 has been computed. In all cases (unless where the contrary is especially indicated in a note, or otherwise as by the letters B.A.C. or A.S.C., Au., &c. inserted in place of a number in the column itself—which indicate that the R.A. is that of a star in one of those catalogues, or rests upon that other authority), the observations used for all objects included in my former catalogues are brought up from the data there registered, to the exclusion of all others; and in such cases (the vast majority) no parenthesis or other distinctive mark is applied. When, however, no satisfactory R.A. is there recorded, or when the R.A. is there expressly stated to have been set down from the "working list," the R.A. adopted is that brought up from the Zone Catalogue of C.H., and in such cases the number of observations used is enclosed in parentheses ( ). Dots attached (:) indicate some uncertainty in the R.A.; (::) a very considerable doubt, extending, perhaps, to a whole minute; ? and ?? express still wider limits of uncertainty. In those nebulæ of my Father's catalogues which have no number corresponding in column 2 (indicating the absence of any observations of my own), the places set down both in R.A. and Declination are those brought up from the Zone Catalogue of Miss C. H., and the numbers of observations on which they rely are set down in the appropriate column without any parenthesis or distinctive marks, the absence of any number in column 2 being a sufficient indication. In the case of M. D'Arrest's nebulæ, the numbers in column 6 enclosed thus [ ] indicate the number of his observations of the nebula employed by himself to give the place.

Columns 8, 9, and 10 contain, in like manner, the North Polar Distance for 1860, the precessions for 1880, and the numbers of observations used for P.D. in the case of each object; and the same remarks apply to these as to columns 5, 6, and 7.

In column 11 is given a short description of the nebula or cluster in abbreviated words, made out from an assemblage and comparison of *all* the descriptions of each object given in my Father's *and* my own observations. As regards the former, recourse was had, not to the printed account in the Philosophical Transactions (which gives only a single description), but to a series of manuscript sheets in the nature of a Register (and *as such cited* in the notes which follow this Catalogue), into which have been transcribed, *verbatim*, from the original sweeps, all the descriptive parts of each and every observation of each cluster or nebula in the order of their dates, and the data for computing their places, derived from the sweeps by applying the index and clock corrections pertaining to each. In this Register the nebulæ are entered, each with its class and number, and each on a separate sheet; the whole series being arranged, however, not in the order of their classes and numbers, but in the order of the dates of their discovery, from No. 1, corresponding to October 28, 1783, to No. 2508, corresponding to September 30, 1802. Of these, the first 2500 only are included in the catalogues com-

municated to the Royal Society; the other 8 are printed in the form of an Appendix to my Cape Catalogue, in p. 128 of the "Results of Observations," &c. A similar and separate Register in sheets has been kept for my Father's observations of MESSIER's nebulæ, and these have in like manner been collated with my own observations of the same objects in framing the ultimate, or, as it may be termed, the average description of each.

In making out these descriptions, it was found to a certain degree practicable, in the particulars of brightness, size, and extension, to make a kind of arithmetical approximation to a mean conclusion, by arranging the degrees of brightness, &c. in a progressive upward scale from 1 to 10, and taking a mean of these numbers in each case, as indicating the designating words to be finally adopted. Thus, taking the extreme degree of faintness when a nebula was declared to be "excessively faint," or "barely visible," or "hardly more than suspected" for 1, and "extremely" or "excessively bright" for 10, the intermediate degrees, such as *very faint*, *faint*, *considerably faint*, *pretty faint*, *pretty bright*, *considerably bright*, *Bright*, *very bright*, were denoted by the intermediate numbers 2, 3, 4, 5, 6, 7, 8, 9; and similarly for the scale of sizes, exchanging the words Small and Large for Faint and Bright. In the case of extension, the scale 1 to 10 was supposed arranged in the order, *Round*, *very little extended*, *elliptic* or *oval*, *considerably extended*, *pretty much extended*, *much extended*, *very much extended*, *extremely extended*, or *a long ray*. It is obvious that the qualifying words, such as "pretty" and "considerably," admit of a good deal of latitude of interpretation, and that, in reference to brightness or faintness, greatness or smallness, their meaning is rather relative than absolute; and especially, that as between *bright* or *faint*, and "considerably bright" or "considerably faint," for instance, there is so little real distinction of an absolute kind, that it is impossible to say which is to be accepted as indicating the superior degree. In the case of extension there is the same indistinctness as to precedence between the qualifying phrases "considerably" and "pretty much." Nicety, however, in this respect would be misplaced, when it is considered that when several descriptions of the same nebula, observed at different times, come to be compared, they can hardly ever be reconciled except by allowing to each qualification a latitude of meaning extending over several degrees of our arbitrary scale. In many instances, indeed, the discordance, or rather contradiction is so great, as to authorize a strong suspicion of variability in the object itself. In a few cases where, from the low altitude of the object in England, coupled with corresponding discordances of description, it was evident that it must have been seen to much greater advantage from the Cape station (as, for example, in that of h. 3375 = H. III. 754), additional weight has been attributed to the Cape observations.

In the descriptions, I have found it absolutely necessary to abstain from any specification of the estimated sizes of nebulæ or clusters in angular measure. In comparing estimations of this kind I find the discordance so great, and (to speak only of my own practice) so little evidence of adherence to any definite standard of estimation, that nothing but confusion would have arisen from introducing such estimates. Nevertheless, as in the use of such a catalogue as the present some guide is necessary for the

observer, to advertise him of what sort of object he may expect to see, the following scale may be taken as conveying a general idea of the magnitudes intended by the conventional words used. Thus, a round nebula of 3″ or 4″ in diameter would be called *extremely small;*

one of 10″ or 12″, *very small;*
20″ or 30″, *small*, or *considerably small;*
50″ or 60″, *pretty small*, or *pretty large;*
3′ or 4′, *considerably large*, or *large;*
8′ or 10′, *very large;*
20′ and upwards, *extremely large.*

In estimating clusters of stars (that is to say, of well separated and scattered stars) a wider acceptation must be understood, so that, for instance, a cluster of only 1′ in extent would be considered *extremely* or *very small*; one of 15′ or 20′ *large*, and one of 30′ or 40′ *very large.* This amplification of scale, however, must not be held applicable to those resolved or resolvable clusters of a "globular" character marked in the descriptions as ⊕, which must be understood as belonging to "nebulæ" and not to "clusters," so far as the conventional terms used in the descriptions are concerned. I should observe also, that when in making out the average appropriate phrase in size I have found any extravagant discordance between the estimate in words and that in figures, as, for instance, where a nebula has been described in words as *very large*, and the diameter then set down as 2′, a compromise has usually been made, and the word modified, as, for instance, to *large* or *considerably large.*

The abbreviations employed in the column of descriptions and elsewhere, in the notes, &c., are as follows:—

ab.	about.
alm.	almost.
am.	among.
app.	appended.
att.	attached.
Auw.	Auwers.
A.S.C.	Astronomical Society's Catalogue.
b.	brighter.
bet.	between.
biN.	binuclear.
bn.	brightest towards the north side.
bs.	brightest towards the south side.
bp.	brightest towards the preceding side.
bf.	brightest towards the following side.
B.	Bright.
Br.	Brisbane (Sir T.'s) Catalogue of Stars.
Bo.	Bode.
B.A.C.	British Association Catalogue.
c.	considerably.
co.	coarse, coarsely.
ch.	chevelure.
com.	cometic.
cont.	in contact.
C.	Compressed.
Cl.	cluster.
C.G.H.	"Results of observations, &c. at the Cape of Good Hope."
C.H.	Miss Carolina Herschel. When it occurs in column 4 it indicates that the object was discovered by her.
d.	diameter.
dist.	distance. distant.
dif.	diffused.
diffic.	difficult.
D.	double.
D'Arr.	D'Arrest.
Δ.	Dunlop.
def.	defined.
e.	extremely.

ee.	excessively.
er.	easily resolvable.
exc.	excentric.
E.	extended.
f.	following.
F.	faint.
g.	gradually.
gr.	group.
H.	Sir William Herschel.
h.	Sir John Herschel.
h.o.n.	list of omitted nebulæ in C.G.H.
i.	irregular.
inv.	involved.
	involving.
iF.	irregular figure.
l.	little (adv.).
	long (adj.).
L.	Large.
Lac.	Lacaille.
Lal.	Lalande.
Lass.	Lassell.
m.	much.
mm.	mixed magnitudes.
mn.	milky nebulosity.
mon.	monograph.
M.	Middle, or in the middle.
M.	(in col. 3) Messier.
Mess.	Messier.
n.	north.
neb.	nebula.
np.	north preceding.
nf.	north following.
nr.	near.
N.	nucleus, or to a nucleus.
o.	omitted.
ON.	omitted nebula.
p.	preceding.
p.	pretty (before F, B, L, S, &c.).
pg.	pretty gradually.
pm.	pretty much.
ps.	pretty suddenly.
P.	poor.
Pi.	Piazzi.
P.T.	Philosophical Transactions.
quad.	quadrilateral.
quar.	quartile.
r.	resolvable, barely (mottled as if with stars).
rr.	partially resolved—some stars visible.
rrr.	well resolved—clearly seen to consist of stars.
R.	round.
RR.	exactly round.
R. nova.	New nebula discovered by Lord Rosse.
R. MS.	Manuscript notes furnished by His Lordship.
Ri.	Rich.
R.	The Earl of Rosse.
s.	suddenly.
s.	south.
sp.	south preceding.
sf.	south following.
sc.	scattered.
st.	stars.
sev.	several
susp.	suspected.
sh.	shaped.
stell.	stellar.
S.	small.
sm.	smaller.
sw.	sweep.
Σ.	Struve.
tri-N.	tri-nuclear.
trap.	trapezium.
v.	very.
vv.	an intensive of v.
var.	variable.
W. H.	Sir W. Herschel.

Besides these abbreviations of words, the following arbitrary signs are used.

* a star; *10 a star of the 10th magnitude.

** a double star; *** a triple star.

! a remarkable object; !! very much so; !!! a magnificent or otherwise exceedingly interesting object.

? doubtful; ?? very doubtful, either as to accuracy of place or reality of existence, according to the column in which it occurs.

: , : : , see explanations already given.

△ a triangle. Forms a triangle with.

⊕ a globular cluster of stars.

○ a planetary nebula.

◎ an annular nebula.

st. 9 . . . . . Stars from the ninth (or other) magnitude downwards.

st. 9 . . . . 13 Stars from the ninth down to the 13th magnitude.

As examples of the interpretation and expansion of these abbreviations some examples are subjoined.

Ex. 1. pB; vL; vg, vsmbMN 15″; pmE 162°·3; "pretty Bright; very Large; at first very gradually, then very suddenly much brighter in the middle to a nucleus 15″ in diameter; pretty much extended—the position of the longer dimension micrometrically measured 162°·3 (*i. e.* reckoned from the north round to 162°·3 in the direction nfsp)."

The angles of position in all cases are to be understood as so reckoned. When decimals of degrees are annexed (or if integer, written decimally thus 151°·0), they have been micrometrically measured. If thus, E 0° or E 45°, E 90°, they mean only in or near the meridian, or parallel or oblique to the meridian from nf to sp, &c., as the case may be. If with a $\pm$ annexed, the position is from a more or less careful estimation.

Ex. 2. R; psbM ill def O; pB*10 125°·4, 70″; "Round, pretty suddenly brighter in the middle to an ill-defined planetary disc; has a pretty Bright star of the 10th magnitude, whose position measured *from* the centre of the nebula is 125°·4, and whose distance also from the centre is 70″ by estimation."

The relative situations of neighbouring stars or nebulæ are *invariably* to be understood as thus reckoned, *i. e.* taking the centre of the nebula or other object described as a starting-point or origin of angle or distance. Thus S*s will mean that a small star is south *of the nebula*, *np nr that a star is *near the nebula* in a north preceding direction from it; $*_*$4ˢf, 3′n, that a double star follows the centre of the nebula 4 seconds of time, and is 3′ to the north of it.

Ex. 3. Cl; pRi; pmC; L; st6, 10 ... 15. "A cluster; pretty rich; pretty much compressed; Large; consisting of stars one of which is of the 6th, and the rest from the 10th to the 15th magnitudes."

Attached or vicinary stars or small nebulæ are always placed at the ends of the descriptions. Thus ⊕sf means that the nebula described "has a globular cluster following and to the southward of it." When, however, the description of a cluster ends abruptly thus, $*_*$, it is to be understood that "the place taken is that of a conspicuous double star."

The 12th column of the Catalogue contains the number of times that each nebula has been observed by both my Father and myself, whether its place were taken or not, comprising all the cases in which the object has been seen, and whether described or not. Since attention has been drawn to the real or supposed variability of nebulæ, and since it can hardly be doubted that comets have occasionally been observed as nebulæ, this enumeration is not without its importance. In this column the abbreviation "mon" occasionally occurs. In such cases the nebulæ have been so often and diligently observed for the purpose of exact delineation or "monographing," that a special enumeration of the observations would be impossible or useless.

Finally, at the end of the line allotted to each nebula occur occasionally one or both of the marks * and †. The former refers to the notes appended to the Catalogue, the latter to the list of figured nebulæ in which the publications wherein are contained figures of the nebulæ are referred to by plate and figure—those at least which seem entitled, in the present state of astronomical instrument-making and pictorial representation, to be pointed out to the observer as conveying any idea of their appearance.

Notes on the Catalogue.

No.

12 h. 5. D'Arrest says, "h. II. positio certe erronea," but gives no indication of the correction required in R.A. or P.D.

29 h. 13; II. 241=II. 243. In P.T. the determining * is omitted, and in the statement of the places of these nebulæ, as well as of II. 239, 240, 242, and III. 199, there is much confusion, for the correction of which see the list of errata subjoined. Auwers has threaded the intricacies of this maze with singular felicity, but has been misled in the case of II. 243 into assigning to it a totally erroneous place ($22^h\ 48^m$ R.A., 73° 37′ P.D. 1830), and, in consequence, has not perceived its identity with II. 241.

78 II. 3. Auwers makes the P.D. of this neb. (1830)=99° 32′, from P.T., which places it 2°± *n* of 17 Ceti. C.H. makes it 1° 51′ *n* of the same star, or for 1830, 99° 42′. In fact H. has two observations of it, neither of them more than eyedrafts with neighbouring stars, and the P.D. is concluded graphically by C.H. from these diagrams.

88 III. 876. The P.D. of Auwers (81° 16′) is 1° wrong. The place given in P.T. is 1° 43′ *n* of 51 Piscium; so also in Register (H. 2296).

119 Auw. N. 4=D'Arr. 6. The place given is that brought up from D'Arrest's observations, the R.A. being set down only roughly in Auw.

132 h. 57=V. 20. Once looked for by Lord Rosse and not seen. Having been observed both by H. and h., there can be no doubt of its existence.

138 h. 61=h. 2345=V. 1. In h.'s sweep 733 the position reading is set down as 324°·5. This is in contradiction with a diagram made at the time, and is an obvious mistake for 234°·5, which =180°+54°·5, agreeing well with the diagram and with 2 obs. of W.H., in both of which it is described as "*nf* to *sp*." There is also an erratum in the C.G.H. Catal., *for* 143°·8 *read* 144°·5, since 324·5−180=144·5.

145 h. 64=II. 621=II. 703. Auwers remarks that A Ceti, the determining star of W.H., does not exist; but C.H. has perceived this, and by using another determining star (13 Ceti, sw. 756, W.H.), has fixed the place of the nebula II. 703 for 1800 at R.A. $0^h\ 37^m\ 47^s$, P.D. 93° 53′ (=93° 43′, 1830), thereby identifying it with II. 621. Auwers, using a conjectural star, sets down the P.D. erroneously as 92° 52′ (1830).

165 h. 2356. This is the main body of the nubecula minor.

169 h. 2359. A complex object with several nuclei. There is an erratum in the R.A. set down in C.G.H. as resulting from sw. 488, *for* $46^m\ 12^s{\cdot}1$ *read* $47^m\ 12^s{\cdot}1$.

177 79, *a*, *b*. In Lord Rosse's diagram, α=h. 79, β=h. 78, γ=nova, accidentally omitted in the body of the Catalogue, but inserted as No. 5058 at the end. The whole Catalogue having been finally numbered before the omission was detected, it could not be inserted in its place. δ is a star; ϵ=h. 79, *a*.

No.

178, 179, 196 h. 4007, 4008, 4012. In the Catalogue of C.G.H. these nebulæ are placed erroneously in the 23^h of R.A., owing to a mistake of a whole hour in reducing.

202, 203, 205, 206, 207, 208, 209, 210 These constitute the group laid down by Lord Rosse as seen in and about the places of h. 84, 85, 86, viz. his α, β, γ, γ', δ, ϵ, ζ, θ. Of these, α is No. 202=h. 84; β=No. 203=h. 85; γ=No. 206=h. 86; γ'=No. 205=86, *a*; δ=No. 209=86, *b*; ϵ=No. 208=D'Arrest No. 10; ζ=No. 207=D'Arr. 9, and θ=86, *c*. In the MS. notes furnished me by Lord R. it is stated that α=h. 84, β=h. 85, and θ=h. 86. The latter identification, however, is incorrect.

214 h. 88=I. 54. This is *not* the I. 54 of the P.T., which proved to be one of Messier's nebulæ, but another subsequently inserted by W.H., so as not to break the order of the numbers, as appears from a MS. correction in P.T., and from Register (H. 570).

275, 276, 277, 280, 290 These constitute Lord Rosse's group seen in or near the place of h. 103, and marked in his diagram as A, β, δ, ϵ, and another unlettered (which call γ). These I identify as follows:—A=No. 276=h. 103; β=No. 277=103, *b*; γ=No. 275=103, *a*; δ=No. 280=103, *c*; and ϵ=No. 290=103, *d*.

297, 311, 317, 319, 325 In reference to M. Auwers's remark on the nebulæ 170, 171, as also 167, 168 (H. class III.), after very careful examination of all the data, I can arrive at no other conclusion than that embodied in the present Catalogue under these Nos.: h. 118 is certainly not III. 171, neither is h. 120. Both places and descriptions disagree.

313, 314 h. 119 was taken for III. 556, but no R.A. was obtained, that set down being the R.A. brought up from C.H. The descriptions differ so materially, especially in the particular of extension, that they are most probably distinct nebulæ.

330 h. 124=VII. 48. Auwers remarks in his 'Verbesserungen zu *h*,' that this cluster, h. 124, is not nova, but VII. 48. This is correct. Re-examining sweep 216, I find an error of 1° committed in reducing the P.D.

358 This is not in M. D'Arrest's final list, communicated to me in MS.; but being set down by M. Auwers as No. 15 in his 'Verzeichniss neuer Nebelflecke,' I felt bound to retain it.

418 h. 160=h. 2442=I. 62. This nebula, though set down by W.H. as of the 1st class (*i. e.* as a bright nebula), could not be seen by D'Arrest with the Leipzig Fraunhofer of 6-feet focus and 4½ inches aperture. It is marked in this Catalogue, however, by a mean of 4 observations, only as "F."

428 55 Andromedæ. Although this star has been eight times examined by Lord Rosse without perceiving any nebulous atmosphere, yet as my observation is corrobo-

No.

rative of Piazzi's designation of it as "Nebulosa," it is retained for occasional future examination.

442, 444 h. 169, II. 221. The places agree almost exactly, but the descriptions are irreconcileable. One makes the nebula round, the other much extended. They are therefore almost certainly distinct nebulæ, and there is therefore probably some error in the R.A. of II. 221. The neighbourhood is rich in nebulæ (see the next note, however).

442, 444, 445, 446, 447 In Lord Rosse's diagram of the group about h. 169, assuming α to be h. 169 =No. 444, the others will be β=No. 445=169, *a*; γ=No. 446=169, *b*; δ=No. 447=169, *c*; and ϵ=II. 221.

462 h. 179=50 Cassiopeiæ. Retained in the Catalogue for future occasional observation. Nothing can be more difficult than to verify or disprove the nebulosity of a considerable star under ordinary atmospheric circumstances.

472 h. 184=III. 583. Though Lord Rosse on one occasion did not find this nebula, its existence cannot be doubted, having been found by h. nearly in the place assigned by C.H.

487 h. 193=I. 152. M. D'Arrest found this nebula too faint for observation with the Leipzig refractor, though placed by W.H. in Class I., and standing in this Catalogue (from a mean of 3 observations) as a "bright" nebula.

501 h. 204=III. 604. C.H. and Auwers make the R.A. 1^m less. Both H. and h. rely on single observations. Sweep 188 h. examined and reduction found correct.

510 h. 206=III. 457. Not found by Lord Rosse; once looked for. See notes on Nos. 472 and 132.

516 h. 210=II. 246. Singularly enough, h. and H. are at issue about the two adjacent stars. h. makes the stars south of the nebula; H., on the contrary, places the nebula south of the stars, and says expressly that both this nebula and III. 201, observed just previously, were similarly situated with regard to their attendant stars. Now in h.'s obs. of III. 201 (No. 513) the attendant star *is* stated to be sf the nebula, and in that of II. 246 the larger of the two stars is south and only a very few degrees preceding. I believe the error to lie on the side of the older observations, as I have a diagram of the small star nearer to II. 246, sf, which shows that I made no mistake of n and s.

536 I. 153. Auwers makes the R.A. for 1830 $1^h\ 28^m\ 45^s$, whereas C.H. makes it $2^h\ 15^m\ 13^s$. The cause of the discordance lies in an erratum in P.T. (see list of errata). In C.H.'s reductions the error is corrected, and I find the correction verified on reference both to the Register (H. 1488) and the original sweep (sw. 596). The nebula *follows* (not precedes) the determining star.

549 h. 226=I. 154. Auwers makes the R.A. of this for 1830, $2^h\ 23^m\ 8^s$; C.H.

No.

$2^h\ 20^m\ 57^s{\cdot}8$, by the observations in different sweeps differing only 18^s in R.A. The latter is the more correct; so that M. Auwers's remarks on this nebula are not confirmed. The cause of the disagreement lies in a misprint in P.T. (See List of Errata.)

557, 558, 559, 561, 563 In Lord Rosse's description of this group, α=No. 557=h. 231; β=No. 563=h. 234; γ=No. 558=231, a; δ=No. 559=231, b. The other nebula, "about 12′ south following," is probably No. 563=h. 234. No. 561=h. 233 seems to have escaped notice.

571 h. 240=II. 238=III. 198. C.H. has overlooked or omitted an obs. of W.H. of III. 198 in sw. 574, which, referred to, confirms Mr. Marth's surmise that the nebulæ are identical.

573 II. 6. This was probably really a comet, as indicated by its description, having been subsequently looked for and not found.

574 h. 244=I. 102. M. D'Arrest found this nebula, when observed with the Leipzig refractor of 4½ inches aperture, inferior to a 1st class nebula. In this Catalogue, from a mean of 5 observations, it ranks as "considerably bright."

591 h. 258=I. 1. M. D'Arrest found this nebula, when examined with the Leipzig refractor, not entitled to rank above the 2nd class. With this our present Catalogue agrees, it being set down from a mean of 8 observations as "pretty faint."

614 This nebula of Bessel was also looked for and not found by D'Arrest, who therefore supposes it to have been a comet.

636 h. 280=II. 502. II. 502 is described by H. as cS; F; stellar. Either then the identity is doubtful, or some change must be suspected. The place, however, agrees well.

639 h. 281=IV. 43. Once looked for by Lord Rosse, but not found. (See notes on 134, 472, 510.)

646 h. 284=III. 578. The same remark. Twice looked for unsuccessfully by Lord Rosse. On one occasion clouds were passing.

654, 655 In Lord Rosse's diagram of this pair and the neighbouring stars γ and δ, the figure is in contradiction with the measures. The position of $\alpha\gamma$, instead of 2°, should, I presume, have been stated thus, $\gamma\alpha$=178°, or, which comes to the same thing, $\alpha\gamma$=−2°. This has been assumed in deducing the place of No. 655=289, a from No. 654=h. 289.

656 h. 291=III. 591. H. makes this nebula to be the nf of two, but both those of h. the sf.

674 h. 293=II. 603. H.'s description is pB; stellar; a pc* with eS, vF chevelure. The place, however, agrees well with that of h. 293.

684 III. 195. Auwers makes the R.A. (1830)=$3^h\ 11^m\ 50^s$ and C.H. $3^h\ 10^m\ 13^s$; but

No.

a misprint in P.T. (see List of Errata) accounts for the difference of the minute at least.

708 } III. 959; I. 60. The catalogued places contradict the described position sf and
709 } np; but this is owing to the error in R.A. of I. 60, which D'Arrest makes less by 40^s, which would place I. 60 at 3^h 19^m 35^s (1860).

710 Au. N. 17. The discovery of this nebula is attributed by Au. to Schönfeld in 1858, but it seems to be identical with that described by Tuttle (Astronom. Notices, xix. p. 224). Auwers's place is preferred, Tuttle's being only approximate.

768 Au. N. 18. The celebrated variable nebula of Tempel, discovered Oct. 19, 1859.

774 II. 594. Auwers considers this as identical with II. 548, with 1° mistaken in P.D.

778 h. 309=I. 155. Auwers makes the R.A. of I. 155 for 1830=3^h 53^m 33^s, destroying the identity of these two nebulæ. But his place is deduced from an erroneous entry in P.T. (see List of Errata). C.H., by 2 observations in sweeps 608, 638 agreeing to 3^s in R.A. and 2′ in P.D., gives a place which, brought up to 1830, gives R.A. 3^h 37^m 58^s; P.D. 94° 29′ 7″.

810. h. 311=IV. 69. M. D'Arrest found the nebulous atmosphere around the central star of this nebula very conspicuous with the Leipzig 4½-inch refractor.

826 h. 2618=IV. 26. D'Arrest's R.A. is preferred, that of h. 2618 being clearly shown to be erroneous.

836 II. 464. The P.D. is given by W.H. as the same with that of 44 Eridani. C.H., using an erroneous place of this star, makes the P.D. 5′ too small. This is here corrected, and the result agrees with Auwers.

839 Auw. N. 20. This is the remarkable variable nebula discovered by Mr. Hind on Oct. 11, 1852. M. D'Arrest testifies to its complete disappearance on the 3rd and 4th of Oct. 1861, "Hujus nebulæ .... ne umbram quidem detegere valeo." —"Cœlo serenissimo regionem summâ curâ perlustravi adjuvante Dr. Schjellerup. *Nebula reverâ deest.*" (In 1855 and 1856 it was found by M. D'Arrest within 2′ of Mr. Hind's original place.) On Dec. 29, 1861, it was seen by M. Otto Struve with the great Pulkowa refractor, but so excessively faint as to be barely within the power of that instrument. On March 22, 1862, with the same telescope, it was again seen, but considerably brighter, so as to bear a faint illumination of the wires.

851 h. 314=III. 587. Not seen by Lord Rosse, once looked for, clouds passing. See notes on Nos. 639, 646, &c.

880 h. 322. The bright star preceding is ν Eridani.

908 h. 333=II. 547. Not seen by Lord Rosse, once looked for. See notes 132, 472, &c.

926 h. 335. Erroneously identified in my Catalogue of 1833 with III. 453 (No. 981). See the note on that nebula.

No.

953 h. 341=D'Arrest 48. Observed by him as "nova," but since recognized as unquestionably =h. 341.

970 VIII. 43. Auwers makes the P.D. of this cluster for 1830 =66° 25′, which is incorrect. The determining star is 109, *n*, Tauri, the cluster being 1° 29′ north of the star. This would give 66° 39′ for the P.D. for 1800, agreeing with C.H., and 66° 36′ for 1830.

975 h. 343. A very large diffused nebulosity, distributed in zigzags. This has been looked for seven times by Lord Rosse and not found. Its existence is therefore very doubtful.

979 h. 2709. The place graphically determined by measurement of a diagram, as compared with h. 2710.

981 III. 453. This was erroneously identified with h. 335 in my Catalogue of 1833. By an unlucky coincidence, its place per working list, roughly brought up from C.H., agreed so well with the latter nebula as taken in sw. 322 (h.), that it was unhesitatingly assumed to be the same. It appears, however, that in C.H.'s reduction an error of 10^m in R.A. has been committed, the star of comparison being 10 Orionis, and the nebula *following* the star by $5^m\ 7^s$ (as ascertained by reference both to the register sheet (H.1160) and the original sweep (sw. 462, H.)). M. Auwers, misled by my erroneous identification, has assumed that the nebula must have *preceded* the star, which would (nearly) account for the difference, and in consequence, his R.A. of this nebula is 10^m too small. C.H.'s error probably arose from misapplying in like manner the sign of the Δ. R.A.

998 III. 268. Auwers's R.A. ($4^h\ 57^m\ 23^s$, 1830) is adopted in preference to $5^h\ 0^m\ 28^s$, that brought up from C.H. to the same epoch. In the sweep 367 (H.) three stars of comparison are given, 58 Eridani, *α* Leporis, and 19 Leporis. The Δ. R.A. of *α* and 19 comes out correct, but that of 58 from each is wrong by $3^m\ 5^s$, so that the star must have been mistaken. C.H. has used 58 and *α*, and has rightly brought out the place of the nebula by the former (the wrong star), and wrongly by the right one; and by an odd coincidence the two results agree well, though both wrong.

1030 h. 349=VII. 4. Described by D'Arrest as "Ein Ausserordentlich reicher Hauf," an extraordinarily rich cluster.

1133 h. 356. Looked for four times by Lord Rosse, in two of which the sky was fancied to have a milky appearance.

1138, 1139 h. 2841. Double nebula. In my Cape Catalogue, sweep 538, *for* "first" *and* "second" *read* "larger" *and* "smaller." The smaller is sp. The position 260° is right. It is very remarkable that in sweeps 508, 522, 658, and 761 the smaller of the two was not noticed. Is it variable?

1167 III. 747. Auwers makes the P.D. 8′ 20″ greater. It is difficult to identify the determining star used by C.H.

No.

1165, 1168, 1171, 1174 h. 2866, 2867, 2868, 2869. $16^{s}\cdot2$ added to all the R.A.'s of these nebulæ in the Cape Catalogue to compensate an error detected in sw. 538. The correction is deduced from a comparison of the diagram fig. 20, Pl. VI. C.G.H. with the place of No. 1171.

1179 h. 360. $3^{s}\cdot3$ added to h.'s P.D. to bring it to the place in B.A.C.

1180 V. 30. The place of V. 30 corrected by $+3^{s}\cdot2$ in R.A. and $+25' \; 45''\cdot4$ in P.D. to bring it to the place of c^{I} 42 Orionis in the B.A.C.

1183 h. 361=V. 31. h.'s place corrected by $+0^{s}\cdot4$ in R.A. and $-0' \; 27''\cdot2$ in P.D. to bring it to that of ι 44 Orionis in B.A.C.

1185 III. 1. ?? There are two observations by H. of III. 1, but they differ enormously. One agrees with M. 43. The place of M. 43 is corrected to agree with its place in the Catalogue of Stars, &c. in the great nebula in Orion, C.G.H. p. 28.

1191 Chacornac's recently discovered nebula. Place from Moigno's "les Mondes," No. 9, p. 241.

1196 III. 269. Auwers gives as the R.A. of this nebula for 1830 $6^{h} \; 27^{m} \; 57^{s}$, which is mistaken by 1^{h}. The Philosophical Transactions says that it precedes 19 Leporis by $32^{m} \; 23^{s}$, and that this is no misprint appears from C.H.'s reductions.

1226 IV. 24. Annular according to Lord Rosse.

1287 III. 270. Auwers places this nebula in R.A. $6^{h} \; 40^{m} \; 20^{s}$, or an hour too late. Its place is very distinctly settled by two determining stars, α Leporis and 19 Leporis, the former of which it followed by $15^{m} \; 4^{s}$, and preceded the latter by $20^{m} \; 0^{s}$.

1425 h. 393=IV. 3. Lord Rosse's account of this nebula is extremely remarkable. "This h. 393," he says, "is an enormous nebulosity which I have traced f and n of it to a great distance—*some degrees*. It narrows at times to a band across the finding eyepiece about 6′ or 8′."

1440 h. 401=V. 27=VIII. 5. Retained as a cluster, though but a poor one. Nine times examined by Lord Rosse for nebulosity, but none seen.

1452 III. 271. Auwers places this nebula in R.A. $8^{h} \; 3^{m} \; 35^{s}$, P.D. 76° 21′ (1830). There has been some mistake. III. 271 is stated to follow 8 (ν3) Canis, $8^{m} \; 0^{s}$, and to be 4′ n of that star, which gives a place agreeing with C.H. and with the present Catalogue.

1454 h. 441=M. 41. This nebula was also observed by Flamsteed.

1455 In Lord Rosse's diagram of this group, α is No. 1457=h. 410; β=No. 1455=410, *a*;
1456 γ=No. 1456=410, *b*; δ=No. 1458=h. 409; and ϵ=No. 1460=410, *c*. But
1457 some suspicion seems to have arisen that the principal nebulæ observed were
1458 not really h. 409, 410, but h. 406, 407. In that case the identification will
1460 stand as follows:—

α=No. 1448=h. 406.

β=h. 406$-5^{s}\cdot2$ in R.A., and $-1' \; 25''$ in P.D.

γ=No. 1449=h. 407.

No.

δ =h. 406+1^s·6 in R.A., and −5′ 6″ in P.D.
ϵ =h. 406+14^s·7 in R.A., and −5′ 2″ in P.D.

1480 h. 423. This nebula is entered by C.H. as VIII. 1. B, with a remark "not in print."

1508 h. 439=VI. 6. The R.A. is nearly 2^m in excess of C.H. and of Auwers. Examined sweep (h.) 393 in which it was observed. Found all clear and correctly reduced.

1527
1528
1530
1531 Compared with Lord Rosse's two diagrams of the nebulæ composing this group. None of them are "novæ." α=h. 449; β=h. 448; γ=h. 447; $\delta=\beta$; $\epsilon=\gamma$; ζ=h. 446.

1533 VIII. 44. Auwers's P.D. is 84°, instead of 82°, owing to an erratum in P.T. (See List of Errata.)

1578 h. 468=III. 479. No nebulosity seen by Lord Rosse in 5 observations. In H.'s single observation the nebula is "suspected," and in those of h. it is not positively ascertained. The object seems therefore to be merely a small resolved cluster of vFst.

1594. M. 47. Auwers assigns a R.A. greater by 4^m. The cluster has not since been observed. It is probably a very loose and poor one.

1611 h. 480=VI. 37. h.'s P.D. corrected by −10′ as the presumed error of reading in the single observation obtained. Harding in 1827 (it appears) observed its P.D.=100° 10′ (for 1830), and W.H.'s place for that epoch is 100° 12′, that of h. being 100° 19′ 4″.

1615 In Lord Rosse's diagram, α=No. 1617=h. 483; β=No. 1616=D'Arr. 51;
1616 γ=No. 1615=483, *a*. D'Arrest's place for β is preferred to that which results
1617 from comparison with the diagram. h. 284 could not have been in the field, being almost a degree distant.

1633 h. 493=II. 719. h.'s R.A. in P.T. diminished by 1^m for an error of 1^m detected in the reduction of the observation. This brings it nearer to Auwers.

1652 h. 3176. *Polarissima Australis.* This nebula is so near the south pole that its precession in R.A. varies from year to year with great rapidity, so that its R.A. cannot be computed correctly by the ordinary approximate method.

1666 The four nebulæ h. 508, 510; 510, *a*; 510, *b* evidently include among them that
1667 third nebula referred to by Lord Rosse as the accompanying "nova" "forming
1668 a triangle with h. 507, 508—of the last degree of faintness." h. 507, however, is 30° distant in P.D., so that in the observation of Feb. 9, 1850, the P.D. of h. 507 must doubtless have been read as 36° instead of 66°, giving rise to a mistaken identity with one of the *two* really new nebulæ at that time in view.

1696 III. 50. I find a memorandum to the effect that this nebula is lost, and was probably a comet; but I cannot recover my authority for the statement. It is described by H. as "of the last degree of faintness," and it is therefore no way

No.

surprising that it should not have been again perceived without some time and trouble bestowed, and in clear weather.

1707 h. 527=II. 48. M. Auwers, owing to an erratum in P.T. (see List of Errata), makes the R.A. of II. 48 two minutes too great, and is thus led to doubt its identity with h. 527. There still remains the rather considerable disagreement of 5′ in P.D. D'Arrest found neither of these nebulæ; but there can be no doubt of the existence of one at least, in or near the place here given. This is *not* the nebula seen by Lord Rosse "nearly in contact with h. 526." This latter (described already by h. as "bi-nuclear") was seen by R. as distinctly double.

1712 h. 531=M. 67. Discovered by Oriani.

1720 h. 535=II. 823. W.H. describes this nebula as "Round;" h. as "much extended," while Lord Rosse saw it as bi-nuclear, or a double nebula joined by faint nebulosity. Is it separating into two, like Biela's comet?

1735, 1736 h. 542 and II. 557. The descriptions are irreconcileable, and they must be two distinct nebulæ. The R.A. of h. 542 was *not* observed, and its P.D. is set down as "hardly more than conjectural," having been looked for by working list as II. 557 and set down as such.

1742 h. 545=II. 834. Misprinted II. 844 by Auwers in the Catalogue, but the number is correct in his general list of the nebulæ by numbers and classes.

1743 h. 546. Not seen by Lord Rosse in one observation. Examined sweep 21 (h.) and found all right.

1756 III. 291=D'Arr. 60. These are assuredly one and the same nebula. Auwers's declination of III. 291 (+27° 7′) should be +26° 7′.

1773 h. 565=III. 61. The P.D. according to H. is 70°.

1788 II. 708. Owing to an erratum in the determining star in Phil. Trans. (see List of Errata), Auwers has given the place of this nebula for 1830 R.A. $9^h\ 12^m\ 39^s$; P.D. 39° 17′, instead of $9^h\ 6^m\ 29^s$; 47° 20′.

1791, 1794 h. 577; h. 578. Not seen by Lord Rosse in one observation. (See next note.)

1792 D'Arrest 62. This nebula must surely be variable, as it is inconceivable else that it should not have been seen by h., when h. 578, to which it is almost close, was observed and its place taken. D'Arrest says, "Fugerat Herschelium necnon me anno 1862." Neither of the three (Nos. 1791, 1792, 1794) were seen by Lord Rosse. Sweep 59 (h.) and the reductions re-examined. Found all clearly written and all correct.

1804 to 1815, 1817, 1818, 1821 h. 581, 582; 581, *a*, *b*, *c*, *d*, 582, *a*, *b*, *c*, *d*, *e*, *f*, *g*; D'Arr. 63. Of this very complex group of 15 nebulæ or "knots" (as they are called by Lord Rosse), six have been determined from his diagram, and six more by the aid of notes subsequently furnished me from the records of the observatory at Birr Castle, containing differences of R.A. and P.D. from one or other of the former. These

No.

are indicated by the letters MS. attached in the column of descriptions. The others I identify as follows:—

α (in Lord R.'s diagram) is No.			1813=582, *c*.
β	"	"	1812=582, *b*.
γ	"	"	1811=h. 582.
δ	"	"	1806=h. 581.
ϵ	"	"	1815=582, *e*.
ζ	"	"	1821=582, *g*.

One of those for which no data are given must have been D'Arr. 63, and the two remaining ones are included under the entries Nos. 1817, 1818 as 582, *f*.

1832 h. 590. Not seen by Lord Rosse; once looked for. Re-examined the sweep and reductions. Found all correct.

1868 h. 3171. In the omitted observations of nebulæ in the last page of the C.G.H. observations, *for* h. 3170 *read* h. 3171; and this observation, combined with the two in the body of the work, gives the mean result for 1830 employed to deduce the place in the present Catalogue.

1911 h. 3185=III. 289. In consequence of a misprint in P.T. (see List of Errata), the P.D. of Auwers is 5′ too small. Corrected by this, his place agrees well with my observation.

1953 M. 81?? A nebula observed by W.H. as described, but differing most materially in place from M. 81. It would certainly be very extraordinary should *three* nebulæ so extremely remarkable as M. 81 and 82 and this be found to lie so near together.

1959 } h. 3198, 3202 are distinct nebulæ, and were observed consecutively in one and
1962 } the same sweep—sw. 561 (h.).

1960 } h. 3199 and 3201 are also distinct nebulæ, and were observed consecutively in
1961 } sweep 562 (h.).

1974 III. 293. M. Auwers makes the place of this nebula $9^h\ 24^m\ 4^s$; 66° 30′ (1830), instead of $9^h\ 48^m\ 48^s$; 60° 13′. The cause of the error is an erratum (see List) in P.T., where the determining star is set down as 23 Leonis instead of 23 Leonis Minoris, another of the instances of confusion arising from the use of this silly and barbarous nomenclature.

2014 h. 669=III. 65. Not seen by Lord Rosse in one observation. It was found by h. in its place *per* working list.

2019 h. 672. Not seen by Lord Rosse in one observation. Examined the sweep and reductions, and found all correct.

2043 h. 250. This nebula is so very close to the North Pole, that its place cannot be calculated by a precession proportional to the time in the usual approximate mode, the R.A. changing from year to year with extreme rapidity.

No.

2054
2055 In Lord Rosse's diagram, α=No. 2058=h. 692=II. 44; β=No. 2061=h. 693
2057 =II. 45; γ=No. 2055=692, *b*; δ=No. 2054=D'Arr. 61; ϵ=No. 2057=692, *c*,
2058 not lettered in the diagram.
2061

2088, 2089 II. 28, 29. Both D'Arrest and Secchi agree in placing this double nebula more to the south than W.H. by 15'±, and D'Arrest supposes the P.D. to have been misread to that extent. As so great a proper motion is most improbable, and the identity is indisputable, I have adopted this supposition and made the necessary correction.

2094 h. 706. Not seen by Lord Rosse in 6 observations. Re-examined the record of the original obs. Sweep 115 (h.), No 68, and the reductions. The entries are all clear and perfectly legible. Reduction in P.D. correct; reduction in R.A. erroneous by $-0^m\ 26^s{\cdot}6$. This, however, could not have caused its non-observation by R. This then was a comet, or is a lost nebula. The error of reduction is corrected in the present Catalogue.

2111 III. 316. C.H.'s reduction of this nebula being affected with a considerable error, Auwers's R.A. is adopted, after verification.

2144 h. 3276. Place approximate, by equatoreal zone review.

2189 h. 745=V. 52. Not seen by Lord Rosse when once looked for (see note on No. 132, &c.).

2192 h. 3294. The minute in R.A. doubtful.

2197 h. 3295. The great nebula about η Argus. According to a letter from Mr. Eyre B. Powell of Madras, a most extraordinary change has taken place in this nebula since my figure of it was delineated. He states that the southern end of the curious oval vacuity close to the great star, which was *decidedly closed* when I depicted it, is now *decidedly* open. Should this be established, it will be the most extraordinary fact that has yet appeared in the history of a nebula.

2201 h. 754=II. 99. M. D'Arrest found this nebula in the Leipzig refractor, bright enough to be ranked in the first class. And it is marked as "very bright" in this Catalogue by a mean of 5 observations. It must have been ill seen in the earlier observation when classed as II.

2231, 2234 IV. 6=II. 131 and h. 777=III. 88. I adopt, on due consideration, the opinion of Auwers, that III. 88 and II. 131 are not the same. Their having been successively observed in the same sweep is decisive. Also, that IV. 6 is not III. 88, but in reality identical with II. 131. The descriptions are made out in conformity with this.

2233, 2236 I. 118 and h. 779. The degree of P.D. is probably mistaken in I. 118. Marth, according to Auw., suggests that the determining star 46 Ursæ (which though

No.

not so called in B.A.C., is doubtless No. 3741 of that catalogue) was mistaken, and should have been called 46 Leonis minoris. Consulting the original sweep (sw. 487, H.), I find this surmise *not* corroborated; for the nebula, when reduced by the star next preceding it (37 Leonis minoris), gives the same Polar distance, and, within a few seconds, the same R.A. But there is some faint indication of the figure 6 in the reading of the Polar distance piece 56° 55′ having been written over a 7, which would have thrown the nebula somewhat below the southern limit of the sweep, and might have caused a suspicion of error at the time. I found no nebula in the catalogued place in my sweep No. 337 (h.), so that the probability of an erroneous degree is strengthened. At the same time, it is not impossible that this nebula may be identical with No. 2236=h. 779, the mistake in the degree lying the other way.

2238 h. 780=I. 172. h., in Ph.Tr., suggests that this nebula may have moved. There is, however, no ground for this supposition, as its place agrees quite remarkably with that brought up from C.H. But query if the double star have not moved, since one of the observations places it "in the middle," and a subsequent one makes the southern extremity of the nebula touch the large star of the double star.

2276 h. 806=II. 101. Found to rank as a first-class nebula by M. D'Arrest with the 4½-in. Leipzig refractor. In this Catalogue it stands described as "very Bright," by a mean of 4 observations. See remark in note 2201.

2310 h. 823=III. 111. There is a strange amount of discordance between the observed and reduced places of this nebula. Auwers makes the P.D. for 1830=84° 29′. C.H. has reduced the single observation of W.H. by two stars 84, τ Leonis and 349 Bode Leonis, and her results differ by 10′; τ, which gives the greater, being stated to be "too far distant in P.D." The several results stand thus:—

		°	′
P.D. 1830,	by Auwers	84	29
„	by τ Leonis (C.H.)	84	20
„	by h. obs.	84	15
„	by 349 B. Leonis (C.H.)	84	9½

My observed P.D. is nearly a mean between those of C.H.

2315 h. 828=II. 42. Not seen by Lord Rosse when once looked for (see notes on No. 132, &c.).

2319 h. 829=III. 351. The observations of this nebula, which are numerous, disagree so very remarkably in the particular of brightness, that a considerable suspicion of variability exists.

2373 h. 854=M. 65. There is a misprint, 45° for 75° np to sf, in the position of extension in my Catalogue of 1833. The diagram in the original sweep also corroborates this, as does also the figure (fig. 53) accompanying that Catalogue.

No.

W.H. twice says mE in merid. (180°)—h.'s position 75° np to sf=165°; a mean of those of Winnecke and Auwers =172°.

2377 h. 857, h. 875; M. 66. No doubt these are the same. fig. 54 P.T. 1833 corroborates their identity. The accompanying stars and their positions agree entirely. The R.A. of h. 875, however, requires to be corrected by -3^m, allowing the seconds and the P.D. observed in that observation their weight.

2382 II. 30. Auwers deduces his R.A. for 1830 ($11^h\ 12^m\ 21^s$) from the statement in P.T. "following 68, δ Leonis, $6^m\ 30^s$." C.H. from the same data concludes R.A. $11^h\ 11^m\ 31^s$ (also for 1830). The latter is (within 2^s) the correct result.

2388 h. 867=h. 861? These are very probably the same. But as, after all, the difference of the observed R.A.'s is sufficient to have allowed one to escape while observing the other, so that they *may* be different, and as moreover one is described as "Round," and the other as "extended," both are retained.

2405 h. 882=I. 20. This nebula would seem to have decreased in brightness. The bright ✱ is 1341. A.S.C.

2411 h. 886=I. 131. Ranked by M. D'Arrest in the second class with the 4½-inch Leipzig telescope. In this Catalogue it stands as "pretty Bright" from a mean of three observations.

2417 III. 112. Auwers has reduced this nebula by the star given in P.T. φ, 74 Leonis. But I find a MS. note that this star was not dependable, and that Mayer's No. 510 is the proper determining star. The nebula was subsequently looked for and found, not in the place given by φ, but 8′ from the P.D. concluded from Mayer 510. A mean of these two determinations is therefore used in this Catalogue.

2440 h. 907=III. 353. Auwers doubts the identity of these nebulæ. But this is in consequence of a misprint in P.T. (see List of Errata), 53^m for 43^m. The error is found also in the Register Sheet (H. 937), but C.H. has avoided it and used 43^m in her reduction so as to give a R.A. agreeing within 35^s with that of h. 907.

2461 h. 918=II. 784. Lord Rosse, in his observation of this nebula, mentions "another brush-like, 20′ np." This was no doubt II. 783=No. 2454.

2501 h. 945=I. 94. W.H. makes this nebula by one observation extended, n to s, by another nf to sp, while h. has two observations agreeing in making it extended in the parallel. Surely it does not rotate?

2540 h. 967. 1^m added to the R.A. It is evidently the first of the group of 4.

2577 III. 113. This nebula is reduced also in Auwers's catalogue by φ Leonis, the star set down in P.T. But C.H. remarks that φ was above the sweep, and otherwise observed under unfavourable circumstances, and Mayer's 510 zod. star. s. 0° 31 is preferred, which gives a result differing by +24′ in P.D. and -48^s in R.A. The place adopted in the present Catalogue is in conformity with this remark. (See note on No. 2417.)

No.

2591 h. 1000=III. 616. The star 6m, 5′ n only noticed by W.H. The other 7m, f in the parallel only by h. Are there really two stars? and are they both variable?

2597 h. 1002=I. 203. Auwers, in consequence of an erratum in P.T. (see List of Errata), makes the R.A. of this nebula 7^m too small. The error is corrected in the Register (H. 1889) and in C.H.'s reduction.

2604 h. 1009=I. 202. The same misprint in P.T. mentioned in the last note on h. 2597 has also vitiated M. Auwers's R.A. of this nebula. It is corrected in the Register Sheet (H. 1886) and in C.H.

2608 h. 1013=III. 381. I adopt Mr. Marth's identification of these nebulæ. The place of III. 381 in the catalogue of C.H., from which my working lists were made out, is vitiated by some great mistake. The P.D. is supposed to be derived from 1 Comæ, the neb. being 1° 12′ south of the star. This, however, would give 68° 9′ 29″ for 1830 instead of 65° 45′ 0″, that brought up from C.H.

2650 h. 1039. This cannot be identical with h. 1036, and its brightness precludes its being accepted as III. 354. But there is extreme uncertainty as to its P.D. The degree may even be wrong.

2652 h. 1041=II. 733. According to W.H. the position of extension is "near the meridian." If *meridian* be not a mistake for *parallel* it has changed. h. has a measure 62°·3, and an estimation 65° in another observation.

2653 h. 1042. This cannot be III. 3, as C.H. has reduced two obs. of this latter well agreeing, and giving a R.A. 2^m exceeding that of h. 1042, which also rests on 2 obs. of h.

2668 h. 1050=I. 253. The difference of descriptions is extraordinary, so that they seem hardly to pertain to the same object; but the places agree.

2683, 2684, 2685, 2686, 2689, 2690, 2693, 2694, 2697, 2699, 2701, 2702 h. 1062, 3, 4, 5, 7, 8, 1070, 1, 3, 5, III. 391, 2, 3, 4, 5, 6. The places set down for the nebulæ of this extensive group are made out by a most careful consideration of all the observations and records in the sweeping books which seem irreconcileable with a group of six nebulæ only. The group, however, needs a thorough re-examination.

2730 II. 14. Owing to an erratum in P.T. (see List of Errata) Auwers gives quite an erroneous place for this nebula (11^h 39^m 27^s R.A., 81° 9′ P.D. 1830).

2747 h. 1103=III. 814. Auwers suspects some error of the press, since his P.D. for 1830 comes out 36° 58′, while that of h. 1103 is 35° 56′. There is, however,

No.

no error, either of printing, registry, or reduction in any part of the older work. The determining star is rightly set down as 5 Canum, whose P.D. for 1800 (the epoch of C.H.'s catalogue) is 37° 19′ 42″, and III. 814 is declared to be 1° 32′ north of it, so that 35° 48′, the P.D. of C.H., is correct, and reduced to 1830 (=35°. 58′) agrees with my place within 2′. Neither is there any error of the press or of reduction, or any apparent mistake of a clerical nature in all the process of h. 1103, and the nebula observed is set down *in* the sweeping book (of course from the working list) *as* III. 814. I consider their identity therefore as fully established.

2771
2773 h. 1211=II. 372. H. says, the most northerly of the pair II. 372, III. 360 the largest: h., "by diagram," makes the following nebula, III. 360=No. 2773, the larger of the two.

2814 II. 109. The reductions of the sweep 187 (H.) in which this occurs are somewhat precarious, and in C.H.'s revision of the sweep the Δ. P.D. from 6 Comæ is set down at 1° 50′, that in the P.T. at 1° 54′ (these changes are never made without good reason), and this accounts for 4′ out of the 5′ difference between her P.D. and that of M. Auwers.

2846 III. 535. In a sweep two years subsequent to the obs. of this nebula by H. it was looked for again but not found. ? if a comet.

2849 D'Arr. 89. M. D'Arrest makes mention in a letter which he has done me the honour to address to me, of a nebula having the same R.A. as this, but a P.D. (1860)=83° 46′ 42″. He does not include it in his final list. It should, however, be looked for.

2852 h. 1183, 7, 9, 1190, 4; II. 568, 9, 570, 1, 2, 3. There cannot be a doubt that
2856 II. 568, 569, 570, 571, are in 82° P.D., and II. 572, 3, in 83°. It is equally
2857 certain that h. 1183, 1189, 1190, 1194 are in 83°. They were observed in two
2862 distinct sweeps (sw. 111 and 238); I observed also II. 572 in sw. 238, and III.
2865 573 in sw. 250. There must be a set of nebulæ, at least 8 in number, hereabouts.
2869 N.B. W.H. makes II. 568, 569, 570, 571, 34′ n. of 11 Virginis. If n. be a mistake for s, these agree with h. 1187, 1189, 1190, 1194.

2855 h. 1186=I. 90=II. 322. Marth's conjecture is right (see Auwers's note on I. 90) as regards II. 322, but not so his conclusion that II. 322=II. 377.

2878 h. 1202=I. 139=M. 61. Discovered by Oriani. N.B. The first discoverers of the nebulæ in Messier's list, when not Messier himself, are mentioned by M. Auwers in his catalogue of those nebulæ (pp. 66–71), except in the cases of Oriani's nebulæ, M. 14 ?, 18 ?, 35 ?, 61, 67.

2884 1202, *a*. Under h. 1196 and 1202, two nebulæ, unidentifiable, are described as companions, but there must be some great error in Lord Rosse's account of them, as the place of one is referred to a scarlet star "10′ south of a scarlet star R.A. 12^h 25′." Now h. 1202 is in R.A. 12^h 14^m. To afford a fair chance

No.

of reobserving them, the companion 10′ nf h. 1202 is entered here as 1202, *a*, and that south of the scarlet star, under No. 3060 as 1196, *a*.

2892 D'Arr. 90. "Reperta a me Mart. 4, 1862. Eandem reperit Schönfeldus, April 1, 1862. Vide Comptes Rendus, &c."

2951 II. 87. This *may* be h. 1240, but 7′ in P.D. is a large error.

2961 h. 1253=M. 86. The nebula of Lord Rosse 14′ sp this is no doubt II. 168.

2976 h. 1261=III. 492. III. 492 was looked for April 11, 1787, by W.H. in the place assigned to it, but was not seen. Auwers, however, makes it identical with h. 1261. Yet the descriptions are radically different, and after all there may be another nebula, the real III. 492, in the neighbourhood.

2992 }
2995 } R. novæ. 1274, *a*; 1275, *a*. Of the eleven "knots" seen by Lord Rosse in this place these two are the only really "novæ." The other 9 were h. 1237, 1244, 1250 (1 & 2), 1253, 1259, 1274, 1275, and Auw. N. 30, numbered in this Catalogue 2931, 2949, 2955, 2956, 2961, 2965, 2974, 2991, 2994. h. 1203, numbered by Lord Rosse as one of the group, seems too far remote in R.A. to have been seen on that occasion.

2999 h. 1279=II. 156. H. says "F;" h. "vB." The latter preferred, since F might arise from fog or haze.

3003 h. 1282. II. 56 and II. 90. Both II. 56 and II. 90 were seen in one sweep, March 1, 1784, at 1^m interval of time (by the same star, 25 Comæ), II. 56 being 1′ more north, and II. 90 3′ more south than the star. This is a case of positive disappearance, for in sweep 334 (h.) the neighbourhood was carefully examined and only one nebula found.

3008 I. 23. By ρ Virginis, sw. 174; n. 1° 31′; ∴ P.D. (1830) 77° 18′ 29″. By 34 Virginis in sw. 199, s. 0° 19′, whence P.D.=77° 25′ 33″, mean 77° 22′. Auwers makes it 77° 16′. This nebula is placed in the 2nd class by M. D'Arrest as seen with the Leipzig refractor. In this Catalogue it is set down from a mean of two observations, as "pretty bright."

3011 h. 1289=II. 212=II. 750. The two nebulæ so designated were not observed by H. in one sweep, and are, no doubt, identical.

3013 h. 1290=II. 122=II. 174. These two nebulæ of the 2nd class were also not observed by H. in the same sweep, and are presumed to be identical, as the places agree.

3021 h. 1294=M. 49. Discovered by Oriani in 1771.

3026 h. 1295=II. 117=II. 629. The same remark applies as in the notes on Nos. 3011, 3013.

3029 II. 116. Not seen by D'Arrest.

3043 h. 1307=I. 83. Not found by Lord Rosse when once looked for. There can be no doubt, however, of its existence in or near this place.

3060 1196, *a*=R. nova. See note on No. 2884.

No.

3075 h. 1329=I. 31=I. 38. H. describes I. 31 as "between two bright stars." The places differ 15′ in P.D.; h. describes I. 38 (the place well agreeing with that of H.) in one observation as having a large star f, and in two others as having a star 9m, p; that is, in effect, as lying between two bright stars. N.B. The star used for I. 31 is 31 *d* 1 Virginis, and for I. 38, 32 *d* 2 Virginis. The declination of 31 *d* 1 is 30′ wrong in A.S.C. (No. 1469). In B.A.C. it is right. The P.D.'s of the two nebulæ of H. differ, as already remarked, by 15′. The R.A.'s agree. They must be identical with a mistake of 15′ in I. 31. D'Arrest says he is sure there are not two nebulæ here.

3078 III. 26. Place as per C.H., $12^h\ 25^m\ 32^s$, 68° 32′ for 1830; as per Auwers, $12^h\ 25^m\ 40^s$, 68° 47′ (see List of Errata). The correction of the place in P.T. is not, properly speaking, an erratum, but the substitution of a good observation for a bad one. In the obs. sw. 177 (H.), where 20 Comæ was used as the determining star, the place is given only by description. In a sweep long subsequent (sw. 944) it was compared with 26 Comæ in the regular form of observation, and this is of course to be preferred. Auwers's place is deduced from the earlier, and that of C.H. from the later observation, rejecting the other.

3079 h. 1322=8 Canum. This very remarkable object occurs among the list of those observed by Lord Rosse in his paper in P.T. 1861, but without a word of remark or description; and it does *not* occur among his list of nebulosities looked for but not perceived. Surely it might be inferred from this that the nebulosity surrounding the star *was* seen, or its absence would have been noticed, as in the instance of 55 Andromedæ. Yet Mr. Lassell saw no nebulosity about 8 Canum.

3097 h. 1348=M. 89. Lord Rosse has h. 1343 and 1348, and in his account of them says, "two others, about 20′ s. of 1348;" one of these must have been h. 1343, and the other h. 1349.

3103 h. 1353=I. 119. This nebula was barely perceptible, with straining the attention, by M. D'Arrest with the 4½-inch Leipzig refractor. It is described in this Catalogue as "considerably bright" by two observations.

3108 } h. 1358, 1359, 1363=IV. 8, 9. The obs. of 1363 in my Catalogue of 1833, in
3109 } which the R.A. is uncertain, undoubtedly refers to the same very remarkable double nebula, IV. 8, 9. D'Arrest is sure that there is no other double nebula in this neighbourhood.

3111 M. 90. The place is from two observations by W.H., as also the description.

3127 h. 1374=I. 273. The descriptions of H. differ so much that it is not impossible there may be another bright nebula near this place.

3138 h. 1379=II. 577. Two diagrams by h. in sweeps 141, 143, agreeing, represent this nebula as making a considerably acute-angled, nearly isosceles triangle with

No.

two following stars. H. says, "Between two Bright stars, making a triangle with them." No one now, looking at those diagrams, would call the situation of the nebula between the stars. A suspicion of proper motion arises in such a case.

3148 h. 1384=II. 148. In my Catalogue of 1833 this nebula is identified with II. 20, and in the Register Sheets (H. 320), under the head of II. 148, there is a memorandum, "Probably the same as II. 20 (H. 47)." But on examining all the observations of both nebulæ, I arrive at the conclusion that they are different, II. 20 being nearly 2^m later in R.A.

3170 h. 1401. Query if not =II. 38, with one degree mistaken in P.D.

3174 See note on 3148, above.

3177, 3179, 3206, 3216, 3224 h. 1406, 1407=II. 794 (1 & 2), III. 778; h. 1428, 1435=II. 795, 796. Auwers remarks, and justly, on the great apparent discordance of the observations of h. and his places of II. 794, 5, 6, and those of W.H. The fact is that the places of these in the P.T. all rest on comparisons with ε Ursæ in sweeps 921 and 1001 (H.); and the observation of that star has been erroneous or mistaken in sw. 921 by about 11′ in P.D., as appears from an obs. of 73 Ursæ in the same sweep. The nebulæ affected by this error are those here enumerated, and it requires very careful consideration to disentangle all the observations of each nebula by both stars, and to decide on their identities. My final conclusions are,—1st, that in these sweeps two distinct nebulæ, II. 794, 1 and II. 794, 2, were observed, and confounded together under one number (=H. 2079 register). These are my h. 1406, 1407. 2ndly, that h. 1407 and III. 778, II. 795, 796 are correctly determined in sw. 1001 (H.). 3rdly, that in sw. 921 (H.) the nebula set down as II. 794 was not the same as that called II. 794 in the reduction of sw. 1001; *i. e.* that it was in fact h. 1406, and that in this observation there is also an error of 6′ in P.D., or that, if not, there must be still another nebula in P.D. 33° 54′ (1860). Finally, that the place of III. 778 given in Phil. Tr., which is affected by the same *general* cause of error, requires a correction of +9′ in P.D.

3180 h. 1405=III. 44. This is the companion of M. 60, and is placed by M. D'Arrest in the first class, even with the $4\frac{1}{2}$-inch Leipzig refractor. Perhaps the very superior light of M. 60 may have led both H. and h. to under-estimate that of its, anyhow, much fainter companion.

3189, 3190 h. 1414, 1415=I. 176, 177. These two, according to Lord Rosse, are connected by faint nebulosity.

3206 III. 778. See note on 3174.

3214 h. 1426=II. 181. Auwers points out a discordance of 19′ in P.D. between my observation and that of II. 181. This is owing mainly, however, to a misprint in Phil. Trans. (See List of Errata.)

No.

3216
3224 } II. 795, 796. See note on 3174.

3228 I, 8=III. 6. The later of these nebulæ is expressly stated in the register (H. 38) to be of the 1st class, though set down (it does not appear why) in the 3rd.

3254 h. 1452=I. 41. The case of this nebula is a very odd one. H. has two observations of it. One on April 5, 1784, where it is described as a "L; B; r neb; sbM; iR Fig; Class I." Another on March 3, 1789, calls it "pB; cL; i Fig; er. Many of the st. visible." So that it may be called a cluster. Both the places of H. and that of h. agree so well, that the object in all must have been the same. Here seems evidence of change.

3256 h. 1453=II. 73. Contradictory descriptions, and possibly two nebulæ differing 1^m in R.A.

3311 h. 1480=I. 141. Query if not changed. h.'s observations are positive as to the clearness of the sky. But query as to the state of the speculum.

3319 h. 1485=II. 384. Not seen by Lord Rosse in two observations (hazy).

3337 h. 1497=I. 68; II. 299; h. 1511=I. 69; h. 1536=II. 301; h. 1574=III. 382.
3338 Auwers finds 5′ Δ.P.D. between H. I. 68 and h. 1497. His place is from P.T.
3358 53 Virginis n. 1° 4′, whereas C.H. in her reductions uses n. 1° 11′, and my
3420 observations of this and the other nebulæ in this list justify the departure. I
3483 subjoin her note on this nebula (in zone 103° C.H.):—

"I. 68, I. 69, III. 282 are each 7′ more north than they are given in the "printed Catalogue. The disagreement is the result of the recalculation, and "is probably owing to my attempting more accuracy in valuing the 'numbers "'to a degree,' &c. &c." (*i. e.* in the index reductions of the Polar distance readings which were parts of an arbitrary scale). And in the next zone (104° C.H.) occurs,

"II. 299 and II. 301 require the same memorandum." In point of fact, comparing my own observations with those reduced by M. Auwers, the differences, as stated by him, run thus:

I. 68	. . . Δ.P.D. H−h=	+5′
I. 69	. . . .	+7′
III. 282	. . . .	+7′
II. 299	. . . .	——
II. 301	. . . .	+6′

so that in each case, where I have observed the object, the alteration is justified. This is only one out of the innumerable instances of painstaking and laborious scrutiny bestowed by her upon these reductions which have occurred to me in the collation of her zone catalogue with the original observations and with my own results.

3356 h. 1509=I. 143. Auwers places this nebula 1° 13′ too much to the south in consequence of an erratum in P.T. (see List of Errata).

No.

3358 See note on 3337.

3363 V. 3. Auwers makes the R.A. of this neb. for 1830 $13^h\ 2^m\ 31^s$, which is 10^m too great. The P.T., which in this instance is correct, makes it follow 75 Leonis $1^h\ 44^m$.

3393 h. 1527. This is not impossibly III. 937, but as both R.A.'s and P.D.'s differ very much, they *may* be different, and are therefore separately stated.

3415 h. 1535. Not seen by Lord Rosse in one observation; clouds passing h. has two observations, both agreeing well.

3420 See note on No. 3337.

3421 II. 185. Auwers, misled by an error in P.T. (see List of Errata), makes the R.A. of this neb. too small by 10^m.

3426 Auw. N. 31. Not visible in the Königsberg Heliometer.

3483 See note on No. 3337.

3506 II. 22. P.D. extremely doubtful.

3512 II. 826. Place re-reduced by the star used by H. and A.S.C.

3527 h. 1597=II. 314. Auwers makes Δ.R.A. H.—h.=$+107^s$, and remarks that there is perhaps some error in P.T. This is the case (see List of Errata), and with the correction there indicated the agreement is satisfactory.

3550 D'Arr. 94. D'Arrest says "not found again, Feb. 19, 1863. Sky perfectly clear. Perhaps a comet."

3588 h. 1633=III. 926. H. says it is sp a considerable star. h. has "a *9m with a very dilute nebulous atmosphere." Has the star or the nebula moved?

3650 III. 946. Auwers makes the declination $+89°\ 17'$, a misprint for $+80°\ 17'$.

3662 h. 1674=I. 255. Evidently ill seen by h. The description of H. preferred.

3664, 3668 h. 1676, 1679=III. 422, 423. Auwers makes the P.D. 12′ too great by reason of an erratum in P.T. (see List of Errata).

3728 h. 1720=III. 666. Auwers finding Δ.R.A. H.—h.=$+52^s$, supposes a mistake of 1^m. Examined sweep 146 (h.), and found all clearly written and right reduced.

3750, 3751 h. 1734, 1735=II. 309, 310. H. says the second is the larger, h. the smaller of the two.

3760, 3762, 3763, 3764, 3766, 3767, 3770, 3771, 3773, 3774 h. 1744=M. 101, and its attendants in more or less intimate nebulous connexion. Of those in Lord Rosse's woodcut, P.T. 1861, p. 729, N, the principal nucleus, is No. 3770=h. 1774; n_1=No. 3774=1744, i; n_2 No. 3773=1744, h. The others are not lettered, and are made out from the joint evidence of this diagram and the measures of position and distance of the stars compared with the copper plate, fig. 35.—1744, *a* is not improbably=III. 787.

No.

3820 h. 1763=III. 804=III. 835. The identity of these nebulæ rests on a memorandum in MS. in my copy of Ph. Tr., supported by the reductions of all the obs. by C.H. in 3 sweeps, each with two determining stars. Auwers makes them differ by 14′ in P.D.

3836 III. 551. Place concluded from h. 1772=III. 552 from H.'s description, viz. that it precedes that nebula by 3′ or 4′ (3′ 30″)=14ˢ of time.

3844 h. 1777=III. 347. Auwers makes Δ.P.D.=−59′, but observes that there must be some misprint. Examining all, I find that such is the case (see List of Errata), which recognized, shows that 1° has been mistaken, and the identity is therefore proved.

3846 h. 1779=I. 144. Auwers makes the P.D. (1830)=86° 30′, and H.—h.=1° 14′. The cause of the discordance is a misprint in P.T. (see List of Errata), in consequence of which the nebula is 1° 13′ north of its printed place.

3858 h. 1789, 1788, 1791=III. 416, 417. Lord Rosse says that of these three only
3859 two were found. The obs. in sw. 28 re-examined—1789 and 1791 were both
3860 observed. Moreover, in sw. 337, III. 417=h. 1791 and h. 1788 were both observed, and 1791 is expressly stated to have been the sf of two seen in moonlight. Now the np of these could not be h. 1789, which is eF and not north, but south preceding, whereas h. 1788 by its place in sw. 338 is np. All three, therefore, really *existed* at the date of these observations. It was h. 1789 (eF) which escaped Lord Rosse's notice, though looked for with greater instrumental power. Perhaps it may have changed.

3863 III. 135. Auwers's P.D. for 1830 is 63° 0′. C.H. reduced to 1830 gives 62° 50′ 20″. Auwers has used (P.T.) 1° 5′ n. of *d*, 12 Bootis; C.H. 1° 16′ n. of the same ⁎. C.H. is to be preferred on every account to P.T. Her Δ.P.D.'s are grounded on a most complete and searching re-examination and recomputation (*according to the then existing star catalogues*) of all the data (in the earlier sweeps most obscure—*foliis sibyllinis obscuriora*) for determining the degrees and minutes of P.D. from the index numbers. In almost every case I find her corrections (or rather interpretations) to be justified; and I have no doubt that in this particular instance such will prove the case, though *here* I confess myself, after consulting the original sweep, unable to perceive the reason for the deviation.

3888 III. 319. Auwers, following P.T., which places the nebula 2° 26′ north of β Ursæ min., makes the P.D. 1830 =12° 46′, and so it stands in the Register sheet (H. 864). But it should be 2° 26′ *south*. So C.H. has used it, and so it proves to be on reference to the original sweep, sw. 391 (H.), giving for the P.D. 17° 36′ 12″.

3920 h. 1832=II. 695. Not seen by Lord Rosse in one observation. See note on No. 132.

3922 h. 3573=Δ. 342. In Auwers's list of Lacaille's nebulæ, he sets down for the

No.

declination of this $-55° 58'·8$. *For* $58'·8$ *read* $48'·8$, if it be the same object, but of that some doubt remains.

3967 VI. 8. Auwers, using χ Virginis, the determining star in P.T., places this cluster in R.A. $14^h 53^m 37^s$ (1830), 99° 55′ P.D. This, however, is declared by a subsequent MS. note to be a mistake for Mayer's 577 zod. star, whence the place in this Catalogue is accordingly derived. But this star, too, must have been mistaken, and on consulting the original sweep (sw. 209, H.) I find no star in the sweep whose identity can be satisfactorily ascertained. All that can be certainly affirmed is that, within a degree one way or the other in P.D., and from 5 to 10 minutes of time in R.A. of the place set down, there exists a fine cluster of the 6th class which should be looked for. Fortunately it is the only nebula observed in the sweep, a very short one.

3977 h. 1866=I. 184. Some suspicion of variability, inasmuch as one description calls it R, another E, and another mE, besides other indications in respect of brightness.

3998 III. 373. C.H., by three distinct observations in three different sweeps (400, 730, 917, H.) from the same determining star 11 Libræ (s. 0° 13′, s. 0° 14′, and s. 0° 15′), deduces a P.D., which reduced to 1830=91° 49′ 39″. Auwers, using the same star, s. 0° 12′ as *per* P.T., places it in P.D. 91° 17′, which, however, is probably a misprint for 91° 47′. Two of H.'s observations place the small star *south*, and one *north* of the nebula.

3999 h. 1881=II. 576. The binuclear character verified by R, who says that it is a close double nebula.

4016 h. 1892=III. 131. Query if not variable in brightness. H. in two observations calls it F and cB; h., in two others, vF and eF.

4025 } II. 756=h. 1898? In the two observations by H. of II. 756 it is described as
4029 }

cF; pL; iF; r;
pB; s; E;

and no mention is made of a double star near it, so that though the places agree within the *possible* limits of discordance, they are most probably two distinct nebulæ.

4043 } 1901, *a*. Two of six seen by Lord Rosse. The others must have been h. 1901,
4044 } h. 1902, II. 541 and III. 511.

4048 } III. 886, 887. Auwers has made an error of −12′ in the declination, or +12′ in
4049 } the P.D. of this double nebula as determined from P.T. (20′ n. of 7 Serpentis). The P.D. here set down is that correctly reduced, C.H. having on her part committed an error of +2′ in P.D.

4051 h. 1905=II. 751. In Auwers's declination, *for* +20° 44′ *read* +20° 14′, an evident misprint.

4065 II. 818. Owing to an erroneous designation of the determining star in P.T. (see

No.

List of Errata), Auwers has given the place of this nebula (1830) as R.A. $14^h\ 41^m\ 3^s$; Decl. $+60°\ 5'$.

4124 h. 1934, &c. In Lord Rosse's diagram of the group h. 1934, A, the most con-
4127 spicuous, would naturally be selected as identical with that nebula, but in that
4128 case II. 766 would not be included in the group. On the other hand, if B be
4131 taken for h. 1934, the identifications will stand as follows:—A=No. 4131
4133 =II. 766; B=No. 4128=h. 1934; C=No. 4127=1934, *b*; D=No. 4124 =1934, *a*. This, however, supposes an error of 45^s of R.A. in H.'s place of II. 766, which is not probable, while on the other hand it is difficult to account otherwise for its not having been noticed at all. All things considered, I have thought it best to enter A as a new nebula, No. 4133=1934, *c*, leaving 766 untouched.

4167 h. 1948=III. 74. Not seen by Lord Rosse, once looked for (see note on No. 132).

4173 h. 3624=M. 80. This is Pogson's globular cluster, with a variable star in the centre, for whose most singular history see the Monthly Notices of the R. Ast. Soc. xxi. pp. 32, 33, by Mr. Pogson. Mr. P. in that statement says that Sir J. Herschel (among others mentioned) had described it as either "cometary" or "nebulous." This is incorrect. In both my observations of this object it stands described as a globular cluster, *all completely resolved into stars.* (See C.G.H. h. 3624.)

4234 h. 1970=Σ. 5. D'Arrest calls this planetary nebula *blue*. The place used is a mean of his observations, that of h. (Catal. of 1833) being only Struve's roughly brought up. M. D'Arrest makes the diameter $=14''\cdot6$.

4247 III. 727. The comparison of the place here set down with that of Auwers is curious for the great number of perfectly accidental errors which have heaped themselves together. The place (C.H.) is rightly reduced by her from σ Herculis, f $16^m\ 11^s$; n $0'\ 14''$, which is that given in P.T., and which, reduced to 1830, gives for the R.A. $16^h\ 44^m\ 46^s\cdot8$ and for the P.D. $47°\ 58'\ 16''$, differing $+8^s\cdot8$ and $+11''$ from the exact result. In M. Auwers's catalogue it is entered thus: III. 127; R.A. $16^h\ 14^m\ 47^s$; Decl. $+43°\ 1'$ (corresponding to P.D. $46°\ 59'$). That is to say, there is a misprint *in each of the three particulars.* This is not to be taken as a specimen of M. Auwers's work, which is an admirable example of painstaking devotion, and far beyond any eulogy in my power to offer. But it is a striking instance of the way in which, in the great run of chances, unlucky coincidences will happen.

4259 h. 1974. Doubtful whether a nebula or a very faint double or triple star.

4294 M. 92 (= also Lalande No. 31544). Not observed by h., but 8 times by H. Place from Wollaston's catalogue, which is almost identical with Auwers (Δ.R.A.$=0^s\cdot1$, Δ.P.D.$=0'\ 3''$).

4302 h. 1981=h. 3686=IV. 11. The annular form only perceived in the southern

No.

observations. Both H. and h., in their northern observations, describe it as of equable light throughout. It appears from Lord Rosse's observations that the annular form is much more common among these "planetary" nebulæ than H. or h. had any idea of.

4364 h. 3723=II. 200. On a ground astonishingly rich.

4368 V. 13. P.D. by Auwers =113° 36′ (1830), owing to an error in P.T. (see List of Errata).

4372 h. 3726=Δ. 473. There is a singular statement respecting this cluster by Cacciatore in No. 113 of the Astronomische Nachrichten. He observed it as a nebula, he says, on the 19th of March, 1826 (of course, therefore, Dunlop has the priority in point of date). But *where* he saw it Lacaille, he says, noted his star 1483 (Cœlum Australe). Also, Piazzi in 1794 and 1801 in the same place saw only a star. Cacciatore in 1809 and 1810 observed the same star, but saw no nebula, only a star 9m following it (P. xvii. 341, 346). In looking for the comet of 1826, however, "fui colpito," he says, "da questa bella nebulosa." Unfortunately for this curious history, the place of Piazzi's star referred to (and which he identifies with 1483 C.A.) differs by no less than 18′ in P.D. from that of the nebula in question, which was therefore out of the field of view, both of his own and of Piazzi's telescope, when observing the star.

4390 h. 2000. Σ. 6. Omitted by Auwers from his catalogue of new nebulæ, which contains many far less remarkable. Diameter, according to D'Arrest, =7″·05. Bessel's place =h.+0s·8, −0′ 22″.

4397 h. 2004=M. 24. H.'s two observations hardly consist with this description, and their deviation in R.A. of nearly 4m from Messier's place makes it very doubtful whether he really saw this object.

4411 M. 69. Piazzi, in a note on xviii. 122 of his catalogue, says that both M. 69 and M. 70 are 1° more to the south. But he is wrong.

4415 Auwers, N. 40. This is the nebula discovered by Tuttle on Sept. 1, 1859, and it would appear to be variable, for M. D'Arrest says (in a letter of May 8, 1863), "La nébuleuse de M. Tuttle (Astron. Nachr. No. 1337. p. 272) était, le 24 Sept. 1862, si brillante et si remarquable dans le chercheur (*grandis et præclara, ovalis, 2′ longa, 80″ lata*), que je suis persuadé qu'elle n'a pas été telle du temps de Messier et de votre père, et de vos propres observations. Voici la position que j'ai obtenue. 1861·0 R.A. 275° 55′·6, N.P.D.=15° 30′·1." The place given in the present Catalogue is that of M. Auwers, and differs somewhat, though not considerably, from this determination.

4428 M. 70. See the note on No. 4411.

4462 III. 742. This agrees too well with M. D'Arrest's place of his No. 113 not to be the same. His description is F; S; R; *10p 12′·6, s 2′ 30″.

4473 Auwers, N. 44. This is the nebula discovered by Mr. Hind on March 30, 1845.

No.

It was observed in May 1852 as a nebula of the first class; subsequently as "pretty faint and diluted." M. Auwers found it "surprisingly faint," and of the 2nd class at the highest.

4487 h. 2037=III. 743. This was seen as a planetary nebula in the twilight by M. D'Arrest with the 4½-inch refractor, and can therefore hardly be ranked so low as Class III.

4536 h. 2062=III. 144. Not seen by Lord Rosse; once looked for. (See note on No. 132, &c.)

4570 h. 2073. Not seen by Lord Rosse; twice looked for. h. has three observations agreeing well. The object is an equivocal one.

4585 }
4586 } h. 2081=I. 103. According to an observation of Olbers, cited by Auwers, this is identical with No. 4585=I. 103, the place of the latter nebula, as assigned by H., being 20′ wrong in P.D. This had escaped my notice until the nebulæ in this Catalogue had been finally numbered and much other work accumulated on them; and it was considered better to let No. 4585 stand, though erroneous, than to hazard confusion by striking it out and altering all the subsequent numbering.

4618 h. 2093. In conformity with Mr. Mason's remarks on my observations of this nebula, and with his elaborate and excellent monograph of the great nebulous system of which it forms a part, I have diminished the P.D. in my Catalogue of 1833 by 1°. It is evident that the index reading must have been mistaken, 1° for 0°. Sweep 8 examined; the writing is clear and the reduction correct, but the conclusion from Mr. Mason's observations is irresistible.

4628 h. 2098=IV. 1. According to Lassell this is annular, an elliptic ring with a star in the centre.

4654 h. 2113. Not seen by Lord Rosse; twice looked for. Examined sw. 86 (h.), in which it was observed. All found apparently correct, the observation clearly written and right reduced: and it is added, "the double star" (h. 934 in my "3rd series of observations, &c. &c.," Mem. Ast. Soc. vol. iii.) "is a good guide." A diagram accompanying the observations, by indicating lines points out the relative situation of the double star and nebula.

4710 h. 2133. Not seen by Lord Rosse in four observations.

4714 h. 3897. Not found by Mr. Lassell within 30′ all round the place.

4723 h. 2137=III. 920. Not seen by Lord Rosse in one observation.

4756 h. 2148. Not seen by Lord Rosse in three observations. In one a cloud passing.

4775 h. 2156=III. 932. H. says, "just sf a S∗ to which it seems almost to be attached, but is free from it." h. says, "has a ∗ 13m at a distance from the edge = 1 diameter by diagram." Sw. 274 (h.). This sweep re-examined. The diagram makes the star north of the nebula. The description says, "Diagram certainly right."

4816 2172, *a*. In this group Lord Rosse has given only measures of relative position,

No.

and none of distance; so that it is impossible to assign specific places to the individuals of which it consists. He speaks of five *near* to h. 2172. The diagram exhibits only four. One may possibly be III. 166.

4848 2184, *a*. In Lord Rosse's diagram of the group to which this belongs, α is h. 2183 = No. 4845; β = D'Arr. 117 = No. 4844; γ = h. 2184 = III. 217 = No. 4846; δ = D'Arr. 118 = No. 4847. That marked as 2184, *a* is not lettered in the diagram, and is "nova."

4892 h. 2205 = I. 55. Placed in the second class only by M. D'Arrest with the 4½-inch Leipzig refractor. In this Catalogue it is set down as only "pretty Bright," from a mean of seven observations.

4894 h. 3971 = h. 3972. These are assuredly identical; but the minute of R.A. being doubtful, that of the earlier 3971 is preferred. The mean of the seconds and the Polar distances is taken, blending the two, and also the descriptions.

4922 h. 2223 = III. 222. Three times called by h. "pretty Bright," and three times by h. and H., eF; vF; eF. Is this a case of variability?

4933 h. 2228 = h. 3982 = I. 104. Placed in the second class by M. D'Arrest. With this the present Catalogue agrees; making it "pretty Faint" by a mean of three observations.

4941 D'Arr. Not included by M. D'Arrest in his final list; but there are four observations of it recorded in his "Resultate," all agreeing well.

4964 h. 2241 = IV. 18. According to Mr. Lassell this superb "planetary nebula" is *bi-annular*, consisting of a nucleus and *two oval rings*.

4966 h. 2242 = III. 226. Called by h. in four observations, pB; pB; pB; pB, and in two by H. eF; vF.

4980 h. 2250 = III. 213. Not seen by Lord Rosse in 4 observations. In my observations of sweep 103, a very short sweep, using the quadrant instead of the index arc, and with no good zero star, both R.A. and P.D. may be a good deal wrong. My place, however, agrees pretty well with that of H. (Δ.R.A. = 5^s, Δ.P.D. = $4'$), and the existence of a nebula as described, *hereabouts*, is certain, but it should be looked for within somewhat wider limits.

4998 h. 2261 = I. 110. H. has two observations in which this nebula is called cB; h. has one where it is called eF; adding "sky quite clear."

5003
5004 h. 2263 = II. 208. These can hardly be the same. The R.A.'s differ by nearly 2^m and the P.D.'s by $6'$. The descriptions also disagree. 255°, the position of the star 14m in h. 2263, is not np but sp, and the estimates of their magnitudes differ materially.

5015 h. 2271 = III. 854. A very problematic object, and in which there is great difficulty in making out its nature. Stars and nebula oddly mixed.

5020
5021
5022 h. 2274 = II. 230; 2274, *a*; h. 2275 = II. 231. In Lord Rosse's diagram of this group, α = h. 2274; β = h. 2275; γ = nova = 2274, *a*. h. sweep 91 makes II. 230 the np of two, and II. 231 "to have II. 230, 45° sp." This is contradicted

No.

by the diagram. There is some confusion among the observations as to whether the two nebulæ II. 230, 231 really lie np or sp from each other, and it might be suspected that the P.D.'s had been read crossways, the R.A.'s being rightly set down; but Lord Rosse's diagram and measures decide the point in favour of the relative situation being here correctly given.

5051 h. 2302. Not seen by Lord Rosse in two observations. Examined the original observation, all clear and apparently correct. The nebula certainly exists in or very near the place here set down.

5061 2849, *a*. A nebula mentioned by M. D'Arrest, but not included in his MS. list of well-determined nebulæ. Should, however, be looked for.

References to Figures of Nebulæ in various works.

In the following list of figured nebulæ, the first column contains the current number of the nebula or cluster in the present Catalogue; the second the number attached to it in my Catalogues in P.T. 1833 and C.G.H.; or if not found in either of these, the class and number in my Father's Catalogues or other sufficient designation. The third contains an abbreviated reference to the publication in which the figure will be found, viz.—

P.T. 33. The volume of the Philosophical Transactions of the Royal Society for A.D. 1833.

P.T. 44. Ditto, Ditto, for 1844 }
P.T. 50. Ditto, Ditto, for 1850 } Lord Rosse's papers.
P.T. 61. Ditto, Ditto, for 1861 }

C.G.H. Results of astronomical observations at the Cape of Good Hope by J.F.W.H.

R. di. The woodcut diagrams in Lord Rosse's paper, Philosophical Transactions, 1861; such only being referred to as express some distinct peculiarity not elsewhere figured.

B.A.A. Professor Bond's Memoirs in vol. iii. N.S. of the Transactions of the American Academy of Arts and Sciences.

M.A.A. Mr. Mason's Memoirs in vol. vii. of the Transactions of the American Academy.

D'Arr. M. D'Arrest's Inaugural dissertation and description of the Copenhagen Equatoreal, 1861.

Lam. Dr. Lamont's "Oeffentliche Vorlesung über die Nebelflecken." München 1837.

Lass. Mr. Lassell's Memoirs in vol. xxiii. of the Transactions of the Royal Astronomical Society.

Column 4 contains the number of the Plate in the volume referred to where the figure will be found, and column 5 the number of the figure in that Plate.

The figures annexed to Mr. Dunlop's catalogue are not included, as for the main part they offer no resemblance to the objects figured (when identifiable), and would serve only

to mislead. The same remark applies to most of the older figures of nebulæ scattered through the volumes of the Histoire de l'Académie Française, and other collections. Of the older figures of the nebula in Orion, however, for curiosity's sake, a list is subjoined. The figures accompanying my Father's memoir in Philosophical Transactions, 1811, are also omitted. They do not profess to be resemblances, and are given rather as types of certain classes of objects into which he there considers the nebulæ to be distributable. At least they are made from very rude diagrams.

References to published figures of Nebulæ.

No. in Catalogue.	h. &c.	Work cited.	No. of plate.	No. of fig.
27	2315	C.G.H.	iv.	8
31	15	P.T. 61	xxv.	1
52	2322	C.G.H.	iii.	1
67	2327	C.G.H.	vi.	19
105	44	B.A.A.	opp. p. 86	
106	45	B.A.A.	Ditto.	
116	50	B.A.A.	Ditto.	
117	51	B.A.A.	Ditto.	
138	61	P.T. 33	vi.	52
169	2359	C.G.H.	v.	10
187	2370	C.G.H.	iv.	6
298	112	P.T. 33	v.	38
303	116	R. di.		
352	131	P.T. 50	xxxvi.	5
		P.T. 61	xxvi.	10
372	142	R. di.		
400	151	P.T. 33	vi.	58
412	156	P.T. 61	xxv.	2
527	218	P.T. 33	ii.	28
		R. di.		
544	223	D'Arr.	ii.	7
560	232	P.T. 61	xxv.	3
567	2487	C.G.H.	vi.	14
572	241	P.T. 61	xxv.	4
575	242	P.T. 33	vi.	56
		P.T. 61	xxv.	5
600	262	P.T. 61	xxv.	6
705	2534	C.G.H.	vi.	7
731	2552	C.G.H.	iv.	1
810	311	P.T. 33	ii.	31
		P.T. 61	xxv.	17
822	2620	C.G.H.	v.	11
823	2621	C.G.H.	v.	11
826	2618	D'Arr.	ii.	9
		Lass.	ii.	4
853	315	P.T. 61	xxv.	8
888	327	P.T. 61	xxv.	9
979	2709	C.G.H.	iii.	3
980	2710	C.G.H.	iii.	3
981	2711	C.G.H.	iii.	3
987	2716	C.G.H.	iii.	3
1057	2775	C.G.H.	vi.	1
1082	2802	C.G.H.	iii.	6
1084	2803	C.G.H.	iii.	6
1085	2804	C.G.H.	iii.	6
1086	2805	C.G.H.	iii.	6
1089	2808	C.G.H.	iii.	6
1090	2810	C.G.H.	iii.	6
1135	2840	C.G.H.	iii.	2
1137	355	P.T. 33	v.	49
1140	2842	C.G.H.	iii.	2
1141	2843	C.G.H.	iii.	2
1142	2844	C.G.H.	iii.	2
1143	2845	C.G.H.	iii.	2
1156	2859	C.G.H.	iv.	7
1157	357	P.T. 33	viii.	81
		P.T. 44	xix.	81
		R. di.		
		D'Arr.	ii.	4
		Lass.	ii.	1
1163	2864	C.G.H.	iv.	7
1164	2865	C.G.H.	iv.	7
1165	2866	C.G.H.	vi.	20
1168	2867	C.G.H.	vi.	20
1171	Δ. 136	C.G.H.	vi.	20
1171	2868	C.G.H.	vi.	20
1174	2872	C.G.H.	iv.	7
1175	2869	C.G.H.	vi.	20
1176	2875	C.G.H.	iv.	7
1177	2876	C.G.H.	iv.	7
1179	360	C.G.H.	viii.	1
		B.A.A.	opp. p. 96	
		Lass.	i.	1
		*	see note	
1180	V. 30	C.G.H.	ii.	3
1183	361	C.G.H.	ii.	3
		P.T. 50	xxxviii.	6
		Lass.	ii.	3
1185	M. 43	C.G.H.	viii.	1
		B.A.A.	opp. p. 96	
		Lass.	i.	1
		*	see note	
1225	365	D'Arr.	ii.	2
		Lass.	ii.	2
1226	iv. 24	D'Arr.	ii.	10
1233	2910	C.G.H.	iii.	5
1235	2913	C.G.H.	iii.	5
1238	2916	C.G.H.	iii.	5
1243	2918	C.G.H.	iii.	5
1248	2923	C.G.H.	iv.	9
1249	2925	C.G.H.	iv.	9
1258	2935	C.G.H.	iv.	9
1259	2933	C.G.H.	iv.	9
1260	2936	C.G.H.	iv.	9
1265	2938	C.G.H.	iv.	9
1266	2939	C.G.H.	iv.	9
1267	368	P.T. 33	iv.	36
		R. di.		
1269	2941	C.G.H.	ii.	4
1276	2948	C.G.H.	iii.	4
1277	2949	C.G.H.	iii.	4
1278	2950	C.G.H.	iii.	4
1279	2951	C.G.H.	iii.	4
1281	2952	C.G.H.	iii.	4
1282	2953	C.G.H.	iii.	4
1283	2954	C.G.H.	iii.	4
1419	390	R. di.		
1425	393	P.T. 61	xxvii.	11
1437	399	P.T. 33	vi.	64
		P.T. 50	xxxvii.	10
		Lass.	ii.	8

Table (continued).

No. in Catalogue.	h. &c.	Work cited.	No. of plate.	No. of fig.	
1467	415	P.T. 33	viii.	91	
1477	421	P.T. 61	xxvii.	12	
1511	3075	C.G.H.	iv.	4	
1519	444	P.T. 33	vii.	72	
		P.T. 50	xxxvii.	6	
1520 1521	445	Lass.	ii.	9	
1532	450	P.T. 50	xxxviii.	15	
		Lass.	ii.	6	
1565	464 3093	P.T. 50	xxxviii.	12	
		Lass.	ii.	5	
1567	3095	Lass.	ii.	7	
1677	3131	C.G.H.	vi.	12	
1721	536	P.T. 33	vi.	61	
		Lam.	i.	8	
1728	537	P.T. 33	vi.	65	
1745	3145	C.G.H.	v.	12	
1801	3154	C.G.H.	v.	8	
1861 1863	604	P.T. 33	vii.	70	
		P.T. 50	xxxvi.	3	
1911	639	P.T. 61	xxvii.	13	
2003	3221	C.G.H.	v.	9	
2017	3228	C.G.H.	vi.	9	
		Lass.	ii.	10	
2058	692	P.T. 61	xxvii.	14	
2063	3241	C.G.H.	vi.	2	
2067	3239	C.G.H.	iv.	3	
2102	3248	C.G.H.	vi.	5	
		Lass.	ii.	11	
2158	731	P.T. 33	v.	40	
2197	3295	C.G.H.	ix.	1	η Argûs.
2216	765	P.T. 61	xxvii.	15	
2217	766	P.T. 61	xxvii.	15	
2333	3324	C.G.H.	iv.	10	
2336	3325	C.G.H.	iv.	10	
2337	3326	C.G.H.	iv.	10	
2338	3327	C.G.H.	iv.	10	
2340	3329	C.G.H.	iv.	10	
2342	3330	C.G.H.	iv.	10	
2343	838	P.T. 33	ii.	32	
		P.T. 50	xxxvii.	11	
2373	854	P.T. 33	vi.	53	
		P.T. 50	xxxvii.	7	
		Lam.	i.	6	
2377	857 875	P.T. 33	vi.	54	
		P.T. 61	xxvi.	16	
2378	859	P.T. 33	vi.	51	
2379	858	R. di.			
2445	910	R. di.			
2486	934 3355	P.T. 33	vii.	79	
2488	936 3356	P.T. 33	vii.	79	
2559	982	R. di.			
2597	1002	R. di.			
2606	1011	P.T. 61	xxvi.	17	
2652	1041	P.T. 50	xxxvii.	7	
2670	1052	P.T. 61	xxvi.	16	
2671	1053	P.T. 61	xxvi.	16	
2680	1061	P.T. 61	xxvii.	19	
2733	1092	R. di.			
2756	1111	P.T. 61	xxvii.	20	
2760	1113	P.T. 61	xxvii.	20	
2804	1146	P.T. 33	vii.	71	
2806	1148	P.T. 33	vi.	59	
2807	1149	P.T. 50	xxxvii.	8	
2838	1173	P.T. 50	xxxv.	2	

No. in Catalogue.	h. &c.	Work cited.	No. of plate.	No. of fig.	
2841	1175	P.T. 33	vi.	55	
2870	1196	P.T. 61	xxvii.	21	
2878	1202	P.T. 33	vii.	69	
2884	1196. a	P.T. 61	xxvii.	21	
2910	1225	P.T. 33	vi.	57	
2950	1245	P.T. 61	xxvii.	22	
2958	1252	P.T. 33	vii.	68	
2962	1252	P.T. 33	vii.	68	
2972	1258	R. di.			
3041	1306	P.T. 61	xxvii.	23	
3042	1308	P.T. 61	xxvii.	23	
3085	1337	P.T. 33	iv.	37	
		P.T. 61	xxviii.	24	
3101	1352	P.T. 33	viii.	83	
3106	1357	P.T. 50	xxxvii.	9	
3108	1358 1363	P.T. 33	vii.	78	
3109	1359 1365	P.T. 33	vii.	78	
3113	1362	P.T. 33	vi.	66	
3132	1376	P.T. 33	vi.	50	
3151 3152	1385	P.T. 61	xxviii.	25	
3165	1397	P.T. 33	vii.	76	
		P.T. 50	xxxvii.	9	
3180	1405	P.T. 33	vii.	74	
3182	1408	P.T. 33	vii.	74	
3189	1414	P.T. 33	vii.	75	
3190	1415	P.T. 61	xxviii.	26	
3240	1441	P.T. 61	xxviii.	27	
3249	1451	R. di.			
3258	1456	P.T. 33	v.	41	
3275	3435	C.G.H.	i.	2	
3278	1466	P.T. 33	viii.	84	
3321	1486	P.T. 33	ii.	27	
3340	1499	P.T. 33	vi.	62	
3356	1509	P.T. 33	vi.	67	
3511	1589	P.T. 61	xxviii.	28	
3525	3501	C.G.H.	iv.	2	
3531	3504	C.G.H.	v.	7	
3570	3514	C.G.H.	vi.	1	
3572	1622	P.T. 33	ii.	25	
		P.T. 50	xxxv.	1	
		R. di.			
3606	3523	C.G.H.	iv.	5	
3614	1649	P.T. 33	v.	39	
3615	1650	P.T. 61	xxviii.	29	
3651	3541	C.G.H.	vi.	15	
3706	3548	C.G.H.	vi.	10	
3717	1713	P.T. 61	xxviii.	30	
3750	1734	R. di.			
3766	III. 787	P.T. 61	xxix.	35	
3770	1744	P.T. 61	xxix.	35	
3778	III. 788	P.T. 61	xxix.	35	
3779	III. 789	P.T. 61	xxix.	35	
4051 4052	1905	P.T. 33	vii.	77	
		P.T. 61	xxviii.	31	
4058	1909	P.T. 50	xxxvii.	8	
4066	3594	C.G.H.	vi.	8	
4083	1916	P.T. 33	viii.	87	
4087	1917	R. di.			
4118	1929	P.T. 33	viii.	89	
4125	3610	C.G.H.	vi.	7	
4160	1946	P.T. 61	xxviii.	32	
4224	3641	C.G.H.	v.	4	
4229	3644	C.G.H.	v.	6	
4230	1968	P.T. 33	viii.	86	
		P.T. 61	xxviii.	33	
4234	1970	Lam.	i.	1	

TABLE (continued).

No. in Catalogue.	h. &c.	Work cited.	No. of plate.	No. of fig.		No. in Catalogue.	h. &c.	Work cited.	No. of plate.	No. of fig.	
4261	3661	C.G.H.	vi.	13		4572	2075	P.T. 33	v.	47	
4284	3675	C.G.H.	vi.	6				P.T. 61	xxviii.	34	
4290	3680	C.G.H.	vi.	3				Lam.	i.	5	
	3680, 2	C.G.H.	v.	8	Milky Way.	4594	2084	P.T. 61	xxx.	36	
4302	{1891 3686}	C.G.H.	vi.	4		4600	2088	P.T. 33	iii.	33	
						4616	2092	P.T. 33	iii.	34	
4305	3688	C.G.H.	vi.	18				M.A.A.	vii.	1	
	3702, 2	C.G.H.	v.	1	Milky Way.	4618	2093	P.T. 33	viii.	82	
4335	3707	C.G.H.	v.	5				M.A.A.	vii.	1	
4342	3713, 2	C.G.H.	v.	2		4627	2099	P.T. 61	xxx.	37	
4343	1989	P.T. 33	v.	42		4628	2098	P.T. 33	v.	44	
4355	{1991 3718}	P.T. 33	viii.	80				P.T. 50	xxxviii.	14	
								D'Arr.	ii.	1	
		C.G.H.	ii.	2				Lam.	i.	4	
		M.A.A.	iv.	1		4678	2125	P.T. 33	viii.	88	
4361	3722	C.G.H.	i.	1				P.T. 44	xviii.	88	
4375	3727	C.G.H.	vi.	16		4687	{2128 3878}	P.T. 33	viii.	90	
4395	2002	P.T. 33	ii.	30							
4403	2008	P.T. 33	iv.	35		4729	3908	C.G.H.	iv.	11	
		C.G.H.	ii.	1		4730	3909	C.G.H.	iv.	11	
		Lam.	i.	10		4731	3910	C.G.H.	iv.	11	
		M.A.A.	vi.	1		4733	3911	C.G.H.	iv.	11	
4437	2019	Lam.	i.	9		4734	2139	P.T. 61	xxx.	38	
4447	2023	P.T. 33	ii.	29		4815	2172	P.T. 61	xxx.	39	
		P.T. 44	xix.	29	†	4876	2197	P.T. 33	vii.	73	
		D'Arr.	ii.	5		4877	2198	P.T. 33	vii.	73	
4487	2037	Lam.	i.	7		4892	2205	P.T. 33	vi.	63	
4510	2047	P.T. 33	v.	46				P.T. 50	xxxvi.	4	
		D'Arr.	ii.	3				D'Arr.	ii.	6	
		Lam.	i.	2		4950	2236	P.T. 33	vi.	60	
4514	2050	P.T. 33	v.	43		4964	2241	P.T. 33	v.	45	
4532	2060	P.T. 33	ii.	26				P.T. 50	xxxviii.	13	
		P.T. 44	xix.	26				P.T. 61	xxx.	40	
		P.T. 50	xxxviii.	17				Lam.	i.	3	
		P.T. 61	xxxi.	43		4971	2245	P.T. 33	viii.	85	
		D'Arr.	ii.	8				P.T. 61	xxx.	41	
4565	2072	P.T. 33	v.	48		5046	2297	P.T. 61	xxx.	42	

* No. 1179=h. 360. Other figures of the great nebula in Orion will be found in Huyghens's Systema Saturnium, 1659; ditto, copied by Le Gentil in Mém. Acad. Sci. Par. 1759, pl. 21. fig. 1; Le Gentil's own figure in do. do. fig. 2; by Picard, do. do. fig. 5; another by Le Gentil, do. do. fig. 6. See also:—

Mairan, "Sur la Lumière Zodiacale," copied in Lalande's 'Astronomy.' These older representations, however, are mere curiosities, and present no points of exact resemblance.

Messier, Hist. de l'Acad. Sci. Par. 1771, p. 435...461. Plate 8 is a careful and (for the time) elaborate figure.

J. F. W. Herschel, Mem. Astron. Soc. ii. 1826.

De Vico, Memoria intorno ad alcune osservazioni fatte nel Collegio Romano nel corrente anno 1838, nebulosa d'Orione osservata al Telescopio di Cauchoix. 1839.

Bond. A very fine engraving—not yet published.

† No. 4447. P.T. 44. xix. fig. 29. There is an erratum in this figure. *For* Decl. 32° 49′ n *read* 22° 49′ n.

The following nebulæ have been indicated by Lord Rosse as being either "of spiral structure (S), having in them dark spaces (D), as knotted (K), or as in the form of rays (*i. e.* much elongated forms) with splits or clefts (R).

No. in Catalogue.	h. &c.		No. in Catalogue.	h. &c.		No. in Catalogue.	h. &c.		No. in Catalogue.	h. &c.	
202	84	K	2158	731	D	2717	1085	S	3249	1451	S
372	142	S	2194	749	S	2733	1092	S	3258	1456	S
594	257	K	2248	788	D	2749	1107	D	3474	1570	S
600	262	S	2373	854	S	2807	1149	R	3572	1622	S
604	264	D	2377	857	D	2870	1196	S	3750	1734	S
888	327	S	2379	858	S	2878	1202	S	3843	1776	S
895	329	K	2413	887	D	2890	1211	S	4045	1901	K
1267	368	D	2445	910	S	2910	1225	D	4058	1909	D
1458	409	K	2499	943	S	2991	1274	K	4087	1917	R
1527	446	K	2559	982	S	3049 3050	1312	S	4572	2075	S
1676	514	D	2597	1002	S				4815	2172	S
1806	581	K	2652	1041	R	3106	1357	R	4964	2241	D
2058	692	D	2670	1052	S	3121	1368	S	4971	2245	S
2066	695	S	2680	1061	S						

List of Errata and Corrigenda in Sir William Herschel's Catalogue of 2500 *Nebulæ in the Philosophical Transactions.*

Class.	No.	No. in Catalogue.	Error and Correction.
I.	6	3702	*for* f. $3^m\ 56^s$ *read* f. $33^m\ 56^s$
	54	214	*for* f. $12^m\ 44^s$; s. 2° 50′ *read* f. $18^m\ 36^s$; s. 1° 26′
	87	2274	*for* f. $9^m\ 30^s$ *read* f. $10^m\ 30^s$
	137	1837	*for* 42 Lyncis *read* 41 Lyncis
	143	3356	*for* s. 2° 7′ *read* s. 0° 54′
	144	3846	*for* n. 0° 24′ *read* n. 2° 7′
	153	536	*for* p. $23^m\ 16^s$ *read* f. $23^m\ 16^s$
	154	549	*for* f. $1^m\ 23^s$ *read* p. $1^m\ 23^s$
	155	778	*for* f. $7^m\ 49^s$ *read* p. $7^m\ 49^s$
	202	2604	*for* f. $0^m\ 47^s$ *read* f. $7^m\ 47^s$
	203	2597	*for* f. $7^m\ 42^s$ *read* f. $14^m\ 42^s$
II.	1	4738	*for* p. 15^m::, s. ½°:: *read* p. $11^m\ 45^s$, n. 0° 17′
	11	2824	*for* f. $1^m\ 24^s$, n. 0° 24′ *read* f. $1^m\ 13^s$, n. 0° 30′
	14	2730	*for* 3 (ν) Virginis f. $2^m\ 20^s$, n. 1° 22′ *read* 59 *e* Virginis p. $69^m\ 0^s$, n. 0° 11′
	48	1707	*for* f. $56^m\ 45^s$ *read* f. $54^m\ 45^s$
	181	3214	*for* s. 0° 48′ *read* s. 1° 15′
	185	3421	*for* p. $11^m\ 0^s$ *read* p. $1^m\ 0^s$
	239	634	*for* 27 (κ) Persei p. $8^m\ 20^s$, n. 0° 2′ *read* 30 Persei p. $14^m\ 41^s$, n. 0° 51′
	240	5046	*for* .... *read* 39 Pisc. p. $2^m\ 24^s$, n. 1° 0′
	241	29	*for* .... *read* 39 Pisc. p. $14^m\ 24^s$, s. 0° 11′
	242	4973	*for* 48 (μ) Pegasi *read* 87 (*u*) Pegasi
	264	1335	*for* 47 (δ) Cancri *read* 25 (δ) Canis
	265	1384	*for* 1 χ Can. *read* ξ 1 Can.
	286	654	*for* .... p. *read* 13 (ζ) Eridani p.
	314	3528	*for* f. $17^m\ 57^s$ *read* f. $15^m\ 57^s$
	372	2771	*for* p. $74^m\ 24^s$ *read* p. $14^m\ 24^s$
	658	1718	*for* 44 Lyncis *read* 43 Lyncis
	708	1788	*for* 37 Lyncis *read* 36 Lyncis
	794	3177 3179	*for* s. 0° 49′ *read* s. 1° 0′ (see note on this No. in Catal.)
	795	3216	*for* s. 1° 13′ *read* s. 1° 24′ (see note on this No. in Catal.)
	796	3224	*for* s. 1° 25′ *read* s. 1° 36′ (see note on this No. in Catal.)
	818	4056	*for* 12 Draconis *read* 12 Draconis Hevelii
	853	14	*for* p. $25^m\ 38^s$ *read* p. $25^m\ 48^s$
II .	6	3228	*for* 59 (*e*) Virginis p. $28^m\ 11^s$ *read* *d* Virginis f. $2^m\ 42^s$, n. 0° 57′. The obs. belongs to I. 8
	26	3078	*for* 20 Comæ f. $4^m\ 30^s$, s. 0° 37′ *read* 26 Comæ p. $5^m\ 5^s$, s. 0° 32′
	112	2417	*for* 74 φ Leonis f. $10^m\ 6^s$, s. 1° 52′ *read* Mayer 510. z. p. $61^m\ 48^s$, s. 1° 10′
	113	2577	*for* φ Leonis f. $34^m\ 18^s$, s. 1° 3′ *read* Mayer 510. z. p. $37^m\ 36^s$, s. 0° 31′
	178	631	*for* 17 (γ) Persei f. $9^m\ 6^s$ *read* 17 (*r*) Persei f. $10^m\ 0^s$
	192	419	*for* 72 Ceti *read* 62 Ceti

Table (continued).

Class.	No.	No. in Cata-logue.	Error and Correction.
III.	195	684	*for* f. 42^{m} 42^{s} *read* f. 41^{m} 6^{s}
	199	628	*for* 27 (κ) Persei p. 8^{m} 27^{s}, n. 0° 2′ *read* 30 Persei p. 14^{m} 44^{s}, n. 0° 55′
	256	1641	*for* s. 0° 48′ *read* s. 0° 58′
	289	1911	*for* s. 0° 25′ *read* s. 0° 31′
	293	1974	*for* 23 Leonis *read* 23 Leonis minoris
	319	3888	*for* n. 2° 26′ *read* s. 2° 26′
	347	3844	*for* s. 1° 17′ *read* s. 0° 17′
	353	2440	*for* f. 53^{m} 4^{s} *read* f. 43^{m} 4^{s}
	369	3618	*for* —— 25^{m} 41^{s} *read* f. 25^{m} 41^{s}
	422	3664	*for* n. 0° 44′ *read* n. 0° 36′
	423	3668	*for* n. 0° 44′ *read* n. 0° 36′
	511	4042	*for* f. 3^{m} 5^{s} *read* f. 3^{m} 11^{s}
	607	1645	*for* p. 12^{m} 33^{s} *read* p. 12^{m} 23^{s}
	627	1820	*for* 43 Lyncis *read* 42 Lyncis
	739	4149	*for* p. 32^{m} 30^{s} *read* p. 32^{m} 47^{s}
	751	1897	*for* 39 Lyncis *read* 38 Lyncis
	778	3206	*for* s. 1° 4′ *read* s. 1° 15′ (see note on this No. in Catal.)
IV.	29	2255	*for* f. 3^{m} 36^{s} *read* f. 3^{m} 46^{s}
	31	4802	*for* —— 0° 37′ *read* s. 0° 37′
V.	13	4368	*for* n. 0° 39′ *read* s. 0° 38′ : :
VI.	8	3967	*for* 26 χ Virginis f. 23^{m} 44^{s}, s. 0° 6′ *read* Mayer 577. z. f. 1^{m} 48^{s}, n. 1° 26′
VII.	6	1509	*for* 50 Geminorum *read* 51 Geminorum
VIII.	11	1534	*for* 50 Geminorum *read* 51 Geminorum
	28	1229	*for* (1 λ) Orionis *read* (1st χ) Orionis
	44	1533	*for* 5 (η) Can. min. *read* 4 (γ) Can. min.

The following nebulæ are declared in MS. notes to be identical.

II. 6=I. 1; II. 119=II. 94; II. 148=II. 20; II. 176=II. 70; II. 243=II. 241; II. 703=II. 621; III. 6=I. 8; III. 198=II. 238; III. 835=III. 804.

Errata and Corrigenda in M. Auwers's Catalogue.

Page.		*For*	*Read*	Page.		*For*	*Read*
19	....	II. 844	II. 834	37	III. 946 in Decl.	89°	80°
20	III. 291 in Decl.	27°	26°	39	III. 347 in Decl.	24°	25°
24	II. 30 in R.A.	11^{h} 12^{m} 21^{s}	11^{h} 11^{m} 33^{s}	40	II. 751 in Decl.	20° 44′	20° 14′
25	IV. 59 under $\Delta\alpha$	−5	−31	40	....	I. 282	I. 182
26	III. 385 in R.A.	16^{h}	11^{h}	42	III. 127.........	III. 127	III. 727
26	III. 388 in R.A.	10^{h}	11^{h}	42	Do. in R.A.	14^{m}	44^{m}
26	II. 342 in R.A.	10^{h}	11^{h}	42	Do. in Decl.	43°	42°
28	III. 814 in Decl.	53°	54°	72	No. 27 Decl.	58′·8	48′·8
32	....	III. 858	III. 850	77	List of Errata, II. 341...	16^{m} & 11^{m}	16^{u} & 11^{u}
33	III. 778 in R.A.	13^{h} 37^{m}	12^{h} 37^{m}	77	Ditto, III. 680...	26^{m} & 16^{m}	26^{u} & 16^{u}

M. Auwers has given a list of errata and corrigenda required in my two previous Catalogues. They are very numerous, but relate almost exclusively to errors of identification with my Father's classes and numbers. They had been, with hardly an exception, detected and rectified during the process of preparing and arranging the present Catalogue, which being therefore expurgated of them, it is unnecessary to annex a list of them here.

One very important erratum, however, must be noticed, not having been set down by M. Auwers. In p. 494, explanation of plates, Phil. Trans. 1833, figs. 13...18, *for* pmbM; vbM; vmbM *read* psbM; sbM; vsbM.

No. of Catalogue.	References to: Sir J. H.'s Catalogues of Nebulæ.	Sir W. H.'s Classes and Nos.	Other Authorities.	Right Ascension for 1860, Jan. 0.	Annual Precession in Right Ascension for 1880.	No. of Obs. used.	North Polar Distance for 1860, Jan. 0.	Annual Precession in N.P.D. for 1880.	No. of Obs. used.	Summary Description from a Comparison of all the Observations, Remarks, &c.	Total No. of times of Obs. by h. and H.
	h.	H.		h m s	s		° ′ ″	″			
1	......	......	D'Arrest, 1	0 0 5	+3·07	[4]	63 4 0	−20·05	[4]	F; S; R; bet*11 and *14 ...	0
2	4014	......	....	0 1 13·8	3·065	3	120 41 11·5	20·05	3	eF; cL; mE; vgvlbM.........	3
3	4015	......	....	0 1 28·3	3·062	2	124 38 54·5	20·05	2	F; cL; vlE; glbM	2
4	1	III. 868	....	0 1 34·2	3·073	2	86 7 38·5	20·05	2	eF; plL; vglbM	3
5	2	III. 866	....	0 1 35·7	3·084	2	57 20 43·5	20·05	2	vF; vS; Sst+neb	3
6	2, a	......	R. nova	0 1	......	...	57 20	......	...	....	0
7	3	II. 591	....	0 1 37·0	3·076	3	74 57 52·5	20·05	3	vF; pS; R; glbM	4
8	4	......	....	0 1 52·1	3·083	1	63 3 27·5	20·05	1	pB; S; R; bM..................	1
9	......	III. 147	....	0 2 33·4	3·081	1	64 49 57·5	20·05	1	3Sst+neb	1
10	2308	III. 461	....	0 2 47·2	3·061	1	115 44 58·5	20·05	1	vF; cL; mE; gbM	2
11	2309	......	....	0 2 57·1	3·053	2	147 48 14·5	20·05	2	vF; S; R	2
12	5	IV. 15	....	0 3 14·6	3·085	(1)	63 4 58·5	20·05	1	vF; vS; stell	2*
13	2310	......	....	0 3 25·2	3·033	1	147 46 24·5	20·05	1	eF; p of 2........................	1
14	6	II. 853	....	0 3 31·8	3·089	2	57 25 28·5	20·05	2	pB; pL; E 0°±	3
15	2311	......	....	0 3 39·0	3·020	2	147 46 21·5	20·05	2	eeF; S; R; f of 2.............	2
16	......	......	Auw. N. 1	0 3 41·7	3·078	...	71 59 0·7	20·05	...	F (Schmidt 1861, Oct. 10) ...	0
17	2312	......	....	0 4 22·0	3·023	1	147 43 45·5	20·05	1	eF; S; R	1
18	7	III. 861	....	0 5 7·5	3·093	2	59 44 3·5	20·05	3	v F; pS; R	4
19	......	III. 456	....	0 5 11·3	3·076	1	84 21 57·5	20·05	1	vF; pS; iF	1
20	8	IV. 58	....	0 5 31·4	3·188	3	18 15 26·5	20·05	3	vF; vS; R; vsmbM*10; *12 241°·4; 25ˢ.	4
21	9	......	....	0 5 45·3	3·095	1	59 51 46·5	20·05	1	eF; *12, 45″, 325°	1
22	10	......	....	0 5 59·4	3·096	1	59 29 20·5	20·05	1	eF; vS	1
23	2313	......	....	0 6 47·8	3·052	1	113 57 14·8	20·04	1	eF; L; vgvlbM; L*cont, f ...	1
24	......	......	Auw. N. 2	0 6 59·2	3·074	...	84 47 28·8	20·04	...	A nebula (Markree Cat. 1852, Oct. 22).	0
25	11	III. 183	....	0 7 47·8	3·089	1	72 14 19·8	20·04	1	vF; S; E	2
26	2314	......	....	0 7 53·1	2·978	1	151 6 13·8	20·04	1	eF; S; R; bM..................	1
27	2315	......	Δ. 507	0 8 0·4	3·028	3	129 59 33·8	20·04	3	vB; vL; vmiE; tri-N	3†
28	12	......	....	0 8 8·9	3·083	1	78 19 58·8	20·04	1 ::	eL; eF; diff.....................	1
29	13	II. 241 = II. 243	....	0 8 18·0	3·088	1	73 26 48·8	20·04	1	F; S; R; sbM	4*
30	14	III. 248	....	0 9 14·6	3·065	1	97 5 46·8	20·04	1	vF; S; iR; psvlbM	3
31	15	V. 16	....	0 11 6·2	3·112	1	60 42 9·1	20·03	1	eF; L; 3 or 4st+neb	2†
32	15, a	......	R. 6 novæ	0 11	......	...	60 42	......	...	Nos. 32...37 incl...............	0
38	16	......	....	0 13 54·8	3·106	1	68 24 45·7	20·01	1	F; S; R; psbM	1
39	17	......	....	0 14 5·3	3·107	1	68 19 15·7	20·01	1	E; bi-N; 3Bst near	1
40, 41, 42	17, a	......	R. 3? novæ	0 14±	......	...	68 19±	......	...	Several F, S (3 novæ at least presumed).	0
43	2316	......	....	0 14 18·5	2·968	2	139 24 41·7	20·01	2	eF; S; R; gbM; 1st of 4	2
44	2317	......	....	0 14 28·2	2·967	2	139 25 13·7	20·01	2	eF; vS; R; 2nd of 4	2
45	2318	......	....	0 14 30·0	2·967	2	139 26 36·7	20·01	2	vF; S; R; gbM; 3rd of 4 ...	2
46	2319	......	....	0 14 36·9	2·966	2	139 24 21·7	20·01	2	F; S; R; gbM; 4th of 4	2
47	19	II. 257	....	0 15 9·8	3·088	1	80 17 48·7	20·01	1	F; pL; R; gbM	3
48	18	......	....	0 15 10·5	3·124	1	61 1 29·7	20·01	1	F; vS; R; gbM	1
49	2320	......	....	0 15 55·7	2·970	1	136 3 13·0	20·00	1	vF; pS; R; bM; r	1
50	2321	......	....	0 16 56·0	3·004	2	123 19 22·3	19·99	2	pB; pL; lE; *14, f............	2
51	20	......	....	0 17 38·2	3·266	1	29 26 39·3	19·99	1	Cl; pS; pC; st11...18.........	1
52	2322	......	Δ. 18 = 47 Toucani	0 17 47·4	2·721	4	162 51 33·3	19·99	4	⊕; !!; vB; vL; vmCM	4†
53	21	III. 148	....	0 18 39·6	3·134	2	61 33 49·6	19·98	2	pF; pL; R; pslbM	3
54	......	......	D'Arrest, 2	0 18 49	3·117	[1]	68 58 20	19·98	[1]	vF; S; 3 st near, making quadr.	0
55	22	......	....	0 19 18·9	3·411	1	19 23 2·6	19·98	1	Cl; pR; lC; st9...12	1
56	2323	......	....	0 19 51·3	2·989	2	124 27 21·9	19·97	2	vF; pL; lE; D*2′, np.........	2
57	2324	......	....	0 20 17·3	2·877	2	147 45 22·9	19·97	2	pB; S; R; mbM................	2
58	2325	......	....	0 20 25·1	2·685	1	162 18 23·9	19·97	1	pB; pS; lE; vgbM	1
59	23	III. 869	....	0 21 41·4	3·123	2	87 56 3·2	19·96	2	vF; S; bM; D*vnr; p of 2...	3
60	23, a	......	R. nova	0 22 0·7	+3·123	...	87 57 38·5	−19·96	...	No descr (MS)	0

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	h.	II.		h m s	s		° ′ ″	″			
61	23, b	......	R. nova	0 22 0·7	+3·123	...	87 54 8·5	−19·95	...	No descr (MS)	0
62	25	II. 854	....	0 22 4·7	3·077	2	87 54 38·5	19·95	2	pB; vS; lE, 0°±; bM; f of 2	3
63	24	VIII. 79	....	0 22 6·2	3·302	1	30 33 4·5	19·95	1	Cl; vL; pR; lC; st 9...13 ...	2
64	23, c	......	R. nova	0 22 8·7	3·123	...	87 54 8·5	19·95	...	No descr (MS)	0
65	2326	......	....	0 22 42·6	2·979	2	124 2 2·5	19·95	2	F; pL; pmE; vgbm; p of 2...	2
66	26	II. 855	....	0 23 0·2	3·076	3	88 41 12·5	19·95	3	pF; cL; R; vglbm; r	6
67	2327	......	....	0 23 27·6	2·976	2	124 1 45·8	19·94	2	vB; L; vmE, 47°·5; psbM; f of 2; ✱10 327°·9 45″.	2†
68	......	VI. 35	....	0 23 48·3	3·332	1	29 16 1·8	19·94	1	⊕; vF; S; eC	1
69	......	II. 471	....	0 24 3·5	3·096	1	80 34 1·8	19·94	1	F; iF; lbM	1
70	{27= 2328}	......	....	0 24 38·0	3·057	2	95 55 33·1	19·93	2	{F; pL; vlE; vgbM; ✱8·9, 75°±; 5′.}	2
71	28	......	....	0 25 12·7	3·367	1	27 29 19·4	19·92	1	Cl; pL; lC; st 11...12; D✱...	1
72	29	......	....	0 25 30·6	3·243	1	42 16 32·4	19·92	1	vF; vL; iR; g, smbM✱11 ...	1
73	2329	......	....	0 26 22·3	2·971	1	122 33 57·7	19·91	1	vB; S; lE 90°; smbM✱11 ...	2
74	{30= 2330}	II. 478	....	0 26 57·8	3·042	2	100 28 30·7	19·91	2	pF; pL; lE 90°; vglbM	4
75	2331	......	....	0 27 3·7	2·513	1	163 53 11·0	19·90	1	vF; L; R; vglbM	1
76	31	III. 467	....	0 27 14·0	3·033	1	103 25 59·0	19·90	1	eF; vS; R	2
77	2332	......	....	0 28 2·9	2·818	2	146 33 23·3	19·89	2	vF; pS; R; glbM; 3stf	2
78	......	II. 3	....	0 28 32·5	3·044	2	99 32 4·3	19·89	2	F; L; mE; bet 2eBst.........	2*
79	32	III. 476	....	0 28 43·0	3·147	1	66 48 49·3	19·89	1	vF; vS; stellar; ✱7, 15°, 5′...	2
80	32, a	......	R. nova	0 28	......	...	66 48	... ..	...	Makes Dneb with h. 32	0
81	......	III. 954	....	0 29 20·1	3·039	1	100 51 4·6	19·88	1	eeF; S	1
82	......	......	D'Arrest, 3	0 29 33	3·15	[1]	66 46 18	19·87	[1]	F; pL; R; ✱6, 3′½ dist	0
83	......	III. 223??	....	0 29 41·2	3·008	1	109 44 5·6	19·88	1	vF; pL; lE; 2pBst sf	1
84	33	III. 871	....	0 30 1·5	3·076	2	88 48 44·9	19·87	2	vF; S; R; vgbM; ✱11, 225°±; 80″.	3
85	2333	......	....	0 30 2·4	2·968	3	120 14 30·9	19·87	3	eF; S; vlE; amBst	3
86	2334	III. 223	....	0 30 20·6	3·004	1	110 42 9·9	19·87	1	pB; pL; E; gbM; r	1
87	2335	......	....	0 30 21·5	2·446	2	163 56 28·9	19·87	2	eF; S; vlE; r; ✱8 near	2
88	......	III. 876	....	0 30 46·6	3·098	1	82 6 6·2	19·86	1	vF; pL; iR; ✱np inv	1*
89	......	III. 870	....	0 31 0·5	3·079	1	88 1 7·2	19·86	1	vF; S; iR; vgbM	1
90	35	II. 707	....	0 31 11·4	3·278	1	42 26 0·2	19·86	1	pB; vL; iR; vgmbM; r	2
91	......	......	D'Arrest, 4	0 31 15	3·08	[2]	87 36 6	19·85	[2]	F; S; R; lbM	0
92	34	......	....	0 31 40·6	5·151	1	5 26 27·6	19·84	1	Cl; vL; R; 150...200st 10... 18.	1
93	36	......	....	0 31 40·7	3·407	1	29 42 31·5	19·85	1	Cl; pL; R; st 11...15	1
94	37	......	....	0 31 49·4	3·081	1::	87 25 4·5	19·85	1	vF; L; p of 2; st 15 close ...	1
95	38	II. 479	....	0 31 53·4	3·039	1	99 46 25·5	19·85	1	pB; pL; iE 0°±...............	3
96	39	III. 872	....	0 32 2·3	3·072	5	89 54 28·5	19·85	5	F; pS; pmE; bM; 1st of 3...	6
97	......	II. 857	....	0 32 4·5	3·079	1	87 57 7·5	19·85	1	F; S; vgbM	1
98	40	II. 856	....	0 32 5·6	3·080	1	87 42 54·8	19·84	1	pB; S; R; vgbM...............	2
99	40, a	......	R. nova	0 32	......	...	87 42	......	...	(?=h. 37 or II. 857)	0
100	41	III. 595	....	0 32 6·8	3·072	1	89 51 23·8	19·84	1	F; pS; R; psmbM; f of 2 ...	2
101	42	II. 860	....	0 32 14·4	3·081	1::	87 27 4·8	19·84	1::	pB; pS; R; vgbM; 2nd of 3	5
102	43	III. 873	....	0 32 21·4	3·072	2	89 55 7·8	19·84	2	vF; cL; E; vglbM; f of 3 ...	3
103	......	......	D'Arrest, 5	0 32 22	3·08	[1]	87 37 24	19·84	[1]	F; vS; ✱8, p 27s·7, ls.........	0
104	......	II. 858	....	0 32 22·5	3·079	1	87 52 7·8	19·84	1	pB; S; vgbM	1
105	44	V. 18	C.H.	0 32 45·4	3·243	1	49 4 49·8	19·84	1	vB; vL; mE 165°; vgvmbM	6†
106	45	V. 36	....	0 32 47·8	3·237	1	50 1 34·8	19·84	1	vF; vL; mE 0°	4†
107	46	II. 452	....	0 33 32·4	3·020	3	104 38 20·1	19·83	3	B; pS; R; psbM; r; ✱90″±	4
108	46, a	......	R. nova	0 33 32	......	...	104 38	......	...	E 0°±	0
109	......	III. 200	....	0 33 48·8	3·129	2	74 17 8·4	19·82	2	F; S; bet 2 Sst	2
110	2236	......	....	0 33 49·9	2·763	2	146 56 14·4	19·82	2	vF; S; R; p of 2...............	2
111	47	II. 209	....	0 34 5·4	3·166	1	65 16 6·4	19·82	2	pF; pS; gvlbM; r	4
112	2337	......	....	0 34 25·7	2·757	2	146 58 27·7	19·81	2	F; S; R; amst; f of 2.........	2
113	49	III. 244	....	0 34 28·9	2·990	1	111 48 59·7	19·81	1	eF; vS; lE 0°...90°............	2
114	48	II. 480	....	0 34 30·2	3·033	1	100 47 1·7	19·81	1	F; S; lE 90°±; glbM	3
5058	......	......	......	0 35 1·0	......	...	89 50 6·6	......	...	See No. 5058.	
115	2338	......	....	0 35 2·1	+2·344	3	164 9 52·7	−19·81	3	F; iR; vgbM; 1st of several	3

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	h.	H.		h m s	s		° ′ ″	″			
116	50	......	M. 31	0 35 3·9	+3·252	1	49 29 45·7	−19·81	1	!!!eeB; eL; vmE; (*Androm. Gt. Neb.*) Bifid (Bond)	13†
117	51	......	M. 32	0 35 5·3	3·250	1	49 54 12·7	19·81	1	! vvB; L; R; psmbMN	8†
118	2339	......	....	0 35 5·4	2·338	1	164 14 1·7	19·81	1	vF; R; 2nd of several.........	1
119	......	......	D'Arrest, 6= Auw. N. 4	0 35 6	3·07	[3]	89 55 0	19·81	[3]	vF; pL; R (Bond, Jan. 1853)	0*
120	52	VIII. 78	C.H.	0 35 19·2	3·457	1	28 58 43·0	19·80	1	Cl; L; lC; st 9...10	3
121	53	......	....	0 35 24·0	3·204	1	58 10 55·0	19·80	1	eF; S; R; *13, 20″ 180° ...	1
122	......	II. 444	....	0 35 33·8	3·064	1	92 18 11·0	19·80	1	F; pL; lbM....................	1
123	2340	......	Δ. 2??	0 35 50·6	2·329	1	164 7 16·6	19·78	1	i train of st and neb	1
124	54	III. 149	....	0 36 5·2	3·196	2	60 10 58·3	19·79	2	F; vS; R; lbM	3
125	......	II. 245	....	0 36 10·9	3·124	4	76 27 54·3	19·79	4	F; pS; ilE; bM	4
126	2341	......	....	0 36 53·3	2·803	1	140 56 42·6	19·78	1	eF; pl; R; gvlbM	1
127	2342	......	....	0 38 13·4	2·278	1	164 11 39·2	19·76	1	vF; R	1
128	2343	......	....	0 38 18·6	2·275	3	164 12 17·2	19·76	3	vF; S; bi-N..................	3
129	55	III. 485	....	0 38 45·1	3·005	1	106 20 58·5	19·75	1	vF; S; iR; r; *10, 5′ s	3
130	......	II. 445	....	0 39 3·7	3·063	1	92 29 13·5	19·75	1	F; pS; iF; er	1
131	56	V. 25	....	0 40 1·4	3·019	2	102 38 24·1	19·73	2	vF; L; 4st in diff n...........	3
132	57	V. 20	....	0 40 4·2	2·979	1	111 30 57·1	19·73	1	F; vL; vmE 172°	2*
133	2344	......	....	0 40 13·4	2·240	2	164 8 36·1	19·73	2	F; S; E or bi-N; vglbM......	2
134	2346	......	Δ. 19? 21?	0 40 17·9	2·255	3	163 51 1·1	19·73	3	F; pL; vlE; r	3
135	58	III. 204?	....	0 40 22·9	3·154	(2)	71 9 48·1	19·73	1	vF; S; R; lbM; 2vSstf; *inv	3
136	59	II. 609	....	0 40 32·9	3·195	5	63 8 35·1	19·73	5	pB; S; R; pmbM; r; ⁑ p ...	6
137	59, *a*	......	R. nova	0 40	......	...	63 8	......	...	One of R.'s novæ; the other =h. 60.	0
138	61= 2345	V. 1	C.H.	0 40 37·6	2·954	3	116 3 40·4	19·72	3	!!; vvB; vvL; vmE 54°·5 gbM; 4st.	9*†
139	60	......	....	0 40 39·3	3·195	1	63 8 14·4	19·72	1	pF; R; bM	1
140	2347	......	....	0 40 40·7	2·920	3	122 11 28·4	19·72	3	vB; pS; lE; smbM; *8, 5′nf	3
141	62	II. 472	....	0 40 43·5	3·020	1	102 14 39·4	19·72	1	F; pS; R; gbM	2
142	2348	......	....	0 40 44·3	2·223	4	164 16 44·4	19·72	4	F; S; R; gbM; *9, 40″nf ...	4
143	......	II. 863	....	0 40 47·1	3·105	1	82 27 14·4	19·72	1	pL; lE; gbM; r	1
144	63	......	....	0 40 55·7	3·057	1	93 37 22·4	19·72	1	vF; Δ 2st & neb	1
145	64	II. 703 = II. 621	....	0 40 56·1	3·058	1	93 32 40·4	19·72	1	F; S; E 135°±; lbM.........	3*
146	2349	......	Δ. 3, 4, 21?	0 41 23·8	2·233	3	163 52 6·7	19·71	3	F; pL; R; gbM*13	3
147	2350	......	....	0 41 41·6	2·872	1	129 0 20·7	19·71	1	F; S; R; vsvmbM*13	1
148	2351	......	....	0 42 3·8	2·198	4	164 15 4·0	19·70	4	F; pS; R......................	4
149	65	III. 153	....	0 42 13·6	3·226	4	58 29 23·0	19·70	4	pB; pS; lE; psbM; r; *8sf4′,	5
150	2352	......	....	0 42 49·8	2·195	1	164 2 37·3	19·69	1	Cl; F; pL; stvS	1
151	66	III. 463	....	0 43 1·3	3·046	2	95 57 59·3	19·69	2	vF; pS: ilE; r.................	4
152	2353	......	....	0 43 20·8	2·169	1	164 18 1·6	19·68	1	vF; S; R......................	1
153	68	III. 955	....	0 43 28·7	3·030	(1)	99 25 39·6	19·68	2	pF; vS; iR; pgbM	3
154	67	II. 446	....	0 43 34·3	3·061	1	92 39 59·6	19·68	1	pF; S; lE; psbM; *8 f 5″·5...	3
155	......	III. 430	....	0 43 47·0	3·038	1	97 39 17·9	19·67	1	vF; vS........................	1
156	69	III. 429	....	0 43 56·8	3·037	1	97 49 22·9	19·67	1	pB; pS; smbM; sp of Dneb .	3
157	70	......	....	0 43 59·1	3·037	1	97 49 2·9	19·67	1	vF; S; R; nf of Dneb	2
158	71	I. 159	....	0 44 10·2	3·352	1	43 12 15·9	19·67	1	cB; pL; R; 2st10nr	4
159	73	III. 439	....	0 44 59·5	3·059	2	92 59 10·5	19·65	2	vF; S; iR; bM; stellar	4
160	72	III. 477	....	0 45 0·3	3·189	1	66 25 23·5	19·65	1	eF; S; R; *15, f30″	3
161	75	......	....	0 45 52·1	3·241	1	58 16 59·8	19·64	1	eF; S; R	1
162	74= 2354	VI. 20	....	0 45 52·1	2·932	2	117 20 41·8	19·64	2	⊕; B; L; lE; st 12...16......	3
163	2355	......	....	0 45 57·6	2·903	3	121 57 53·1	19·63	3	vB; L; pmE; glbM; *11np	4
164	2357	......	....	0 46 16·9	2·135	1	163 54 48·1	19·63	1	eF	1
165	2356	......	....	0 46 33·5	2·115	2	164 6 33·1	19·63	2	Cl; F; eeL; R; st 12...18 ...	2*
166	2358	......	Δ. 5, 6?	0 47 14·8	2·102	1	164 8 28·7	19·61	1	vF; pL; R; vglbM; r.........	2
167	......	II. 214	....	0 47 51·9	+3·242	1	59 11 21·0	−19·60	1	F; E; aB*f, vnr	1

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	h.	H.		h m s	s		° ′ ″	″			
168	2360	......	....	0 48 23·0	+ 2·151	4	162 57 22·3	− 19·59	4	pB; vS; R; gvlbM; r	4
169	2359	......	....	0 48 24·5	2·844	3	128 27 8·3	19·59	4	pB; vL; vmiE; vgpmbM ...	5*†
170	76	......	....	0 48 59·6	3·131	1	78 40 51·6	19·58	1	Cl; S; scst	1
171	2361	......	....	0 49 15·8	2·132	1	163 0 0·9	19·57	1	F; vS	1
172	77	......	....	0 49 25·0	3·060	1	92 31 27·9	19·57	1	pF; S; E	1
173	78	......	....	0 49 55·7	3·240	2	60 28 21·2	19·56	2	pF; vS; R; gbM	2
174	2363	......	....	0 50 3·5	2·675	2	143 32 6·2	19·56	2	F; S; R; *12 f 90°	2
175	2362	......	....	0 50 10·7	2·883	2	122 43 13·2	19·56	2	eF; vS; R; pB* f 2′	2
176	79	II. 210	....	0 50 11·5	3·241	3	60 24 25·2	19·56	3	pB; pL; R; gbM; *9, 3′ 135°	4
177 5059	} 79, a, b	......	R. 2 novæ	0 50 22·3	3·241	...	60 21 31·2	19·56	...	{F; S; R (ε of Lord R.). For b, see No. 5059.}	0*
178	4007	......	....	0 50 29·3	2·780	1	134 35 56·5	19·55	1	eF; vS; R; lbM	1*
179	4008	......	....	0 50 36·2	2·780	1	134 30 1·5	19·55	1	vF; vS; R; lbM; 3stp	1*
180	2365	......	....	0 50 36·7	2·668	2	143 43 58·5	19·55	2	pF; S; R; bM; p of 2	2
181	2364	......	....	0 50 40·4	2·810	1	131 12 36·5	19·55	1	(?)F; S; stellar	1
182	2366	......	....	0 50 51·5	2·667	1	143 39 58·8	19·54	1	vF; lE; vgbM; f of 2	1
183	2367	......	Δ. 23	0 51 27·2	2·078	5	163 13 35·1	19·53	5	⊕; vB; S; lE; st 13...15 ...	5
184	2368	......	....	0 52 10·5	2·850	2	125 53 21·4	19·52	2	vF; S; R; glbM; 2st11s	2
185	80	II. 433	....	0 52 46·1	3·027	...	98 19 51·0	19·50	...	pF; L; E 0°±; glbM; *10, f 20s·5.	2
186	2369	......	....	0 53 13·3	1·902	1	165 12 27·0	19·50	1	F; L; R; vgbM	1
187	2370	......	Δ. 25	0 54 19·9	2·043	5	162 56 9·3	19·49	5	B; L; viF; mbMD*; r	5†
188	2371	......	....	0 54 45·4	2·632	2	143 59 49·6	19·48	2	eF; S; R	2
189	81	III. 191	....	0 55 5·2	3·044	2	94 59 49·5	19·45	2	pF; S; iE; *8f97s	4
190	82	II. 434	....	0 56 16·4	3·032	1	97 5 49·1	19·43	3	F; S; iR; sbM; *14 nf 20″...	4
191	2372	......	....	0 57 26·8	2·308	1	156 21 30·0	19·40	1	eF; vmE 145°·4; vlbM	1
192	2374	......	Δ. 55 ??	0 57 28·7	2·022	2	162 22 33·0	19·40	2	vvF; pL; vlE; vgbM	2
193	2375	......	Δ. 62	0 57 28·8	2·069	3	161 35 59·0	19·40	3	⊕; vB; vL; vC; vmbM; st 13...14.	4
194	2373	......	....	0 57 44·3	2·828	2	125 53 46·0	19·40	2	F; S; R; glbM	2
195	83	......	....	0 57 45·8	3·700	1	28 34 5·0	19·40	1	Cl; S	1
196	4012	......	....	0 57 57·6	2·745	1	134 1 50·3	19·39	1	eF; vS; *7·8 sp 3′	1*
197	......	......	D'Arrest, 7	0 58 53	3·28	[1]	58 20 18	19·37	[1]	vF; *13, s 15″; m diff	0
198	2376	......	Δ. 31 ??	0 58 56·4	1·969	5	162 48 27·9	19·37	5	Cl; F; L; R; pC; st 14...16.	5
199	......	......	D'Arrest, 8	0 59 21	3·29	[1]	57 57 18	19·36	...	F; S; bet 2 st 15	0
200	2378	......	Δ. 36 ??	0 59 27·3	1·904	2	163 34 23·2	19·36	2	⊕; B; S; R	2
201	2377	......	....	0 59 31·7	2·864	1	120 55 53·2	19·36	1	vF; S; R; gbM	1
202	84	II. 215	....	0 59 32·3	3·289	3	58 14 9·2	19·36	3	pF; S; R; bM; 1st of 3	4*
203	85	II. 216	....	0 59 34·3	3·289	3	58 16 41·2	19·36	3	pF; S; R; sbM; 2nd of 3 ...	4*
204	......	VIII. 64	C.H.	0 59 37·4	3·703	1	29 9 36·2	19·36	1	Cl; pC	1
205	86, a	......	R. nova	0 59 39·6	3·289	...	58 20 48·2	19·36	...	γ′ in Lord R.'s diagram	0*
206	86	II. 217	....	0 59 40·5	3·289	3	58 20 16·2	19·36	3	pF; pL; R; gbM; 3rd of 3...	4*
207	......	......	D'Arrest, 9	0 59 42	3·29	[3]	58 26 0	19·36	[3]	p of D neb; vF; pS; {Δ.R.A.=0, Δ.P.D.=93″}	0*
208	......	......	D'Arrest, 10	0 59 42	3·29	[3]	58 25 18	19·36	[3]	vF; R; pS: f of D neb	0*
209	86, b	......	R. nova	0 59 46·3	3·288	...	58 23 20·2	19·35	...	δ in Lord R.'s diagram	0*
210	86, c	......	R. nova	1 0 2·3	3·288	...	58 27 16·2	19·34	...	θ in Lord R.'s diagram	0*
211	......	......	Auw. N. 9	1 0 15·1	3·071	...	89 48 55·8	19·34	...	F nebula (Bond, Jan. 1853) .	0*
212	87	II. 218	....	1 0 39·6	3·299	1	57 37 10·1	19·33	1	F; vS; R; mbM; bet 2 st ...	2
213	87, a	......	R. nova	1 0	......	...	59 37	......	...	makes a D neb with h. 87 ...	0
214	88	I. 54	....	1 0 43·4	3·361	2	51 5 25·1	19·33	2	F; vS; vlE; gbM; 4Sstnr ...	4*
215	......	......	D'Arrest, 11	1 0 45	3·30	[1]	57 36 18	19·33	[1]	F; S; pos from h. 87=40°; dist 47″.	0
216	2379	......	....	1 0 45·8	1·940	1	162 44 38·1	19·33	1	vF; pL; R; glbM	1
217	......	......	D'Arrest, 12	1 1 29	3·30	[1]	57 59 48	19·31	[1]	vF; S; R; *10 f 1s·8, s 80″ ...	0
218	89	II. 224	....	1 1 39·0	3·326	2	55 2 13·7	19·31	2	pB; cL; R; gbM; β *Androm.* nr	3
219	2380	......	....	1 2 5·5	2·679	1	137 25 34·0	19·30	1	eS; stellar; =*7m	1
220	2381	......	....	1 2 39·6	2·041	1	160 37 28·3	19·29	1	F; vL; R; vglbM	1
221	......	II. 219	....	1 2 44·2	3·306	(1)	57 34 0·3	19·29	(1)	eS; F; p of D neb	1
222	......	II. 220	....	1 2 44·2	+3·306	(1)	57 34 0·3	−19·29	(1)	pL; f of D neb	1

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	h.	H.		h m s	s		° ′ ″	″			
223	2382	......	....	1 3 9·2	+2·800	1	126 31 25·9	−19·27	1	eF; S; R; vS* nr	1
224	2384	......	....	1 3 27·6	1·908	1	162 30 42·9	19·27	1	cF; pL; R; gvlbM	2
225	2383	......	....	1 3 37·2	2·801	2	126 14 35·2	19·26	2	vF; S; R; glbM	2
226	2386	......	....	1 3 39·2	1·861	4	163 6 23·2	19·26	4	F; pS; R; gbM	4
227	2385	......	....	1 3 57·5	2·849	3	120 57 48·5	19·25	3	F; pL; R; vglbM; p of 2 ...	3
228	2387	......	Δ. 36 ?	1 4 4·2	1·814	1	163 37 59·5	19·25	1	....	4
229	90	III. 15	....	1 4 25·7	3·303	1	58 37 12·8	19·24	1	F; pS; R; bM.................	2
230	......	III. 15⁴	....	1 4 26·1	3·303	1::	58 36 22·8	19·24	1::	eF; vS	1
231	[162]	......	....	1 4 28·3	1·890	1	162 30 54·8	19·24	1	vF	1
232	2388	......	....	1 4 41·6	2·855	2	119 58 44·8	19·24	2	eF; S; E; glbM; f of 2	2
233	2389	......	....	1 4 57·6	2·768	1	128 49 39·1	19·23	1	vF; S; R; glbM	1
234	91	III. 592	....	1 5 41·2	3·066	1	91 3 7·7	19·21	1	vF; vS; R	2
235	91, *a*	......	R. nova	1 5	......	...	91 3	......	...	Forms a Δ with h. 91 & 92...	0
236	2390	......	....	1 5 43·8	2·826	2	122 49 44·7	19·21	2	2 vSst+F neb	2
237	92	III. 593	....	1 5 46·4	3·065	1	91 4 22·7	19·21	1	eF; eS	2
238	......	II. 622	....	1 5 49·2	3·074	1	89 45 44·0	19·20	1	F; L; R; bM; er..............	1
239	93	II. 447	....	1 5 51·7	3·066	1	90 59 22·7	19·21	1	F; vS; R; vsbM*	3
240	95	......	....	1 6 16·1	3·324	1	57 1 50·0	19·20	1	F; S; vsbM.....................	1
241	2391	......	....	1 6 16·3	2·340	1	152 20 39·0	19·20	1	F; S; R; gbM; *12 f	2
242	94	......	....	1 6 27·6	3·732	1	30 36 52·3	19·19	1	Cl; S; lC	1
243	2392	......	....	1 6 36·9	2·430	2	148 59 52·3	19·19	2	B; S; R; psbM	2
244	......	VII. 45	....	1 6 53·4	3·703	2	31 56 6·6	19·18	2	Cl; S; iF; pC	2
245	2393	......	....	1 7 7·0	2·761	2	128 38 59·9	19·17	2	pF; S; R; glbM	2
246	2394	......	....	1 7 12·4	2·823	2	122 29 25·9	19·17	2	pB; S; R; gbM	2
247	2396	......	....	1 7 13·1	2·423	2	149 1 44·9	19·17	2	F; vS; R	2
248	2395	......	....	1 7 16·6	2·823	2	122 32 55·9	19·17	2	pF; S; R; gbM	2
249	......	......	D'Arrest, 13	1 7 21	3·32	[1]	57 31 18	19·16	[1]	F; S; R; *15 p, 8ˢ·3, 270° ...	0
250	......	......	D'Arrest, 14	1 7 51	3·32	[2]	57 40 42	19·15	[2]	F; pL; bM; *11 nr...........	0
251	96, *a*	......	R. nova	1 8	......	...	59 43	......	...	vF; mE 135°±; lbM; np h.96	0
252	96	......	....	1 8 29·3	3·306	1	59 42 41·8	19·14	1	vF; E; *9np; S*nf, vnr......	1
253	2397	......	....	1 8 35·1	2·480	1	146 8 37·8	19·14	1	vF; S; R; bM.................	1
254	......	III. 440	....	1 8 48·8	3·061	1	91 35 19·1	19·13	1	vF; vL............................	1
255	2399	......	Δ. 7, 10 ?	1 9 50·6	1·668	3	164 2 15·0	19·10	4	pF; pL; iR; r; 1st of sev, neb and st.	3
256	97	VII. 42	....	1 10 24·8	3·722	1	32 24 35·3	19·09	1	Cl; B; L; pR; st 7, 8, 10 ...	3
257	2401	......	Δ. 60 ?	1 10 40·8	1·799	2	162 17 13·6	19·08	2	pF; L; R; vgbM	2
258	......	III. 205	....	1 10 41·4	3·198	1	73 4 51·6	19·08	1	eF	1
259	2402	......	Δ. 8, 10 ?	1 10 48·6	1·650	4	164 2 41·6	19·08	4	F; pL; iR; gbM; r; 2nd of sev.	4
260	2400	......	....	1 10 54·4	2·794	1	124 6 11·6	19·08	1	pB; R; glbM; ? 1° in P.D....	1
261	2404	......	Δ. 9 ?	1 11 42·3	1·629	1	164 4 36·5	19·05	1	pB; pL; iF; 3rd of sev.	1
262	2403	......	....	1 11 43·3	2·365	1	149 38 46·5	19·05	1	vF; R; gbM; 30	1
263	99	III. 250	....	1 11 56·6	3·091	1	87 25 16·5	19·05	1	pB; pL; R; gmbM; p of 2...	4
264	......	I. 108	....	1 12 1·0	3·091	1	87 30 [illegible]·5	19·05	1	cB; vL; iR; pB*f	1
265	98	......	....	1 12 2·9	3·335	1	58 1 55·5	19·05	1	vF; eS; stellar.................	1
266	......	......	D'Arrest, 15	1 12 41	3·33	[2]	58 2 0	19·03	[2]	eF; vS; *9p14ˢ; v diffic.......	0
267	......	III. 206	....	1 12 41·2	3·192	1	74 14 55·1	19·03	1	eF; S	1
268	100	III. 577	....	1 13 19·9	3·428	1	50 14 18·7	19·01	1	vF; pS; vlE; vglbM	2
269	......	III. 251	....	1 13 21·4	3·092	3:	87 22 9·7	19·01	3:	pB; S; smbM; f of 2	3
270	101	......	....	1 13 47·2	3·119	1::	83 43 2	19·00	1	cF; pL; R; red * 7·8, 225°...	1
271	2405	......	....	1 14 3·5	2·691	1	131 42 52·3	18·99	1	eF; lE	1
272	102	III. 156	....	1 14 6·0	3·350	1	57 17 30·3	18·99	1::	vF; eS; 1st of 3	3
273	2406	......	....	1 14 7·3	2·353	2	149 15 33·3	18·99	2	vB; S; lE; psmbM	2
274	......	......	D'Arrest, 16	1 14 29	3·14	[1]	81 31 48	18·97	[1]	pB; S; E.........................	0
275	103, *a*	......	R. nova	1 14 29	3·107	...	85 24 26·6	18·98	...	γ in Lord R.'s diagram	0
276	103	III. 252	....	1 14 31·2	3·107	2	85 28 25·6	18·98	2	pB; L; R; svmbM; *7 f 1ᵐ...	3
277	103, *b*	......	R. nova	1 14 46·4	3·107	...	85 21 47·6	18·98	...	β in Lord R.'s diagram	0
278	......	III 157	....	1 14 51·3	3·351	2	57 15 28·9	18·97	2	vF; S; 2nd of 3	2
279	2407	......	....	1 14 53·5	2·772	2	124 48 15·9	18·97	2	B; S; vlE; bM; vS*nr	2
280	103, *c*	......	R. nova	1 14 57·1	3·107	...	85 18 37·6	18·97	...	δ in Lord R.'s diagram	0
281	105	III. 594	....	1 14 59·5	3·075	1	89 47 7·2	18·96	1	vF; L; mE 60°±; lbM	2
282	104	......	....	1 15 4·4	+3·352	1	57 33 18·2	−18·96	1	vF; E; ⁑s	1

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	h.	H.		h m s	s		° ′ ″	″			
......	104, a	......	R. 5 novæ	1 15	+......	...	57 33	−18·95	...	No descr Nos. 283...287 incl	0
288	106, a	......	R. nova	1 15	......	...	57 16	18·95	...	No description	0
289	106	III. 158	....	1 15 19·6	3·354	2	57 16 46·5	18·95	2	pB; pL; R; 3rd of 3	4
290	103, d	......	R. nova	1 15 23·2	3·107	...	85 20 17·6	18·95	...	ε in Lord R.'s diagram	0
291	......	......	D'Arrest, 17	1 15 35	3·35	[3]	57 31 18	18·94	[3]	vF; S; obs. with H. 157, 158, 159, 160.	0
292	107	......	....	1 15 36·6	3·352	1:	57 36 31·8	18·94	1::	No description	1
293	......	......	D'Arrest, 18	1 15 38	3·13	[2]	81 41 6	18·94	[2]	cB; S; R; bMN	0
294	108	III. 159	....	1 15 48·0	3·354	2	57 28 37·8	18·94	2	vF; pL; R; bM; p of 2	3
295	109	III. 160	....	1 15 49·0	3·354	1	57 26 52·8	18·94	1	vF; S; f of 2	2
296	110	......	....	1 16 7·8	3·362	1	56 49 53·1	18·93	1	vF; vS	1
297	111	III. 169	....	1 16 34·4	3·362	1	56 56 28·4	18·92	1	F; S; stellar	2
298	112	II. 252	....	1 16 35·8	3·169	2	77 49 10·7	18·91	2	F; L; lE; vglbM; ⁑ f	3†
299	113	III. 167	....	1 16 45·3	3·360	1	57 16 37·7	18·91	1	Stellar; p of 2	2
300	......	......	D'Arrest, 19	1 16 48	3·14	[2]	81 10 6	18·91	[2]	eF; S; v diffic; I 151 f41ˢ ...	0
301	114	III. 168	....	1 16 48·8	3·360	1	57 18 37·7	18·91	1	pB; R; stellar; f of 2	2
302	114, a	......	R. nova	1 16	......	...	57 18	......	...	S; R; bM	0
303	116	III. 253	....	1 17 19·4	3·097	1	86 55 16·3	18·89	1	F; cL; E 135°±	3†
304	115	II. 461	....	1 17 19·8	3·078	2	89 0 21·3	18·89	2	F; pL; R; gbM	5
305	......	......	D'Arrest, 20	1 17 22	3·14	[2]	80 44 24	18·89	[2]	eF; pL; iF; ?Cl+neb	0
306	......	......	D'Arrest, 21	1 17 25	3·37	[1]	56 42 18	18·89	[1]	D neb; vF; 90° pos	0
307	117	I. 151	....	1 17 26·8	3·142	1	81 11 50·3	18·89	1	vB; pL; mbM; 4S st nr	2
308	......	......	D'Arrest, 22	1 17 31	3·14	[2]	81 1 12	18·88	[2]	vF; vS; ✱ 11·12 p 5ˢ	0
309	2408	......	....	1 17 35·1	2·750	3	125 48 8·3	18·89	3	F; S; lE; p of 2	3
310	2409	......	....	1 17 37·5	2·748	3	125 51 0·3	18·89	3	F; S; lE; bM; f of 2	3
311	118	......	....	1 17 44·5	3·377	1	56 1 31·3	18·89	1	pB; vS; sbM; p of 2	1*
312	118, a	......	R. nova	1 17	......	...	56 1	......	...	One of 4 neb nr h. 120	0
313	......	III. 556	....	1 17 51·6	3·140	1	81 28 43·6	18·88	1	vF; pL; mE 15°±	1*
314	119	......	....	1 17 51·6	3·140	(1)?	81 29 30·6	18·88	1	Not vF; L; R; bM	2*
315	121	II. 462	....	1 18 19·5	3·081	2	88 58 4·9	18·87	2	pB; pL; R; gmbM	3
316	2410	......	....	1 18 22·9	2·708	2	128 52 11·9	18·87	2	eeF; S; R; vgbM; 1st of 4...	2
317	120	......	....	1 18 24·6	3·379	1	56 2 19	18·87	1	pB; pL; gbM; f of 2	1*
318	120, a	......	R. nova	1 18	......	...	56 2	......	...	One of 4, see h. 118, 120 ...	0
319	......	III. 170	....	1 18 26·6	3·373	1	56 39 5·2	18·86	1	Stellar	1*
320	2411	......	....	1 18 51·2	2·707	2	128 48 57·5	18·85	2	eeF; S; R; vgbM; 2nd of 4...	2
321	2412	......	....	1 18 52·4	2·707	2	128 47 17·5	18·85	2	eeF; S; R; vgbM; 3rd of 4...	2
322	......	II. 448	....	1 18 55·2	3·056	1	92 3 54·5	18·85	1	Stellar; p of D neb	1
323	......	II. 449	....	1 18 55·2	3·056	1	92 3 54·5	18·85	1	Stellar; f of D neb	1
324	2413	......	....	1 19 6·8	2·707	1	128 44 40·8	18·84	1	eeF; S; R; vgbM; 4th of 4...	1
325	......	III. 171	....	1 19 27·2	3·383	1	56 4 7·1	18·83	1	Stellar	1*
326	122	II. 463	....	1 19 33·1	3·083	2	88 42 53·1	18·83	2	F; S; E 90°; bM; r	5
327	123	III. 560	....	1 19 42·9	3·415	1	53 32 28·1	18·83	1	vF; S; E; vglbM; ✱13 nr ...	2
328	......	III. 172	....	1 19 56·5	3·371	1	57 16 7·4	18·82	1	vS; stellar; p of 2	1
329	......	III. 173	....	1 19 56·5	3·371	1	57 16 7·4	18·82	1	vS; stellar; f of 2	1
330	124	VII. 48	....	1 20 10·3	3·969	1	27 25 54·7	18·81	1	Cl; B; pL; pRi; st mm	2*
331	......	......	D'Arrest, 23	1 20 21	3·38	[1]	56 25 18	18·80	[1]	eF; pL; R	0
332	......	III. 441	....	1 20 34·8	3·051	1	92 40 8·0	18·80	1	vF; vS; iE; p of 2	1
333	......	III. 442	....	1 20 39·9	3·052	1	92 37 8·0	18·80	1	vF; vS; iF; f of 2	1
334	125	......	....	1 21 10·0	3·362	1	58 23 57·6	18·78	1	vF; S; R	1
335	2414	......	....	1 21 37·4	2·726	1	126 27 1·2	18·76	1	vF; S; R	1
336	2415	......	....	1 22 20·5	2·675	1	130 2 23·8	18·74	1	eF; S; att to S✱; B✱nr	1
337	2416	......	....	1 22 43·5	2·723	1	126 19 27·1	18·73	2	vS; ⁑ pos 225° inv	3
338	2417	......	....	1 23 15·2	2·453	1	142 18 52·7	18·71	1	F; S; R; bM; am st 11	1
339	2418	......	....	1 23 43·9	2·864	1	113 23 44·0	18·70	1	B; L; pmE; gpmbM	1
340	127	......	....	1 23 47·8	3·390	1	57 6 32·0	18·70	1	vF; pL; gbM	1
341	126	......	Σ.131=M.103	1 23 59·8	3·916	2	30 2 9·3	18·69	2	Cl; B; R; Ri; pL; st 10...11	5
342	128	I. 100	....	1 24 18·7	3·008	3	97 35 25·6	18·68	3	vB; pL; R; mbM; p of 2 ...	4
343	128, a	......	R. nova	1 24	......	...	97 35	......	...	No description	0
344	......	III. 431	....	1 24 22·2	3·008	(1)	97 36 39·6	18·68	(1)	eF; S; f of 2	1
345	129	......	....	1 24 33·4	3·059	1	91 38 46·6	18·68	1	vF; S; R; bM	1
346	130	......	....	1 24 35·7	+3·008	1	97 36 47·9	−18·67	1	vF; vS; R	1

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	h.	H.		h m s	s		° ′ ″	″			
347	......	......	D'Arrest, 24	1 24 40	+3·41	[1]	55 26 18	−18·66	[1]	vvF; S; ?rr	0
348	......	......	D'Arrest, 25	1 25 0	3·35	[2]	60 5 0	18·65	[2]	F; M. 33, f65ˢ; another f28ˢ ..	0
349	......	......	D'Arrest, 26	1 25 28	3·35	[2]	60 5 0	18·64	[2]	F; pL, f of 2......................	0
350	2419	......	....	1 25 53·4	2·737	2	124 13 32·1	18·63	2	F; S; R; bM	2
351	132	II. 4	....	1 25 54·1	3·006	1	97 44 13·1	18·63	1	pB; R; bM; r; ✱6, f47ˢ·5 ...	7
352	131	V. 17	M. 33	1 25 56·3	3·358	1	60 3 50·1	18·63	1	!; eB; eL; R; vRi; vgbMN; rr.	14†
353	......	II. 473	....	1 26 1·4	2·960	1	102 53 18·1	18·63	1	F; S; iF; er......................	1
354	......	III. 432	....	1 26 15·0	3·003	1	98 2 38·4	18·62	1	eeF	1
355	133	III. 150	....	1 26 17·0	3·361	1	59 57 47·4	18·62	1	vS; R; vvlbM	5
356	2421	......	Δ. 17?	1 26 24·6	1·327	2	164 16 37·4	18·62	2	B; S; R; psbM✱; r............	2
357	......	......	R. nova	1 26 33·5	3·355	: :	60 31 53·9	18·61	: :	S, a neb or Cl with 3stinv...	0
358	......	......	{ D'Arr. = Auw. N. 15 }	1 27 18·1	3·002	...	98 5 42·6	18·58	...	{ Nebulous ✱11, m (D'Arr. Resultate). }	0*
359	134	......	....	1 27 23·6	3·399	1	57 4 11·6	18·58	1	vF; psbM; stellar	1
360	2423	......	....	1 27 44·5	2·690	1	127 13 26·9	18·57	1	F; vS; R; ✱12, p	1
361	{ 139 = 2422 }	I. 281	....	1 27 47·4	2·780	3	120 7 37·9	18·57	3	{ vB; vL; vmE, 118°·3; sbM; ✱34°·5, 6ˢ·5. }	5
362	135	III. 174	....	1 27 53·1	3·401	1	57 2 16·9	18·57	1	pF; psbM; stellar	2
363	137	II. 282	....	1 28 18·0	3·001	1 :	98 2 43·5	18·55	1	pB; pL; ilE; gmbM; r; ✱8, np10′.	4
364	136	......	....	1 28 19·5	3·399	1	57 19 26·5	18·55	1	pB; pL; bM; ✱f, 2ᵐ 51ˢ	1
365	2424	......	....	1 28 37·9	2·687	1	127 12 19·8	18·54	1	eeeF; vS; R; p of 2............	1
366	138	III. 454	....	1 28 48·9	3·072	1	90 2 25·1	18·53	2	eF; pL; not bM	2
367	2425	......	....	1 28 50·4	2·686	1	127 12 40·1	18·53	1	F; S; R; f of 2	1
368	140	III. 471	....	1 28 55·4	2·976	1	100 43 34·1	18·53	1	eF; S; am vSst	2
369	2426	......	Δ. 479	1 28 57·2	2·612	1	132 9 23·1	18·53	2	B; pL; mE; gpmbM	3
370	2427	......	....	1 29 2·6	2·647	1	129 51 50·1	18·53	1	pF; S; R; bM..................	1
371	141	......	....	1 29 9·2	3·405	1 ?	57 7 44·4	18·52	1 : :	vF; R; f of 2	1
372	142	......	M. 74	1 29 11·1	3·211	2	74 55 46·4	18·52	2	⊕; F; vL; R; vg, psmbM; rr	11†
373	......	......	Auw. N. 16	1 29 14·4	4·681	...	17 49 45·0	18·50	...	iF; 3st + neb (Struve, Σ. 2)...	0
374	2428	......	....	1 29 27·8	2·642	1	130 3 42·7	18·51	1	pF; S; R; bM..................	1
375	143	......	....	1 29 59·6	3·119	1	84 50 14·0	18·50	1	pB; S; R; psbM................	1
376	2429	......	....	1 30 9·9	2·669	2	128 2 13·0	18·50	2	pB; S; R; gbM; ⁑np.........	2
377	144	II. 283	....	1 32 7·6	2·997	2	98 13 12·4	18·42	2	pB; vS; R; mbM; r	4
378	......	VII. 49	....	1 32 8·2	4·132	1	26 40 32·4	18·42	1	Cl; pS; L & vSst..............	1
379	2430	......	....	1 32 32·0	2·759	3	120 38 10·7	18·41	3	vF; vS; p of 2	3
380	2432	......	....	1 32 38·7	2·575	2	133 14 25·0	18·40	3	F; S; R; gpmbM; p of 2 ...	3
381	2431	......	....	1 32 39·3	2·759	3	120 37 15·0	18·40	3	vF; pS; R; gbM; ✱f, nr......	3
382	2435	......	....	1 32 40·7	0·929	2	166 16 4·0	18·40	2	vF; pS; R; vglbM	2
383	2433	......	....	1 32 50·9	2·573	2	133 18 22·0	18·40	2	F; S; vlE; glbM; f of 2	2
384	2434	......	....	1 32 55·1	1·907	1	155 36 37·3	18·39	1	vF; iR; vglbM	1
385	......	......	M. 76	1 33 28·5	3·334	1	39 8 52·4	18·37	1	vB; p of D neb	2
386	......	I. 193	....	1 33 37·5	3·734	1	39 7 27·4	18·37	1	vB; f of D neb..................	1
387	145	VII. 46	....	1 34 28·0	4·062	2	28 49 19·8	18·34	2	Cl; iF; Ri; one ✱ 6·7; st 11 ...14.	4
388	146	......	....	1 34 41·6	3·856	1	34 49 52·1	18·33	1	Cl; pRi; st 12, m..............	1
389	......	VIII. 65	C.H.	1 34 45·0	4·018	1	30 0 37·1	18·33	1	Cl; S; lRi; stL	1
390	......	II. 253	....	1 35 35·6	3·199	1	77 4 36·0	18·30	1	pB; pL; E; bM; r	1
391	147	II. 610	....	1 36 23·8	3·366	1	62 0 24·9	18·27	1	F; S; R; bM; r	3
392	......	VI. 31	....	1 36 29·2	4·055	1	29 27 40·9	18·27	1	Cl; B; L; eR; st pL	1
393	148	......	....	1 36 30·8	3·107	1	86 28 36·9	18·27	1	vF; S; R........................	1
394	......	II. 588	....	1 37 15·0	3·165	2	80 17 10·8	18·24	2	F; S; lE; bM; r	2
395	149	II. 611	....	1 39 33·1	3·365	2	62 48 58·2	18·16	2	F; S; lE 0°±	3
396	150	I. 157	....	1 40 0·8	3·360	2	63 16 25·8	18·14	3	F; E 0°...90°; bet 2 st.........	4
397	......	II. 589	....	1 40 35·6	3·183	2	79 10 46·7	18·11	2	F; pL; E; lbM; ✱nf, 2′	2
398	......	......	D'Arrest, 27	1 41 29	3·30	[1]	68 21 3	18·08	[1]	pB; vmE; ✱14, f8ˢ	0
399	......	II. 228	....	1 41 34·5	3·299	2	68 42 50·6	18·08	2	pB; S; iR; mbM; 1st of 2 ...	2
400	151	IV. 42	....	1 41 37·9	3·126	2	84 47 12·6	18·08	2	vF; vmE, 165°±; sbM✱9 ...	3†
401	......	III. 175	....	1 41 51·1	+3·482	1	54 53 51·9	−18·07	1	F; stellar	1

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	h.	•H.		h m s	s		° ′ ″	″			
402	......	II. 229	....	1 41 57·7	+3·299	2	68 44 49·2	−18·06	2	pB; S; iR; mbM; 2nd of 2...	2
403	152	II. 612	....	1 42 3·5	3·369	(1)	63 3 33·2	18·06	1	F; vlE, 90°; *15, nr	2
404	2436	II. 481	....	1 42 17·0	2·959	1	101 7 2·5	18·05	1	pF; cL; R; glbM; S*p, 90″..	2
405	153	......	....	1 42 17·6	3·186	1::	79 0 17·5	18·05	1	eF	1
406	154	II. 501	....	1 42 18·3	2·910	1	105 39 54·5	18·05	1	cF; S; R; gvlbMN............	2
407	2438	......	....	1 42 20·7	2·290	1	143 29 3·5	18·05	1	F; vL; R; vgvlbM	1
408	155 = 2437	III. 459	....	1 42 21·3	2·808	1	114 29 31·5	18·05	1	vF; vS; R; gbM; er; 2stnr..	4
409	......	III. 561	....	1 42 25·7	3·492	1	54 21 52·8	18·04	1	vF; stellar	1
410	......	II. 617	....	1 42 55·7	3·299	1	68 56 54·1	18·03	1	F; cL; vglbM..................	1
411	2439	......	....	1 43 2·1	2·393	2	139 19 59·1	18·03	2	B; S; R; gbM	2
412	156	II. 859	....	1 43 10·9	3·129	1	84 33 6·4	18·02	1	pF; S; E90°; vglbM; *10,nf	2†
413	......	......	D'Arrest, 28	1 43 13	3·30	[1]	68 42 18	18·01	[1]	F; S; R; bet 2 st 15............	0
414	......	II. 618	....	1 43 29·0	3·309	1	68 7 54·7	18·01	1	vS; stellar	1
415	2440	......	....	1 43 32·7	2·652	1	125 38 52·7	18·01	1	F; S; R	1
416	......	III. 179	....	1 43 39·3	3·307	2	68 20 25·0	18·00	2	F; cL; E; mbM	2
417	2441	......	....	1 43 42·0	2·653	1	125 34 5·0	18·00	1	eF; S	1
418	160 = 2442	I. 62	....	1 44 8·0	2·964	1	100 23 45·6	17·98	2	F; pL; E; vgvlbM; r.........	4*
419	158	III. 192	....	1 44 17·0	3·024	2	94 44 57·9	17·97	3	eF; vlE 0°±, *13, s, 90″......	4
420	......	III. 564	....	1 44 25·9	3·496	1	54 35 56·9	17·97	1	Stellar; 3rd of 4	1
421	......	III. 565	....	1 44 25·9	3·496	1	54 35 56·9	17·97	1	Stellar; last of 4	1
422	157	III. 562	....	1 44 27·3	3·497	1	54 31 58·9	17·97	1	vF; stellar; 1st of 4...........	2
423, 424, 425	157, a	......	R. 3 novæ	1 44	......	...	54 32	......	...	Near h. 157, 159	0
426	161	II. 596	....	1 44 30·7	3·131	1	84 24 17·9	17·97	1	F; S; bM; *13 1′, n	3
427	159	III. 563	....	1 44 34·3	3·497	1	54 33 1·9	17·97	1	F; pL; bM; 2nd of 4	2
428	162	......	55 Androm.	1 44 55·9	3·575	1	49 57 41·5	17·95	1	Fine nebulous * with strong atm.	1*
429	163	......	....	1 44 56·2	3·509	1	53 52 9·5	17·95	1	vF; R; am pBst	1
430	164	II. 270	....	1 45 56·9	3·110	2	86 29 23·7	17·91	2	pB; S; iR; psmbM............	3
431	165 = 2443	I. 105	....	1 46 12·4	2·918	2	104 25 39·0	17·90	2	cB; pL; lE; psmbM	3
432	......	......	D'Arrest, 29	1 46 19	3·28	[1]	70 50 18	17·89	[1]	eF; R; *19, f	0
433	......	......	D'Arrest, 30	1 46 32	3·56	[1]	51 18 3	17·88	[1]	eF; pL...........................	0
434	......	......	D'Arrest, 31	1 47 5	3·29	[3]	69 59 18	17·86	[3]	vF; vS; R; β Arietis in field	0
435	166 = 2444	III. 460	....	1 47 13·9	2·797	2	114 26 46·2	17·86	2	pF; vS; R; vgbM	3
436	167	......	....	1 47 14·6	2·795	1	114 33 21·2	17·86	1	vF; pL; R; gbM; S*195°...	1
437	2445	......	....	1 47 43·1	2·621	1	126 34 24·8	17·84	1	F; S; R; bM	1
438	168	......	....	1 47 47·9	3·110	1	86 29 14·8	17·84	1	Suspected neb..................	1
439	2446	......	....	1 47 50·1	2·621	1	126 32 0·8	17·84	1	eeeF; S; R	1
440	......	III. 266	....	1 48 6·0	2·969	1	99 38 5·4	17·82	1	eF; stellar	1
441	177 = 2447	III. 265	....	1 48 17·0	2·968	1	99 44 29·4	17·82	1	vF; pS; vlE	3
442	......	II. 221	....	1 48 26·0	3·463	1	57 38 16·7	17·81	1	F; pL; mE; r	1*
443	......	III. 176	....	1 48 33·4	3·483	1	56 23 6·7	17·81	1	eeeF; stellar	1
444	169	......	....	1 48 33·9	3·463	3	57 38 53·7	17·81	3	pB; R; bM; *13, np	3*
445	169, a	......	R. nova	1 48 34·5	3·463	...	57 38 25·7	17·81		β, γ, δ of Lord R.'s diag. ε=II. 221 H.	0*
446	169, b	......	R. nova	1 48 38·4	3·463	...	57 38 0·7	17·81			0*
447	169, c	......	R. nova	1 48 47·9	3·463	...	57 40 30·7	17·81			0*
448	172	II. 271	....	1 49 2·8	3·126	1	85 3 29·3	17·79	1	pF; S; R; p of 2; pos=102°·4	1
449	173	II. 272	....	1 49 3·2	3·126	1	85 3 41·3	17·79	1	vF; vS; R; sbM; f of 2	4
450	170	......	....	1 49 6·3	4·127	1	30 30 49·6	17·78	1	Cl; not Ri; *	1
451	171	......	....	1 49 7·6	+3·953	1	35 13 13·6	−17·78	1	Cl; pL; pRi; iF; st 11...13..	1

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	h.	H.		h m s	s		° ′ ″	″			
452	2449	......	....	1 49 12·4	+2·112	1	147 22 44·6	−17·78	1	pB; S; R; gbM	1
453	176	III. 193	....	1 49 20·4	3·017	2	95 9 13·6	17·78	2	eF; *9, 315°±	3
454	2448	......	....	1 49 21·5	2·707	3	120 36 23·6	17·78	3	pB; S; E; lM..............	3
455	175	II. 222	....	1 49 23·6	3·468	1	57 29 41·6	17·78	1	F; pL; mE; r; f of 2	3
456	175, a	......	R. nova	1 49	......	...	57 29	......	...	nf h. 175	0
457	174	VII. 32	....	1 49 25·8	3·541	1	53 1 14·9	17·77	1	Cl; vvL; Ri; st L and sc ...	5
458	2450	......	....	1 49 27·7	2·107	1	147 26 49·9	17·77	1	vF; S; R; bM	1
459	178 = 2451	III. 464	....	1 49 57·4	3·006	3	96 5 19·5	17·75	3	vF; S; lE; vglbM	4
460	180	......	....	1 51 17·8	3·158	1	82 20 13·3	17·69	1	vF; S; R; *10, 2′ 285°......	1
461	......	......	D'Arrest, 32	1 51 35	3·28	[2]	71 42 48	17·68	[2]	vF; S; R; nr I. 112 H.	0
462	179	......	50 Cassiop.	1 51 36·4	5·005	1	18 15 36·6	17·68	1	Suspected nebulous *.........	1*
463	181	112	....	1 51 39·0	3·283	2	71 40 12·6	17·68	2	B; cL; R; gbM; r	3
464	181, a	......	R. nova	1 51 39	3·283	::	71 45±	......	::	5 or 6′ s of h. 181	0
465	2452	III. 468	....	1 52 0·8	2·936	1	102 10 50·9	17·67	1	cF; pL; E 0°±; glbM	2
466	......	III. 214	....	1 52 2·0	3·223	1	76 41 15·9	17·67	1	vF; stellar	1
467	2453	......	....	1 52 3·7	2·751	2	116 58 27·9	17·67	2	pF; S; R; glbM..............	1
468	......	......	D'Arrest, 33	1 52 5	3·34	[1]	67 2 18	17·66	[1]	F; pL	0
469	182	II. 223	....	1 52 10·0	3·451	1	59 15 3·2	17·66	1	pB; pL; R; glbM	2
470	183	I. 101	....	1 52 40·6	2·999	3	96 38 24·8	17·64	3	cB; L; mE 163°·0; mbM ...	6
471	......	III. 215	....	1 52 43·3	3·209	1	77 59 17·8	17·64	1	eF; stellar	1
472	184	III. 583	....	1 52 52·3	3·406	1	62 26 29·1	17·63	1	vF; vS; E; 3 stp; *250° ...	2*
473	2454	......	....	1 52 57·3	2·039	1	148 28 9·1	17·63	1	pB; pL; lE; *12 att	1
474	185	II. 435	....	1 54 2·1	2·989	(1)	97 30 22·6	17·58	1	pF; pS; R; bM	2
475	186	III. 433	....	1 54 20·6	3·005	1	96 3 38·9	17·57	1	cF; cS; R; bM	3
476	......	......	D'Arrest, 34	1 54 25	3·16	[1]	82 10 18	17·56	[1]	vF; S; * 14 f 90°; 11s·65 ...	0
477	187	......	....	1 54 39·4	3·247	1	74 57 35·2	17·56	1	eF; S; R; * 11 75°	1
478	188	III. 207	....	1 54 49·2	3·280	1	72 17 54·5	17·55	1	vF; cS; R; stellar	2
479	2455	......	....	1 54 55·4	2·100	1	146 30 26·8	17·54	1	pF; S; R; 2 st 11, nr	1
480	2456	......	....	1 55 1·3	0·699	3	164 54 18·8	17·54	3	eF; vS; R; *12, 25″ 315°....	3
481	189	III. 566	....	1 55 8·5	3·571	1	52 33 52·1	17·53	1	vF; S; iR; sbM; * nr	3
482	2457	......	....	1 55 46·1	1·428	1	158 32 40·7	17·51	1	eeF; vS; R; *13 p 100″ ...	1
483	190	III. 208	....	1 56 8·7	3·253	1	74 38 1·3	17·49	1	vF; S; iR; glbM; *10 p 3s·5	2
484	191	III. 151	....	1 56 54·9	3·428	1	61 40 47·2	17·46	1	vF; vS; iR; bet2stn and sp..	3
485	192 = 2458	......	....	1 57 28·5	2·780	2	113 58 18·1	17·43	2	vF; pS; vlE	2
486	2459	......	....	1 58 26·4	1·343	1	159 7 19·3	17·39	1	pF; S; R; gbM	1
487	193	I. 152	....	1 59 51·2	3·196	1	79 40 47·1	17·33	1	B; vS; vlE; svmbM; *10, 55″ 320°.	3*
488	194	II. 604	....	2 0 19·0	3·604	1	51 54 21·7	17·31	1	pB; cL; lE; mbM	2
489	195	......	....	2 0 49·6	3·238	1	76 18 58·6	17·28	1	F; R; vS; bM	1
490	2461	......	....	2 0 55·4	2·468	2	131 49 14·6	17·28	2	cF; vS; R; sbM; r............	2
491	196 = 2640	......	....	2 0 56·9	2·741	2	116 7 18·6	17·28	2	vF; vF ** inv	2
492	2462	......	....	2 0 58·2	2·561	1	127 9 11·6	17·28	2	F; S; R; vsvmbM*13	2
493	198	III. 227	....	2 1 34·0	3·161	1	82 41 44·2	17·26	1	vF; S; R; bM; am st.........	2
494	197	II. 605	....	2 1 40·3	3·617	1	51 28 53·5	17·25	1	pB; S; iR; ** f 15s	2
495	199 = 2463	II. 482	....	2 2 30·1	2·942	3	100 47 38·7	17·21	3	F; S; R; 1st of 4	4
496	......	III. 567	....	2 2 32·5	3·591	1	53 0 41·7	17·21	1	vF; S; lE	1
497	200 = 2464	II. 483	....	2 2 32·8	2·942	3	100 47 50·7	17·21	3	F; S; R; 2nd of 4	4
498	201 = 2465	II. 484	....	2 2 47·1	+2·942	3	100 48 31·0	−17·20	3	vF; vS; R; 3rd of 4	4

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	h.	H.		h m s	s		° ′ ″	″			
499	{ 202 = 2466 }	II. 485	....	2 2 51·4	+2·941	2	100 51 11·0	−17·20	3	vF; pS; R; 4th of 4	4
500	203	......	....	2 2 55·0	2·971	1	98 25 7·3	17·19	1	vF; vS; R; psbM	1
501	204	III. 604	....	2 3 56·7	3·593	1	53 10 55·5	17·15	1	vF; iF; stellar.................	2*
502	2467	......	....	2 4 15·8	1·992	1	147 23 34·1	17·13	1	pF; pS; R; glbM; r	1
503	......	III. 259	....	2 4 27·5	3·047	1	92 9 47·4	17·12	1	eF; eS; iF	1
504	......	II. 486	....	2 4 45·7	2·951	1	99 58 5·7	17·11	1	F; S; E........................	1
505	2468	......	....	2 5 36·7	2·555	3	126 30 33·9	17·07	4	cF; pS; lE 0°; gbM	4
506	......	II. 613	....	2 5 51·5	3·435	1	62 46 51·2	17·06	1	F; S, lE 90°; bM	1
507	2469	......	....	2 6 30·8	2·623	2	122 36 11·1	17·03	2	cB; S; E; psmbM	2
508	2470	......	....	2 7 24·0	2·420	2	132 41 12·3	16·99	2	F; vS; svmbM	2
509	205	III. 260	....	2 7 24·4	3·056	1	91 24 41·3	16·99	1	vF; R; bM; stellar	2
510	206	III. 457	....	2 8 6·4	3·140	1	84 39 32·5	16·95	1	eF; cL; R; gbM; *12 sf att.	2*
511	......	III. 2	....	2 8 41·3	3·078	1	89 36 2·1	16·93	1:	eF; vS; R; bM	1
512	207	VI. 33	....	2 9 15·3	4·166	2	33 29 55·0	16·90	2	!; Cl; vvL; vRi; st7...14 ...	5
513	208	III. 201	....	2 9 33·1	3·252	1	76 5 52·3	16·89	1	vF; vS; E; *10 sf 4′	2
514	208, a	......	R. nova	2 9	......	...	76 5+	......	...	neb s of h. 208	0
515	{ 209 = 2471 }	II. 474	....	2 9 43·2	2·920	5	101 59 58·6	16·88	5	F; pL; R; vglbM	7
516	210	II. 246	....	2 10 22·6	3·253	1	76 6 11·5	16·85	1	pF; pL; lE; pgbM; { *9, 185°± 5′ S* sf 1′ }	2*
517	210, a	......	R. nova	2 10	......	...	76 6	......	...	neb s of h. 210	0
518	211	II. 436	....	2 11 14·5	2·980	1	97 17 24·7	16·81	1	F; pS; E; bM; 2 or 3 st nr...	2
519	213	......	....	2 11 59·3	3·272	1	74 48 57·9	16·77	1	eF; R; gbM; * 16 nr.........	1
520	215	II. 437	....	2 12 8·7	2·978	2	97 25 44·2	16·75	2	pF; pS; vlE; bM; ⁑ nr	3
521	212	VI. 34	....	2 12 34·2	4·188	2	33 32 49·5	16·75	2	!; Cl; vL; vRi; ruby * M ...	5
522	214	......	....	2 12 46·6	4·536	1	26 52 20·1	16·73	1	Cl; L; lC; sc st 9...13	1
523	216	III. 486	....	2 12 54·3	2·852	1	106 42 4·1	16·73	1	F; S; iR; pgbM..............	2
524	2473	......	....	2 13 20·1	1·774	1	150 30 8·7	16·71	1	eF; S; R; 2 or 3 vSst nr ...	1
525	2472	......	....	2 13 30·8	2·400	2	132 23 1·0	16·70	2	vF; vS; R; bM; *7 sf and 6 more.	2
526	217	II. 225	....	2 13 41·2	3·548	4	57 22 48·3	16·69	4	B; S; R; bM; 3S st sp	5
527	218	V. 19	....	2 13 50·4	3·737	1	48 16 47·6	16·68	1	!; B; vL; vmE 22°·3	5†
528	2474	......	....	2 14 21·4	2·404	2	132 2 58·2	16·66	2	pF; pS; R; lbM; *8 90°, 4′	2
529	219	II. 438	....	2 14 35·7	2·993	1	96 10 8·8	16·64	2	F; vL; iR; gbM..............	4
530	219, a	......	R. nova	2 14	......	...	96 10	......	...	E; F; bM; makes D neb with h. 219; both E.	0
531	......	III. 695	....	2 15 1·0	4·448	1	28 40 47·1	16·63	1	eF; pL; iF	1
532	2475	......	....	2 15 9·4	2·563	1	124 21 30·4	16·62	1	pB; S; R; psbM; *10 f 90°·0 35″.	1
533	......	III. 570	....	2 15 14·0	3·732	2	48 42 17·4	16·62	1	eF; vS; lE	1
534	2476	......	....	2 15 26·4	2·779	1	111 27 12·0	16·60	1	pB; S; gbM; r; ⁑p	1
535	2477	III. 224	....	2 16 34·0	2·778	1	111 20 33·5	16·55	1	F; S; E 90°; gbM	3
536	......	I. 153	....	2 16 36·1	2·770	1	111 52 23·5	16·55	1	cB; vL; E 0°...90°...........	1*
537	......	III. 571	....	2 16 44·2	3·736	1	48 49 21·5	16·55	1	eF; stellar	1
538	221	......	....	2 17 49·4	3·319	1	72 7 46·3	16·49	1	pF; L; R; *10 sf 3′	1
539	220	......	....	2 17 50·6	3·543	1	58 23 33·3	16·49	1	vF; S; R; 4 st nr	1
540	2478	III. 239	....	2 18 44·7	2·710	1	115 26 13·5	16·45	1	cF; pL; R; gpmbM	2
541	......	III. 474	....	2 18 50·8	3·350	1	70 7 27·8	16·44	1	eF; vS; iR	1
542	222	III. 177	....	2 18 55·7	3·571	2	57 3 25·1	16·43	2	cF; cL; E; vgbM; 2st13np..	3
543	......	II. 489	....	2 19 58·5	3·350	1	70 15 30·6	16·38	1	F; S; lE; 3 st inv	1
544	223	IV. 23	....	2 20 29·1	3·049	1	91 46 18·5	16·35	1	vB; vL; R; mbMN	2†
545	2479	......	....	2 20 59·9	2·295	2	135 4 17·1	16·33	2	vvF; S; R; gvlbM............	2
546	224	III. 261	....	2 21 21·5	3·049	1	91 47 36·7	16·31	1	vF; cL; R; f of 2	2
547	{ 225 = 2480 }	II. 487	....	2 21 48·4	2·919	2	101 10 10·3	16·29	2	vF; L; iR; glbM	3
548	2481	......	....	2 22 1·5	+2·795	1	109 40 3·6	−16·28	1	pB; E; gbM	1

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	h.	H.		h m s	s		° ′ ″	″			
549	226	I. 154	....	2 22 18·6	+3·654	1	53 29 51·2	−16·26	1	cB; L; E; vgbM	3*
550	2482	......	....	2 23 20·3	2·366	4	132 1 30·7	16·21	4	vF; pL; lE; gbM; *8 sf 3′...	4
551	229	II. 278	....	2 23 24·6	3·050	1	91 43 1·7	16·21	1	pB; S; E; psbM...............	3
552	228	......	....	2 23 27·5	3·837	1	45 59 37·0	16·20	1	Cl; pRi; st 9...15	1
553	227	......	....	2 23 28·9	4·288	1	33 5 46·0	16·20	1	Cl; pL; pRi; st 13...15	1
554	230	II. 237?	....	2 23 38·8	3·024	1	93 33 56·3	16·19	1	pF; ilE 0°±; bM	2
555	2483	......	....	2 25 21·5	2·482	2	126 39 0·0	16·10	4	pB; pS; mE 215°·7	4
556	2484	......	....	2 25 38·1	2·817	2	107 49 36·8	16·09	2	F; S; iR; gbM	2
557	231	......	....	2 25 44·2	3·580	1	57 40 44·3	16·09	1	S; R; psbM; 1st of 3.........	1*
558	231, a	......	R. nova	2 25 47·7	3·580	...	57 38 59·3	16·09	...	γ and δ of Lord R.'s diag.	0*
559	231, b	......	R. nova	2 25 52·0	3·580	...	57 38 50·3	16·09	...		0*
560	232	II. 211	....	2 25 52·4	3·512	(2)	61 17 57·6	16·08	(2)	pB; cL; lE 0...90°; gmbM; 3st s.	3†
561	233	......	....	2 26 0·7	3·580	1	57 40 14·9	16·07	1:	vF; R; bM; 2nd of 3.........	1*
562	2485	III. 472	....	2 26 16·8	2·912	1	101 22 37·5	16·05	1	eF; pS; R; vlbM; am sc st...	2
563	234	......	....	2 26 21·2	3·580	1	57 46 23·5	16·05	1	pB; R; 3rd of 3	1*
564	2486	......	....	2 26 26·9	2·270	2	135 8 21·8	16·04	2	F; S; R; bet 2 st in par......	2
565	235	III. 572	....	2 26 33·1	3·753	1:	49 47 23·8	16·04	1:	vF; pS; p of 2; 210″; 157°...	2
566	236	III. 573	....	2 26 37·4	3·754	1	49 44 28·1	16·03	1	F; S; f of 2; 210″; 337° ...	2
567	2487	......	Δ. 519??	2 28 0·1	2·404	2	129 39 17·2	15·96	2	pB; L; pmE; smbM; bi-N...	2†
568	237	III. 161	....	2 28 23·1	3·595	2	57 17 16·5	15·95	2	F; S; vlE; bM; r; 2 st 14 np	4
569	238	III. 557	....	2 28 43·8	3·232	1	78 58 34·1	15·93	1	F; S; vlE; psbM; r	2
570	239	III. 434	....	2 28 46·2	2·963	1	97 46 30·4	15·92	1	vF; cL; iF; vlbM	2
571	240	II. 238 = III. 198	....	2 30 24·9	3·761	1	49 43 53·8	15·84	1	pF; L; E 90°±; mbM; r ...	4*
572	241	III. 152	....	2 30 56·6	3·540	1:	60 27 46·0	15·80	1	F; pS; iR; bM; st inv	3†
573	......	II. 6	....	2 31 35·3	3·077	1	89 44 6·9	15·77	1	S; cometic	1*
574	244	I. 102	....	2 31 36·0	2·968	3	97 17 18·9	15·77	3	cB; pL; vR; mbM............	5*
575	242	I. 156	....	2 31 38·3	3·731	1	51 32 45·9	15·77	1	vB; vL; vmE; vvmbM	3†
576	243	II. 592	....	2 31 38·5	3·222	1	79 45 46·9	15·77	1	pF; S; lE; bM; *11, 25″ 50°	2
577	2488	......	....	2 31 43·4	1·877	1	145 28 45·9	15·77	1	eF; S; R; p of 2...............	1
578	......	VIII. 66	....	2 32 1·1	4·572	2	29 3 28·5	15·75	2	Cl; L; sc st, one 10...........	2
579	245	III. 581	....	2 32 2·2	3·333	1:	72 34 8·5	15·75	1::	vF; iE...........................	2
580	2490	......	....	2 32 8·3	1·874	1	145 28 16·8	15·74	1	F; S; R; gbM; *11, s 2′ ...	1
581	246	II. 5	....	2 32 11·6	3·080	2	89 30 24·8	15·74	2	pB; S; vlE 0...90°; bm; 3st trap.	10
582	249 = 2489	II. 284	....	2 32 36·5	2·945	2	98 44 27·7	15·71	2	pF; L; mE; r; *17, att sf ...	4
583	247	III. 475	....	2 32 42·1	3·354	(1)	71 19 38·7	15·71	1	F; S; R; lbM..................	2
584	248	......	M. 34	2 33 2·3	3·829	2	47 49 25·0	15·70	2	Cl; B; vL; lC; sc st 9	8
585	251	III. 228	....	2 33 36·3	3·193	1	81 52 7·2	15·66	1	vF; vS; p of 2; *10 p	2
586	253 = 2491	II. 488	....	2 33 42·9	2·898	2	101 53 28·2	15·66	2	F; S; R; bM	3
587	252	III. 229	....	2 33 43·8	3·192	1	81 53 22·2	15·66	1	eF; vS; f of 2.................	2
588	2492	......	....	2 33 57·9	2·491	1	124 52 20·8	15·64	1	pB; S; R; stellar	1
589	254 = 2493	I. 63	....	2 34 12·1	2·943	3	98 51 19·1	15·63	3	B; pL; R; mbM*12	4
590	256	III. 584	....	2 34 35·0	3·520	1:	62 1 17·0	15·60	1	F; S; R; psbM	2
591	258	I. 1	....	2 34 35·1	3·072	2	90 9 14·0	15·60	2	pF; cL; iE 80°; bM; pB*nr	8*
592	255	II. 633	....	2 34 39·9	3·701	1	53 16 21·0	15·60	1	pF; cL; R; glbM	3
593	259	......	....	2 34 44·5	3·337	1	72 35 27·0	15·60	1	eF;?	1
594	257	III. 162	....	2 34 49·1	3·596	2	58 10 18·3	15·59	2	F; pL; R; lbM; *7·8 p 43″·5	4
595 596 597	257, a	......	R. 3 novæ	2 34	......	...	58 10 .	......	...	6 seen (including ∴ h. 257, 260, 261).	0
598	260	III. 163	....	2 35 23·6	3·598	1	58 7 34·9	15·57	1	vF; pL; R; lbM; sp of 2 ...	2
599	261	......	....	2 35 26·2	+3·599	1	58 4 35·2	−15·56	1	eF; S; nf of 2................	1

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	h.	H.		h m s	s		° ′ ″	″			
600	262	......	M. 77	2 35 30·2	+3·065	1	90 35 56·2	−15·56	1	vB; pL; iR; sbMrrN; ✱130°, 2′.	13†
601	263	II. 273	....	2 36 0·3	3·138	1	85 37 19·1	15·53	1	pF; S; iR; gbM	2
602	......	III. 455	....	2 36 33·3	3·085	2	89 13 22·0	15·50	2	vF; L; lbM; er	2
603	2494	......	....	2 37 43·7	2·589	1	119 35 49·8	15·44	1	B; pL; pmE; sbM	1
604	264	I. 64	....	2 39 6·9	2·950	1	98 9 52·5	15·35	1	vB; pL; E; gpmbM	3
605	265	II. 466	....	2 39 15·4	3·057	1	91 5 53·8	15·34	1	pB; cL; lE; mbM	4
606	266	II. 465	....	2 39 21·1	3·061	1	90 49 5·8	15·34	1	vF; pL; iR; bM	4
607	......	III. 582	....	2 39 22·7	3·313	1	74 25 31·8	15·34	1	vF; S; iF	1
608	267	III. 462	....	2 40 18·4	3·061	1	90 50 57·3	15·29	1	vF; S; R; 2Sst p	2
609	2496	......	....	2 40 20·8	1·552	1	150 30 14·3	15·29	1	F; pS; R; glbM	1
610	2495	V. 48	....	2 40 21·5	2·557	2	120 51 30·3	15·29	2	vB; L; vmE 151°·1; vbMN	3
611	{269 = 2497}	III. 449	....	2 42 35·2	2·796	2	107 34 23·5	15·15	2	pF; pL; pmE; glbM	3
612	268	......	....	2 42 39·0	3·839	1	48 55 4·5	15·15	1	neb or vSCl of vSst	1
613	270	II. 601	....	2 43 44·5	3·858	1	48 22 21·3	15·09	1	cF; S; iR; vgbM; r	2
614	......	......	Bessel	2 44 6·5	3·738	...	53 4 14·9	15·07	...	? a comet	0*
615	272	III. 450	....	2 45 8·9	2·799	1	107 12 50·7	15·01	1	vF; S; lE; gbM	3
616	271	II. 602	....	2 45 25·7	3·847	1	49 0 2·3	14·99	1	cF; pS; iR; vglbM	2
617, 618	271, *a*	......	R. 2 novæ	2 45	......	...	49	......	...	h. 271 is D; another near ...	0
619	273	......	....	2 45 44·4	3·044	1	91 51 24·6	14·98	1	eF; pL; gbM; ✱8f	1
620	......	II. 254	....	2 46 0·9	3·269	1	77 34 54·2	14·96	1	F; S; iR; r	1
621	2498	......	....	2 46 35·8	1·778	1	145 32 30·4	14·92	1	F; R; gbM	1
622	2499	......	....	2 46 44·5	1·772	1	145 38 31·4	14·92	1	F; R; gbM	1
623	274	III. 580	....	2 47 29·7	3·897	1	47 31 21·6	14·88	1	vF; vS; R; gbM; 2Sst Δ ...	2
624	{275 = 2500}	II. 470	....	2 47 46·2	2·905	2	100 36 9·2	14·86	2	vF; S; R; stellar	4
625	2501	......	....	2 48 7·7	2·760	1	109 13 6·8	14·84	1	F; pL; vmE; 2Sst f	1
626	276	II. 274	....	2 50 52·8	3·118	1	87 11 7·6	14·68	1	F; vS; ilE; sbM; er	2
627	......	II. 619	....	2 51 35·0	3·493	1	65 21 16·1	14·63	1	pB; cL; pmE 0°; r; ✱n 1′...	1
628	277	III. 199	....	2 51 54·7	3·949	1	45 40 24·7	14·61	1	F; pS; lE; SbM; ⁑p 6ˢ·5 ...	4
629	277, *a*	......	R. nova	2 52 ±	......	...	45 40 ±	......	...	R (nisi=H. II. 239)	0
630	2502	III. 469	....	2 52 16·6	2·863	1	102 57 33·3	14·59	1	F; R; glbM; stellar	2
631	......	III. 178	....	2 52 47·8	3·711	1	55 18 19·2	14·56	1	vF; pL; R; spmbM	1
632	278	......	....	2 52 51·3	3·908	1	47 58 42·2	14·56	1	eF; vS	1
633	2503	......	....	2 52 56·7	2·484	2	122 39 22·5	14·55	2	vF; pL; E; vlbM	2
634	......	II. 239?	....	2 53 0·1	3·977	2	45 35 20·5	14·55	2	pB; pL; iR; mbM	3
635	279	II. 620	....	2 53 28·9	4·025	(2)	44 10 52·4	14·52	(2)	pF; pS; iF; sbM	2
636	280	II. 502	....	2 55 0·2	2·818	1	105 23 8·1	14·43	1	pF; pL; R; psbM	2*
637	......	II. 607	....	2 55 21·2	3·906	1	48 13 26·7	14·41	1	F; cL; E	1
638	......	II. 704	....	2 55 44·9	8·608	1	9 42 40·3	14·39	1	F; pL; mE 90°...180°	2
639	281	IV. 43	....	2 56 20·1	3·925	1	47 43 32·5	14·35	1	F; mE; smbMS✱	3*
640	2504	III. 245	....	2 56 25·0	2·669	1	113 25 4·8	14·34	1	pF; cL; pmE; gbM✱16; r...	2
641	......	II. 608	....	2 56 32·2	3·975	1	46 10 29·0	14·40	1	F; cL; er	1
642	2505	......	....	2 56 55·1	2·864	1	102 38 19·0	14·30	1	vF; sp of 2	2
643	282	II. 503	....	2 57 5·8	2·802	1	106 8 59·3	14·29	1	cB; pS; iR; smbM	2
644	2506	II. 475	....	2 57 14·1	2·865	1	102 33 6·3	14·29	1	pF; cL; iR; bM; nf of 2 ...	3
645	283	I. 109	....	2 58 3·7	2·603	1	116 35 45·8	14·24	1	cB; pS; vlE 0°; r; S✱nr ...	4
646	284	III. 578	....	2 59 19·7	3·810	1	52 9 24·2	14·16	1	cF; vS; R; psbM	2*
647	{285 = 2507}	II. 285	....	2 59 25·2	2·905	3	100 5 8·5	14·15	3	pB; S; lE 80°±; lbM	7
648	286	II. 504	....	2 59 30·2	2·799	1	106 8 39·5	14·15	1	B; S; cE; psbM	2
649	2508	......	....	3 0 48·0	2·288	1	129 34 26·9	14·07	1	pF; S; R; psbM	2
650	287	......	....	3 1 29·2	4·337	1	37 11 56·1	14·03	1	Cl; vS of Sst	1
651	2509	II. 258	....	3 3 27·9	2·702	3	111 7 0·0	13·90	3	pB; cL; R; gbM; r	7
652	288	III. 262	....	3 4 20·1	3·022	(1)	93 3 4·5	13·85	1::	stellar; difficult	2
653	......	III. 164	....	3 4 23·3	+3·634	1	59 57 41·8	−13·84	1	eF; vS; ? vSst	1

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	h.	H.		h m s	s		° ′ ″	″			
654	289 = 2510	II. 286	....	3 4 26·8	+2·912	3	99 27 28·8	−13·84	3	F; pL; R; vglbM; *9np ...	5
655	289, *a*	......	R. nova	3 4 31·4	2·912	...	99 26 25·8	13·84	...	No description..................	0*
656	291 = 2511	III. 591	....	3 4 36·2	2·912	2	99 29 11·6	13·78	2	eF; vS; R; stell; sf of 2......	3*
657	2512	......	Δ. 205??	3 4 41·9	0·771	2	157 18 47·1	13·83	2	F; S; pmE; gbM	3
658	290	VI. 25	....	3 5 6·3	4·106	1	43 17 22·3	13·79	1	Cl; pL; Ri; C; iR; st 12...15	3
659	2513	......	....	3 5 18·2	0·747	2	157 29 13·6	13·78	2	pF; S; R; glbM..............	2
660	......	II. 900	....	3 5 38·6	2·884	1	101 0 7·2	13·76	1	F; pL; E 80°±	1
661	292	III. 443	....	3 5 49·8	2·975	3	95 45 4·5	13·75	3	cF; S; lE; bM; *9, n 5′ ...	4
662	2514	......	....	3 5 57·7	1·775	1	143 52 15·8	13·74	1	B; L; vmE 80°; vgbM	2
5060	......	......	....	3 6 55·2	......	...	89 4 27·5	......	...	See No. 5060.	
663	2515	......	....	3 7 7·5	1·474	1	148 40 18·9	13·67	1	Cl of 18 or 20 st...............	1
664	......	IV. 17	....	3 7 7·6	3·016	1	93 27 10·9	13·67	1	* with neb att 90″ l	1
665	2516	......	....	3 7 46·9	2·667	1	112 30 46·1	13·63	1	F; S; E; alm stell; *8, np...	1
666	2517	......	Δ. 337	3 8 25·7	1·635	2	145 44 50·6	13·58	2	⊕; B; L; R; rr	2
667	......	III. 194	....	3 8 43·9	3·020	1	93 8 18·9	13·57	1	eF; eS	1
668	......	......	D'Arrest, 35	3 9 31	3·93	[2]	49 2 30	13·49	[2]	F; vS; R; stellar; 1st of 7...	0
669	......	......	D'Arrest, 36	3 9 31	3·93	[2]	49 1 42	13·49	[2]	eF; S; lE; cometary; 2d of 7	0
670	2518	......	....	3 9 41·5	2·198	1	131 36 25·7	13·51	1	vB; R; gmbM	1
671	......	......	D'Arrest, 37	3 9 45	3·93	[2]	49 1 54	13·48	[2]	vF; S; R; 3rd of 7............	0
672	......	......	D'Arrest, 38	3 10 8	3·93	[2]	49 1 36	13·46	[2]	F; S; R; 4th of 7	0
673	......	......	D'Arrest, 39	3 10 11	3·94	[2]	48 58 30	13·45	[2]	vF; vS; 5th of 7..............	0
674	293	II. 603	....	3 10 32·7	3·937	1	49 0 5·5	13·45	1	pB; pS; R; bM; 6th of 7 ...	2*
675	......	......	D'Arrest, 40	3 10 35	3·97	[2]	49 0 3	13·42	[2]	F; S; *17; 7th of 7	0
676	2519	III. 956	....	3 11 2·0	2·884	1	100 48 41·4	13·42	1	eF; vS; 2 st 2′ or 3′ s.........	2
677, 678, 679, 680, 681, 682	293, *a*	......	R. 6 novæ	3 11 ±	......	...	49 ±	......	...	6 of 15 (including probably h. 294, 295)	0
683	2520	......	....	3 11 34·2	2·425	1	123 5 43·3	13·39	1	vF; L; R; vglbM	1
684	......	III. 195	....	3 11 43·7	3·017	1	93 14 30·6	13·38	1	eeF; eeS	1*
685	2521	......	Δ. 487	3 12 15·0	2·189	2	131 36 47·1	13·33	2	⊕; vB; pL; R; mbM *er ...	2
686	294	III. 574	....	3 12 23·9	3·938	1	49 11 42·1	13·33	1	vF; R; bM; p & sm of 2 ...	2
687	295	III. 575	....	3 12 25·0	3·938	1	49 10 4·4	13·32	1	vF; R; bM; f of 2; 100″, 352°·4.	2
688	296	II. 287	....	3 13 15·0	2·954	4	96 45 57·9	13·27	4	vF; S; vlE; gbM; er.........	7
689	2522	......	....	3 13 22·5	2·710	2	110 55 8·9	13·27	2	cB; vL; vmE; psvmbM......	2
690	......	III. 444	....	3 14 7·5	2·983	1	95 8 11·7	13·21	1	eF; vS..........................	1
691	......	III. 568	....	3 15 25·7	3·016	1	93 15 44·1	13·13	1	eF; S; iF; am 3 or 4 st	1
692	2523	I. 106	....	3 15 36·2	2·786	1	105 53 51·7	13·11	1	pB; cL; iR; gbM; *7, f 7^s·5, 211°·0.	3
693	2524	......	....	3 15 41·5	2·295	3	127 38 38·7	13·11	3	⊕; vF; pL; R; vgvlbM......	3
694	2525	......	....	3 16 0·3	1·749	1	142 41 0·3	13·09	1	F; pL; mE 37°·3; gbm	1
5061	......	......	....	3 16 29·0	......	...	89 19 3·0	......	...	See No. 5061.	
695	2528	......	Δ. 206	3 16 34·1	0·698	1	156 59 37·2	13·06	1	pB; L; lE; vgbM; r	1
696	2526	......	....	3 16 54·8	2·666	1	111 52 19·1	13·03	1	pB; S; R; gbM	1
697	2527	......	Δ. 548	3 17 20·4	2·288	2	127 43 40·0	13·00	2	vB; cL; vlE; vsvmbMN ...	2
698	2529	......	Δ. 547	3 17 23·1	2·291	2	127 36 24·0	13·00	2	pB; pS; psbM.................	2
699	2533	......	....	3 17 45·6	2·662	1	112 1 0·9	12·97	1	F; S; R; bM; p of 2	1
700	298 = 2530	III. 197	....	3 17 48·3	3·011	1	93 31 34·9	12·97	1	vF; S; R; bM; 1st of 3......	3
701	297 = 2531	III. 196	....	3 17 48·8	3·011	1	93 30 34·9	12·97	1	vF; vS; E; ? neb *; 2nd of 3	3
702	2532	......	....	3 17 50·9	+3·012	1	93 25 54·9	−12·97	1	F; vS; R; bM; 3rd of 3......	1

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	h.	H.		h m s	s		° ′ ″	″			
703	297, a	......	R. nova	3 17 53·6	+ 3·011	...	93 16 41·9	−12·97	...	....	0
704	299	III. 445	....	3 18 4·9	2·962	2	96 14 11·2	12·96	2	vF; pS; pmE	3
705	2534	IV. 77	....	3 18 18·0	2·661	2	112 1 49·8	12·94	2	F; mE; 239°·1; com; *9, 10 att.	3†
706	2535	......	....	3 18 37·6	2·305	1	126 58 12·7	12·91	1	O? pS; vsvmbMN	1
707	2536	......	....	3 19 39·5	2·739	1	108 4 59·5	12·85	1	F; pS; R; glbM..............	1
708	......	III. 959	....	3 19 53·5	2·662	2	111 51 1·8	12·84	2	vF; vS; sf of 2	1*
709	......	I. 60	....	3 20 15·0	2·662	2	111 50 36·0	12·80	2	vB; S; E 90°...180°; smbMN; np of 2.	2*
710	......	......	Auw. N. 17	3 20 41·7	3·690	...	59 6 31·2	12·76	...	F; L; *10f 4ˢ; n 2′·5 (Schönfeld, 1858).	0*
711	......	......	D'Arrest, 41	3 20 47	3·98	[1]	48 39 6	12·72	[1]	eF; pL; lbM	0
712	2537	......	....	3 21 10·0	2·319	3	126 12 33·5	12·75	3	vF; S; vlE; gbM	3
713	2538	......	....	3 22 29·8	2·407	2	122 46 15·5	12·65	2	cB; pS; R; psbM; ⁑ p	2
714	2542	I. 257	....	3 22 38·1	2·437	(1)	121 34 40·8	12·64	1::	cB; pL; iR; vgbM...........	2
715	2539	......	....	3 22 38·5	2·441	1	121 23 12·8	12·64	1	vB; pS; lE; psbM	1
716	2540	......	....	3 22 39·6	2·275	1	127 38 11·8	12·64	1	F; S; R; *12, sf..............	1
717	301	VIII. 88	....	3 22 41·0	3·852	1	53 9 36·8	12·64	1	Cl; vL; ab 60 st	3
718	300	III. 694	....	3 22 54·4	6·312	2	17 54 18·1	12·63	2	F; vS; iR; gbM; ⁑ vnr	3
719	2541	......	....	3 23 10·3	2·732	1	108 16 10·7	12·61	1	vF; S; R; pslbM	1
720	......	VIII. 84	....	3 23 42·3	4·361	1	39 3 16·6	12·58	1	Cl; lRi; stL..................	1
721	2545	......	Δ. 591	3 24 11·0	2·365	1?	124 12 43·8	12·54	1::	B; L; mE; vmbMRN	1
722	2544	......	....	3 25 8·3	2·332	1	125 20 9·9	12·47	1	pB; pS; R; psbM	1
723	2543	......	....	3 25 16·8	2·698	1	109 45 38·2	12·46	1	eF; pslbM; v diff *8, sf	1
724	2546	III. 246	....	3 25 52·1	2·665	2	111 17 46·1	12·43	2	pB; cL; iE; mbM	5
725	2547	III. 487	....	3 26 2·0	2·778	1	105 41 32·4	12·42	1	vF; S; lE; glbM.............	2
726	2548	II. 290	....	3 26 44·0	2·808	2	104 9 24·9	12·37	2	pF; pL; R; lbM; pL*nf 5′...	5
727	302	III. 446	....	3 26 44·3	2·971	2	95 33 27·9	12·37	2	vF; S; bet 2st..............	3
728	2549	......	....	3 26 30·7	1·789	2	140 45 56·6	12·38	2	vF; pL; iR; gbM; *nr	2
729	2550	......	....	3 27 28·6	2·690	1	109 58 38·4	12·32	1	F; L; R; vglbM..............	1
730	2551	III. 960	....	3 27 38·5	2·674	2	110 46 3·7	12·31	2	vF; S; R......................	3
731	2552	......	.. ..	3 28 18·0	2·289	2	126 36 31·9	12·27	2	!! vB; vL; mE; rN in vLE Halo.	2†
732	2553	III. 857	....	3 28 18·2	2·421	(1)	121 40 37·9	12·27	(1)	vF; S; iF; lbM	2
733	2554	III. 559	....	3 29 0·5	2·670	3	110 50 45·4	12·22	3	vF; S; R; bet 2st 14	4
734	2555	II. 262	....	3 29 1·6	2·570	1	115 24 21·4	12·22	1	pB; pL; vlE; psbM	2
735	2556	......	....	3 29 50·6	2·310	1	125 42 24·2	12·16	1	eF; vS; p of 3................	1
736	2557	......	....	3 30 0·6	2·311	1	125 41 4·5	12·15	1	vB; pL; lE; gmbM; 2nd of 3	1
737	2558	......	....	3 30 0·6	2·309	1	125 43 44·5	12·15	1	B; S; lE; pmbM; 3rd of 3...	1
738	303	II. 288	....	3 30 7·0	2·971	2	95 30 0·1	12·13	2	eF; pL; iR; bM; r............	5
739	2559	......	Δ. 574	3 30 9·5	2·316	1?	125 28 56·8	12·14	1::	vB; L; R; psbM..............	1
740	2560	III. 961	....	3 30 28·7	2·657	2	111 21 44·4	12·12	2	F; S; R; gbM.................	3
741	2561	......	....	3 30 42·7	2·307	1	125 55 1·7	12·11	1	⊕; B; pL; R; gpmbM	2
742	2562	......	....	3 31 19·8	2·710	1	108 48 18·5	12·05	1	pF; S; R; psmbM	1
743	2563	II. 263	....	3 31 26·9	2·576	1	114 58 4·5	12·05	1	pB; pS; R; gpmbM	2
744	2564	......	....	3 31 34·7	2·298	1	125 58 33·5	12·05	1	⊕; vB; pL; R; gmbM	2
745	2565	III. 451	....	3 32 19·8	2·706	1	108 53 34·6	11·98	1	F; S; R; glbM	2
746	2566	I. 58	....	3 32 24·1	2·608	2	113 28 55·6	11·98	2	B; pS; E; psmbM	4
747	2567	II. 593	....	3 33 11·0	2·700	1	109 9 29·4	11·92	1	cB; pS; R; psmbM	2
748	2569	......	....	3 33 11·6	2·296	2	125 54 53·1	11·93	2	⊕; vB; pL; psbM; rr	3
749	2568	III. 247	....	3 33 14·1	2·613	1	113 10 47·4	11·92	1	vF; vS; R	2
750	2571	......	....	3 33 43·1	2·291	1	126 2 48·0	11·90	1	vB; pL; R; psmbM	2
751	2572	......	....	3 33 49·8	2·407	1	121 46 25·6	11·88	1	F; cL; vmE; vglbM; *7np	1
752	2570	I. 107	....	3 33 55·4	2·702	1	109 2 21·9	11·87	1	vB; L; R; svmbMN	3
753	304	III. 263	....	3 34 4·2	3·040	(2)	91 45 34·5	11·85	(2)	eF; stellar or lE	2
754	304, a	......	R. nova	3 34	......	...	91 45	......	...	makes D neb with h. 304 ...	0
755	2573	......	....	3 34 5·2	2·014	2	134 38 18·9	11·87	2	B; pS; R; smbM	2
756	305	III. 569	....	3 34 32·3	2·977	1	95 7 5·4	11·82	1::	eF; lE; er; 1st of 3	2
757	2574	......	....	3 34 36·5	2·531	1	116 40 8·4	11·82	1	F; S; E; gbMbM; *sf 2′ ...	1
758	306	II. 455	....	3 34 41·5	2·976	1	95 8 51·7	11·81	1	pF; pL; lE; lbM; *sf; 2d of 3	3
759	2575	II. 267	....	3 34 51·1	+ 2·615	3	113 0 43·7	−11·81	3	pB; S; lE; pglbM; *sf 2′ ...	4

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	h.	H.		h m s	s		° ′ ″	″			
760	307	II. 456	....	3 35 2·3	+ 2·976	3	95 9 45·6	−11·78	2	vF; S; E; B* 135°, 1′; 3d of 3	4
761	2576	......	....	3 35 29·9	2·229	2	127 58 34·9	11·77	2	pF; pS; R; psbM	2
762	2577	II. 291	....	3 35 56·9	2·804	2	103 56 56·1	11·73	2	F; cL; mE 0°±; r............	4
763	307, a	......	R. nova	3 35 58·6	2·976	::	95 9 45·6	11·73	::	No description	0
764	......	II. 852	....	3 36 30·1	2·438	1	120 21 11·3	11·69	1	F; pL; iR; gbM...............	1
765	2578	III. 248	....	3 36 42·1	2·622	2	112 33 21·6	11·68	2	pF; S; lE; bM	3
766	2579	......	....	3 37 0·0	2·288	1	125 50 50·2	11·66	1	pF; S; R; psmbM	1
767	2580	......	Δ. 426	3 37 37·8	1·879	2	137 40 33·4	11·62	2	vB; L; pmE; vsvmbM*10...	2
768	......	......	Auw. N. 18	3 37 52·3	3·542	...	66 40 12·9	11·57	...	!!! B; vL; iF; VAR. (Tempel)	0*
769	2581	......	Δ. 562	3 38 7·9	2·264	1?	126 34 12·6	11·58	1?	⊕; vB; pmE; pgbM	2
770	2582	......	....	3 38 20·0	2·272	1	126 18 1·9	11·57	1	F; vL; R; glbM...............	1
771	2584	III. 249	....	3 38 43·4	2·623	3	112 21 40·1	11·53	3	F; pS; gpmbM	5
772	......	II. 597	....	3 38 44·4	2·986	2	94 31 51·1	11·53	2	vF; S; iE; *nr	1
773	2583	II. 458	....	3 38 44·4	2·703	2	108 43 32·1	11·53	2	pB; pS; R; smbM*13	3
774	......	II. 594	....	3 38 46·3	2·682	2	109 41 52·1	11·53	2	pB; vS; bM	1*
775	308	VIII. 80	....	3 38 52·0	4·490	2	37 46 18·6	11·48	2	Cl of ab 30st 12...14	3
776	2585	......	....	3 39 4·9	1·977	1	135 5 20·4	11·52	1	pB; L; vmE 221°·6............	1
777	......	II. 459	....	3 39 12·4	2·696	1	109 0 25·3	11·49	1	F; R; lbM	1
778	309	I. 155	....	3 39 32·4	2·989	1	94 24 37·9	11·47	1	pB; S; R; *17M	3*
779	2586	......	....	3 39 56·6	1·974	3	135 5 22·2	11·46	3	pF; pL; eE 42°·3; vgpmbM	3
780	2587	......	....	3 40 59·9	2·241	1	127 7 51·9	11·37	1	F; S; R; *att...................	1
781	2588	II. 460	....	3 42 24·8	2·739	1	106 49 22·2	11·26	1	pB; S; lE; mbMN!............	2
782	2589	......	....	3 43 32·5	+1·139	2	150 14 27·4	11·22	2	cF; S; R; glbM; am 7Bst ...	2
783	2590	......	....	3 45 15·1	−0·360	1	162 6 36·1	11·13	1	pF; pS; iR; glbM; *7f......	2
784	2592	......	....	3 46 48·5	+0·224	2	158 38 24·7	11·01	2	cF; pL; R; gvlbM	2
785	2591	......	....	3 47 34·5	1·955	2	134 56 47·0	10·90	2	cF; S; E 90°; gbM............	2
786	2593	......	....	3 48 18·8	2·644	1	110 52 29·8	10·84	1	eF; S; R; 2Bst f; p of 2.......	1
787	2594	III. 962	....	3 48 28·0	2·643	3	110 54 59·4	10·82	2	F; S; vlE; 2st 10nr; f of 2...	4
788	2595	......	Δ. 427? 428?	3 48 31·1	1·830	2	137 53 54·8	10·84	2	cF; pL; R; vglbM	2
789	2596	......	....	3 49 9·1	2·212	1	127 24 18·6	10·78	1	vF; L; E; vgvlbM	1
790	2597	......	Δ. 480	3 51 4·2	2·029	3	132 46 40·8	10·64	3	pB; pL; R; gbM; 2st Δ	3
791	......	......	Auw. N. 19	3 52 1·3	3·444	...	71 49 55·8	10·54	...	*12 inv in neb (Markree Cat. Nov. 24, 1854).	0
792	2599	......	....	3 52 22·9	0·478	2	156 25 42·3	10·59	2	pB; S; vlE; pmbM............	2
793	......	I. 258	....	3 52 32·8	4·482	1	39 1 19·4	10·52	1	vB; S; iF; bM; r; *inv......	1
794	2598	......	....	3 52 59·7	2·251	1	125 51 46·3	10·49	1	vF; vS; R	1
795	2600	......	Δ. 438	3 53 5·8	1·870	2	136 36 53·0	10·50	2	F; cL; R; vglbM	2
796	2601	......	....	3 53 36·3	1·748	2	139 18 42·9	10·47	2	F; L; R; vgvlbM; 3st n ...	2
797	2602	......	....	3 53 47·2	1·721	3	134 52 57·5	10·45	3	eF; S; lE 90°; vgvlbM	3
798	310	......	....	3 53 47·4	4·551	1	37 45 45·6	10·38	1	Cl; segment of a ring.........	1
799	......	VII. 3	....	3 53 52·2	2·821	1	102 25 27·4	10·42	1	Cl; S; C	1
800	2603	......	Δ. 369?	3 54 33·6	1·573	1	142 43 35·3	10·39	1	F; vS; R; pmbM; *8 np ...	1
801	......	IV. 53	....	3 54 57·0	5·109	2	29 27 30·6	10·28	2	O; pB; pS; vlE; 1′ diam ...	2
802	......	VII. 47	....	3 55 9·9	5·232	2	28 3 51·2	10·26	2	Cl; pRi; cC; iF	2
803	2604	......	....	3 55 25·3	0·456	1	156 25 44·2	10·36	1	eF; pS; R; *10 np...........	1
804	2605	......	....	3 56 44·9	1·552	1	142 58 5·1	10·23	2	eeeF; S; R; bet 2st 12 & 13..	2
805	......	II. 279	....	3 57 19·5	3·021	2	92 35 1·5	10·15	2	vF; pL; mE; vlbM; er	2
806	2606	......	....	3 59 0·1	1·965	2	133 47 35·5	10·05	2	F; pL; R; vgmbM............	2
807	2608	......	....	3 59 3·1	0·218	3	158 1 23·7	10·09	3	pB; pS; mE 121°·5; gbM ...	3
808	2607	......	Δ. 466	3 59 20·3	1·966	3	133 44 23·1	10·03	3	⊕; B; cL; R; bM; rr	3
809	......	VII. 60	....	3 59 35·9	4·416	1	40 51 49·5	9·95	1	Cl; L; vRi; pC; st vL	1
810	311	IV. 69	....	4 0 28·6	3·755	2	59 35 29·2	9·96	2	*8m in neb 3′ diam...........	3*†
811	2609	......	Δ. 348	4 0 39·4	1·448	2	144 29 25·8	9·94	2	B; L; vmE 10°; bM	2
812	2610	III. 499	....	4 1 24·0	2·884	2	99 12 21·5	9·85	2	eeF; S; E; psmbM; er	3
813	2611	......	....	4 1 44·0	+2·613	1	111 33 0·1	9·83	1	B; L; pmE; gbM; *8 sp ...	1
814	2615	......	....	4 2 8·1	−2·026	1	167 13 0·4	9·92	1	Cl; pL; lRi; st 9...10.........	1
815	2612	......	....	4 2 10·3	+2·616	1	111 25 36·0	9·80	1	pB; R; bM......................	1
816	2613	......	....	4 2 35·2	1·525	2	143 2 40·3	9·79	2	eF; vS; R; vlbM	2
817	2614	......	....	4 2 48·1	1·440	1	144 28 35·9	9·77	1	vF; R	1
818	2617	......	....	4 4 8·8	0·425	2	156 12 37·0	9·70	2	eF; vS; R; glbM	2
819	2616	......	....	4 4 17·7	+1·761	2	138 15 52·5	− 9·65	2	pB; pS; E 77°; vsmbMRN...	3

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	h.	H.		h m s	s		° ′ ″	″			
820	......	VII. 61	....	4 4 37·1	+4·520	1	39 7 14·2	−9·56	1	Cl; B; vRi; cC	1
821	2619	......	....	4 5 33·4	0·746	1	153 16 3·3	9·59	1	vF; S; R; gbM	1
822	2620	......	....	4 6 34·6	2·302	3	123 12 36·2	9·46	3	pB; pL; R; bM; np of 2 ...	3†
823	2621	......	Δ. 600	4 6 39·6	2·301	3	123 14 8·2	9·46	3	B; vL; vmE 32°·2; psmbM...	3†
824	2622	......	....	4 6 51·0	1·297	2	146 29 15·9	9·47	2	vB; vL; R; smbM; 2st *10nf	2
825	2623	......	....	4 6 59·8	0·749	2	153 9 21·9	9·47	2	F; S; R; vS* ¾d sf	2
826	2618	IV. 26	....	4 7 50·8	2·792	A	103 5 32·2	9·36	1	⊕; vB; S; R; ps, vsbM; r...	5*†
827	2625	......	....	4 8 4·3	1·267	1	146 50 24·6	9·38	1	vF; R; pL; vlbM	1
828	2624	......	....	4 8 17·9	2·337	1	121 54 41·8	9·34	1	vB; pS; lE; psvmbM	1
829	2626	......	....	4 9 32·7	2·420	2	118 50 16·1	9·23	2	vF; vS; E; gvlbM; r	2
830	2627	......	....	4 9 58·6	1·170	1	148 5 33·1	9·23	1	B; pL; E; smbMN=*11 ...	1
831	......	VIII. 85	....	4 10 25·3	4·489	1	40 5 39·7	9·11	1	Cl; pRi; lC; stL	1
832	2628	......	....	4 11 37·5	1·282	2	146 24 57·7	9·11	2	pB; lE; gbMEN; ⁑p.........	2
833	312	......	....	4 11 45·8	3·959	1	53 25 50·1	9·03	1	Cl; vL; lRi; lC; st10...12...	1
834	2629	......	....	4 12 9·4	1·313	1 ::	145 55 46·2	9·06	1	B; pS; R	1
835	......	......	D'Arrest, 42	4 12 21	3·11	[1]	87 56 12	8·99	[1]	vF; S; R; *13nr............	0
836	......	II. 464	....	4 12 22·5	3·097	1	88 55 34·0	9·00	1	F; vS; R	1*
837	313	III. 490	....	4 13 10·4	3·052	2	91 2 17·1	8·93	2	cF; pS; lE; vgbM; *11sf ...	3
838	2630	......	....	4 13 11·1	1·295	1	146 7 50·6	8·98	2	vB; pS; R; gmbM; am 3st...	2
839	......	......	Auw. N. 20	4 13 47·7	3·488	...	70 48 46·0	8·87	...	!!!; vF; S; *variable* (Hind)...	0*
840	2631	......	....	4 13 56·5	+1·622	2	140 30 9·7	8·91	2	cF; S; R; vglbM	2
841	2633	......	....	4 14 0·8	−0·349	1	160 46 26·2	8·96	1	Cl; vlC; ab 20 sc st	1
842	2632	......	....	4 14 56·3	+1·858	1	135 21 58·1	8·83	1	pF; S; E; gbM	2
843	2634	......	....	4 15 56·3	0·704	2	153 7 47·6	8·78	2	vB; vL; mE; vgpmbM; *14 att n.	2
844	2635	......	Δ. 338 ? ?	4 16 52·1	1·337	2	145 16 34·3	8·69	2	B; vL; vg, svmbM; 15ˢ d in R.A.	2
845	2636	......	....	4 17 9·1	1·707	1	138 35 23·2	8·66	1	F; S; R; bM	2
846	2637	......	....	4 17 42·3	1·916	1	133 47 30·7	8·61	1	F; S; R; gbM............	1
847	......	II. 768	....	4 17 43·5	5·621	1	25 27 13·7	8·51	1	pB; S; lE; bNM; pB*n ...	1
848	2638	......	....	4 17 45·5	1·910	1	133 57 0·7	8·61	1	vF; S; R; gbM; ⁑nf	1
849	2639	.....	....	4 17 59·3	2·026	1	130 55 4·6	8·58	1	pF; S; R; *13 nf 1′	1
850	2640	......	....	4 19 13·4	1·190	1	147 17 51·7	8·51	1	pB; S; R; pgbM; 2S st sf ...	1
851	314	III. 587	....	4 19 24·3	2·989	1	93 56 58·5	8·45	1	eF; bM; bet 2 st	2*
852	2641	......	....	4 20 12·4	1·526	3	141 55 9·4	8·42	3	pF; S; R; bM	3
853	315	I. 217	....	4 21 2·8	3·925	3	55 2 25·3	8·29	3	pB; vL; iR; mbM; *8, 350°, 2′.	6†
854	2642	......	....	4 21 37·2	1·321	2	145 15 47·7	8·31	2	F; S; E; glbM	2
855	......	VIII. 70	....	4 22 9·3	4·233	1	46 27 35·0	8·20	1	Cl; vL; pRi; lC; stL	1
856	2643	......	....	4 22 58·1	1·957	1	132 27 41·3	8·19	1	pF; S; R; gbm; *12, 287°·8	1
857	......	......	D'Arrest, 43	4 23 5	3·05	[1]	90 50 6	8·15	[1]	vF; iF; vlbM; bet ⁑ & *14..	0
858	316	II. 8	....	4 23 29·1	3·081	1	89 39 14·4	8·12	1	F; pS; R; r; p of D neb	5
859	317	II. 9	....	4 23 32·6	3·081	1	89 38 36·7	8·11	1	F; vS; R; r; f of D neb	5
860	318	II. 7	....	4 23 33·5	3·085	1	89 26 33·7	8·11	1	F; pL; lE 132°; *42°, 80″ ...	5
861	2644	......	....	4 23 48·7	2·450	3	117 0 49·0	8·10	3	pF; pS; R; gbM	3
862	2645	......	....	4 23 52·7	2·443	1	117 15 57·0	8·10	1 ::	vF; vS............	1
863	2646	......	....	4 24 24·8	1·710	1	138 7 6·6	8·08	1	vF; S; R; bM	1
864	2648	......	....	4 24 35·0	1·305	2	145 20 17·6	8·08	2	B; pL; mE15°·0; smbM; p of 2.	2
865	2647	......	....	4 24 36·3	1·711	1	138 5 22·2	8·06	1	F; S; R; bM	1
866	319	I. 158	....	4 24 45·0	2·958	4	95 23 3·4	8·02	4	pB; pL; R; gmbM............	6
867 868 869 }	319, *a*	......	R. 3 novæ	4 24 ±	......	...	95 23 ±	......	...	....	0
870	2649	......	....	4 24 48·1	1·303	2	145 22 1·2	8·06	2	eF; pL; lE; f of 2	2
871	......	VI. 26	....	4 24 59·0	4·298	1	45 3 48·9	7·97	1	Cl; vF; pS; C; steS	1
872	......	III. 585	....	4 25 45·2	2·973	1	94 38 56·8	7·94	1	Susp in hazy weather	1
873	......	III. 586	....	4 26 10·8	2·975	2	94 33 58·0	7·90	2	eF; S; E90°±	2
874	2650	......	....	4 28 20·2	1·883	3	134 0 32·2	7·76	3	F; S; E; vglbM	3
875	2651	......	Δ. 339 ? ?	4 28 33·6	+1·321	2	144 54 14·2	−7·76	2	B; L; mE 105°·8; vg, vsmbMN5″. }	2

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	h.	H.		h m s	s		° ′ ″	″			
876	320	II. 524	....	4 29 6·8	+2·999	1	93 26 37·9	−7·67	1	F; S; iF; lbM; 2st sf	2
877	321	II. 514	....	4 29 26·0	3·064	2	90 25 50·1	7·63	2	vF; pL; mE0°...90°; B ⁑nf..	3
878	320, a	......	D'Arrest, 44	4 29 34	3·00	[2]	93 28 48	7·63	[2]	vF; S; *20, 270°, 5ˢ; II. 524 p (R).	0
879	......	V. 49	....	4 29 47·5	4·558	1	39 50 13·9	7·57	1	F; cL; iF; 6 or 7 st+neb	1
880	322	......	....	4 30 7·9	2·995	1	93 35 45·6	7·58	1	vF; E 90°...180°; sbM; B*p 40ˢ.	1*
881	322, a	......	R. nova	4 30 19·9	+2·995	::	90 30 45·6	7·58	::	MS	0
882	2653	......	....	4 31 14·0	−0·756	1	162 8 7·3	7·59	1	vF; pL; R; glbM	1
883	2652	......	....	4 32 20·0	+2·598	1	110 55 34·4	7·42	1	Neb. No description	1
884	323	III. 952	....	4 32 36·3	3·227	2	82 56 7·6	7·38	2	eF; S; R; *8 sp; p of D neb	3
885	324	III. 953	....	4 32 36·5	3·226	1	82 57 2·6	7·38	1	eF; vS; f of D neb	2
886	325	II. 515	....	4 33 1·4	3·055	1	90 49 38·5	7·35	1	F; S; R; bM; *9 nf 12ˢ·5	2
887	326 = 2654	II. 522	....	4 33 57·4	2·878	2	98 52 48·9	7·27	2	vF; pS; R; vgbM; r; *nf 1′.	4
888	327	I. 122	....	4 34 26·6	3·005	2	93 8 53·1	7·23	2	cB; L; R; vgbM; er	3†
889	2655	......	....	4 34 36·9	2·686	1	107 16 3·1	7·23	1	eF; vS; R; bet 2 st	1
890	......	II. 525	....	4 35 29·3	3·028	1	92 2 39·5	7·15	1	F; pL; lE	1
891	2656	......	....	4 35 30·1	0·266	1	156 4 46·4	7·22	1	Cl; pL; pRi; pmC; st 11...16	1
892	......	......	D'Arrest, 45	4 35 45	3·08	[1]	89 39 1	7·12	[1]	F; R; cometary; Δ with 2 st 18, f.	0
893	328	III. 588	....	4 36 51·6	2·950	1	95 35 42·8	7·04	1	eF; vS; iR; bM	2
894	2657	......	....	4 37 7·2	0·206	3	156 27 51·3	7·09	3	F; S; R; gbM	3
895	329	II. 523	....	4 37 41·4	2·879	1	98 47 13·9	6·97	1	F; vS; iR; bM; *7 np	2
896	......	VIII. 8	....	4 37 54·5	+3·500	2	71 11 21·8	6·94	2	Cl; vL; st L, sc	2
897	2660	......	....	4 38 37.3	−0·201	1	159 5 7·6	6·98	1	F; pS; R; gbM	1
898	2662	......	....	4 38 42·3	−0·531	3	160 51 32·6	6·98	3	pF; L; vlE; vglbM	3
899	2661	......	....	4 38 43·9	−0·177	3	158 56 26·9	6·97	3	vF; S; R; glbM	3
900	......	II. 526	....	4 38 49·7	+3·015	1	92 38 59·9	6·87	1	F; cS; R; lbM	1
901	330	......	....	4 39 0·9	2·954	1	95 24 18·2	6·86	1	eF; iF; ?	1
902	2658	......	....	4 39 28·8	1·952	1	131 45 5·5	6·85	1	F; pS; pmE; glbM	1
903	331	III. 589	....	4 39 36·7	2·962	1	95 2 37·7	6·81	1	pF; pS; iE90°±; bM	2
904	2659	......	....	4 39 37·6	1·950	1	131 46 55·1	6·83	1	vF; S; lE; glbM	1
905	332	VII. 1	....	4 40 44·9	3·310	1	79 19 18·7	6·71	1	Cl of L & S sc st	3
906	......	VIII. 7	....	4 40 45·1	3·361	2	77 6 7·7	6·71	2	Cl; lRi; st L & S	3
907	......	VIII. 59	....	4 41 2·4	4·268	1	46 33 17·5	6·65	1	Cl; lRi; lC; pL	1
908	333	II. 547	....	4 41 23·7	2·947	1	95 41 12·2	6·66	1	eF; pL; R; lbM	3*
909	2663	......	....	4 41 59·7	1·811	1	135 2 6·8	6·64	1	eF; R; att to *14	1
910	2664	......	....	4 42 20·4	0·236	1	156 4 5·2	6·66	1	eF; S; R	1
911	......	III. 501	....	4 42 46·2	3·007	1	93 0 18·8	6·54	1	vF; vS	1
912	2665	......	Δ. 296 ??	4 43 32·0	+0·930	2	149 30 3·8	6·54	2	B; L; smbMN	2
913	2667	......	....	4 43 34·7	−0·404	1	160 4 2·6	6·58	1	vF; S; att to *10	1
914	2669	......	....	4 44 19·2	−0·228	1	159 4 32·7	6·51	1	vF; pL; iR; r	1
915	......	III. 502	....	4 44 35·4	+3·010	1	92 51 28·0	6·40	1	vF; S	1
916	2666	......	....	4 44 39·1	2·276	2	122 12 45·4	6·42	2	vB; L; iR; 4 st inv	2
917	2668	......	....	4 44 41·5	1·665	3	138 3 31·4	6·42	3	vF; S; R; r or st inv	3
918	334, a	......	R. nova	4 45 16·3	2·999	::	93 14 46·4	6·32	::	R, MS	0
919	......	......	D'Arrest, 46	4 45 22	3·00	[2]	93 21 0	6·33	[2]	vF; vS; II. 527, f 12ˢ±	0
920	334	II. 527	....	4 45 32·3	2·999	2	93 20 46·4	6·32	2	pF; S; R; bM; *7, 225°±	4
921	334, b	......	R. nova	4 45 44·3	2·999	::	93 20 46·4	6·32	::	MS } No description	0
922	334, c	......	R. nova	4 45 44·3	2·999	::	93 8 46·4	6·32	::	MS } No description	0
923	2670	......	....	4 46 9·1	2·212	2	124 10 25·0	6·30	2	vF; S; R; vglbM	2
924	......	II. 528	....	4 46 12·6	2·999	1	93 18 35·2	6·26	1	F; S; lbM	1
925	2671	......	....	4 46 18·6	0·875	3	150 2 6·7	6·31	3	pB; pL; iR; pgmbM	3
926	335	......	....	4 47 6·7	+3·104	1	88 35 49·3	6·19	1	vF; vS; am vS st; L*sp	1*
927	2672	......	....	4 48 23·2	−0·339	2	159 35 14·9	6·17	2	F; S; R	2
928	2673	......	....	4 48 30·2	0·345	2	159 37 5·2	6·16	2	F; S; R	2
929	2674	......	....	4 48 47·0	0·146	1	158 27 26·8	6·14	1	vF; E; vlbM	1
930	2675	......	....	4 49 1·9	0·203	5	158 47 31·4	6·12	5	⊕; pB; L; R; rr	5
931	2677	......	....	4 49 42·0	−0·302	1	159 20 44·2	−6·06	1	pB; pS; R; glbM	1

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	h.	H.		h m s	s		° ′ ″	″			
932	336	IV. 32	....	4 50 1·9	+2·959	1	95 5 43·5	−5·95	1	cB; S; mbM*	3
933	2676	......	....	4 50 25·3	+2·333	3	120 6 11·1	5·93	3	F; S; vlE; glbM; *10, 75″	3
934	2680	......	Δ. 73?	4 50 27·0	−0·443	1	160 5 32·0	6·00	1	Cl; vF; S	1
935	2678	......	....	4 50 48·7	+0·866	2	149 58 0·1	5·93	2	F; L; R; vglbM; ⁑ att	2
936	2683	......	....	4 50 53·7	−0·427	2	159 59 14·9	5·97	2	F; pS; lE; r	2
937	2679	......	....	4 51 8·5	+1·342	3	143 35 14·3	5·89	3	pF; S; R; pmbM	3
938	2682	......	....	4 51 15·3	0·544	1	153 13 27·7	5·91	1	F; pS; R; vglbM	1
939	338	......	....	4 51 15·6	3·253	1	81 58 39·1	5·83	1	S; R; rrr	1
940	337	......	....	4 51 32·4	+4·753	1	37 19 51·9	5·77	1	Cl; vL; pRi; lC; stL and S.	1
941	2684	......	Δ. 76?	4 51 44·5	−0·473	2	160 13 0·0	5·90	2	⊕; B; S; iR; rrr; st14	2
942	2685	......	....	4 51 46·1	−0·370	1	159 39 49·3	5·89	1	Cl; pB; S	1
943	339	II. 516	....	4 51 47·1	+3·057	2	90 42 45·3	5·79	2	F; S; R; bM; p of 2	2
944	339, a	......	R. nova	4 51±	......	...	90 42±	......	...	No description	0
945	2686	......	....	4 52 0·4	0·043	5	157 8 55·2	5·86	5	vB; S; E or bi-N; bM; sp of 2	5
946	2687	......	....	4 52 2·6	0·046	3	157 7 48·2	5·86	3	vF; S; R; sbM; 2st nr; nf of 2	3
947	2681	......	....	4 52 16·4	2·592	1	110 34 42·9	5·77	1	pF; pL; R; glbM	1
948	340	......	....	4 52 18·9	+3·063	1::	90 28 7·5	5·75	1::	pF; S; iR; pslbM	2
949	340, a	......	R. nova	4 52±	......	...	90 28±	......	...	No description	1
950	2688	......	....	4 52 20·4	+0·022	3	157 16 42·8	5·84	3	F; pS; R; vglbM	3
951	......	......	D'Arrest, 47	4 52 39	+2·89	[2]	98 4 0	5·73	[2]	pF; pL; lbM; h. 341 nr	0
952	2689	......	....	4 52 41·7	−0·366	3	159 37 11·4	5·82	3	Cl; pF; S; R; 2nd of 3	3
953	341	......	....	4 52 57·2	+2·893	1	97 58 11·7	5·71	1	F; R; *13, s	1*
954	2690	......	....	4 53 0·9	−0·358	3	159 34 7·3	5·79	3	Cl; pB; pS; pmE; st12	3
955	......	III. 503	....	4 53 21·2	+2·994	1	93 32 9·9	5·67	1	vF; pL; 2B st v nr	1
956	2691	......	....	4 53 22·6	+0·038	3	157 8 36·5	5·75	3	Cl; pL; lRi; lC; st10...15	3
957	2694	......	....	4 53 43·2	−0·238	2	158 52 23·4	5·72	2	S; R; close ⁑ in M	2
958	2693	......	....	4 53 49·2	+0·073	1	156 53 44·7	5·71	1	eF; pS; R; gbM	1
959	2695	......	....	4 54 8·4	−0·259	1	158 59 17·0	5·70	1	pB; L; R; gmbM	1
960	2696	......	....	4 54 18·5	+0·009	1	157 18 56·9	5·67	2	pF; pS; R; 2st att	2
961	2697	......	....	4 54 19·0	−0·143	1	158 17 20·9	5·67	1	B; R; r	1
962	2698	......	....	4 54 31·9	−0·335	1::	159 24 10·2	5·66	1::	vF; S; 1st of 4	1
963	2699	......	Δ. 114	4 54 49·4	−0·339	2	159 25 17·8	5·64	2	B; pL; R; gbM; r; 2nd of 4	2
964	2692	......	....	4 54 51·1	+2·440	1	116 14 38·2	5·56	1	F; vL; vmE; vgvlbM	1
965	342	......	....	4 54 58·2	+2·994	1	93 30 26·1	5·53	1	eF; vS; *12, sf	1
966	2702	......	....	4 55 7·9	−0·335	1::	159 23 11·7	5·61	1::	F; S; 3rd of 4	1
967	2701	......	....	4 55 12·5	−0·006	2	157 23 32·0	5·60	2	Cl; pS; lRi; stvS	2
968	2704	......	....	4 55 13·8	−0·340	1	159 25 6·0	5·60	1	pB; vS; R; 4th of 4	1
969	2703	......	....	4 55 16·5	−0·166	1	158 24 26·3	5·59	1	vF; R; p of 2	1
970	......	VIII. 43	....	4 55 18·9	+3·630	1	66 33 18·3	5·49	1	Cl; stL, vesc	1*
971	2705	......	....	4 55 19·4	−0·456	1	160 1 59·0	5·60	1	eF; pL; iR	1
972	......	......	D'Arrest, 49	4 55 26	+2·88	[2]	98 26 48	5·49	2	F; pL; pmE; 2 or 3st llnf	0
973	2708	......	....	4 55 36·9	−0·581	1	160 39 49·9	5·57	1	F; S; R; *13att, 135°	1
974	2707	......	....	4 55 37·8	−0·350	1	159 27 21·9	5·57	1	vF; S; R	1
975	343	......	....	4 55 45·0	+2·962	1	94 55 35·9	5·47	1	vLdiff neb in zigzags??	1*
976	2706	......	Δ. 167	4 55 49·5	0·266	1	155 25 43·1	5·53	1	vB; pL; R; gbM; f of 2	1
977	......	VII. 21	....	4 55 55·7	3·634	1	66 25 21·1	5·43	1	Cl; pC; stL and S	1
978	2700	......	....	4 56 5·5	2·037	1	128 55 2·2	5·46	1	vF; pL; vglbM	1
979	2709	......	....	4 56 15·6	0·087	1::	156 43 53·0	5·50	1::	vF; S; 3vSst inv	1*†
980	2710	......	....	4 56 19·0	0·093	3	156 41 5·0	5·50	3	Cl; L; mC; ⁑ 9	3†
981	......	III. 453	....	4 56 24·5	3·104	1	88 34 31·3	5·39	1	vS; vF	1*
982	2711	......	....	4 56 31·9	+0·102	5	156 37 13·3	5·49	5	vB; vL; vimE	5†
983	2713	......	....	4 56 42·0	−0·092	1	157 54 38·9	5·47	1	vF; S; R	1
984	2712	......	....	4 56 51·7	+0·631	2	152 14 11·8	5·44	2	cF; S; R; glbM	2
985	2717	......	....	4 57 15·1	−0·543	2	160 26 16·8	5·44	2	cF; S; gbM	2
5062	......	......	....	4 57 18·4	......	...	159 36 32·1	......	...	See No. 5062	1
986	2718	......	....	4 57 22·2	−0·185	1	158 28 4·4	5·42	1	F; S; R; gbM	1
987	2716	......	....	4 57 29·6	+0·091	5	156 39 56·0	5·40	5	B; L; iR; vsmbM ⁑ 10	5†
988	2715	......	Δ. 169	4 57 39·7	−0·213	2	158 37 42·3	5·39	2	Cl+neb; pL; pRi; st11...18	2
989	2720	......	....	4 57 48·0	+0·509	1	153 20 44·2	5·36	1	vF; mE; glbM; *7, 8np	1
990	2722	......	....	4 57 48·9	−0·416	3	159 46 2·3	5·39	3	pB; pS; iR; rr	3
991	2721	......	....	4 57 50·3	+0·105	1	156 33 59·9	−5·37	1	pF; pL; iR; 2 or 3Bst nr	1

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	h.	H.		h m s	s		° ′ ″	″			
992	2723	......	....	4 58 11·0	−0·027	3	157 27 6·5	−5·35	3	B; S; R; smbM; ⁑+neb ...	3
993	2725	......	....	4 58 18·6	−0·590	1	160 38 26·5	5·35	1	eF; pL; iR	3
994	2724	......	....	4 58 23·2	+0·093	1	156 38 1·4	5·32	1	vF; S; R; gbM	1
995	2728	......	....	4 58 28·2	−1·562	1	164 29 44·2	5·36	1	eF; E; ⁑ 9att, f	2
996	344	VIII. 61	....	4 58 35·7	+4·041	1	53 8 20·3	5·19	1	Cl; pC; lRi; iF; stL	2
997	345 = 2714	III. 500	....	4 58 37·6	+2·860	3	99 20 30·1	5·23	3	pB; S; R; gpmbM............	4
998	......	III. 268	....	4 58 42·9	+2·643	1	108 22 53·0	5·31	1	eF; vS; stellar..................	1*
999	2727	......	....	4 58 43·5	−0·390	2	159 36 16·7	5·31	2	⊕; pB; S; R; pmbM; rr ...	2
1000	2726	......	....	4 58 44·0	+0·153	2	156 11 37·6	5·28	2	cB; L; R; vgpmbM; r	2
1001	2719	......	....	4 58 56·6	+2·797	1	102 4 3·7	5·21	1	pB; pL; vlE; vgbM; am st ..	1
1002	(147)	......	....	4 59 12·2	−0·294	1	159 3 36·9	5·27	1	No description	1
1003	2729	......	....	4 59 20·7	−0·109	3	157 57 2·5	5·25	3	vB; pS; lE; vsvmbM*9 ...	3
1004	2731	......	....	4 59 52·1	+0·170	1	156 2 39·3	5·19	1	Cl; vL; pRi	1
1005	......	V. 32	....	4 59 55·9	+2·993	2	93 32 59·7	5·11	2	B; cL; R; bM ⁑* 15; *10, 318°.	4
1006	2733	......	....	4 59 57·5	−0·926	1	162 5 37·4	5·22	1	vF; pS; R; vglbM	1
1007	346	......	....	5 0 12·6	+4·723	1	38 7 8·8	5·04	1	Cl group of 8 or 9 st10	1
1008	2734	......	....	5 0 20·2	−0·542	1	160 21 49·6	5·18	1	eF; S; R	1
1009	2730	......	Δ. 531?	5 0 23·9	+2·056	2	128 11 33·0	5·10	3	vB; vL; mE314°; glbM; rr .	3
1010	2736	......	....	5 0 36·4	−0·426	2	159 45 22·5	5·15	2	F; S; R; glbM	2
1011	2738	......	Δ. 81	5 0 58·7	−0·472	1	159 59 38·4	5·12	1	F; pL; lE	1
1012	2735	......	....	5 1 1·0	+0·710	2	151 19 31·6	5·08	2	pF; pS; pmE; vglbM.........	2
1013	2732*	......	....	5 1 10·9	+2·260	1	122 8 40·1	5·03	1	pB; pmE; gpmbM; *13f ...	1
1014	2739	......	....	5 1 34·1	−0·440	4	159 48 27·9	5·07	4	F; pL; R; vglbM; p of 2 ...	4
1015	......	VIII. 41	....	5 1 44·4	+3·647	1	66 4 49·8	4·94	1	Cl; st c sc	1
1016	2737	......	....	5 1 51·5	+1·544	1	139 45 48·3	4·99	1	F; S; R; vglbM; *11sf; ? neb	1
1017	2742	......	....	5 1 51·9	−0·344	2	159 16 55·8	5·04	2	F; S; R; bM	2
1018	2741	......	Δ. 233?	5 2 1·9	+0·128	5	156 18 4·4	5·02	5	B; vS; vsmbM; st+neb......	5
1019	2745	......	....	5 2 31·5	−0·159	1	158 11 17·6	4·98	1	pB; L; gbM	1
1020	348	......	....	5 2 37·5	+3·453	1	73 39 38·9	4·87	1	Cl; pRi; stL and S...........	1
1021	2740	......	Δ. 549	5 2 50·1	+2·071	2	127 41 47·0	4·90	2	B; L; E; psbM	2
1022	2747	......	....	5 3 4·7	−0·448	1	159 49 11·8	4·94	1	pF; S; R; gbM; 2nd of 2 ...	1
1023	2746	......	Δ. 235	5 3 9·1	+0·086	5	156 34 15·4	4·92	5	cF; S; R; lbM; ⊕f	5
1024	2743	......	....	5 3 16·9	+2·340	2	119 27 56·5	4·85	2	cF; S; lE; p of 2	2
1025	2744	......	....	5 3 27·3	+2·341	2	119 26 7·1	4·83	2	F; S; R; glbM; f of 2	2
1026	2752	......	....	5 3 54·8	−0·586	1	160 30 43·6	4·88	1	vF; S; R; r.....................	1
1027	2748	......	....	5 3 55·4	−0·054	2	157 29 41·2	4·86	2	vF; R; s of 2 in Cl...........	2
1028	2753	......	....	5 3 56·0	−0·648	2	160 48 33·6	4·88	2	F; vS; R; vlbM; am st	2
1029	2750	......	....	5 3 58·7	−0·048	1	157 27 2·5	4·85	1	vF; R; 2nd neb in Cl.........	1
1030	349	VII. 4	....	5 3 59·1	+3·458	2	73 28 42·5	4·75	2	Cl; L; Ri; lC; st11...14 ...	5*
1031	2749	......	Δ. 236	5 4 2·0	+0·076	6	156 37 3·5	4·85	6	⊕; vB; pL; R; vmC; rr ...	6
1032	2754	......	....	5 4 22·1	−0·049	1	157 26 48·4	4·82	1	Cl; pL; Ri; C; iF	1
1033	2756	......	....	5 4 47·9	+0·104	1	156 23 46·6	4·78	1	vF; S; p of 2	1
1034	2758	......	....	5 4 51·8	−0·595	1	160 31 53·0	4·80	1	Cl; pF; L; iF; st12...15 ...	1
1035	2755	......	....	5 4 53·3	+0·834	2	149 54 12·5	4·75	2	vF; pL; vmE162°·0	2
1036	(199)	......	....	5 5 2·2	−0·328	1::	159 7 51·0	4·80	1::	No description.................	1
1037	2757	......	....	5 5 10·7	+0·101	1	156 24 57·5	4·75	1	vF; S; f of 2	1
1038	2751	......	....	5 5 12·8	+2·088	1	127 9 17·3	4·69	1	vF; vmE; long ray; *11inv .	1
1039	2761	......	....	5 5 16·9	−0·411	4	159 34 33·2	4·76	4	F; S; R; 1st of 3	4
1040	2760	......	....	5 5 22·7	−0·177	1	158 14 14·8	4·74	1	F; pL; R; r	1
1041	2762	......	....	5 5 33·9	−0·403	4	159 31 45·1	4·73	4	F; pS; R; 2nd of 3	4
1042	2759	......	Δ. 246	5 5 35·8	+0·275	2	155 6 40·7	4·71	2	B; L; R; glbM; r	2
1043	......	II. 292	....	5 5 52·9	+2·702	1	105 53 13·4	4·62	1	pB; iR; mbM; cSnf1′	1
1044	2765	......	....	5 5 56·2	−0·679	1	160 55 5·7	4·71	1	vF; pL; 1st of sev	1
1045	2764	......	....	5 6 3·3	0·381	1	159 23 56·5	4·68	1	O ? B; eS; lE	1
1046	2763	......	....	5 6 3·7	0·415	4	159 35 14·6	4·68	4	cB; S; R; gmbM; 3rd of 3..	4
1047	2766	......	....	5 6 12·5	0·278	2	158 48 40·9	4·67	2	st+neb; 1st of sev............	2
1048	2769	......	....	5 6 28·0	0·677	2	160 53 56·2	4·66	2	Cl; L; Ri; st sc	2
1049	2767	......	Br. 895	5 6 32·8	−0·246	1	158 37 28·8	−4·64	1	Cl; L; vlC	1

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	h.	H.		h m s	s		° ′ ″	″			
1050	2768	......	Δ. 170 ?	5 6 38·4	−0·278	2	158 48 37·5	−4·65	2	st+neb; pB; iF; 2nd of sev	2
1051	2771	......	....	5 6 46·2	0·909	1	161 56 6·8	4·64	1	F; R; bM; r(? min. of R.A.)	1
1052	2788	......	....	5 7 1·6	9·684	1	174 12 38·6	4·88	1	pF; L; iR; vsbM; r	1
1053	2772	......	....	5 7 20·8	0·060	2	157 27 15·9	4·57	2	vvF; R; p of 2	2
1054	2773	......	....	5 7 36·6	0·068	4	157 29 56·8	4·54	5	pF; pL; R; gbM; f of 2......	5
1055	2770	......	....	5 7 38·3	0·651	2	160 44 41·2	4·56	2	Cl; vlCM; st 9, 11...16	2
1056	2774	......	....	5 7 45·1	0·089	3	157 37 57·8	4·54	4	pB; cL; R; vglbM; r.........	4
1057	2775	......	....	5 7 56·4	0·343	2	159 8 54·1	4·53	2	B; S; lE; ⁑ in M	2†
1058	2776	......	....	5 8 5·8	−0·785	1	161 21 52·1	4·53	1	Cl; vlC; st 9, ...	1
1059	2778	......	....	5 9 20·8	+0·078	1	156 28 44·3	4·39	1	vF; S; lE; glbM...............	1
1060	2780	......	Δ. 170 ?	5 9 25·2	−0·309	6	158 55 47·0	4·40	6	⊕! vB; L; lE; vmCM; rr...	6
1061	2777	......	Δ. 508	5 9 29·8	+1·970	2	130 12 20·1	4·33	2	⊕! vB; vL; R; vsvvbM; rrr.	3
1062	2781	......	....	5 9 41·5	−0·145	4	157 56 58·9	4·37	4	F; pL; R; vglbM	4
1063	2779	......	....	5 9 51·6	+1·019	3	147 33 40·1	4·33	3	F; S; mE 45°; vgvlbM; *11 nf.	3
1064	2782	......	....	5 10 2·3	−0·327	5	159 1 12·5	4·35	5	⊕; cB; S; R; gbm; 2d of 3	5
1065	2783	......	....	5 10 8·1	0·326	1	159 0 38·8	4·34	1	Cl; vB; L; R; st12............	1
1066	2784	......	....	5 10 20·3	−0·379	3	159 17 59·1	4·33	3	B; pL; R; gbM; 12ˢ diam R.A.	3
1067	350	VII. 33	....	5 10 26·0	+4·138	1	50 48 30·6	4·18	1	Cl; pRi; pC; st 7, ...	2
1068	2785	......	....	5 10 37·2	−0·337	7	159 3 44·0	4·30	7	B; L; iE; biN; Cl+neb ...	7
1069	2786	......	....	5 10 59·4	+0·220	1	155 24 46·2	4·26	1	F; S; R; vgbM; * 7 nf 6′ ...	1
1070	2787	......	Δ. 172?	5 11 21·6	−0·316	1	158 55 38·1	4·23	1	F; pL; R; vgbM	1
1071	2790	......	....	5 12 2·3	−0·708	1	160 56 56·6	4·18	1	eF; pL; R; gvlbM............	1
1072	2789	......	....	5 12 6·1	+0·094	1	156 18 45·2	4·16	1	pF; L; iR; vg[illegible]M; r	1
1073	2791	......	Δ. 173?	5 12 19·1	−0·312	5	158 53 22·5	4·15	5	vB; vS; R; r or stellar	5
1074	2792	......	....	5 12 56·3	0·130	2	157 47 15·6	4·08	2	F; pS; iR; bM; r or stellar...	2
1075	2794	......	Δ. 173??	5 13 6·0	−0·321	3	158 55 37·6	4·08	3	vF; pL; R; vglbM............	3
1076	2793	......	Δ. 247? 248?	5 13 11·3	+0·185	1	155 37 35·2	4·06	1	vB; L; R; vgmbM; r.........	1
1077	2795	......	....	5 13 27·1	+0·071	1	156 27 4·8	4·04	1	eF; pL; R	1
1078	2796	......	....	5 13 41·5	+0·376	2	154 6 36·7	4·01	2	pB; pL; R; vglbM............	2
1079	2798	......	Δ. 210	5 13 58·4	−0·092	3	157 32 7·0	4·00	2	Cl; L; pRi; st sc.............	2
1080	2799	......	....	5 13 59·0	0·386	2	159 16 29·7	4·01	2	B; S; R; glbM	2
1081	2800	......	....	5 14 5·5	0·105	2	157 36 46·0	4·00	2	Cl; lRi; 2nd of sev...........	2
1082	2802	......	....	5 14 7·8	0·423	4	159 28 12·0	4·00	4	pB; R; gbM; 1st of group...	4†
1083	2801	......	....	5 14 8·8	0·086	1	157 29 23·3	3·99	1	Cl; 3rd of sev	1
1084	2803	......	....	5 14 9·6	0·434	1	159 31 55·0	4·00	1	neb & Cl; biN.................	1†
1085	2804	......	....	5 14 16·2	0·432	4	159 31 1·6	3·98	4	pB; iR; biN; 2nd in group...	4†
1086	2805	......	....	5 14 19·0	0·434	1	159 31 50·6	3·98	1	vF; 3rd of group in Cl	1†
1087	2807	......	....	5 14 26·2	−0·651	1	160 37 52·6	3·98	1	vF; lE; gvlbM; r	1
1088	2797	......	....	5 14 37·2	+2·244	1	122 17 45·6	3·88	1	vF; L; R; vgvlbM; *12p ...	1
1089	2808	......	....	5 14 37·9	−0·435	1	159 31 47·2	3·96	1	4th N, of neb in Cl	1†
1090	2810	......	....	5 15 3·1	−0·421	1	159 27 2·4	3·92	1	vF; ⁑ p	1†
1091	2809	......	....	5 15 15·7	+0·091	1	156 16 34·6	3·88	1	pF; R; vgvlbM; r	1
1092	......	VII. 34	....	5 15 28·7	4·452	1	43 35 54·8	3·74	1	Cl; vF; pRi; pC; iF	1
1093	2812	......	....	5 15 45·0	0·084	1	156 18 45·8	3·84	1	eF; pL............................	1
1094	2814	......	....	5 15 47·4	0·362	1	159 7 24·2	3·86	1	pB; vS; R; bM	1
1095	2813	......	....	5 15 53·7	0·061	1	156 28 16·1	3·83	1	vF; vS; R; *p25″	1
1096	{ 352 = 2806 }	II. 289	....	5 16 3·7	+2·802	2	101 38 3·8	3·74	2	pB; pL; R; r	2
1097	352, a	......	R. nova	5 16	......	...	101 38	......	...	Makes a close D neb with h. 352.	0
1098	2816	......	....	5 16 5·8	−1·017	1	162 13 55·5	3·85	1	vF; S; R; glbM	1
1099	2811	......	....	5 16 20·8	+2·124	1	125 51 33·8	3·74	1	Cl; L; Sc; ⁑ taken............	1
1100	2815	......	....	5 16 31·9	+0·245	2	155 6 43·9	3·77	2	cF; pL; E 90°±; vglbM ...	2
1101	351	......	....	5 16 32·3	+3·935	1	56 44 33·9	3·67	1	Cl; L; Ri; lC	1
1102	2818	......	....	5 16 56·7	−0·457	1	159 36 42·2	3·76	1	F; pL; R; sbM; r; st inv ...	1
1103	2817	......	....	5 17 3·0	−0·092	4	157 28 36·8	3·74	4	pF; pL; R; gvlbM............	4
1104	353	VIII. 4	....	5 17 8·1	+3·802	1::	69 58 11·1	−3·63	1::	Cl; vlRi; vlC; st 9...12	3

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	h.	H.		h m s	s		° ′ ″	″			
1105	2820	......	....	5 17 44·0	−0·112	1	157 35 20·6	−3·68	1	eF; S; R	1
1106	2822	......	....	5 17 51·1	−0·495	1	159 47 44·6	3·68	1	F; pS; R	1
1107	2821	......	....	5 17 58·4	−0·180	1	158 1 7·2	3·66	1	F; pS; R; vglbM; 3st10p ...	1
1108	2819	......	....	5 18 1·2	+0·475	2	153 10 14·8	3·64	2	F; pL; lE; vgvlbM; ✱7np...	2
1109	2824	......	....	5 18 7·8	−0·302	1	158 44 6·2	3·66	1	Cl; BM; lRi; st7	1
1110	2823	......	....	5 18 14·1	+0·013	5	156 46 1·1	3·63	5	⊕; pB; pL; R; pmbM; rr...	5
1111	2825	......	....	5 18 22·1	−0·436	5	159 28 39·8	3·64	5	vB; S; R; gmbM	5
1112	......	......	M. 79	5 18 25·6	+2·469	A	114 39 39·5	3·55	A	⊕; pL; eRi; eC; rrr	4
1113	4016	......	....	5 18 36·3	−0·087	1	157 25 24·7	3·61	1	F; S; R; r	1
1114	354	VII. 39	....	5 18 42·6	+4·001	3	54 48 28·6	3·48	3	Cl; pRi; pC; R; st 9...12 ...	5
1115	......	V. 33	....	5 18 52·3	3·011	1	92 39 22·0	3·30	1	v diffused neb susp............	1
1116	......	V. 38	....	5 19 10·4	+2·881	1	98 15 15·9	3·47	1::	eL; strongly susp (2° in P.D.).	1
1117	2827	......	Δ. 129	5 19 10·4	−0·416	4	159 21 30·9	3·57	4	Cl; L; pRi; iR; st 11...16 ...	4
1118	2826	......	....	5 19 13·2	−0·010	1	156 54 4·5	3·55	1	F; R; gbM; am st	1
1119	......	......	M. 38	5 19 17·0	+4·020	(2)	54 17 36·1	3·43	(2)	Cl; B; vL; vRi; iF; st L & S	7
1120	(356)	......	....	5 19 33·6	−0·480	1::	159 41 31·8	3·54	1::	No description............	1
1121	2830	......	....	5 19 35·7	0·833	2	161 23 42·8	3·54	2	F; L; iE	2
1122	2828	......	....	5 19 36·7	0·016	1	156 56 22·4	3·52	1	eF; pL............	1
1123	2829	......	....	5 19 39·1	0·453	3	159 32 47·1	3·53	3	B; S; R; vgvmbM; r.........	3
1124	2831	......	....	5 19 53·3	0·379	1	159 8 15·0	3·50	1	vF; L; R; vglbM	1
1125	(369)	......	....	5 20 10·1	0·496	1::	159 46 33·6	3·48	1::	No description	1
1126	2832	......	....	5 20 15·1	0·030	1	157 0 57·2	3·46	1	Cl; eF; L; iR; mC; rr	1
1127	2833	......	....	5 20 33·0	0·014	7	156 54 44·8	3·44	7	pB; pL; R; vgbM	7
1128	2834	......	....	5 20 33·9	0·527	1	159 55 8·5	3·45	1	vF; pS; lE; r	1
5063	......	●	....	5 20 52·9	−0·469	1::	159 36 35·4	3·42	1::	(See No. 5063)	1
1129	2835	......	....	5 21 6·5	+0·168	1	155 36 58·6	3·38	1	vF; pS; R	1
1130	......	III. 447	....	5 21 8·1	2·947	2	95 26 56·0	3·30	2	vF; pL; iR; st nr	2
1131	2837	......	....	5 21 22·2	+0·113	1	156 0 17·2	3·36	1	Cl; vlRi; lC; st 10	1
1132	2838	......	....	5 21 40·7	−0·480	3	159 39 51·5	3·35	3	pB; pL; iR; r; in diff n......	3
1133	356	......	....	5 22 0·7	+2·875	1	98 29 41·4	3·22	1	Diffused nebulosity............	1*
1134	2839	......	Δ. 131	5 22 1·5	−0·473	2	159 36 55·4	3·32	2	pF; pL; R; gbM	2
1135	2840	......	....	5 22 1·9	−0·198	1	158 3 33·7	3·31	1	F; p of group	1†
1136	2836	......	....	5 22 3·0	+1·666	2	136 51 6·2	3·26	2	pF; S; R; bM; 4B st p......	2
1137	355	I. 261	....	5 22 10·0	3·968	3	55 52 3·6	3·18	3	vB; L; R; b ⁂ in M	4†
1138 1139	} 2841	......	....	5 22 14·9	+0·074	5	156 16 28·3	3·29	5	{pB; S; R; smbM eF; R; stellar} D neb 26°, 80″	} 5*
1140	2842	......	....	5 22 21·3	−0·197	1	158 2 52·6	3·28	1	2nd neb of group	1†
1141	2843	......	....	5 22 24·9	0·204	4	158 5 19·6	3·28	4	pF; S; R; 3rd of group......	4†
1142	2844	......	Δ. 175	5 22 41·6	0·207	6	158 6 22·5	3·25	6	!; pB; S; R; 4th of group ...	6†
1143	2845	......	....	5 22 43·6	0·195	1	158 1 49·5	3·25	1	vF; pL; follows a group......	1†
1144 1145	} 2848	......	Δ. 89?	5 22 45·6	0·564	4	160 4 39·2	3·26	4	{pB; pS; R; glbM F; S; R; glbM} D neb 339°·1, 50″	} 4
1146	2847	......	....	5 22 54·5	−0·082	2	157 18 47·1	3·23	2	pB; vS; R; bM; 2st 9 & 10 f	2
1147	2846	......	....	5 22 55·3	+0·039	1	156 30 33·4	3·22	1	vS; neb+st............	1
1148	2849	......	....	5 23 51·4	+0·358	2	154 4 25·8	3·14	2::	eF; stell; ✱14+neb	2
1149	2850	......	Δ. 90	5 23 53·2	−0·608	3	160 16 48·9	3·17	3	pF; pS; iR; vglbM; ✱15, 190°·6, 60″	3
1150	2852	......	....	5 24 26·6	−1·138	1	162 36 15·8	3·14	1	pB; pL; R; bM	1
1151	2851	......	....	5 24 48·3	+0·025	1	156 34 43·9	3·07	1	eeeF; vvL; irr diff	1
1152	2854	......	Δ. 237?	5 25 8·5	0·034	1	156 30 29·8	3·04	1	pF; R; gbM; r	1
1153	2855	......	....	5 25 37·5	0·255	1	154 52 39·3	2·99	1	pB; L; R; glbM; ✱9np......	1
1154	2856	......	....	5 25 40·3	+0·051	1	156 23 6·3	2·99	1	Cl; cL; Ri; st 13............	1
1155	2857	......	....	5 25 47·4	−0·296	1	158 35 32·3	2·99	1	pB; S; R; psbM............	1
1156	2859	......	....	5 25 50·5	−0·561	2	160 1 23·3	2·99	2	The 1st of a group of 7!......	2†
1157	357	......	M. 1	5 26 3·9	+3·605	1	68 5 10·5	2·85	1	vB; vL; E135°±; vglbM; r	12†
1158	2858	......	....	5 26 4·1	+0·003	2	156 42 31·2	2·96	2	B; lE; sbM ⁑ 10 & 11	2
1159	2862	......	....	5 26 16·7	−0·362	2	158 57 2·5	2·95	2	pB; S; R; glbM	2
1160	2853	III. 590	....	5 26 24·5	+2·738	1	104 9 58·5	2·85	1	vF; S; R; smbM	2
1161	2863	......	Δ. 211	5 26 31·0	−0·137	1	157 37 22·4	2·92	1	Cl; Ri; 2nd of sev	1
1162	2874	......	....	5 26 39·9	−3·085	2	167 51 34·3	−2·99	2	....	2

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	h.	H.		h m s	s		° ′ ″	″			
1163	2864	......	....	5 26 46·8	−0·550	3	159 57 22·7	−2·91	3	F; pL; iR; vgbM; 2nd of group!	3†
1164	2865	......	....	5 26 50·2	−0·569	1	160 2 51·7	2·91	1	F; vL; vgbM; 3rd of group!	1†
1165	2866	......	Δ. 136?	5 27 15·3	−0·363	1::	158 57 7·3	2·89	1	vF; pL; R; 1st of 4!	1*†
1166	358	......	M. 36	5 27 3·1	+3·966	3	55 57 28·2	2·76	3	Cl; B; vL; vRi; lC; st9...11sc	9
1167	......	III. 747	....	5 27 11·2	+6·681	1	20 35 59·9	2·67	1	cF; pL; iF; mbM; er; * inv (? P.D.).	1*
1168	2867	......	Δ. 136?	5 27 32·7	−0·358	2::	158 55 26·2	2·86	2::	F; S; 2nd of 4!	2*†
1169	2861	......	....	5 27 17·2	+2·094	1	126 28 48·3	2·79	1	Cl; st 8...11	1
1170	2860	IV. 21	....	5 27 26·0	+2·536	1	112 2 39·9	2·77	1	F; vS; R; vsvmbM*12; 3st inv.	2
1171	2868	......	Δ. 136	5 27 37·6	−0·361	4	158 56 0·7	2·84	4	{Cl; pL; iF; 1st 9; + group of 4n neb pB; R; pslbM; 3rd of 4}	3*† 1
1172	(456)	......	....	5 27 43·2	0·411	2:	159 12 33·1	2·83	2:	No description..................	2
1173	2870	......	....	5 27 44·2	0·132	1	157 34 9·4	2·82	1	Cl; Ri; 3rd of sev	1
1174	2872	......	....	5 27 46·7	0·553	1	159 57 36·7	2·82	1	F; S; 4th of gr of 7	1†
1175	2869	......	....	5 27 50·4	0·361	1::	158 56 34·5	2·84	1::	4th of 4	1*†
1176	2875	......	....	5 28 1·0	0·556	1	159 58 24·7	2·81	1	5th of gr of 7	1†
1177	2876	......	....	5 28 2·0	0·555	1	159 57 34·7	2·81	1	6th of gr of 7! D; a vS neb np	1†
1178	2877	......	Δ. 213	5 28 16·4	−0·129	1	157 32 34·9	2·77	1	Cl; L; irr........................	1
1179	360	......	{M. 42 = θ^1 Orionis}	5 28 24·0	+2·945	B.A.C.	95 29 10·9	2·68	B.A.C.	!!!; θ^1 Orionis & the great neb	Mon.*†
1180	......	V. 30	42, c^1 Orionis	5 28 29·2	+2·958	B.A.C.	94 56 2·3	2·67	B.A.C.	!!; c^1 42 Orionis & neb.........	2
1181	2878	......	Δ. 238??	5 28 32·1	−0·019	3	156 20 18·8	2·74	3	vB; vL; lE; vgpmbM	3
1182	......	III. 240	....	5 28 32·9	+2·498	1	113 26 9·6	2·68	1	vF; vS; stellar*...............	1
1183	361	V. 31	44, ι Orionis	5 28 35·3	2·933	B.A.C.	96 0 21·0	2·66	B.A.C.	vF; vvL; ι 44 Orionis inv ...	3
1184	362	......	....	5 28 36·5	2·969	1	94 26 50·5	2·65	1	Cl; vB; lRi; stL, sc	1
1185	......	III. 1??	{M. 43 = 144 Bo. Orionis}	5 28 38·4	+2·948	...	95 21 48·7	2·65	...	{!vB; vL; R, with tail; mbM*8·9}	Mon.*†
1186	2881	......	....	5 28 39·5	−0·393	1	159 5 49·5	2·75	1	Cl; vL; pRi; iF	1
1187	2882	......	....	5 28 39·8	−0·419	1	159 14 12·5	2·75	1	Cl; place of **	2
1188	359	III. 865	....	5 28 41·1	+3·897	1	58 6 38·4	2·62	1	cF; S; R; psbM...............	2
1189	2883	......	....	5 28 57·6	−0·581	4	160 4 57·4	2·72	4	B; pL; R; gbM	4
1190	2885	......	....	5 28 59·3	−0·739	2	160 50 53·1	2·73	2	F; L; iR; 3stp	2
1191	......	......	Chacornac	5 29 4·0	+3·581	...	68 52 20·4	......	...	!!!; variable (Chacornac) ...	0*
1192	2871	......	....	5 29 5·5	2·279	3	120 53 59·1	2·63	3	vF; S; R; lbM; st nr	3
1193	363	V. 34	s Orionis	5 29 6·6	+3·042	1	91 17 44·7	2·61	1	!!!; eL; s Orionis inv	2
1194	2884	......	....	5 29 12·9	−0·130	1	157 32 21·3	2·69	1	Cl; 4th of sev	1
1195	2873	......	....	5 29 13·9	+2·276	1	120 59 31·4	2·62	1	eeF; vS	1
1196	......	III. 269	....	5 29 20·4	+2·644	1	107 55 16·0	2·60	1	eF; vS; stellar...................	1*
1197	2887	......	....	5 29 21·2	−0·423	2	159 15 1·3	2·69	2	Cl; eS; st 11...16	2
1198	2879	......	....	5 29 24·4	+1·559	1	138 46 56·4	2·62	1	eeF; R; bM; diffic; p of 2....	1
1199	......	VIII. 42	....	5 29 32·9	3·711	2	64 15 45·5	2·55	2	Cl; L; lC; lRi...................	2
1200	2886	......	....	5 29 35·3	0·437	1	153 18 40·5	2·65	1	eF; cS; R	1
1201	2880	......	....	5 29 37·1	1·559	2	138 47 31·0	2·60	2	vF; R; gbM; st s; f of 2 ...	2
1202	......	IV. 33	....	5 29 37·6	+2·914	3	96 48 42·9	2·57	4	B* inv in N......................	4
1203	2889	......	....	5 29 45·0	−0·997	1	161 58 34·9	2·67	1	F; pL; R; vlbM	1
1204	2888	......	Δ. 178??	5 29 51·1	0·350	2	158 50 55·5	2·65	2	Cl; st 13m	2
1205	2890	......	Δ. 214?	5 30 25·2	−0·048	2	156 59 13·3	2·59	2	vB; S; R; ** + neb in vLCl...	2
1206	2981	......	....	5 30 49·7	+0·014	1	156 34 8·5	2·55	1	B; S; stellar; r	1
1207	2893	......	Δ. 215	5 30 54·5	−0·110	6	157 23 18·5	2·55	6	⊕; B; pL; pRi; C; st 12 ...	6
1208	(509)	......	....	5 31 22·8	0·541	1::	159 51 17·4	2·52	1::	No description..................	1
1209	2895	......	....	5 31 26·5	−0·061	2	157 4 8·3	2·49	2	Cl; eL; vRi; vBvSNM	2
1210	2892	......	....	5 31 36·8	+1·433	1	141 1 12·1	2·43	1	eF; pL; R	1
1211	2894	......	..?.........	5 31 44·8	+1·431	1	141 2 52·1	2·43	1	eF; pL; R; vlbM	1
1212	2897	......	....	5 31 59·0	−0·434	4	159 16 53·2	2·46	4	pF; pS; R; glbM; in Cl......	4
1213	2898	......	....	5 32 15·9	0·764	1	160 55 58·8	2·44	1	F; cL; R; vglbM	1
1214	2899	......	....	5 32 40·9	0·149	1	157 37 1·3	2·39	1	vB; S; R; psmbM	1
1215	2907	......	....	5 32 41·8	−4·411	1	169 57 24·4	−2·52	1	vF; S; lE; bM; 2 st 9nf	1

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	h.	H.		h m s	s		° ′ ″	″			
1216	364	......	....	5 32 43·1	+6·02	1	24 16 50·7	−2·21	1	Cl; vlRi; st 11..................	1*
1217	2900	......	....	5 32 48·0	−0·177	3	157 47 24·3	2·39	3	Cl; pL; pC; iF; st 9...15 ...	3
1218	2901	......	....	5 32 52·0	0·448	1	159 21 16·3	2·39	1	Cl; vL; Ri; vlC	1
1219	2902	......	....	5 32 57·5	−0·583	1	160 3 5·6	2·38	1	F; vL; iR; gbM..............	1
1220	2896	......	....	5 33 7·7	+2·643	1	107 55 45·9	2·27	1	Cl of Lst	1
1221	2904	......	....	5 33 16·8	−0·819	1	161 10 19·2	2·36	1	pB; pL; R; pglbM; *10pinv	1
1222	2905	......	Δ. 98	5 33 29·3	0·624	2	160 15 3·1	2·33	2	B; pL; gbM	2
1223	2903	......	Δ. 218	5 33 31·3	0·180	2	157 48 16·7	2·31	2	F; vL; vlE; vglbM	2
1224	2906	......	....	5 33 46·9	−0·139	1	157 32 58·0	2·30	1	vF; S; R; in pLCl	1
1225	365	IV. 34	....	5 34 26·4	+3·283	2	80 59 3·8	2·14	2	○; pB; vS; vlE; r?	4†
1226	......	IV. 24	....	5 34 40·0	3·019	1	92 18 43·1	2·13	1	B* in M of L, lE neb	1*†
1227	......	V. 28	....	5 34 47·2	+3·028	2	91 55 43·7	2·11	2	!irr; B; vvL; black sp incl ...	2
1228	2909	......	....	5 34 47·8	−0·964	1	161 47 23·1	2·23	1	vB; vS; lE; gmbM; r	1
1229	......	VIII. 28	....	5 34 48·5	+3·557	1	69 57 43·0	2·10	1	Cl; lRi; lC; st pL	1
1230	2908	......	Δ. 241	5 35 4·3	−0·058	3	157 0 29·6	2·18	3	Cl; vL; Ri; st 9...11	3
1231	2912	......	Δ. 100?	5 35 11·2	0·585	1	160 2 26·6	2·18	1	vF	1
1232	2911	......	Δ. 240	5 35 20·3	−0·158	2	157 38 43·5	2·15	2	pB; pL; R; gbM; in cLCl...	2
1233	2910	......	....	5 35 26·6	+0·073	2	156 6 50·8	2·14	2	pB; L; iR; gbM; 1st of 3 ...	2†
1234	2915	......	....	5 35 33·9	−0·804	2	161 5 22·5	2·15	2	⊕; B; pL; R; gbM; rr	2
1235	2913	......	Δ. 219?	5 35 39·8	0·162	5	157 39 47·1	2·13	5	B; L; E; 2nd of 3	5†
1236	(579)	......	....	5 35 45·4	0·553	1::	159 52 28·8	2·14	1::	Cl; no description	1
1237	2914	......	....	5 35 51·8	0·055	1	156 58 42·7	2·11	1	Cl; vL; Ri	1
1238	2916	......	Δ. 220	5 35 52·8	0·163	3	157 40 20·7	2·11	3	B; L; R; bM; 3rd of 3	3†
1239	2917	......	....	5 35 58·0	0·416	1	159 8 38·7	2·11	1	vF; pL; R; gbM	1
1240	(593)	......	....	5 36 18·4	0·553	1::	159 51 30·6	2·08	1::	Cl; no description	1
1241	2920	......	....	5 36 23·5	−0·710	3	160 38 22·6	2·08	3	pB; S; R; gbM; *9, np 5′...	3
1242	366	......	....	5 36 25·7	+3·273	1	81 25 37·9	1·97	1	Cl; vL; lRi; lC	1
1243	2918	......	....	5 36 27·9	−0·160	5	157 39 3·2	2·06	5	F; L; iR; glbM; r............	5†
1244	2919	......	....	5 36 35·9	0·070	4	157 4 23·8	2·04	4	B; S; R; vglbM..	4
1245	2922	......	....	5 37 1·6	0·391	1	159 0 17·4	2·02	1	Cl; vL; Ri; st 12...15	1
1246	(608)	......	....	5 37 7·7	−0·444	1::	159 17 32·7	2·01	1::	Cl; no description	1
1247	367	......	Lal. 10842	5 37 8·3	+3·375	1	77 10 43·0	1·90	1	*8, 9, with Fneb	1
1248	2923	......	....	5 37 11·1	−0·644	1	160 18 59·7	2·01	1	vF; R; gbM; 1st of 7.........	1†
1249	2925	......	....	5 37 27·0	0·635	1	160 16 15·3	1·99	1	F; S; lE; 2nd of 7	1†
1250	2926	......	....	5 37 35·8	−0·522	1	159 41 40·9	1·97	1	vF; L; pmE	1
1251	2921	......	....	5 37 53·2	+2·300	2	120 8 42·2	1·86	2	vF; S; R; bM..................	2
1252	2928	......	....	5 37 54·6	−0·479	1	159 28 11·8	1·94	1	Cl+neb; mC; iF; st vS.......	1
1253	2930	......	....	5 38 2·8	0·810	1	161 5 21·8	1·94	1	pB; S; R; gbM	1
1254	2929	......	....	5 38 3·3	0·557	1:	159 52 11·8	1·94	1	eF; vvS; vglbM	1
1255	2927	......	....	5 38 3·9	0·139	1	157 30 8·4	1·92	1	F; pL; lE; gbM................	1
1256	2931	......	....	5 38 14·7	0·485	1	159 29 59·7	1·91	1	Cl; vL; Ri; st 10...15	1
1257	2932	......	....	5 38 17·9	0·755	1	160 45 15·4	1·92	1	pB; R; bM; p of 2; *9 bet...	1
1258	2935	......	....	5 38 24·3	0·652	1	160 20 40·7	1·91	2	pF; S; R; gbM; 4th of 7 ...	2†
1259	2933	......	Δ. 102	5 38 24·6	0·631	2	160 14 33·7	1·91	2	vB; pL; R; gbM; 3rd of 7...	2†
1260	2936	......	....	5 38 27·3	0·624	1::	160 12 26·7	1·91	1	vF; 5th of 7.....................	2†
1261	(642)	......	....	5 38 42·9	−0·438	1::	159 14 36·6	1·88	1::	neb; no description..........	1
1262	2924	......	....	5 38 49·3	+2·174	1	124 1 5·3	1·79	1	Cl; L; lC; st13	1
1263	2937	......	....	5 38 57·3	−0·055	1	156 56 50·8	1·84	1	vF; pS; E; glbM; 2st 10, s...	1
1264	......	VIII. 2	....	5 39 6·6	+3·276	1	81 16 35·1	1·73	2	Cl; poor; S sc st	3
1265	2938	......	Δ. 103?	5 39 10·9	−0·646	2	160 18 26·8	1·84	2	B; R; 6th of 7..................	2†
1266	2939	......	....	5 39 21·0	−0·634	1::	160 15 5·4	1·82	1::	vF; vS; E; 7th of 7	1†
1267	368	......	M. 78	5 39 34·1	+3·072	2	90 0 15·7	1·61	2	B; L; wisp-sh; vgmbN; 3st inv; r.	8†
1268	2940	......	Δ. 143	5 39 37·8	−0·409	1	159 4 43·3	1·79	1	F; L; E	1
1269	2941	......	Δ. 142	5 39 40·7	−0·423	8	159 10 18·3	1·79	8	!!!vB; vL; looped	8†
1270	......	IV. 36	....	5 39 49·8	+3·077	3	89 46 49·6	1·68	3	* with vF, Lchev..............	3
1271	2934	III. 241	....	5 39 59·5	+2·532	1	112 3 59·6	1·68	1	eF; vS; R; gbM................	2
1272	2942	......	....	5 40 1·0	−0·499	1	159 33 35·2	1·76	1	pB; pL; mE; 5st inv	1
1273	2943	......	....	5 40 9·0	−0·740	1	160 45 20·2	1·76	1	B; R; bM; rr; f of 2	1
1274	......	III. 267	....	5 40 35·0	+2·670	1	106 48 12·4	1·62	1	vF; pS; iE; bM	1
1275	2947	......	....	5 40 49·6	−0·534	1	159 43 48·3	−1·69	1	F; R; p of D neb	1

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	h.	H.		h m s	s		° ′ ″	″			
1276	2948	......	....	5 40 53·3	−0·550	1	159 48 52·3	−1·69	1	neb; np of gr of 4	1†
1277	2949	......	Δ. 152??	5 40 54·2	0·556	1	159 50 42·3	1·69	1	neb; sp of gr of 4	1†
1278	2950	......	....	5 40 56·7	0·532	1	159 43 22·6	1·68	1	B; R; f of D neb	1†
1279	2951	......	....	5 41 10·5	−0·484	1	159 28 20·2	1·66	1	Cl; vF; mC; st+neb.........	1†
1280	2945	......	....	5 41 11·3	+0·297	3	154 21 38·1	1·63	3	pF; L; R; glbM...............	3
1281	2952	......	....	5 41 11·8	−0·549	1 :	159 48 24·2	1·66	1 :	neb; nf of gr of 7.............	1†
1282	2953	......	....	5 41 15·6	0·554	1 :	159 49 51·5	1·65	1 :	neb; sf of gr of 7	1†
1283	2954	......	....	5 41 18·8	0·537	1 :	159 44 57·5	1·65	1 :	vF; R; *10vnr	1†
1284	2956	......	....	5 41 33·6	−0·536	1 :	159 44 30·1	1·63	1 :	B; pS; R; lbM; *10, p	1
1285	2946	......	....	5 41 38·9	+1·127	1	145 35 32·9	1·57	1	eF; pS; R; vlbM	1
1286	2955	......	....	5 41 39·6	−0·314	2	158 31 43·4	1·62	2	vF; S; R	2
1287	......	III. 270	....	5 41 43·7	+2·648	1	107 39 20·4	1·52	1	vF; eS; stellar..................	1*
1288	2944	......	Δ. 594	5 41 56·8	+2·163	2	124 18 21·4	1·52	2	⊕; B; pL; iR; gbM	2
1289	2957	......	....	5 42 5·1	−0·495	1	159 31 7·6	1·58	1	vF; S; mE; glbM; ?D	1
1290	2962	......	....	5 42 38·5	0·450	1	159 16 34·1	1·53	1	vF; pL; R; rr..................	1
1291	2963	......	Δ. 184??	5 42 43·0	0·397	1	158 59 18·4	1·52	1	vF; S; R........................	1
1292	2959	......	....	5 42 45·6	0·298	1	158 25 39·4	1·52	2	vF; S; R........................	1
1293	2961	......	....	5 42 47·9	0·128	2	157 23 26·7	1·51	2	Cl; F; cS; irr	2
1294	(725)	......	....	5 43 2·4	−0·319	1::	158 32 44·3	1·49	1::	neb; no description	1
1295	369	......	M. 37	5 43 7·5	+3·922	3	57 29 38·3	1·49	3	Cl; Ri; pCM; st L & S	8
1296	2960	......	....	5 43 6·1	+0·474	3	152 50 34·2	1·46	3	vF; pS; iR; pslbM*16	3
1297	2965	......	Δ. 185??	5 43 7·3	−0·282	1	158 20 10·3	1·49	2	⊕; B; S; rr	2
1298	2966	......	Δ. 147? 151? 154?	5 43 9·6	−0·450	5	159 16 26·3	1·49	5	⊕; B; pL; irrR; rr	5
1299	2958	......	....	5 43 9·8	+1·358	1	142 8 17·1	1·43	1	eF; pS; R; 3st10 sf...........	1
1300	(730)	......	....	5 43 24·9	−0·503	1::	159 33 29·2	1·46	1::	neb; no description...........	1
1301	2968	......	....	5 43 44·7	−0·888	1	161 23 47·8	1·44	1	pB; L; pmE; gbM*13	1
1302	2964	......	....	5 43 49·9	+1·390	1	141 36 16·9	1·37	1	pB; pS; R; glbM	1
1303	2969	......	....	5 44 23·6	−0·064	1	156 58 0·9	1·37	2	F; pS; R; gbM	2
1304	2967	......	....	5 44 49·2	+2·544	1	111 36 16·5	1·25	1	vF; S; vlE; gbM	1
1305	2971	......	....	5 44 54·6	−0·736	2	160 42 13·8	1·34	2	pB; pS; R; gbM	2
1306	2970	......	Δ. 153?	5 44 58·2	0·444	1	159 14 0·1	1·33	1	eF; pL; lE	1
1307	2972	......	....	5 45 6·7	−0·331	3	158 35 56·7	1·31	3	F; pS; R; vglbM	3
1308	370	III. 448 = III. 510	....	5 45 30·3	+2·897	(4)	97 30 3·2	1·16	1	eF; cS; lE; pslbM; er	5
1309	2973	......	....	5 46 27·5	−0·817	2	161 2 46·7	1·21	2	vF; S; R; gbM	2
1310	371	VII. 24	....	5 46 37·1	+3·080	1	89 38 56·6	1·08	1	Cl; pL; lRi; pC; stS	3
1311	2975	......	....	5 46 40·1	−0·560	4	159 49 44·6	1·18	5	Cl; F; S; iF; vlC; rr	5
1312	2974	......	....	5 46 44·7	−0·246	1	158 5 35·9	1·17	1	eF; pL; iR	1
1313	2976	......	....	5 47 56·7	+1·446	1	140 37 4·7	1·01	1	eeF; vS; 3st10 sp	1
1314	2977	......	....	5 47 57·0	−0·326	1	158 33 11·2	1·06	1	F; S; R; *11p	1
1315	2978	......	....	5 48 7·4	0·149	5	157 29 33·8	1·04	5	F; pL; iR; vlbM; rrr.........	5
1316	2979	......	....	5 48 39·2	−0·438	2	159 10 22·7	1·01	2	⊕; vB; vS; vsmbM; rr	2
1317	2980	......	....	5 49 41·3	+0·369	2	153 42 41·3	0·89	2	eF; pL; R; vglbM	2
1318	2982	......	....	5 50 15·5	−0·922	1	161 30 44·6	0·88	1	vF; cL; vgbM	1
1319	2981	......	Δ. 106	5 50 16·8	−0·619	5	160 6 17·6	0·88	5	Cl; pB; iF; gvmCM; st15 ...	5
1320	2983	......	....	5 51 17·8	+0·162	3	155 20 50·2	0·76	3	pB; vS; R; gbM...............	3
1321	......	III. 225	....	5 51 46·8	+2·584	1	110 3 13·8	0·64	1	eeF; pS; E; r	1
1322	2985	......	....	5 51 59·6	−0·505	1	159 31 16·4	0·72	1	vF; pS; R; gbM...............	1
1323	......	VIII. 68	....	5 52 7·3	+4·659	1	40 6 11·5	0·55	1	Cl, not Ri; 1*7m	1
1324	2986	......	....	5 52 28·5	−0·481	3	159 23 33·9	0·67	3	pB; vS; R; gmbM	3
1325	372	VIII. 26	....	5 52 34·6	+3·647	1	66 42 23·8	0·54	1	Cl; pL; 40 or 50 st 8...15 ...	2
1326	2987	......	....	5 52 41·1	−0·131	7	157 21 35·8	0·64	7	F; pS; R; glbM	7
1327	2984	......	....	5 53 11·9	+2·401	1	116 39 52·1	0·53	1	vF; pS; R; gbM	1
1328	2988	......	....	5 53 22·1	+0·765	1	149 55 53·5	0·55	1	Cl; vlC; st L & S...............	1
1329	2989	......	....	5 53 26·1	−0·854	2	161 12 17·0	0·60	2	F; pL; R; gpmbM	2
1330	2991	......	....	5 53 53·9	0·837	4	161 7 52·5	0·55	4	⊕; B; pL; R; gmbM; r ...	4
1331	2990	......	....	5 53 54·3	0·146	5	157 27 0·8	0·54	5	F; pS; R; r; am st	5
1332	2992	......	Δ. 160	5 54 7·4	−0·504	4	159 30 51·1	−0·53	4	⊕; pB; R; gmbM; rr; st 14...16.	4

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	h.	H.		h m s	s		° ′ ″	″			
1333	2994	......	....	5 54 23·3	−0·503	2	159 30 24·0	−0·50	2	vF; S; R; f of 2	2
1334	2993	......	....	5 54 34·6	+0·090	1	155 51 30·9	0·47	1	eF; S; R	1
1335	......	II. 264	....	5 54 55·9	+2·482	1	113 49 28·9	0·37	1	F; S............	1
1336	2995	......	....	5 55 1·4	−0·342	3	158 37 0·8	0·44	3	pF; pS; iR; bM	3
1337	373	......	....	5 55 15·5	+2·822	1	100 36 14·8	0·34	1	* (3 Monoc) inv in pL, F, n .	1
1338	374	......	....	5 55 39·0	+3·206	1	84 16 44·3	0·29	1	Cl; L; pRi; vlC; st10, ...	1
1339	3009	......	....	5 56 0·0	−6·638	2	172 9 27·8	0·54	2	F; pS; iR; bM	2
1340	2998	......	....	5 56 13·4	0·798	1	160 56 36·5	0·35	1	F; lE; r	1
1341	2997	......	....	5 56 22·5	−0·274	2	158 13 12·4	0·32	2	eF; S; R; bM	2
1342	2996	......	....	5 56 50·7	+0·836	1	149 7 34·5	0·25	1	eF; S; R; *12 vnr	1
1343	3000	......	....	5 57 0·8	−0·519	1	159 34 54·6	0·28	1	F; vS; R; vsmbM; stellar ...	1
1344	3001	......	....	5 57 13·7	−0·416	1	159 2 0·5	0·25	1	F; pS; R; bM	1
1345	2999	......	....	5 57 34·0	+1·437	2	140 44 5·6	0·18	2	eeF; R; *15 att	2
1346	3002	......	....	5 57 46·1	+0·010	1	156 24 55·3	0·19	1	eeF; lE; *16 att............	1
1347	3003	......	....	5 57 52·5	−0·119	4	157 16 2·3	0·19	4	F; pL; R; vglbM	4
1348	3004	......	....	5 58 9·8	+0·141	3	155 28 41·2	0·16	3	F; pL; R; vglbM	3
1349	3005	......	Δ. 196	5 58 32·0	−0·316	5	158 28 3·8	0·14	5	pB; S; R; gbM; 1st of 3 ...	5
1350	3006	......	Δ. 161?	5 58 35·9	−0·447	4	159 11 59·1	−0·13	4	⊕; vB; S; R; vgvmbM; rr..	4
1351	375	VI. 17	....	5 58 49·3	+3·670	1	65 53 46·0	0·00	1	Cl; pS; mC; vRi; nrΔ; steS.	3
1352	3007	......	Δ. 193	5 58 50·0	−0·345	4	158 38 5·4	−0·12	4	pF; S; R; gbM; *15 att nf..	4
1353	3008	......	....	5 58 50·7	0·287	1	158 17 48·7	0·11	1	pF; pS; R; gbM	1
1354	3013	......	....	5 59 33·7	−1·702	1	164 21 22·3	0·09	1	F; pL; R; gpmbM............	1
1355	3010	......	....	5 59 36·3	+0·365	3	153 43 19·4	0·02	3	F; pL; R; vglbM	3
1356	3011	......	Δ 194	5 59 39·1	−0·325	5	158 31 0·8	−0·04	5	⊕; vB; R; mCM; rr.........	5
1357	376	......	....	5 59 59·5	+4·765	1	38 17 55·2	+0·14	1	Cl; pL; poor; st11	1
1358	3012	......	Δ. 223?	6 0 4·7	−0·229	1	157 56 54·0	0·00	1	F; S; R; gbM	1
1359	378	IV. 44	....	6 0 8·7	+2·927	1	96 11 46·0	+0·10	1	Nebulous *7; am 3 st	2
1360	377	......	M. 35	6 0 12·5	3·677	1	65 39 16·9	0·13	1	Cl; vL; cRi; pC; st 9...16...	8
1361	379	VIII. 24	....	6 0 33·4	3·405	1	76 1 38·5	0·15	1	Cl; S; lRi; pmC; ⁑ Σ. 848...	3
1362	......	IV. 19	....	6 0 44·0	+2·923	3	96 22 39·5	0·15	3	*9 in vF, pLneb; E 170° ...	3
1363	3016	......	....	6 0 44·6	−0·750	1	160 43 11·2	0·04	1	eF; L; R; glbM	1
1364	3015	......	....	6 0 52·1	0·348	2	158 38 48·1	0·07	2	F; cL; R; lbM	2
1365	3018	......	....	6 0 56·3	−1·295	1	162 58 41·2	0·04	1	pF; pL; R; gmbM	1
1366	......	......	Auw. N. 21	6 1 19·1	+3·569	...	69 29 42·5	0·01	...	*8m in neb (Bruhns).........	0
1367	3017	......	....	6 1 27·2	−0·056	1	156 51 19·9	0·13	1	eeF; pL; R; gbM	1
1368	3020	......	....	6 1 42·0	−0·192	2	157 43 40·5	0·15	2	F; vS; iR; lbM; r	2
1369	3019	......	....	6 1 56·1	+0·359	1	153 45 54·4	0·18	1	eF; vS; R	1
1370	3014	......	....	6 2 6·2	2·539	1	111 43 53·8	0·26	1	F; pS; vmE; glbM	1
1371	380	VIII. 6	....	6 2 12·0	3·182	1	85 15 56·7	0·29	1	Cl; pRi; lC; st L & S.........	4
1372	3021	......	....	6 2 28·9	0·171	1	155 15 14·6	0·22	1	vF; S; R............	1
1373	381	IV. 38	....	6 2 41·6	2·924	1	96 18 57·6	0·32	1	pB ⁑; L*neb; E 90°±	3
1374	382	......	....	6 3 59·9	2·990	1	93 30 5·2	0·44	1	Cl; L; vlC	1
1375	383	IV. 20	....	6 4 17·9	2·927	2	96 12 6·1	0·47	2	*11&4 S st in vF, L neb	5
1376	384	VII. 25	....	6 4 39·8	+3·200	1	84 31 53·0	0·50	1	Cl; pL; pRi; pC; st L & S...	2
1377	3025	......	....	6 5 3·8	−0·516	3	159 33 49·6	0·42	3	{pB; pS; R; gbM / vF; R; glbM} D neb; 12°·5	3
1378	3022	......	....	6 5 5·6	+2·168	1	124 4 30·3	0·51	1	pF; pL; vmE; gvlbM.........	1
5064	......	......	....	6 5 7	......	...	88 50 39	......	...	See No. 5064.	
1379	3027	......	....	6 5 7·1	−1·817	1	164 42 26·7	0·39	1	vF; pL; R; glbM	1
1380	3023	......	....	6 5 16·5	+1·331	1	142 29 20·0	0·50	1	pB; vS; E; vsbM; *9 p 5ˢ...	1
1381	......	VII. 57	....	6 5 25·2	4·189	1	50 6 21·0	0·60	1	Cl; cL; C; iF; st vS	1
1382	3026	......	....	6 5 47·6	0·196	1	155 4 9·3	0·51	1	F; iF; glbM; 2 or 3 st inv ...	1
1383	......	VI. 5	....	6 6 2·3	3·377	2	77 9 55·9	0·63	2	Cl; L; Ri; gvmCM	2
1384	3024	II. 265	....	6 6 14·4	+2·539	4	111 46 26·6	0·62	4	pF; pS; vlE; pmbM; st nr....	5
1385	3028	......	....	6 6 21·3	−0·087	2	157 4 22·5	+0·55	2	vF; pS; R; gbM............	2
5065	......	......	....	6 6 40·9	......	...	88 58 11·2	......	...	See 5065.	
1386	3031	......	....	6 7 57·9	−1·404	1	163 22 10·8	+0·66	1	F; vS; R; bM............	1
1387	3029	......	....	6 9 5·0	+1·799	2	133 37 23·5	0·85	2	eF; pS; R; vlbM; ?134° PD.	2
1388	3030	......	....	6 9 18·6	1·798	2	133 39 27·1	0·87	2	eF; S; R; pslbM; ?134° PD.	2
1389	385	......	Σ. 885	6 9 23·0	+3·213	1	83 58 10·6	0·92	1	⁑ Chief of Cl	1
1390	3035	......	....	6 9 26·6	−2·061	1	165 24 21·8	+0·76	1	pB; pL; iR; vgpmbM; r ...	1

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	Sir J. H.'s Catalogues of Nebulæ.	Sir W. H.'s Classes and Nos.	Other Authorities.								
	h.	H.		h m s	s		° ′ ″	″			
1391	......	VII. 13	....	6 9 33·3	+2·622	1	108 36 47·3	+0·91	1	Cl; L; pRi; lC	1
1392	3034	......	....	6 10 13·2	0·506	1::	152 29 50·0	0·90	1	pF; S; R; bM.................	1
1393	3032	......	....	6 10 22·8	2·551	1	111 19 46·4	0·98	1	pB; pL; mE, 87°; pslbMRN	1
1394	3033	......	....	6 10 23·9	+2·399	2	116 43 35·4	0·98	2	F; pS; vlE; pslbM	3
1395	3037	......	....	6 12 5·0	−1·526	1	163 47 55·3	1·01	1	vF; cL; R; gvlbM	1
1396	3036	......	....	6 12 33·1	0·423	3	159 5 2·4	1·08	3	vB; pL; R; mbM; r	3
1397	3038	......	....	6 12 45·1	0·917	1	161 29 30·7	1·09	1	vF; S; R; glbM; ⁑* p	1
1398	3039	......	Δ. 201	6 13 34·4	−0·270	2	158 13 12·4	1·18	2	B; pS; lE; gbM; rrr	2
1399	386	VII. 20	....	6 14 5·0	+2·902	2	97 14 22·3	1·31	2	Cl; cL; pRi; pC; st 11...15 .	5
1400	3040	......	....	6 15 36·9	2·532	1	112 0 58·2	1·44	1	vF; pL; R; vglbM	1
1401	3041	......	....	6 16 4·8	2·387	2	117 10 38·1	1·47	2	vB; S; R; psmbM; r	2
1402	......	......	Auw. N. 22	6 16 25·2	3·539	...	70 35 28·2	1·54	...	F Cl (Markree Obs. Jan. 13, 1853).	0
1403	387	......	....	6 16 28·9	2·964	1	94 37 19·9	1·53	1	Cl; P; vlC; st 6, 11...12 ...	1
1404	3042	......	....	6 17 5·4	1·753	1	134 41 54·5	1·55	1	Cl; B; P; st 8, ...	1
1405	3044	......	....	6 18 1·5	0·980	1	147 29 35·0	1·60	1	vF; lE; vgbM; p of 2.........	1
1406	3045	......	....	6 18 1·9	0·983	1	147 27 30·0	1·60	1	vF; lE; vgvlbM; f of 2	1
1407	3043	......	....	6 18 43·9	2·513	3	112 46 8·0	1·70	3	F; pL; R; vglbM; 2st inv...	3
1408	......	VII. 35	....	6 19 44·5	3·372	1	77 16 45·6	1·82	1	Cl; pC; with neb?	1
1409	388	VII. 26	....	6 19 57·6	2·847	1	99 34 30·9	1·83	1	Cl; P; lCM; st 12...15	2
1410	3046	......	....	6 20 1·4	2·536	1	111 55 24·9	1·83	1	eF; R; ⁑ p 270°, 90″	1
1411	3047	......	....	6 20 33·0	0·293	1	154 23 21·3	1·81	1	F; S; R; glbM	1
1412	3048	......	....	6 20 49·4	0·231	1	154 52 49·6	1·82	2	eF; vS; R; 1st of 3...........	2
1413	3049	......	....	6 20 55·3	+0·226	1	154 55 25·2	1·84	1	eF; S; lE; 2nd of 3	1
1414	3050	......	....	6 21 0·8	−0·136	3	157 27 7·2	1·84	3	F; pL; R; gvlbM; ⁑ f	3
1415	......	VIII. 25	....	6 21 1·6	+2·963	1	94 40 44·6	1·92	1	B* (10 Monoc) + Cl	1
1416	3051	......	....	6 21 11·0	0·220	1	154 58 9·5	1·85	1	eF; S; 3rd of 3	1
1417	389	VIII. 9	....	6 21 13·5	3·473	(1)	73 13 33·5	1·95	1	Cl; eL; pRi; lC; st L & S...	2
1418	3052	......	....	6 21 49·5	0·235	1	154 51 45·3	1·91	1	vF; S; R; *12 nr	1
1419	390	VII. 5	....	6 22 11·0	3·233	1	83 4 23·2	[2·04	1	Cl; pRi; pC; st 10, 12...15...	2†
1420	392	......	....	6 23 28·8	3·189	1	84 57 32·2	2·14	1	*8 in L; P; BCl.............	4
1421	391	VIII. 49	....	6 23 31·7	+4·013	2	54 42 17·1	2·17	2:	Cl; pL; P; vlC; st 7, 10...15	3
1422	3054	......	....	6 23 42·4	−0·366	3	158 50 33·8	2·06	3	vF; pL; R; glbM	3
1423	3053	......	Δ. 616?	6 24 13·8	+2·267	2	121 11 28·4	2·18	2	pB; cL; R; vlgbM; 4′	2
1424	......	VII. 2	12 Monoc. B.A.C.	6 24 53·4	3·189	B.A.C.	85 2 14·5	2·27	B.A.C.	Cl; beautiful; st sc	
1425	393	IV. 3	....	6 24 58·0	3·312	2	79 44 43·1	2·27	2	pL; com; mbNsf alm*; *7·8 nf.	7*†
1426	......	......	Auw. N. 23	6 25 51·9	+3·729	...	63 35 16·9	2·36	...	Small cluster (Markree Obs. Dec. 23, 1853).	0
1427	3055	......	....	6 26 38·1	−0·360	4	158 50 0·3	2·31	4	pB; pL; R; vgbM; ⁑ p	4
1428	394	......	....	6 27 5·0	+2·956	1	94 57 53·2	2·44	1	Cl; pRi; lC; iF; st 8, 12...14	1
1429	395	VIII. 3	....	6 27 8·4	3·269	(2)	81 32 16·8	2·46	(2)	Cl; vL; E; Ri; lC	4
1430	396	VIII. 50	....	6 27 23·7	3·199	(1)	84 32 14·7	2·49	1::	Cl; vL; pRi; lC; st S	3
1431	......	VII. 54	....	6 27 56·0	6·043	1	24 2 15·6	2·62	1	vF; st eS	1
1432	397	VII. 22	....	6 28 25·1	3·253	1	82 13 44·4	2·58	1	Cl; S; pC; iF; st 11...15 ...	2
1433	3056	......	....	6 28 59·1	2·153	2	124 42 52·7	2·59	2	eF; S; lE; vlbM	2
1434	3057	......	....	6 29 25·9	0·325	3	154 13 23·4	2·58	3	F; cL; R; vglbM; r; 17m·0d	3
1435	......	VI. 28	....	6 30 47·9	3·329	1	79 0 33·4	2·78	1	Cl; cRi; eC; iF; st eS	1
1436	398	VIII. 48	....	6 31 2·3	3·041	1	91 20 42·7	2·79	1	Cl; vL; P; vlC; st L & S ...	2
1437	399	IV. 2	....	6 31 31·4	3·278	3	81 8 20·5	2·85	3	B; vmE 330°; Ncom = *11	7†
1438	400	VII. 37	....	6 32 24·0	3·101	1	88 43 47·6	2·92	1	Cl; vC; iR; bM; st eS	2
1439	3058	......	....	6 32 41·8	2·462	1	114 43 55·6	2·92	1	pF; lE; bet 2 vS st; pslbM...	1
1440	401	V. 27 / VIII. 5	15 Monoc.	6 33 16·0	3·305	1	79 59 24·7	2·99	1	15 Monoc; Cl; ⁑; ? neb......	6*
1441	402	......	....	6 33 40·5	3·354	1	77 57 3·2	3·04	1	Cl; P; 30 or 40 st 12...13 ...	1
1442	403	VI. 21	....	6 34 32·6	3·748	1	62 53 35·6	3·12	1	Cl; pS; eC; Ri; st 11...15 ...	2
1443	3059	......	....	6 35 31·5	2·236	1	122 20 49·8	3·16	1	pB; S; R; 2 or 3 st v nr	1
1444	404	VI. 3	....	6 36 26·7	3·180	(1)	85 17 41·1	3·27	1	Cl; vmC; not Ri; st vS	2
1445	405	VII. 36	....	6 36 35·5	3·154	1	86 24 51·4	3·28	1	Cl; lC; not Ri	2
1446	3060	......	....	6 37 3·1	2·593	3	113 20 15·0	3·30	3	pF; S; R; gbM; am st	3
1447	3061	......	....	6 37 7·0	+2·390	3	117 19 37·3	+3·31	3	pF; pS; vlE; bM; r	3

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	h.	H.		h m s	s		° ′ ″	″			
1448	406	II. 615	....	6 38 2·9	+3·950	2	56 17 42·9	+3·43	2	F; S; bM........................	4
1449	407	II. 614	....	6 38 2·9	3·951	2	56 15 56·9	3·43	2	eF; vS........................	4
1450	3062	......	....	6 39 16·6	2·386	1	117 30 7·7	3·49	1	pF; pL; lE; gbM	1
1451	......	VIII. 71	....	6 39 30·5	4·223	1	48 47 17·8	3·56	1	Cl; pRi; vlC; st pL	1
1452	......	III. 271	....	6 39 46·2	2·642	1	108 3 23·8	3·56	1	3 or 4 S st + neb..............	1*
1453	408	VIII. 31	....	6 40 39·4	3·002	1	93 1 23·9	3·63	1	Cl; L; C; ab 100 st 9...15...	3
1454	411	......	M. 41	6 41 0·3	2·578	1:	110 36 2·2	3·64	1	vL; B; lC; st 8,...	3*
1455	410, a	......	R. nova	6 41 34·1	3·944	...	56 23 51·2	3·74	...	No desc.; β of Lord R.'s diag.	0 }*
1456	410, b	......	R. nova	6 41 35·9	3·944	...	56 22 58·2	3·74	...	No desc.; γ of Lord R.'s diag.	0 }
1457	410	III. 898	....	6 41 39·3	3·944	1:	56 25 16·2	3·74	1	eF; vS........................	2
1458	409	III. 897	....	6 41 39·4	3·946	1:	56 21 16·2	3·74	1	eF; vS........................	2
1459	3063	......	....	6 41 44·2	2·414	1	116 35 49·0	3·70	1	{pB; R; gbM / eF; R; gbM} D neb; am st	1
1460	410, c	......	R. nova	6 41 54·0	3·944	...	56 19 55·2	3·74	...	No descr.; ε of Lord R.'s diag.	0*
1461	3064	......	....	6 42 3·3	2·414	1	116 34 19·9	3·73	1	eF; S; R; bet st; D neb p ...	1
1462	3066	......	....	6 43 23·6	0·428	1	153 34 16·4	8·78	1	vF; S; R; vglbM	1
1463	3065	......	Δ. 578	6 44 1·7	2·124	4	125 50 56·4	3·88	4	⊕; B; pL; iR; gbM; rr......	4
1464	412	......	....	6 44 17·3	2·915	1	96 49 35·2	3·94	1	Cl of 30 or 40 st.............	1
1465	413	VI. 27	....	6 44 35·1	3·086	2	89 22 44·1	3·97	3	Cl; Ri; L; iF; st L & S	5
1466	414	VIII. 39	....	6 45 3·1	2·912	3	96 55 13·0	4·00	3	Cl; L; P; lC	6
1467	415	VI. 2	....	6 46 55·4	3·501	2	71 49 12·7	4·09	2	Cl; pL; Ri; mC; st vS	5†
1468	3067	......	....	6 47 45·6	0·375	1	154 6 43·8	4·16	1	vF; vS; R; 2 st Δ	1
1469	416	VIII. 51	....	6 47 47·1	2·910	1	97 1 26·2	4·24	1	Cl; P; vlC	4
1470	3068	......	....	6 47 58·3	0·369	2	154 9 52·4	4·18	2	vF; pS; vlE 90°...............	2
1471	417	VI. 18	....	6 49 15·4	2·911	3	97 1 24·8	4·36	3	Cl; pL; pRi; mC; st 13... ...	7
1472	3069	......	....	6 49 23·0	1·949	2	130 41 29·2	4·34	2	pB; pL; vmE 44°·8; pslbM..	2
1473	418	VIII. 60	....	6 50 51·9	2·971	1	94 24 15·0	4·50	1	Cl; lC; not Ri................	2
1474	419	......	....	6 51 6·4	3·311	1	79 33 25·9	4·53	1	Cl; P	1
1475	......	......	D'Arrest, 50	6 51 23	2·89	[3]	97 45 24	4·51	[3]	F; vS; R	0
1476	420	......	....	6 51 49·9	4·665	2	39 12 51·9	4·63	2	eF	2
1477	421	II. 304	....	6 52 55·6	2·898	3	97 35 39·1	4·67	3	pF; S; R; r; S st inv	6†
1478	421, a	......	R. nova	6 52	......	...	97 35	......	...	Makes a close D neb with h. 421.	0
1479	{422 = 3070}	VII. 14	....	6 53 4·4	2·759	2	103 30 44·4	4·68	2	Cl; L; Sc; st 8...9	3
1480	423	VIII. 1B	....	6 53 48·2	3·145	2	86 44 53·5	4·75	2	Cl of sc st; st 8, 9,...	3*
1481	......	III. 874	....	6 54 46·7	4·658	1	39 15 39·4	4·88	1	vF; vS; lE	1
1482	424	II. 861	....	6 54 57·4	4·661	1	39 13 10·3	4·81	1	pB; S; iR; gbM; *8, 120°...	2
1483	425	......	M. 50	6 56 12·5	2·886	4	98 8 46·5	4·95	4	! Cl; vl; Ri; pC; E; st 12...16	8
1484	427	VII. 38	....	6 56 57·0	3·100	2	88 44 31·6	5·02	3	Cl; L; Ri; cC; st 12...16 ...	5
1485	3071	......	....	6 57 7·4	2·368	1	118 30 13·3	5·01	1	pB; pL; lE; gbM	1
1486	426	II. 734	....	6 57 28·6	4·663	1	39 5 50·6	5·02	1	vF; pL; iR; psmbM; st p ...	2
1487	428	IV. 25	....	6 57 33·2	2·817	1	101 6 47·8	5·06	1	pB * inv in S, vF, neb	3
1488	3072	......	....	6 58 6·8	1·912	2	131 51 55·4	5·08	2	vF; S; vlE; bM; am st	2
1489	429	{II. 735 = III. 875}	....	6 58 33·7	4·547	1	41 10 42·0	5·20	1	vF; vS; stellar.................	4
1490	432	VIII. 40	....	6 58 47·5	3·741	2	62 35 40·7	5·19	2	Cl; L; vlC; Scl inv...........	3
1491	430	II. 862	....	6 58 49·2	4·631	1	39 36 2·6	5·22	1	F; S; R; psbM	2
1492	430, a	......	R. nova	6 58 ±	......	...	39 36 ±	......	...	Several near h. 430 (? 426, 433 & 1 nov).	0
1493	431	III. 899	....	6 58 54·3	3·986	(1)	54 39 36·3	5·21	1::	vF; S; R; bM..................	2
1494	......	VIII. 32	....	6 59 54·6	2·847	1	99 52 4·8	5·26	1	Cl; L; lC......................	1
1495	435	......	....	7 0 6·6	2·949	1	95 24 37·7	5·29	1	Cl; vlC	1
1496	434	II. 769	....	7 0 9·7	3·515	1	71 0 17·0	5·30	1	pB; pL; R; glbM	2
1497	433	II. 736	....	7 0 27·9	4·627	2	39 36 26·8	5·36	2	pF; S; R; glbM; r...........	4
1498	......	VIII. 33	....	7 1 35·7	2·834	1	100 26 14·0	5·40	1	Cl; cL; P; lC	1
1499	3073	......	....	7 1 52·0	2·775	1	102 57 4·9	5·43	1	Cl; pL; pRi; gbM; st 10...14	1
1500	......	IV. 65	....	7 2 14·0	3·055	2	90 30 8·1	5·47	1	*9 aff with S, vF, neb	1
1501	......	III. 746	....	7 2 47·5	+5·827	1	24 57 49·7	+5·59	1	vF; S; R; lbM	1

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	h.	H.		h m s	s		° ′ ″	″			
1502	3074	......	....	7 3 7·1	+0·009	1	157 11 8·8	+5·46	1	Cl; P; lC; 30 st ±	1
1503	436	VII. 27	C. H.	7 3 16·3	2·881	1	98 23 59·5	5·55	1	Cl; cL; P; cC	3
1504	437	......	....	7 6 55·9	2·817	1	101 15 9·5	5·85	1	Cl; lC; ⁑ taken	1
1505	......	VII. 15	....	7 7 48·2	2·506	1	113 51 10·6	5·92	1	Cl; pRi; pC	1
1506	......	VIII. 34	....	7 7 50·5	2·845	1	100 3 44·9	5·93	1	Cl; L; lC; one vB*	1
1507	438	VII. 16	....	7 8 28·2	2·463	1	115 29 29·4	5·98	1	Cl; cRi; lC	2
1508	439	VI. 6	....	7 9 3·8	3·390	1	75 58 53·5	6·05	1	Cl; pS; pRi; mC; st 15...16	2*
1509	......	VII. 6	....	7 9 12·4	3·394	1	75 46 51·8	6·06	1	Cl; lC	1
1510	......	VIII. 45	....	7 10 31·8	2·687	2	106 47 59·8	6·16	2	Cl; P; lC	1
1511	3075	V. 21	....	7 11 3·0	2·778	1	102 57 54·0	6·20	1	!!; vF; vvL; viF	3†
1512	440 = 3076	VII. 12	C. H.	7 11 23·4	2·721	2	105 23 19·9	6·23	2	Cl; vL; Ri; pC; st 9...12 ...	5
1513	441 = 3077	VII. 17	A.S.C. 905	7 12 54·6	2·488	2	114 42 14·2	6·34	2	Cl; pL; Ri	4
1514	442	......	....	7 13 59·7	2·908	1	97 18 9·5	6·05	1	Cl; pC; st pL; bifid	1
1515	......	III. 748	....	7 14 2·3	6·410	1	20 42 16·2	6·54	1	vB; pL; R; mbM; r; vS* inv.	1
1516	......	VIII. 27	....	7 14 8·9	2·568	1	111 40 18·5	6·45	1	Cl; S; P; lC	1
1517	443	......	....	7 14 20·5	2·845	1	100 7 38·4	6·48	1	Cl; S; pRi; st 15	1
1518	3078	......	....	7 15 0·5	0·679	2	152 5 45·1	6·47	2	pB; pL; iE; glbM	2
1519	444	II. 316	....	7 16 42·9	3·792	1	60 14 41·0	6·70	1	B; S; R; bMN; p of D neb, 45°, 60″.	3†
1520	445	II. 317	....	7 16 44·7	3·792	1	60 14 21·0	6·70	1	pB; S; R; bMN; f of D neb	3†
1521	3080	VIII. 35	....	7 17 31·6	2·781	1	102 59 33·2	6·74	1	Cl; vL; pRi; lC; st L	5
1522	3079	......	....	7 18 18·9	2·423	1	157 15 54·7	6·79	1	pF; pS; R; vsmbM; am st ...	1
1523	3084	......	....	7 18 27·0	0·617	1	112 48 48·5	6·75	1	vF; vS; R; am st	1
1524	3082	......	....	7 18 32·8	2·427	1?	117 6 24·0	6·80	1::	pF; S; R; bM	1
1525	3081	......	....	7 18 39·7	2·597	1	110 39 57·6	6·82	1	Cl; pS; pmC; st 12	2
1526	3083	......	....	7 18 59·7	2·595	2	110 44 57·5	6·85	2	Cl; lC; bifid; ⁑	2
1527	446	......	....	7 19 17·6	3·920	1?	55 54 27·3	6·91	1::	neb; 1st of 4	1*
1528	447	III. 703	....	7 19 23·1	3·921	1	55 53 27·3	6·91	1::	vF; vS; R; bM	3*
1529	......	II. 820	....	7 19 44·8	4·017	1	52 58 44·5	6·95	1	pB; S; stellar	1
1530	448	III. 900	....	7 19 45·1	3·920	1	55 55 8·2	6·95	1	vF; S; R; bM	2*
1531	449	III. 901	....	7 19 57·6	3·921	1	55 51 47·1	6·97	1	vF; S; R; psbM	2*
1532	450	IV. 45	....	7 20 54·4	3·557	1	68 48 33·2	7·04	1	B; S; R; *8 M	4†
1533	......	VIII. 44	....	7 21 2·5	3·232	1	82 41 1·2	7·04	1	Cl; L; P; vlC; st L	1*
1534	......	VIII. 11	....	7 21 11·7	3·384	1	75 56 51·5	7·05	1	Cl; pRi; C	1
1535	451	VIII. 36	....	7 21 38·5	+2·818	1?	101 27 32·1	7·07	1?	Cl; vL; vlC	2
1536	3085	......	....	7 21 42·1	−0·151	2	158 43 56·0	7·00	2	pB; cL; cE 117°, lbM	2
1537	......	......	Auw. N. 24	7 22 32·1	+3·070	::	89 55 49·5	7·15	...	Two B neb (Bond, Feb. 1853).	0
1538	......	......	Auw. N. 25	7 22 32·1	3·070	::	89 55 49·5	7·15	...		0
1539	454	VII. 65	....	7 22 57·2	2·767	1	103 41 23·1	7·17	1	Cl; S; cRi; cC; st vS	2
1540	453	III. 19	....	7 23 6·9	3·290	2	80 3 25·0	7·20	3	eF; S; R; lbM; * inv	4
1541	......	V. 44	....	7 23 18·6	5·864	1	24 0 3·0	7·30	1	!!; cB; cL; vmE; vgmbMN 7′	2
1542	452	......	....	7 24 35·7	6·898	1	18 1 37·9	7·43	1	Cl; vlC	1
1543	3086	......	....	7 25 21·7	2·693	1	106 54 16·1	7·37	1	Cl; S but B; st 8...10	1
1544	......	VIII. 52	....	7 26 49·5	2·789	1	102 47 49·7	7·49	1	Cl; vL; P; vlC	1
1545	455	VIII. 37	....	7 26 49·5	2·735	1	105 8 46·7	7·49	1	Cl; P; lC; st 9, &c.	3
1546	456	II. 821	....	7 27 43·4	3·954	1	54 28 22·7	7·59	2	pB; cS; R; vgvlbM; r; alm○	3
1547	3087	......	....	7 28 25·2	0·743	1	151 57 47·1	7·57	1	vF; L; R; gbM; r	1
1548	457	I. 218	....	7 28 37·8	4·077	1	50 48 47·4	7·68	1	pB; pL; lE 90°; vgbM; *7, 8, 19°.	3
1549	458	VI. 1	....	7 30 5·7	3·566	1	68 7 10·4	7·78	1	Cl; cL; Ri; C; st 11...18 ...	10
1550	3089	VII. 67	....	7 30 10·0	2·615	1	110 18 0·8	7·76	1	Cl; L; cRi; st 11...13	3
1551	459 = 3088	VIII. 38	....	7 30 10·8	2·760	2	104 10 31·8	7·76	2	Cl; B; vL; pRi; st L & S ...	4
1552	3090	VII. 28	....	7 30 38·0	2·774	1	103 32 5[illegible]·0	7·80	1	Cl; vL; Ri; pC; st vS	2
1553	......	VIII. 87	....	7 31 51·7	+2·751	1	104 35 14·0	+7·90	1	Cl; P; S; st vS	1

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	h.	H.		h m s	s		° ′ ″	″			
1554	460	II. 822	....	7 32 26·8	+4·676	1	37 20 50·0	+8·00	1	cF; R; vgbM; r; ✱8 p	2
1555	3091	......	....	7 32 31·5	1·739	1	137 18 28·9	7·93	1	eF; L; pmE; gmbM; 2 st inv	1
1556	......	VIII. 47	....	7 32 55·5	2·715	1	106 11 49·4	7·98	1	Cl; vL; vlC....................	1
1557	......	VIII. 46	....	7 33 4·6	2·719	1	106 1 50·0	8·00	1	Cl; vL; vlC....................	1
1558	......	III. 829	....	7 34 26·7	4·715	1	36 34 59·8	8·16	1	eF; vS; R; bM	1
1559	3092	VI. 36	....	7 34 41·6	2·656	1	108 45 38·6	8·12	1	Cl; pL; pC; E 0°; st L & S...	3
1560	462	......	....	7 35 4·7	+3·277	1	80 24 50·1	8·17	1	eF; ✱15, 300°·0, 90″	1
1561	3096	......	....	7 35 9·9	−0·117	5	158 58 3·7	8·09	5	pB; S; R; pmbM; 3 st 11 n...	5
1562	......	II. 616	....	7 35 11·2	+3·833	1	57 59 59·0	8·20	1	F; S; lbM	1
1563	461	......	....	7 35 19·3	4·653	1	37 35 56·6	8·22	1	vF; vS; R; bM	1
1564	463	......	M. 46	7 35 24·3	2·755	1	104 29 50·4	8·18	1	!; Cl; vB; vRi; vL; inv ○...	4
1565	464 = 3093	IV. 39	....	7 35 25·4	2·757	2	104 24 39·4	8·18	3	○; pB; pS; elE; r; 3″·75 d...	4†
1566	3094	......	....	7 35 26·2	2·328	1	121 19 45·1	8·17	1	Cl; B; pRi; pL; lC; st 9, 12...14.	1
1567	3095	IV. 64	....	7 35 41·2	+2·677	1	107 53 22·3	8·21	1	○; cB; not v well def	3†
1568, 1569	3097	......	....	7 36 43·9	−0·149	4	159 12 49·3	8·21	4	{cL; vF; R / pL; vF; R} D neb; 40°; ✱ inv M ...	4
1570	465	......	....	7 37 36·4	+4·803	1	35 3 14·3	8·41	1	F; am 4 st	1
1571	3098	......	M. 93	7 38 39·2	2·542	1	113 32 43·2	8·44	1	Cl; L; pRi; lC; st 8...13 ...	2
1572	466	......	Lal. 15134	7 38 41·4	2·522	1	114 21 12·2	8·44	1	Cl of 18 or 20 st 11...13 ...	1
1573	3099	......	....	7 40 19·2	2·138	1	127 38 15·8	8·56	1	Cl; vvL; vlC; 1✱ 4·5 m......	1
1574	3100	......	....	7 41 47·4	2·457	2	117 0 6·7	8·69	2	○; F; S; lE; am 60 st	2
1575	3101	......	....	7 41 56·8	2·459	1::	116 54 21·0	8·70	1::	Cl; S; pRi; pC	1
1576	3102	......	....	7 42 52·2	2·611	1	110 57 4·1	8·77	1	Cl; cL; pRi; lC; st 12	1
1577	467	......	....	7 42 56·7	4·838	1	34 9 26·9	8·83	1	vF; R; vgbM	1
1578	468	III. 479	....	7 44 24·4	3·280	2	80 5 29·5	8·91	2	vF; S; rr group + neb	3*.
1579	469	......	....	7 45 47·6	+4·908	1	32 57 44·1	9·07	1	eF; R; p of 2	1
1580	3104	......	....	7 46 20·9	−0·423	1	161 3 33·8	8·96	1	vF; S; R; lbM	1
1581–1588	469, *a*	......	R. 8 novæ	7 47 ±	......	...	32 57 ±	......	...	8 of 10 neb, in line with h. 469, 470.	0
1589	472	IV. 22	....	7 46 40·1	+2·488	1	116 2 1·1	9·07	1	pB; vL; R; er; ✱8 M.........	3
1590	470	III. 836	....	7 46 42·0	4·905	2	32 57 5·9	9·13	2	F; vS; R; ✱9 sf; f of 2	3
1591	471	III. 830	....	7 47 11·2	4·660	1	36 46 9·8	9·16	1	F; pS; E?; bM vS✱? L✱ nf	2
1592	471, *a*	......	R. nova	7 47	......	...	36 46	......	...	Makes D neb with h. 871 ...	0
1593	3103	......	Δ. 535	7 47 18·9	2·133	2	128 11 8·3	9·11	2	!; Cl; B; Ri; L; lC; st 12 ...	3
1594	......	......	M. 47	7 48 20·5	2·751	W.	105 3 19·3	9·21	W.	Place from Wollaston's Cat.	0*
1595	......	VII. 58	....	7 48 39·8	2·700	1	107 21 4·6	9·22	1	Cl; pL; pRi; pC; st S.........	1
1596	473	II. 302	....	7 48 50·3	3·602	1	65 52 5·8	9·26	1	F; S; lE; bM; er	3
1597	473, *a*	......	R. nova	7 48 ±	......	...	65 52 ±	......	...	vF; E; ✱ inv near N	0
1598	474 = 3106	VII. 10	....	7 49 0·6	2·544	2	113 56 10·5	9·25	2	Cl; L; cRi; vlC	5
1599	3105	......	....	7 49 1·8	2·453	1	117 29 59·5	9·25	1	Cl; L; lC........................	1
1600	475	III. 837	....	7 50 26·3	4·881	1	33 3 54·6	9·42	1	vF; vS; R; glbM	2
1601	479 = 3107	VII. 23	Δ. 626	7 50 36·9	2·396	3	119 41 58·1	9·37	3	Cl; pL; cRi; pC; st 11...13	4
1602	477	......	....	7 50 53·9	3·682	1	62 35 59·9	9·43	1	vF; S; R; bM..................	1
1603	477, *a*	......	R. nova	7 50	......	...	62 35	......	...	F; S..............................	0
1604	476	III. 750	....	7 50 55·0	4·067	(1)	49 47 50·2	9·44	(1)	cB; S; R; sbM	2
1605	476, *a*	......	R. nova	7 50 +	......	...	49 47 ±	......	...	Follows III. 750 (h. 476) ...	0
1606	......	III. 838	....	7 51 11·3	4·903	1	32 42 17·4	9·48	1	eF; vS...........................	1
1607	478	III. 709	....	7 51 25·4	+4·530	1	38 52 4·7	+9·49	1	F; L; R; vgbM; r; am st ...	3

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	h.	H.		h m s	s		° ′ ″	″			
1608	3108	......	....	7 52 3·2	+2·778	2	103 59 2·7	+ 9·49	2	cF; S; vlE 90°; glbM; am st.	2
1609	3109	......	....	7 52 42·0	1·565	1	141 55 10·1	9·47	1	pF; S; R; vgpmbM	1
1610	......	III. 839	....	7 53 15·3	4·672	1	36 10 28·9	9·63	1	eF; vS	1
1611	480	VI. 37	....	7 53 15·8	2·858	1	100 14 21·4	9·58	1	Cl; pL; vRi; C; st 11...20...	2*
1612	481	II. 544	....	7 53 39·5	3·409	1	73 54 23·9	9·63	1	pB; pL; iR; vgbM; er; *225°·5, 60″.	3
1613	......	VIII. 1	....	7 54 31·8	2·675	3	108 41 12·4	9·68	3	Cl; B; pRi; lC; stS	4
1614	482	III. 605	....	7 54 45·3	3·586	1	66 13 22·6	9·72	1	vF; S; iR	2
1615	483, *a*	......	R. nova	7 54 32·6	3·273	...	80 7 43·3	9·71	...	γ in Lord Rosse's diagram ..	0*
1616	......	......	D'Arrest, 51	7 54 40·8	3·27	[2]	80 12 48	9·70	[2]	eF; III. 512 f 10s·5; n 50″ ...	0
1617	483	III. 512	....	7 54 47·2	3·273	2	80 11 58·3	9·71	2	F; S; R; psmbM; r	3
1618	484	III. 7	....	7 54 48·7	3·255	1	81 4 16·3	9·71	1	F; vS; vlE; 2 st p	3
5066	......	......	....	7 55 12·5	......	...	65 25 19·4	......	...	See No. 5066.	
1619	3111	......	....	7 56 1·4	1·004	6	150 29 10·5	9·75	6	Cl; vB; vL; pRi; st 7...13...	6
1620	3110	......	....	7 56 17·4	2·825	1	101 54 45·3	9·81	1	F; vS; R; bet 3 st 13, 14 ...	1
1621	3112	......	....	7 56 48·9	2·457	1	117 47 40·5	9·85	1	Cl; B; pRi; pC	1
1622	485	......	....	7 57 22·1	4·938	1	31 49 50·5	9·95	1	pF; pL; R; psbM; *9, np 3′.	1
1623	486	III. 877	....	7 59 0·5	2·845	1	101 2 6·9	10·03	1	cF; pL; R; vgvlbM; am st...	2
1624	488	VIII. 30	....	7 59 28·2	2·461	(2)	117 46 23·5	10·05	1	Cl; vL; pRi; lC; st 10...15...	3
1625	487	III. 752	....	7 59 54·2	3·451	1	71 46 29·0	10·10	1	eF; lE; vS*n	2
1626	489	II. 726	....	8 1 15·2	3·857	2	55 38 18·6	10·22	2	pB; pL; R; vglbM; r; 2 st nf	4
1627	3113	......	....	8 1 22·2	2·419	2	119 29 53·7	10·19	2	Cl; pL; Ri; C; st 9, 13...14..	2
1628	490	III. 840	....	8 1 50·7	4·775	1	33 55 30·7	10·29	1	pF; pL; R; psbM; *8, 164°·3	2
1629	491	IV. 55	....	8 3 18·1	4·271	1	43 35 28·4	10·38	1	⊕; pB; pL; R; rrr st 20......	3
1630	3114	VII. 11	....	8 4 7·8	2·819	2	102 24 59·0	10·40	2	Cl; vL; Ri; lC; st 11...13 ...	3
1631	492	III. 710	....	8 4 26·2	4·404	1	40 30 20·4	10·48	1	F; L; E; vgbM	2
1632	3115	......	B.A.C. 3073	8 4 42·2	2·817	1	102 30 54·5	10·45	1	Nebulous * 6·7	1
1633	493	II. 719	....	8 4 51·0	3·919	1	53 19 26·7	10·49	1	F; pL; iR; vgbM; ⁑ nr	3*
1634	494	II. 627	....	8 6 2·9	3·524	1	68 13 36·1	10·57	1	F; S; lE 45°; *8, np 4′	4
1635	3116	......	Δ. 563	8 6 26·4	2·215	3	126 58 24·1	10·57	3	Cl; B; L; lC; iE; st 9...12...	3
1636	3117	......	Δ. 411	8 6 31·6	1·769	2	138 51 3·1	10·57	2	Cl; B; L; lC; st 7...16	2
1637	496	VI. 22	C. H.	8 6 50·1	2·964	2	95 22 30·6	10·62	3	Cl; vL; pRi; pmC; st 9...13.	7
1638	495	......	....	8 7 37·5	4·892	1	31 46 48·9	10·73	1	pB; S; mE 0°; psmbM	1
1639	......	III. 711	....	8 8 46·0	4·438	1	39 32 33·9	10·73	1	eF; cL; lE 45° ±	1
1640	497	II. 308	....	8 10 33·7	3·568	1	66 6 1·0	10·90	1	F; S; R; mbM; r	4
1641	498	III. 256	....	8 10 43·9	3·094	1	88 48 46·3	10·91	1	vF; cS; iF; 3S st inv?	3
1642	499	III. 606	....	8 11 6·8	3·500	1	69 3 9·5	10·95	1	vF; S; R; sbM; stellar	3
1643	3118	......	....	8 11 21·1	2·499	1	117 2 10·9	10·93	1	F; pL; gmbM; am 60 st	1
1644	......	......	D'Arrest, 52	8 11 42	3·51	[3]	68 34 36	10·98	[3]	F; pL	0
1645	500	III. 607	....	8 12 13·6	3·512	1	68 26 14·6	11·02	1	vF; cS; R	2
1646	501	II. 634	....	8 12 26·5	3·510	1	68 30 37·2	11·04	1	cF; S; R; bM.....................	2
1647	3119	......	....	8 12 27·9	2·633	1	111 22 33·6	11·02	1	vF; S; R; gbM; am 60 ± st ..	1
1648	......	III. 288	....	8 12 48·8	2·549	1	115 1 53·2	11·04	1	vF; cL; er	1
1649	{503 = 3120}	VII. 64	....	8 12 58·1	2·421	3	120 12 19·8	11·06	3	{Cl; pL; pRi; lC; iR; st 11...14.}	4
1650	......	......	D'Arrest, 53	8 13 10	3·50	[2]	68 41 48	10·98	[2]	vF; cE; 3vS st f	0
1651	502	VI. 39	....	8 13 15·1	+2·444	1	119 18 33·1	11·07	1	Cl; vL; cRi; lC; st 9, ...	3
1652	3176	......		8 13 25·4	−140·624	1	179 41 7·5	7·74	1	F; S; R; glbM; Polaris. Austr.	1*
1653	......	II. 259	....	8 14 16·8	+3·543	1	66 57 59·4	11·18	1	F; S; iF; r	1
1654	3121	III. 902	....	8 14 49·0	2·818	2	102 52 35·0	11·20	2	F; vlE; gbM; r; am 50 st ...	3
1655	3122	......	....	8 15 35·1	2·272	4	125 46 36·2	11·24	4	⁑ = h. 4083 in pS neb; am 70 st.	4
1656	3123	......	....	8 15 44·3	2·435	1	119 52 8·5	11·25	1	Cl; cL; pRi; pC; R; st 12...	1
1657	504	III. 753	....	8 17 3·0	3·488	2	69 13 21·1	11·37	3	vF; pS; R; glbM; * p 75″...	4
1658	3124	......	....	8 17 39·0	2·461	1	119 2 32·8	11·46	1	Cl; pmCM; iF; st 9, 10...13.	1
1659	3125	......	....	8 17 39·2	2·369	1	122 31 10·1	11·47	1	Cl; F; S; R; gbM; st 10 ...	1
1660	505	II. 315	....	8 18 41·2	3·615	1	63 35 7·7	11·49	1	pF; S; R; vsbM*	2
1661	506	III. 599	....	8 19 34·0	3·511	1	68 3 46·8	11·56	1	vF; pL; iF; r; ⁑ sp 2′.........	2
1662	507	III. 234	....	8 23 58·8	+3·529	1	66 58 8·1	+11·87	1	vF; S; stellar	2

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	h.	H.		h m s	s		° ' "	"			
1663	3126	......	....	8 24 27·7	+0·438	1	157 39 23·2	+11·84	1	F; pS; R; gbM	1
1664	508	......	....	8 24 32·8	4·510	2	36 41 28·9	11·93	2	eF; S; R; *95°; p of 2	2
1665	509	III. 292	....	8 24 48·6	3·691	1	59 59 13·9	11·93	2	vF; pL; R; lbM; r; ⁑ nr ...	4
1666	510	......	....	8 25 8·2	4·500	2	36 44 52·5	12·05	2	cF; S; R; f of 2; *310°	2
1667	510, a	......	R. nova	8 25 8·2	4·500	::	36 40 52·5	12·05	::	Place from 510 h. by MS. ...	0*
1668	510, b	......	R. nova	8 25 ±	......	...	36 44 ±	......	...	No description or place	0
1669	511	......	....	8 25 27·2	3·628	1	62 32 51·4	11·98	1	eF	1
1670	512	II. 318	....	8 26 43·8	3·661	2	61 3 35·1	12·07	2	F; vlE; mbM; r	3
1671	3130	......	....	8 26 48·7	1·170	2	150 38 29·3	12·01	2	Cl; pS; lRi; lC	2
1672	513 = 3127	IV. 35	....	8 26 57·1	2·771	2	105 39 51·8	12·06	2	F; S; att to *13; *7 nf, 10s...	3
1673	3128	......	....	8 27 10·4	2·830	1	102 41 44·4	12·08	1	B; S; E; psbM; bet 2 st	1
1674	3129	II. 266	....	8 27 14·9	2·628	3	112 29 47·4	12·08	3	cB; L; vmE 110°·3	5
1675	515	III. 257	....	8 28 48·7	3·093	1	88 48 50·0	12·20	1	eF; pL; iF	2
1676	514	II. 319	....	8 29 0·5	3·662	3	60 49 18·6	12·22	3	F; pS; R; bM; r	4
1677	3131	......	....	8 30 27·5	2·178	3	130 11 3·7	12·29	3	*9 inv in pB, pL, R, neb ...	3†
1678	516 = 3132	VII. 63	....	8 31 28·3	2·476	2	119 28 1·1	12·37	2	Cl; cL; pRi; pC; st 11...13..	4
1679	......	III. 982	H. O. N.	8 31 32·3	6·536	1	16 45 45·1	12·47	1	vF; S; stellar	1
1680	......	III. 235	....	8 31 51·1	3·539	1	65 57 15·6	12·42	1	eF; S	1
1681	517	......	M. 44	8 32 9·0	3·462	1	69 32 36·2	12·44	1	Præsepe Cancri	3
1682	......	III. 983	H. O. N.	8 32 33·3	6·501	1	16 51 47·2	12·54	1	vF; S; stellar	1
1683	3133	......	....	8 32 58·2	2·357	1	124 16 9·1	12·47	1	Cl; pmC; irr Δ; st 13	1
1684	518	I. 204	....	8 33 37·9	4·342	4	39 17 45·8	12·56	4	cB; S; E 130°±; psmbM*?..	5
1685	3134	......	....	8 33 41·5	1·596	2	144 37 47·0	12·50	2	pB; S; R; 3 or 4 vS st p nr...	2
1686	519	......	....	8 34 20·5	3·005	1	93 37 53·4	12·50	1	vF; pL; gbM; r; 2 pB st s, sf	1
1687	3136	......	....	8 34 20·5	2·000	1	135 43 55·5	12·55	1	Cl; S; st L & S	1
1688	521 = 3135	III. 49	....	8 34 51·4	3·345	3	75 13 2·6	12·62	3	F; S; vlE 135°±; psbM	5
1689	522	II. 727	....	8 35 17·8	3·802	2	54 47 16·5	12·65	3	F; L; R; r	4
1690	......	II. 908	....	8 35 58·9	6·024	1	19 11 53·8	12·76	1	pB; pL; iF; er	1
1691	520	I. 288	....	8 37 4·6	8·188	1::	11 15 26·7	12·89	1	vB; cL; lE 90°±; g, svmbM.	2
1692	523	......	....	8 37 28·1	4·503	1	35 36 46·6	12·82	1	eF; psbM	1
1693	4017	......	Δ. 609	8 37 45·6	2·423	1	122 9 4·7	12·79	1	Cl; pS; lRi; lC; viF; st 12, 13	1
1694	3137	......	....	8 37 50·2	2·060	2	134 27 15·7	12·79	2	Cl; L; Ri; pmE; st 11...14...	2
1695	3138	......	....	8 38 1·3	1·977	2	136 42 16·0	12·80	2	Cl; pS; mC; iR; gbM; st 13...15.	2
1696	......	III. 50	....	8 38 43·9	3·309	1	76 54 43·4	12·88	1	eF; cL; R; lbM	1*
1697	3139	......	....	8 38 59·6	2·800	1	104 47 19·7	12·89	1	vF; vS; R; bM; *15m nr ...	1
1698	524	......	....	8 39 29·7	3·309	1	76 53 9·2	12·94	1	Cl; st 9...10	1
1699	......	......	D'Arrest, 54	8 40 20	3·43	[1]	70 27 48	12·99	[1]	eF	0
1700	525	......	....	8 40 20·3	4·183	1	42 25 39·3	13·01	1	Cl; lC	1
1701	3140	......	....	8 40 50·8	1·694	1::	143 27 29·7	12·99	1::	Cl; L; P; lC; st 10...13	1
1702	3142	......	....	8 41 3·4	1·928	1	138 16 31·0	13·00	1	Cl; pL; P; lC; st 13	1
1703	3141	......	Δ. 489? 490?	8 41 7·1	2·174	1	131 22 30·3	13·01	1	Cl; pRi; lCM; st 12...13 ...	1
1704	526	II. 80	....	8 41 23·5	3·432	3	70 24 45·8	13·06	3	B; pL; lE 10° or biN; mbM*	5
1705	526 a	......	R. nova	8 41 23±	......	...	70 24 ±	......	...	Nearly in contact with h. 526 (see description of h. 526).	?
1706	......	......	D'Arrest, 55	8 41 52	4·46	[1]	35 58 48	13·11	[1]	vF; R; *15 p 12s, 270°	0
1707	527	II. 48	....	8 42 1·3	3·430	1	70 28 19·0	13·10	1	eeF; pL; lbM; r	2*
1708	528	VIII. 10	....	8 42 32·9	3·283	3	78 8 45·2	13·14	4	Cl; vlC; P	5
1709	529	III. 294	....	8 42 59·4	3·685	1	58 36 38·1	13·17	1	pF; vS; R; bM	2
1710	529, a	......	R. nova	8 43 ±	......	...	58 36 ±	......	...	Makes e close D neb with h. 529.	0
1711	530	I. 242	....	8 43 26·1	4·348	2	38 9 35·3	13·21	2	vB; L; vg, vsmbM*10	3
1712	531	......	M. 67	8 43 34·3	3·291	4	77 40 36·0	13·20	5	!; Cl; vB; vL; eRi; lC; st 10...15.	8*
1713	532	I. 200	....	8 43 58·0	+3·746	3	56 3 38·9	+13·23	3	vB; vL; vmE 40°·9; gmbM ..	5

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	h.	H.		h m s	s		° ′ ″	″			
1714	533	III. 712	....	8 45 2·1	+4·247	2	40 18 38·6	+13·32	2	F; pL; R; gbM; 4 S st nr ...	3
1715, 1716, 1717	533, a	......	R. 3 novæ	8 45 ±	......	...	40 18 ±	......	...	4 (incl h. 533) nearly in a line	0
1718	......	II. 658	....	8 45 42·6	3·909	1	49 55 12·8	13·36	1	pF; vS; mbM	1
1719	534	III. 831	....	8 46 48·6	4·366	1	37 23 56·2	13·44	1	vF; S; R; psbM	2
1720	535	II. 823	....	8 46 53·6	4·333	1	38 7 1·2	13·44	1	pB; mE 0°±; psmbM	2*
1721	536	II. 280	....	8 47 20·4	3·027	1	92 32 2·2	13·44	1	pF; cS; E 90°±; bet 2 st ...	2†
1722, 1723, 1724	536, a	......	R. 3 novæ	8 47 ±	......	...	92 32 ±	13·48	...	No description	0
1725	536, b	......	R. nova	8 48 0·0	3·025	...	92 28 2·2	......	...	MS. No description	0
1726	538	......	....	8 48 33·4	3·025	1	92 39 34·6	13·52	1	vF; pS; R; r; *9 p............	1
1727	......	......	D'Arrest, 56	8 48 43	3·025	[2]	92 34 48	13·53	[2]	vF; S; R; *15 p, 44″ n; h. 538 nr.	0
1728	537	IV. 66	....	8 48 46·5	4·438	1	35 41 54·1	13·57	1	pB; fan-shaped; *11 att......	2†
1729	......	III. 625	....	8 48 54·1	3·892	1	50 7 22·8	13·56	1	vF; vS...........................	1
1730	......	II. 281	....	8 48 58·2	3·022	2	92 50 24·5	13·55	1	vF; pS, R	2
1731	......	III. 841	....	8 49 15·0	4·536	1	33 46 55·0	13·60	1	vF; S	1
1732	540	......	....	8 50 3·8	4·065	1	44 33 26·5	13·65	1	pB; L; E; vgbM*18	1
1733	3143	......	....	8 50 5·7	1·451	1	148 41 21·4	13·58	1	eF; S; R; pslbM...............	1
1734	3144	......	....	8 50 52·5	2·632	2	114 8 0·1	13·67	2	pF; S; R; vgpmbM	2
1735	542	......	....	8 51 19·6	3·189	? ?	83 7 51·0	13·70	1 ::	F; pL; R........................	1*
1736	......	II. 557	....	8 51 19·6	3·189	1	83 9 32·0	13·70	1	F; pL; mE	1*
1737	541	III. 540	....	8 51 25·7	3·785	1	53 44 3·6	13·72	1	vF; S; E 110°±; 2 vF st inv..	4
1738	539	......	....	8 51 28·6	8·388	1 :	10 14 54·9	13·83	1 ::	pB; S; E 45°±; *nf	1
1739	543	II. 529	....	8 51 59·2	2·997	2	94 21 30·2	13·74	2	cF; pL; R; vgbM	3
1740	......	III. 264	....	8 52 29·4	3·017	2	93 11 6·1	13·77	2	vF; vS; stellar	2
1741	544	......	....	8 53 6·7	3·781	1	53 42 41·9	13·83	1	eF; S; stellar	1
1742	545	II. 834	....	8 53 51·4	4·769	1	29 30 44·0	13·90	1	cF; pS; iR; er..................	2*
1743	546	......	....	8 54 2·0	3·023	1	92 50 15·1	13·87	1	vF; L; R; bM	1*
1744	547	......	....	8 55 0·5	3·018	1 ::	93 10 51·9	13·93	1	eF; R	1
1745	3145	......	....	8 55 30·8	2·097	1	135 20 55·2	13·94	1	!; eeF; vL; vvmE 19°.........	1†
1746	......	......	D'Arrest, 57	8 55 58	3·47	[2]	67 28 48	14·00	[2]	D neb; pB; S, not R; comes s 4′.	0
1747	......	......	D'Arrest, 58	8 55 59	3·46	[2]	67 32 48	14·00	[2]	vF; vS	0
1748	549	......	....	8 56 5·6	4·305	1	37 41 23·9	14·03	1	4S st in neb	1
1749	549, a	......	R. nova	8 56	......	...	37 41	......	...	Makes D neb with h. 549 ...	0
1750	550	I. 249	....	8 56 26·3	4·788	1	28 58 21·8	14·06	1	cB; cL; E 90°±; er	3
1751	......	III. 608	....	8 56 34·0	3·527	1	64 26 53·5	14·05	1	eF; S; R; vlbM	1
1752	551	III. 60	....	8 56 43·6	3·399	2	70 59 30·5	14·05	2	vF; S; R; r; ⁑ nr	3
1753	548	......	....	8 57 9·1	7·170	1 :	12 56 4·8	14·16	1 ::	pB; pL; E; vglbM	1
1754	552	III. 825	....	8 57 15·0	3·760	4	54 3 51·7	14·09	4	eF; S; R; vglbM; *12, 345°, 50″.	5
1755	......	......	D'Arrest, 59	8 57 28	3·39	[4]	71 7 5	14·09	[4]	pF; S; R; bMN=*15	0
1756	......	III. 291	....	8 57 34·4	3·514	3	64 0 27·3	14·11	3	vF; cL; R; bMN; 2 c st p ...	3*
		......	D'Arrest, 60	8 57 35	3·53	[2]	64 0 24	14·10	[2]	*15·16 inv in pB; pL neb 40″ diam.	0*
1757	......	......	D'Arrest, 61	8 57 55	3·53	[1]	64 6 12	14·12	[1]	vF; vS	0
1758	......	III. 626	....	8 58 51·7	3·928	2	47 44 29·7	14·19	2	vF; S; iF; lbM; r	2
1759	553	II. 828	....	8 58 52·0	4·387	1	35 35 23·3	14·21	1	pB; pS; E; vgbM	2
1760	554	III. 647	....	8 59 32·7	3·811	1	51 48 43·9	14·23	1	vF; cS; R	3
1761	560	III. 275	....	9 0 13·6	2·820	1	104 56 21·5	14·25	1	vF; pS; bM; S* 30″ n	2
1762	557	III. 236	....	9 0 16·2	3·450	1	67 59 30·1	14·27	1	cF; vS; R; er; bet 2 pB st ...	3
1763	558	II. 520	....	9 0 16·9	3·136	1	86 2 37·8	14·26	1	vF; pL; E; gbM; er	3
1764	556	......	....	9 0 29·6	4·225	1	39 2 40·0	14·30	1	eF; sbM*15; 1st of 3.........	1
1765	555	I. 250	....	9 0 41·1	4·728	1	29 23 40·6	14·32	1	cB; cL; lE; psmbMLBN ...	2
1766	559	......	....	9 0 51·3	4·224	2	39 0 48·6	14·32	2	pF; S; E; pslbM; 2nd of 3...	2
1767	559, a	......	R. nova	9 0 ±	......	...	39 ±	......	...	One of 4 (h. 556, 559, 561); one vF; one E.	0
1768	562	II. 490	....	9 0 56·7	+3·694	3	56 18 26·3	+14·31	3	F; L; mE 150°; r; 2 st n......	4

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	h.	H.		h m s	s		° ′ ″	″			
1769	561	......	....	9 1 0·0	+4·226	2	39 4 21·1	+14·27	2	vF; vS; lE; 3rd of 3	2
1770	3146	......	....	9 1 27·6	2·671	1	113 5 19·6	14·32	1	eF; lE; lbM?	1
1771	564	I. 2	....	9 2 52·2	3·195	2	82 23 38·9	14·43	2	cB; cL; R; vg, vsmbM; r? ..	7
1772	563	......	....	9 2 55·9	4·014	1	44 28 15·5	14·45	1	pB; L; R; vgbM; r	1
1773	565	III. 61	....	9 3 1·4	3·377	1	71 44 55·2	14·44	1	vF; S; R; am 5S st; (?PD 70°)	2*
1774	566	II. 564	....	9 3 44·1	3·732	4	54 24 3·7	14·49	4	pB; S; R; psmbM	6
1775	566, *a*	......	R. nova	9 3	......	...	54 24	......	...	eF; companion of h. 566, 567.	0
1776	567	III. 826	....	9 4 3·5	3·728	1	54 29 46·3	14·51	1	vF; S; R; S ** 7·5 p	2
1777	569 = 3147	I. 66	....	9 4 50·4	2·837	2	104 14 58·2	14·54	2	B; S; pmE 90°±; psmbM...	3
1778	568	I. 167	....	9 5 16·1	3·858	(1)	49 21 24·4	14·58	(1)	cB; R; mbMBN...............	2
1779	......	III. 295	....	9 5 21·2	3·613	1	59 23 25·4	14·58	1	vF; vS; R; 2pB st sp	1
1780	571 = 3148	I. 59	....	9 6 5·8	2·670	3	113 36 36·3	14·61	3	B; L; mE63°·7; gmbM	5
1781	570	I. 216	....	9 6 25·9	5·535	1	20 12 11·7	14·69	1	B; pL; lE90°±; mbM; r; vS*sf inv.	4
1782	3150	......	....	9 6 36·0	0·866	1::	157 21 50·0	14·60	1	vF; vS; mE105°...............	1
1783	3149	......	....	9 7 10·6	2·253	3	131 51 37·1	14·67	3	!; ○; pB=*9; vS; R; am st.	4
1784	572	......	....	9 8 11·6	3·705	2	54 59 4·8	14·76	2	vF; S; R; ** pl°, n5′	2
1785	573	III. 296	....	9 8 17·0	3·624	1	58 31 50·8	14·76	1	eF; S; R; lbM	2
1786	575	III. 62	....	9 8 21·5	3·392	(1)	70 27 55·8	14·76	1	vF; S; R; r.....................	2
1787	575	III. 63	....	9 8 21·8	3·392	(1):	70 28 25·8	14·76	1	vF; S; R; r.....................	2
1788	......	II. 708	....	9 8 31·6	3·899	1	47 27 36·4	14·78	1	pB; S; stellar	1*
1789	574	III. 832	....	9 8 39·8	4·271	1	36 54 12·0	14·80	1	vF; S; lE; * att; * inv	2
1790	......	III. 878	....	9 8 45·5	4·971	2	25 18 38·6	14·82	2	vF; L; R; mbM...............	2
1791	577	......	....	9 8 54·8	3·413	1	69 13 19·7	14·79	1	vF; S; R; np of 2	1*
1792	......	......	D'Arrest, 62	9 9 5	3·34	[3]	69 22 9	14·80	[3]	vF; vS; h. 578 f7s·5; Δ.P.D. 118″.	0*
1793	3152	......	Δ. 265	9 9 9·9	1·185	4	154 17 18·8	14·76	4	!; ⊕; vL; eRi; vgeCM; 45s d; st 13...15.	4
1794	578	......	....	9 9 12·3	3·410	1:	69 21 43·3	14·81	1	vF; S; R; sf of 2	1
1795	......	III. 749	....	9 9 35·0	5·953	1	17 34 40·4	14·88	1	F; cS; bM	2
1796	580 = 3151	II. 505	....	9 9 37·1	2·817	2	105 43 14·6	14·82	2	pB; pS; E 45°±; psmbM ...	3
1797	......	II. 868	....	9 9 39·0	4·975	(1)	25 9 42·1	14·87	(1)	F; S; iF; 1st of 2	1
1798	576	II. 869	....	9 9 41·2	4·974	1	25 10 11·1	14·87	1	F; S; E; 2nd of 2	2
1799	3153	III. 242	....	9 10 3·6	2·688	1	113 2 16·5	14·85	1	F; S; lE; gbM	3
1800	579	......	....	9 10 17·0	4·684	1	28 57 59·7	14·89	1	F; pmE	1
1801	3154	......	Δ. 564	9 10 21·5	2·417	2	126 1 53·8	14·86	2	!; ○pB; pL; R; vglbM; in L, C, Cl.	2†
1802	3155	......	....	9 10 39·7	2·628	1	116 14 46·4	14·88	1	eF; *11 att	1
1803	3156	......	....	9 10 41·4	0·713	1::	159 3 42·5	14·85	1	pF; vS; R; glbM	1
1804	581, *a*	......	R. nova	9 10 43·8	3·680	: :	55 49 34·9	14·88	: :	R.MS. No description	0*
1805	581, *b*	......	R. nova	9 10 43·8	3·680	: :	55 19 34·9	14·88	: :	R.MS. No description	0*
1806	581	......	....	9 10 50·9	3·680	1	55 40 39·3	14·91	2	vF; E; I. 113 f	2*
1807	......	......	D'Arrest, 63	9 10 52	3·68	[2]	55 47 48	14·91	[2]	vF; vS; R; h. 581, 6′ n	0*
1808	581, *c*	......	R. nova	9 10 53·4	3·680	: :	55 29 4·9	14·92	: :	R.MS. No description	0*
1809	581, *d*	......	R. nova	9 10 58·4	3·680	: :	55 47 34·9	14·93	: :	R.MS. No description	0*
1810	582, *a*	......	R. nova	9 11 3·1	3·680	: :	55 28 34·9	14·93	: :	R.MS. No description	0*
1811	582	I. 113	....	9 11 12·9	3·680	2	55 39 34·9	14·93	2	cB; cL; lE; mbf; 3 st s	4*
1812	582, *b*	......	R. nova	9 11 14·5	3·680	: :	55 40 20·3	14·92	: :	β in Lord R.'s diag. } v nr I.	0*
1813	582, *c*	......	R. nova	9 11 16·0	3·680	: :	55 27 34·9	14·92	: :	α in Lord R.'s diag. } 113.	0*
1814	582, *d*	......	R. nova	9 11 17·6	3·680	: :	55 27 34·9	14·92	: :	R.MS. No description	0*
1815	582, *e*	......	R. nova	9 11 32·6	3·680	: :	55 42 29·3	14·92	: :	s of Lord R.'s diagram	0*
1816	3157	......	....	9 11 41·7	+0·760	1	158 45 49·0	+14·90	1	F; pS; R; glbM	1

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	h.	H.		h m s	s		° ′ ″	″			
1817, 1818	582, f	......	R. 2 novæ	9 11±	......	...	55 40±	......	...	2 of 15 seen	0*
1819	585	......	....	9 11 45·3	+2·817	1	105 53 11·2	+14·94	1	eF; R; bM; *f 8s·5	1
1820	583	III. 627	....	9 11 50·3	3·813	1	50 7 15·1	14·97	1	vF; vS; R	3
1821	582, g	......	R. nova	9 12 6·0	3·680	: :	55 46 35·3	14·98	: :	ζ of Lord R.'s diagram	0*
1822	586	III. 827	....	9 12 13·5	3·714	1	54 2 26·7	14·99	1	cF; S; R; *10 np 2′	3
1823	584	I. 205	....	9 12 19·6	4·187	1	38 25 33·3	15·01	1	vB; L; vmE 150°·8; vsmbM = *10.	3
1824	3158	......	....	9 12 29·8	1·357	1	152 28 51·8	14·96	1	F; vS; bet 2 st	1
1825	......	III. 64	....	9 12 41·2	3·385	1	70 28 52·3	15·01	1	S* and neb	1
1826	......	III. 628	....	9 12 53·2	3·832	1	49 15 51·9	15·03	1	cF; cS	1
1827	3159	......	....	9 13 2·9	2·392	1	127 25 37·3	15·01	1	vF; S; R; *12 att sf	1
1828	587, a	......	R. nova	9 13 30	......	...	105 55	......	...	np 587 h.; close	0
1829	587	III. 488	....	9 13 30·4	2·819	1	105 55 14·5	15·05	1	vF; cL; E 45°±; glbM; *11 sf 9s.	2
1830	3160	......	....	9 13 52·9	2·330	1	129 57 21·8	15·06	1	eF; cL; R; vglbM rr	1
1831	588	III. 629	....	9 14 20·7	3·826	1	49 16 29·6	15·12	1	vF; cS; R; *10 p 2′; 1st of 2	2
1832	590	III. 630	....	9 14 24·2	3·827	1	49 14 39·6	15·12	1	vF; S; vgbM; 2nd of 2	2*
1833	589	III. 714	....	9 14 30·9	4·108	2	40 11 39·9	15·13	2	cF; cS; vlE; pglbM; 1st of 2	3
1834	589, a	......	R. nova	9 14±	......	...	40 11±	......	...	Seen with h. 589, 591	0
1835	592	I. 132	....	9 14 41·5	2·895	1	101 18 57·6	15·12	1	pB; pL; R; gmbMN	4
1836	591	III. 713	....	9 14 44·2	4·108	2	40 9 20·2	15·14	2	cF; cS; lE; bM; 2nd of 2	3
1837	593	I. 137	....	9 15 45·1	3·684	3	54 53 13·7	15·19	3	vB; pL; R; smbM	4
1838	594	III. 520	....	9 16 48·2	2·920	1	99 50 3·2	15·24	1	cF; S; E; bet 2 st 12, 16	2
1839	......	II. 57	....	9 17 8·3	3·259	1	77 46 1·8	15·26	1	F; vS, p of 2	1
1840	......	II. 58	....	9 17 11·2	3·258	1	77 46 16·1	15·27	1	pF; S, f of 2	1
1841	3161	......	....	9 17 11·3	2·710	2	112 34 33·8	15·26	2	B; S; R; gbM	2
1842	3162	......	....	9 17 21·9	2·016	1	140 30 21·5	15·25	1	Cl; lC	1
1843	3163	......	....	9 17 28·1	1·694	7	147 42 57·5	15·25	7	!!; ◯ = *8; vS; R; *15, 59°·13, 13″.	8
1844	595	III. 846	....	9 17 37·9	4·454	1	32 1 44·8	15·26	1	cF; S; E; vglbM	2
1845	597	II. 546	....	9 18 8·2	3·255	5	77 57 53·6	15·32	6	pF; pS; R; bM; p of 2, 109°	7
1846	597, a	......	R. nova	9 18	......	...	77 58	......	...	Forms Δ with 2 E neb	0
1847	598	II. 547	....	9 18 13·1	3·255	6	77 58 34·9	15·33	6	vF; pL; R; bM; f of 2	7
1848	596	I. 260	....	9 18 33·5	4·758	2	26 54 27·1	15·37	2	B; cS; R; mbM; am st	3
1849	3164	......	....	9 19 30·4	2·502	1	123 29 36·4	15·38	1	vF; S; vglbM; rrr; st 11m...	1
1850	599	......	....	9 19 42·4	3·445	1::	66 22 59·3	15·41	1	eF; vS; E 90°±	1
1851	3165	......	....	9 20 8·5	2·740	1	111 8 37·6	15·42	1	eeF; pL	1
1852	3168	......	....	9 20 11·4	1·370	2	153 12 29·0	15·40	3	F; S; R; pmbM; B*nr	3
1853	3166	......	....	9 20 12·5	2·627	2	117 25 24·9	15·43	2	cF; S; R; gmbM	2
1854	600	II. 555	....	9 20 24·7	2·904	1	101 2 12·2	15·44	1	pF; pS; vlE; vglbM; r	2
1855	3167	......	....	9 20 41·0	2·688	1	114 11 43·5	15·45	1	F; S; R; bM	1
1856	602	III. 297	....	9 21 59·9	3·562	2	59 49 57·2	15·54	2	vF; S; R; vsbM*12	3
1857	603	III. 8	....	9 22 2·3	3·195	1	81 39 42·2	15·54	1	vF; E; er; 2 or 3 st inv	4
1858	601	......	....	9 22 12·0	4·421	1	31 54 24·1	15·57	1	vF; vS; R; vgbM; ⁑ 7′s ...	1
1859	3169	......	....	9 22 42·2	1·839	1	145 30 15·5	15·55	1	F; pL; R; gmbM; am 80 st	2
1860	......	III. 276	....	9 24 5·4	2·861	1	104 6 31·2	15·64	1	vF; vS; stellar	1
1861	604·1	I. 56	....	9 24 14·6	3·409	2	67 53 46·8	15·66	3	cB; vL; E; gmbM; r; sp of 2	4†
1862	3170	......	....	9 24 15·4	2·592	1	119 46 47·5	15·65	1	F; S; lE; psbM	1
1863	604·2	I. 57	....	9 24 16·3	3·410	2:	67 52 50·1	15·67	2:	vF; cL; R; psbM; r; nf of 2	4†
1864	606	II. 495	....	9 24 37·6	3·203	2	80 57 11·4	15·68	2	F; pS; lE; gbM	3
1865	607	II. 506	....	9 25 2·9	2·830	1	106 7 27·0	15·70	1	pF; S; lE 90...180; mbsf ...	2
1866	......	III. 977	....	9 25 15·9	7·880	1	9 37 30·0	15·80	1	eF; vS	1
1867	605	......	....	9 25 26·7	4·972	1	23 26 15·8	15·76	1	eF; S; psbM	1
1868	3171	......	....	9 25 37·0	1·992	3	142 17 26·0	15·70	3	Cl; eL; pRi; pC; st 10...14.	3*
1869	608	II. 40	....	9 26 15·3	3·228	1	79 13 53·1	15·77	1	F; pL; R; gbM; p of 2	4
1870	609	III. 513	....	9 26 33·1	+3·227	1	79 16 23·4	15·78	1	vF; S; R; bMN, f of 2	2
1871	3174	......	....	9 26 54·2	−0·275	1	166 0 46·2	15·74	1	pF; pL; R; gbM	1
1872	610	II. 260	....	9 26 57·2	+3·408	2	67 40 32·3	15·81	2	F; S; vlE	3
1873	611	III. 298	....	9 27 24·1	3·589	1	57 40 57·2	15·84	1	vF; cS; R; sbMN	3
1874	3172	......	....	9 27 49·1	+2·769	1::	110 13 55·2	+15·84	1::	eF; S; R; p of 2	1

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	h.	H.		h m s	s		° ′ ″	″			
1875	3173	III. 597	....	9 28 2·3	+2·768	2	110 17 59·8	+15·86	2	vF; pS; lE; vglbM; f of 2...	3
1876	3175	......	....	9 28 32·8	2·840	1	105 46 48·4	15·88	1	pB; S; R..........................	1
1877	3177	......	....	9 28 57·4	1·993	1	142 49 22·7	15·89	1	Cl; pRi; pC; ⁑ taken...........	1
1878	......	......	D'Arrest, 64	9 29 32	3·43	[1]	66 10 24	15·94	[1]	eF; vS; lE; vlbM; 1st of 3...	0
1879	......	......	D'Arrest, 65	9 29 34	3·43	[1]::	66 12 12	15·94	[1]	eF; S; 2nd of 3	0
1880	......	......	D'Arrest, 66	9 29 42	3·43	[1]	66 8 18	15·95	[1]	eF; vS; 3rd of 3..............	0
1881	3179	......	....	9 30 6·6	2·221	1::	136 18 58·8	15·96	1::	Cl; eL; vRi; st L & S	1
1882	3178	II. 556	....	9 30 13·5	2·768	2	110 30 23·1	15·97	2	pB; pS; vlE; gmbM	5
1883	612	III. 963	....	9 30 28·4	6·574	1	12 47 47·8	16·06	1	eF; S; iFig; ⁑ f 3′	2
1884	614	III. 4	....	9 30 39·3	3·215	2	79 51 27·0	16·00	3	vF; S; vlE; bM; Δ st nf......	5
1885	613	......	....	9 30 42·6	3·626	2	55 21 52·3	16·01	2	F; pL; vl E0°; vglbM.........	2
1886	3180	......	....	9 31 15·5	2·756	2	111 24 53·9	16·03	2	F; S; R; glbM; 2 or 3 S st nr	2
1887	615	III. 519	....	9 31 34·7	3·178	(1)	82 24 1·5	16·05	1::	vF; pL; vgbM	2
1888	616	IV. 68	....	9 32 17·7	4·410	2	30 31 15·3	16·11	2	cF; vS; R; vgvmbMN	3
1889	3182	......	....	9 32 38·7	3·291	1::	74 32 3·3	16·11	1::	eeF; susp	1
1890	3181	......	....	9 32 44·3	3·292	1	74 26 8·3	16·11	1	vF; S; R; n of 2................	1
1891	620	III. 541	....	9 32 45·9	3·659	1	53 29 0·6	16·12	1	cF; pS; iR; glbM; r	4
1892	617	......	....	9 33 7·8	5·746	1	16 22 12·4	16·18	1	eF; ✱13 nr	1
1893	618	......	....	9 33 12·5	5·149	1	20 45 24·1	16·17	1	F; pL; R; vglbM; ✱ n	1
1894	621	......	....	9 33 24·7	3·131	1	85 46 2·7	16·19	1	vF; R; gbM	1
1895	619	III. 315	....	9 33 39·4	5·734	1	16 23 43·3	16·21	1	vF; vS; R; bM	2
1896	622	I. 114	....	9 34 34·3	3·571	4	57 31 54·6	16·22	3	B; vL; lE; vgbM; p of 2 ...	5
1897	623	III. 751	....	9 34 46·9	3·660	1	53 6 33·9	16·23	1	eF; vS; R; bM; r	3
1898	626	II. 275	....	9 34 49·3	3·094	1	89 1 56·6	16·22	2	pF; pL; R; vglbM..............	4
1899	624	II. 491	....	9 34 51·5	3·572	4	57 25 34·9	16·23	4	pB; pL; lE; vglbM; f of 2...	5
1900	628	III. 527	....	9 34 59·1	2·961	1	97 57 26·9	16·23	1	vF; pS; iR; vglbM............	3
1901	627	......	....	9 35 7·4	3·573	1	57 21 1·2	16·24	1	F; nf of 3........................	1
1902	3183	......	Δ. 397	9 35 16·9	2·143	1	139 41 32·9	16·23	1	Cl; S; lRi; pC; st 13	1
1903	4018	......	....	9 35 22·9	2·628	1	119 24 42·2	16·24	1	eF; pS; B✱8m f	1
1904	630	I. 61	....	9 35 27·7	3·029	1	93 4 16·5	16·25	1	B; cS; iR; bM; ✱9 sp 3ˢ......	3
1905	625	I. 285	....	9 35 35·6	5·048	1	21 26 21·7	16·29	1	B; vL; mE 152°·4; st inv ...	2
1906	......	I. 282	....	9 35 47·6	6·111	1	14 15 5·9	16·33	1	cB; pL; iF	1
1907	631	III. 521	....	9 36 17·2	2·937	1	99 44 57·7	16·29	1	pF; pS; vlE; psbM.............	2
1908	632	III. 528	....	9 36 18·4	2·948	1	98 58 2·7	16·29	1	vF; pS; lE 0°±; vglbM......	3
1909	629	I. 78	....	9 36 41·2	5·576	1	17 4 29·8	16·36	1	vB; cL; R; psmbM; ✱ inv f..	3
1910	3184	......	....	9 36 52·2	2·330	1	133 33 40·3	16·31	1	Cl; P; E; st 10...11	1
1911	3185	III. 289	....	9 37 7·5	2·791	1	109 49 54·9	16·33	1	F; pS; R; bM; r; stellar ...	4*
1912	633	III. 34	....	9 37 23·0	3·231	(1)	78 20 10·5	16·35	2::	eF; vS; R; bM (? P.D. 15′)..	3
1913	......	II. 311	....	9 38 10·3	2·780	3	110 37 58·7	16·39	3	pB; pS; iR; mbM	3
1914	......	II. 624	....	9 38 39·5	3·156	1	83 41 19·3	16·41	1	F; pS; lE 90°±	1
1915	3186	......	....	9 38 52·0	2·825	1	107 44 26·6	16·42	1	F; R; gbM; ⁑ f	1
1916	634, a	......	R. nova	9 38	......	...	67 20	......	...	Makes a D neb with h. 634; which follows it.	0
1917	634	......	....	9 38 55·9	3·391	1	67 20 5·9	16·43	1	F; vS; bM; sp of 2	1
1918	635	III. 277	....	9 38 58·1	2·884	1	103 41 8·9	16·43	1	cF; S; R; bM; stellar; p of 2	2
1919	637	III. 278	....	9 39 4·6	2·884	1	103 43 23·9	16·43	1	cF; S; R; bM; stellar; f of 2	2
1920	636	......	....	9 39 18·1	3·391	1	67 16 32·5	16·45	1	F; S; R; bM; nf of 2.........	1
1921	3189	......	....	9 39 21·6	2·008	1	144 8 1·9	16·43	1	Cl; P; lC; st mm..............	1
1922	3187	......	....	9 39 28·0	2·778	1	110 56 47·5	16·45	1	vF; S; ✱20 f 1′	1
1923	3188	V. 50	....	9 39 32·5	2·619	2	120 32 50·5	16·45	2	!; vF; vL; vg, vsbMN 4″; 19″·5 d.	3
1924	638	II. 717	....	9 40 2·0	3·823	1	45 15 27·0	16·50	1	pF; pL; iR; bM; r............	2
1925	638, a	......	R. nova	9 40 4·8	3·823	...	45 16 27·0	16·48	...	RMS	0
1926	638, b	......	R. nova	9 40 4·8	3·823	...	45 16 57·0	16·48	...	Suspected; MS	0
1927	3191	......	Δ. 397	9 40 5·0	2·167	1?	139 47 14·1	16·47	1?	Cl; S; lRi; iF; st 12...15 ...	1
1928	638, c	......	R. nova	9 40 7·6	3·823	...	45 12 27·0	16·48	...	MS	0
1929	3190	......	....	9 40 10·4	2·634	1	119 48 2·4	16·48	1	F; S; R; ✱12 att 320°	1
1930	638, d	......	R. nova	9 40 13·3	3·823	...	45 21 27·0	16·49	...	MS	0
1931	639	V. 26	....	9 40 15·0	3·584	1	55 56 14·0	16·50	1	!; cB; L; vimE 90°...........	3†
1932	638, e	......	R. nova	9 40 30·2	3·823	...	45 13 27·0	16·52	...	MS	0
1933	640	......	....	9 41 1·2	+3·825	1	45 1 50·5	+16·55	1	pF; R; bM; r; p of 2.........	1

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	h.	H.		h m s	s		° ′ ″	″			
1934	638, f	......	R. nova	9 41 4·0	+3·823	...	45 15 27·0	+16·55	...	MS	0
1935	641	......	....	9 41 7·2	3·825	1	45 0 16·5	16·55	1	F; psbM; rr; f of 2	1
1936, 1937	641, a	......	R. novæ	9 41 ±	......	...	45 0 ±	......	...	Several near	0
1938	......	......	D'Arrest, 67	9 41 32	3·49	[1]	54 38 7	16·56	[1]	vF; pL; R; cometary	0
1939	642	......	....	9 41 32·1	3·251	1	76 32 0·8	16·56	1	F; pL; R; glbM	1
1940	642, a	......	R. nova	9 41	......	...	76 32	......	...	3 "novæ," with 642 (Vide h. 646, 648).	0
1941	644	......	....	9 42 3·2	3·011	1	94 33 38·4	16·58	1	eF; L; p of 2	1
1942	646	III. 51	....	9 42 31·0	3·250	3	76 31 58·3	16·61	3	eF; pS; lE 0°±; r	4
1943	647	......	....	9 42 38·9	3·012	1	94 31 9·3	16·61	1	F; R; vglbM; f of 2	1
1944	645	I. 115	....	9 42 39·6	3·579	2	55 47 35·6	16·62	2	pB; pS; vlE; mbM; *10 sf 100°.	4
1945	648	III. 52	....	9 42 54·1	3·249	1	76 36 18·9	16·63	1	eF; pL; E; r	2
1946	3192	......	....	9 42 58·1	2·782	1	111 5 19·6	16·62	1	eF; vS; R; *9 s	1
1947	643	V. 23	....	9 43 9·9	5·466	1	17 8 23·4	16·68	1	vF; vL; lE; r	3
1948	3193	......	....	9 43 17·7	2·821	1	108 31 38·2	16·64	1	F; S; R; lbM	1
1949	649	......	M. 81	9 43 48·9	5·066	1	20 16 10·0	16·70	1	!; eB; eL; E 156°·0; g, svmbMBrN.	4
1950	......	IV. 79 = 4H. ON	M. 82	9 43 52·3	5·142	1	19 34 16·3	16·71	1	vB; vL; vmE "a beautiful ray."	2*
1951	650	......	....	9 44 0·3	3·497	2	60 7 7·4	16·68	2	F; S; sbM *12; bet 2B st ...	2
1952	3194	......	B. 2686	9 44 1·8	1·975	2	145 45 42·8	16·66	2	Cl; pL; pRi; iF; st 11...12...	2
1953	......	W. H. nova?	M. 81??	9 44 38·0	5·064	1	20 12 18·9	16·73	1	vB; cL; mE; 5 or 6 st (?) inv	1*
1954	3197	......	....	9 45 5·8	1·674	1	152 2 12·3	16·71	1	Cl; cL; lC	1
1955	3195	......	....	9 45 6·2	2·705	1	116 22 6·6	16·72	1	F; pS; R; lbM	1
1956	3196	II. 98	....	9 45 25·6	3·300	1	72 39 56·5	16·75	1	⊕; F; L; R; vglbM; rr; 2B st sp.	4
1957	651	II. 835	....	9 46 19·3	4·327	2	30 2 40·3	16·81	2	cF; pS; lE; vgbM; *10 n 7′	4
1958	652	III. 254	....	9 46 26·5	3·097	1	87 55 58·0	16·80	1	vF; vL; vmE 111°·5	3
1959	3198	......	....	9 46 38·8	2·834	1	107 59 3·0	16·80	1	vF; pS; R; lbM	1*
1960	3199	......	....	9 47 1·9	2·704	1	116 40 25·6	16·82	1	pF; R	1*
1961	3201	......	....	9 47 41·3	2·707	1	116 37 56·5	16·85	1	pF; S; R; gbM	1*
1962	3202	III. 272	....	9 47 50·8	2·836	1	107 58 44·8	16·86	1	F; pL; R; glbM	2*
1963	3200	III. 600	....	9 47 53·6	3·293	1	72 54 4·1	16·87	1	vF; S; vlE; gbM	2
1964	656	VI. 4	....	9 47 59·1	3·133	2	85 4 19·1	16·87	2	F; pL; vlE; vgbM; rr; *7f 90s.	4
1965	3203	......	....	9 48 15·8	2·692	2	117 38 37·4	16·88	2	pB; S; R; vgmbM; *11 att 203°·8.	2
1966	......	III. 978	....	9 48 24·1	7·497	1	9 3 41·8	16·96	1	eF; pL; vlbM; 2 S st s	1
1967	3205	......	....	9 48 31·2	0·647	1	163 16 15·5	16·85	1	F; L; iR; glbM; S * inv	1
1968	3204	III. 601	....	9 48 39·8	3·297	1	72 30 4·3	16·91	1	vF; cS; vlE; er	2
1969	654	II. 333	....	9 48 42·7	5·376	1	17 9 25·2	16·94	1	pF; vS; R; bM; *11 nr	3
1970	653	II. 903	....	9 48 44·4	6·102	1	13 10 9·5	16·95	1	vF; pL; r	2
1971	655	II. 334	....	9 48 57·4	5·366	1	17 12 16·5	16·95	1	vF; vS; vglbM	3
1972	......	II. 909	H. ON 5	9 49 50·5	5·382	1	17 12 24·7	16·99	1	F; pL; R; 3rd of 3	1
1973	657	II. 492	....	9 50 7·9	3·533	3	56 57 50·4	16·98	3	pB; pL; E 90±; gbM; *9 nf	4
1974	......	III. 293	....	9 50 32·8	3·474	1	60 21 51·7	16·99	1	eeF; eS; stellar (?)	1*
1975	659	II. 59	....	9 50 39·1	3·209	1	78 58 42·7	16·99	1	pB; pS; R; gmbMN; 3 st nr	2
1976	3206	III. 273	....	9 50 47·0	2·831	2	108 40 49·7	16·99	2	vF; pS; lE; glbM	3
1977	......	III. 853	....	9 51 10·7	4·126	1	33 42 23·9	17·03	1	vF; S; vglbM	1
1978	660	III. 542	....	9 51 18·7	3·585	1	53 55 47·9	17·03	1	vF; pL; iR; vgvlbM	4
1979	3207	......	....	9 51 19·5	3·261	1	74 53 56·9	17·03	1	vvF; *14 att; *11 f	1
1980	3208	......	....	9 51 32·2	2·848	1	107 30 25·9	17·03	1	eF; S; R	1
1981	3209	II. 268	....	9 52 4·6	2·731	1	116 15 22·5	17·05	1	pB; S; R; mbM	2
1982	658	I. 286	....	9 52 7·4	4·925	(2)	20 35 17·7	17·09	1	cB; cL; mbM; R with ray	3
1983	......	V. 47	....	9 52 24·3	4·119	1	33 38 27·7	17·09	1	vB; L; mE 135°±	1
1984	......	III. 934	....	9 52 34·6	3·241	1	76 19 59·7	17·09	1	vF	1
1985	......	III. 596	....	9 52 37·0	2·786	2	112 7 58·4	17·08	2	vF; cS; lbM; ΔS st np	2
1986	3210	......	....	9 52 40·5	2·671	1	119 41 50·4	17·08	1	vF; S; R; ** att	1
1987	3211	......	....	9 52 45·4	+2·722	1	116 28 7·7	+17·09	1	vF; S; R; *13 att sf	1

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	h.	H.		h m s	s		° ′ ″	″			
1988	3212	......	....	9 52 50·7	+2·832	1	108 50 56·7	+17·09	1	vF; S; R........................	1
1989	3213	......	....	9 53 4·5	2·607	1	123 33 46·0	17·10	1	pB; S; R; pmbM; bet 2 st...	1
1990	661	III. 24	....	9 53 17·7	3·367	1	66 55 52·6	17·12	1	vF; S	2
1991	3214	......	....	9 53 19·7	2·705	1	117 38 50·3	17·11	1	pF; pS; R; vS st inv	1
1992	3215	II. 293	....	9 53 38·9	2·831	1	108 57 52·9	17·13	1	pB; pS; iR; bM; p of 2......	2
1993	3216	......	....	9 53 53·3	2·655	1	120 52 53·2	17·14	1	F; L; E; vgvlbM	1
1994	3217	......	....	9 54 2·5	2·832	1	108 57 58·5	17·15	1	eF; R; lbM; f of 2............	1
1995	663	......	....	9 54 22·3	3·398	2	64 37 15·1	17·17	2	pB; S; mE 90°±; psbMN...	2
1996	664	III. 478	....	9 54 25·4	3·525	1	56 37 10·4	17·18	1	eF; S	3
1997	3218	......	....	9 54 27·6	2·655	1	120 59 37·8	17·16	1	pB; pS; R; gpmbM	1
1998	662	III. 916	....	9 54 36·7	4·298	1	29 13 26·7	17·19	1	vF; vS; R; bM; *11, 142°·2	2
1999	665	IV. 48	....	9 55 19·4	3·675	1:	48 35 27·3	17·21	1:	eF; pL; E 45°±; vF * inv...	2
2000	3219	......	....	9 55 46·4	2·120	2	144 6 16·3	17·21	2	Cl; C; lE; st 13...16	2
2001	666	II. 320	....	9 55 58·4	3·504	1	58 8 19·2	17·24	1	F; S; R; sbM..................	2
2002	3220	......	....	9 56 17·6	2·660	2	121 0 33·2	17·24	2	F; S; R; glbM	2
2003	3221	......	....	9 56 40·1	2·746	2	115 29 25·8	17·26	2	cF; vL; vmE 82°·3; lbM ...	2†
2004	......	II. 898	....	9 56 54·3	3·242	1	75 49 0·4	17·28	1	F; L red * n 3′	1
2005	667	......	....	9 57 14·8	3·818	3	42 3 28·0	17·30	3	pB; S; R; SmbM *12	3
2006	3222	......	....	9 58 3·2	2·716	1	117 46 13·6	17·32	1	eF; L; Δ 2 st 8m	1
2007	3224	......	Δ. 297	9 58 11·2	1·934	3	149 26 46·6	17·32	3	Cl; eL; lC; B; st 9...14......	3
2008	{668 = 3223}	I. 163	....	9 58 14·3	2·988	2	97 2 32·9	17·33	2	vB; L; vmE 45°; vg, vsmbMEN.	3
2009	3225	......	....	9 59 14·8	2·627	1	123 32 36·4	17·38	1	F; pS; R; gbM	1
2010	......	......	Auw. N. 26	9 59 18·4	3·247	...	74 55 35·7	17·39	...	F; (Lassell, Mar. 31, 1848)...	0
2011	......	II. 305	....	9 59 26·0	3·003	1	95 51 17·9	17·43	1	F; S; lE; er	2
5067	......	......	....	9 59 51·1	......	...	89 15 7·3	......	...	See No. 5067.	...
2012	3226	......	....	10 0 2·2	2·848	1	108 33 36·3	17·41	1	F; pL; R; lbM; ⁑ s	1
2013	3227	......	....	10 0 14·8	2·700	2	119 15 12·6	17·42	2	cF; S; R; vgbM..............	2
2014	669	III. 65	....	10 0 40·4	3·299	(1)	70 53 43·5	17·45	1::	eF; cS; vlE; r	2*
2015	670	......	....	10 0 45·2	3·195	1	79 20 43·5	17·45	1	eF; S; psbM; 31 Leon sf 100″	1
2016	671	......	....	10 0 54·9	3·296	1	71 4 49·8	17·46	1	pB; pS; pmE; gbM	1
2017	3228	......	....	10 1 8·2	2·523	4	129 45 5·5	17·45	4	!!; ○; vB; vL; lE; *9M; 4ˢ·0 d.	4†
2018	3229	......	....	10 1 46·2	1·540	1?	156 41 44·1	17·47	1?	B; R; bM	1
2019	672	......	....	10 2 9·8	3·759	1	43 21 12·6	17·52	1	F; S; R; gbM	1*
2020	3231	......	....	10 2 9·9	1·545	2	156 41 30·4	17·48	2	pB; pS; R; gbM; * 13 n ...	2
2021	3230	......	....	10 2 30·4	2·719	1	118 22 22·3	17·51	1	vF; S; lE	1
2022	3232	......	....	10 3 9·9	2·982	1	97 47 52·5	17·55	1	F; R	1
2023	673	III. 518	....	10 3 17·7	2·936	1	101 44 15·8	17·56	1	F; pL; R; vg, slbM; f of 2...	4
2024	674	I. 79	....	10 4 47·5	5·285	1	15 54 35·5	17·65	1	vB; L; R; vg, vsvmbM......	2
2025	675	......	....	10 4 48·3	3·861	1	38 49 21·9	17·63	1	* 7 m in photosphere 2′ or 3′ d	1
2026	677	III. 53	....	10 5 8·4	+3·222	1	76 38 47·9	17·63	1	eF; pL; vlE; r; st inv	3
2027	3234	......	....	10 5 10·7	−0·511	1	169 43 54·4	17·58	1	F; S; lE; vlbM; * 15 inv ...	1
2028	680	III. 255	....	10 5 25·3	+3·113	1	86 10 27·2	17·64	1	F; cS; R; psbM; Δ B st f ...	3
2029	3233	......	....	10 5 27·7	2·700	1	120 16 4·2	*17·64	1	vF; pS; E; * 8·9 sp	1
2030	678	II. 639	....	10 5 31·3	3·592	1:	50 33 28·5	17·65	1:	cB; cS; R; psbM; r	2
2031 2032	678, a	......	R. 2 novæ	10 5 ±	......	...	50 33 ±	......	...	3 seen; one (? which) = h. 678	0
2033	676	......	....	10 5 42·7	5·424	1?	14 53 50·7	17·69	1::	vF; S; R........................	1
2034	682	II. 43	....	10 5 43·5	3·345	2	66 34 30·8	17·66	3	pF; cL; R; vglbM; r; S * inv	5
2035	681	II. 640	....	10 5 48·2	3·589	1:	50 40 11·1	17·67	1:	F; S; R; gbM..................	2
2036	679	......	....	10 5 49·2	4·054	1	32 38 47·1	17·67	1	eF; S; R; vglbM	1
2037	684, a	......	R. nova	10 6 15·4	3·116	::	85 56 15·7	17·69	::	vvF; mE 0°±	0
2038	684	I. 3	....	10 6 29·6	3·116	3	85 52 53·7	17·69	4	B; pS; R; psmbM; p of 2...	8
2039	......	......	D'Arrest, 68	10 6 36	3·34	[1]	59 42 7	17·69	[1]	F; S; ?? Cl of vS st	0
2040	683	......	....	10 6 45·3	4·194	1	29 4 36·3	17·71	1	F; psbM; stellar; * 7·8 np 5′	1
2041	685	I. 4	....	10 6 58·5	3·116	3	85 50 20·3	17·71	4	B; pL; vlE; pgmbM; * 11, 78°·2, 80″.	8
2042	686	......	....	10 7 32·0	3·745	1	42 42 39·2	17·74	1	F; S; R	1
2043	250	......	....	10 8 7·9	+87·502	1::	0 6 46·2	+19·47	1::	vF; R; gbM; *11, 2′ s; *Polarissima Borealis.*	1*

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	h.	H.		h m s	s		° ′ ″	″			
2044	3235	……	…………	10 8 9·4	+2·754	1	116 59 43·5	+17·75	1	eF; S; R; 2 B st f …………	1
2045	……	III. 964	…………	10 8 16·1	5·415	1	14 38 38·0	17·80	1	cF; S; stellar; S ✱ f nr ……	1
2046	3236	……	…………	10 8 18·9	2·738	2	118 11 3·8	17·76	2	cB; L; mE 50°·5; vglbM …	2
2047	687	III. 25	…………	10 8 51·5	3·318	3	68 10 42·7	17·79	3	cF; S; R; psBM…………	4
2048	3237	……	…………	10 9 23·5	2·905	1	105 5 56·0	17·80	1	pB; pL; gpmbM…………	1
2049	688, a	……	R. nova	10 9 44·5	3·623	…	47 53 13·9	17·83	…	MS; no description…………	0
2050	688, b	……	R. nova	10 9 47·2	3·623	…	47 54 1·9	17·83	…	MS; no description…………	0
2051	……	I. 265	…………	10 9 47·5	4·082	1	31 5 44·2	17·84	1	cB; cL; iR; vgbM…………	1
2052	688	I. 168	…………	10 9 49·9	3·623	1	47 53 1·9	17·83	1	pB; vL; R; vgbM …………	4
2053	689	……	…………	10 9 53·3	3·627	1	47 41 6·2	17·84	1	pF; vL; R; vgbM; 12s·5 d; ✱ 11 n 2′.	1
2054	692, a	……	D'Arrest, 69	10 9 57	3·32	[2]	67 35 48	17·83	[2]	pF; pL; gmbM (δ in Lord R.'s diagram).	0
2055	692, b	……	R. nova	10 10 4·7	3·324	…	67 25 19·5	17·85	…	Marked γ in Lord R.'s diagr.	0
2056	690	III. 910	…………	10 10 17·0	4·047	1	31 53 7·8	17·86	1	vF; pL; r …………………	2
2057	692, c	……	R. nova	10 10	……	…	67 28	……	…	mE, parallel to h. 692, with which it forms D neb.	0
2058	692	II. 44	…………	10 10 23·3	3·324	3	67 28 10·5	17·85	3	B; pS; E; psbMN; sp of 2	4†
2059	691	……	…………	10 10 25·8	3·726	1	42 51 47·8	17·86	1	F; S; R; bM ………………	1
2060	……	III. 704	…………	10 10 27·6	3·728	1	42 43 44·8	17·86	1	eF; vS; (?) ………………	1
2061	693	II. 45	…………	10 10 41·6	3·324	3	67 24 24·8	17·86	3	B; S; vlE; pslbM; r; ✱ 9, 352°·0, 75″ nf of 2.	4
2062	……	III. 695	…………	10 10 46·8	+5·391	1	14 29 43·0	17·90	1	vF; vS………………………	1
2063	3241	……	…………	10 10 51·5	−0·506	2	170 10 11·6	17·82	2	!; ○; pB; S; lE; 13″·0 d; 3S st nr.	2†
2064	694	III. 348	…………	10 10 53·6	+3·398	1:	61 38 27·1	17·87	1:	eeF; pS; lE…………………	2
2065	……	III. 966	…………	10 10 57·9	6·114	1	11 4 53·6	17·92	1	vF; vS………………………	1
2066	695	I. 199	…………	10 11 14·4	3·702	1	43 44 15·7	17·89	1	pB; vL; mE 45°±; vgbM…	3
2067	3239	……	…………	10 11 46·1	2·128	4	147 15 39·7	17·89	4	!; vB; vL; falcate; ✱✱ N……	4†
2068	3238	……	Δ. 445	10 11 52·8	2·452	2	135 42 6·0	17·90	2	⊕; vL; iR; lCM; gbM; st 13…16.	2
2069	696	II. 720	…………	10 12 5·3	3·643	1::	46 18 42·6	17·92	1:	cF; S; R; vgbM; 1st of 3…	2
2070	3240	……	…………	10 12 5·5	2·777	2	116 0 17·3	17·91	2	pB; S; cE; gbM…………	2
2071	698	……	…………	10 12 12·9	3·396	1	61 28 54·6	17·92	1	eF; pL; gbM ………………	1
2072	699	II. 721	…………	10 12 21·7	3·641	1	46 20 50·2	17·94	1	cF; S; R; vgbM; 2nd of 3…	2
2073	697	I. 266	…………	10 12 30·9	4·011	1	32 22 5·2	17·94	1	pB; cL; E; vglbM…………	2
2074	700	II. 722	…………	10 12 33·3	3·641	1	46 18 50·2	17·94	1	cF; S; R; stellar; 3rd of 3…	2
2075	701	……	…………	10 12 47·0	3·365	1	63 47 46·5	17·95	1	F; S; R; has a ✱ …………	1
2076	3242	……	…………	10 13 17·6	1·946	2	151 58 35·5	17·95	2	○; = ✱ 10 m; R; am 150 st	2
2077	……	III. 979	H. ON	10 13 34·0	6·559	1	9 27 5·6	18·02	1	Stellar; 1st of 3 ……………	1
2078	……	III. 980	H. ON	10 13 34·2	6·565	1	9 26 5·6	18·02	1	vF; S; 2nd of 3 ……………	1
2079	……	III. 981	H. ON	10 13 34·3	6·571	1	9 25 5·9	18·03	1	vF; S; 3rd of 3 ……………	1
2080	702	III. 330	…………	10 13 55·0	3·342	1	65 21 53·7	17·99	1	vF; pS; R; bM ……………	2
2081	……	I. 283	…………	10 14 23·0	5·297	1	14 37 52·9	18·03	1	cB; cL; eR………………	1
2082	……	III. 911	…………	10 14 26·4	3·998	1	32 15 55·6	18·02	1	vF; cL; iF ………………	1
2083	……	……	D'Arrest, 70	10 14 39	3·31	[2]	67 42 12	18·01	[2]	eF; mE; a ray ……………	0
2084	……	……	Auw. N. 27	10 14 55·0	3·288	…	69 24 42·6	18·02	…	F; lbMr (Winnecke, June 1855).	0
2085	3243	……	…………	10 15 20·5	2·683	1	123 32 59·9	18·03	1	pB; vL; vlE; pslbMN ……	1
2086	3244	……	…………	10 15 27·7	2·677	1	123 59 28·2	18·04	1	vF; pS; R; vgmbM ………	1
2087	703	II. 882	…………	10 15 43·0	4·028	1	31 9 19·1	18·07	1	cF; pL; lE; vgbM…………	2
2088	……	II. 28	…………	10 15 55·1	3·289	1	69 23 27·8	18·06	1	vF; cL; R } D neb; 45°, 2′	1*
2089	……	II. 29	…………	10 15 58·5	3·289	1	69 22 28·1	18·07	1	vF; cL; R }	1*
2090	3245	……	Δ. 386	10 16 10·6	2·352	1::	141 1 1·8	18·06	1::	Cl; 9 L & a few S st ………	1
5068	……	……	…………	10 16 14·1	……	…	89 13 46·0	……	…	See No. 5068.	
2091	705	……	…………	10 16 15·7	3·207	3	76 44 21·4	18·08	3	!; ✱✱ or ✱✱✱ in neb …………	3
2092	704	……	…………	10 16 25·6	4·456	1	22 29 6·0	18·10	1	Cl; cL; P; lC; st 10…12 …	1
2093	……	……	D'Arrest, 71	10 16 33	3·39	[1]	61 16 42	18·09	[1]	eeF; ✱ 11 p, ls, 150″ p of 2…	0
2094	706	……	…………	10 17 5·1	3·374	1	62 16 5·0	18·10	1	pB; pS; R; psbM …………	1*
2095	……	……	D'Arrest, 72	10 17 7	3·38	[2]	61 16 12	18·11	[2]	F; S; f of 2………………	0
2096	707	……	…………	10 17 16·6	+4·142	1	28 1 16·9	+18·13	1	eF; vS; psbM; 2 st 11, 12, f	1

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	h.	H.		h m s	s		° ′ ″	″			
2097	709	III. 631	…………	10 17 27·7	+3·556	1:	49 39 49·9	+18·13	1:	vF; vS; R; pgbM …………	3
2098	708	III. 883	…………	10 17 28·5	3·980	1	32 4 25·9	18·13	1	F; S; R; pslbM …………	2
2099	{710 = 3246}	IV. 10	…………	10 17 31·0	3·255	3	72 8 18·9	18·13	3	vF; * 9 inv nr M …………	5
2100	3247	……	…………	10 17 53·2	2·851	2	111 4 56·9	18·13	2	eF; S; R; * nr …………	2
2101	3249	……	…………	10 17 56·8	2·717	1	121 45 28·9	18·13	1	F; pmE; glbM; * 11 np …	1
2102	3248	IV. 27	Lal. 20204	10 18 2·2	2·886	4	107 55 50·2	18·14	4	!; ○; vB; lE, 135°; 32″ d±; *blue.*	7†
2103	4019	……	…………	10 19 20·9	2·612	1	129 6 23·4	18·18	1	vF; * 11 m 90″ n …………	1
2104	711	I. 86	…………	10 19 24·9	3·385	4	60 47 1·7	18·19	5	vB; pL; E; smbMEN ……	6
2105	712	……	…………	10 19 26·0	3·116	1	85 26 24·7	18·19	2	eF; S; R; 2 st Δ; * 6, 300°, 8′	3
2106	3250	……	…………	10 19 37·9	2·196	1::	147 10 35·7	18·19	1	st inv in neb …………	3
2107	713	II. 347	…………	10 20 2·0	3·314	1	66 26 28·6	18·22	3	pB; S; R; psbM…………	4
2108	3251	……	…………	10 20 6·9	2·690	1	124 14 38·3	18·21	1	eF; pL; R; vgvlbM ………	1
2109	3252	……	…………	10 20 25·6	2·625	4	129 13 45·9	18·23	4	pB; pL; R; vg, psbM; *13, 45°.	5
2110	……	……	D'Arrest, 73	10 20 29	3·35	[1]	63 11 42	18·23	[1]	vF; pL; 3 B st sp …………	0
2111	……	III. 316	…………	10 20 51·1	5·054	1	15 27 11·9	18·27	1	eF; pS; mE; r …………	1*
2112	714	I. 72	…………	10 21 27·2	3·392	2	59 47 27·1	18·27	3	cB; L; E 45°±; psmbMN…	5
2113	3253	……	…………	10 21 32·2	2·111	1	149 57 49·8	18·26	1	Cl; pS; vC; st 15 …………	1
2114	3254	……	…………	10 21 53·0	2·553	1	133 11 7·1	18·27	1	cB; S; R; gmbM …………	2
2115	3255	……	…………	10 22 29·8	2·689	3	124 56 49·0	18·30	2	vF; vS; R; psbM; 1st of 4…	3
2116	3256	……	…………	10 22 36·9	2·689	4	124 53 9·0	18·30	4	cF; S; R; pslbM; 2nd of 4…	4
2117	715	II. 870	…………	10 22 42·9	4·272	1	24 14 50·9	18·33	1	F; S; R; gbM…………	2
2118	3257	……	…………	10 22 52·7	2·689	4	124 52 39·0	18·32	4	vvF; vS; R; pslbM; 3rd of 4	4
2119	3258	……	…………	10 23 3·0	2·546	1	133 56 12·6	18·32	1	F; S; R; am st …………	1
2120	3260	……	…………	10 23 8·2	2·554	1	133 29 7·6	18·32	1	eF; S; R…………	1
2121	3259	……	…………	10 23 11·4	2·692	2	124 53 48·9	18·33	2	vF; vS; R; pslbM; 4th of 4	2
2122	3261	……	…………	10 23 13·8	2·556	1	133 24 32·9	18·33	1	F; S; mE 280°±; psbM …	1
2123	718	III. 349	…………	10 23 15·9	3·377	1	60 29 34·9	18·33	1	pF; S; R; psbM; * sf nr …	5
2124	716	……	…………	10 23 18·3	3·895	1	33 11 37·5	18·35	1	eF; bet 2 S st…………	1
2125	717	II. 871	…………	10 23 29·5	4·246	1	24 32 16·8	18·36	1	cF; vS; R; psmbM *………	2
2126	3262	……	…………	10 23 33·9	2·696	1::	124 38 10·2	18·34	1::	eF; vS; R; 1st of 4…………	1
2127	3263	……	…………	10 23 37·9	2·697	2?	124 39 10·2	18·34	2?	F; S; R; 2nd of 4 …………	2
2128	3264	……	…………	10 23 41·1	2·699	1	124 30 10·5	18·35	2	F; S; R; bM; 3rd of 4 ……	3
2129	719	III. 331	…………	10 23 44·1	3·329	1	64 24 28·5	18·35	1	cF; vS; E; glbM …………	2
2130	3265	……	…………	10 23 45·9	2·697	1	124 39 3·5	18·35	1	pF; S; E; pmbM; 4th of 4…	2
2131	720	II. 358	…………	10 24 27·7	3·360	4	61 36 59·4	18·38	4	F; pL; R; glbM; ** f…………	5
2132	3266	……	…………	10 24 34·0	2·681	2	126 1 22·1	18·37	2	F; L; vlE; pslbM …………	3
2133	3267	……	…………	10 24 57·7	2·635	1	129 13 51·7	18·39	1	F; S; * 8 p…………	1
2134	721	II. 359	…………	10 25 4·1	3·369	3	60 46 20·7	18·39	3	cB; cS; R; pgmbM ………	4
2135	3268	……	…………	10 25 26·1	2·637	3	129 13 52·0	18·40	3	F; S; R; ** nf…………	2
2136	3269	……	…………	10 25 34·3	2·711	1	124 8 4·3	18·41	1	eF; pL; E; glbM …………	1
2137	3271	……	…………	10 26 51·1	2·536	1	135 22 3·5	18·45	1	pF; S; R; gbM …………	1
2138	……	III. 912	…………	10 26 58·6	3·944	1	30 44 23·1	18·47	1	eF; vS…………	1
2139	3270	……	…………	10 27 1·3	2·808	4	116 44 11·8	18·46	4	pB; S; lE; gbM; 1st of 9 …	4
2140	722	III. 917	…………	10 27 9·3	3·945	1	30 40 11·4	18·45	1	vF; pS; R; pslbM …………	2
2141	……	……	D'Arrest, 74	10 27 11	3·28	[2]	67 37 18	18·47	[2]	F; pL; ** p 24ˢ, 225″ s ……	0
2142	723	III. 918	…………	10 27 12·7	3·942	1	30 43 31·4	18·48	1	eF; cS; R; vglbM …………	2
2143	3272	……	…………	10 27 47·4	2·713	1	124 34 31·4	18·48	1	eF; vS; R …………	1
2144	3276	……	…………	10 28 7·7	2·258	1::	147 28 14·7	18·49	1::	Cl; B; Ri; pL…………	1*
2145	724	I. 164	…………	10 28 12·8	3·474	1	51 57 17·3	18·51	1	cB; L; mE 135°±; glbM …	4
2146	725	III. 767	…………	10 28 44·6	3·698	1	39 9 51·9	18·53	1	vF; pS; iE …………	2
2147	726	III. 54	…………	10 28 57·8	3·193	1	76 34 38·9	18·53	1	eF; cL; R; vgbM; r ………	2
2148	{727 = 3273}	III. 55	…………	10 29 10·6	3·206	3	75 6 29·2	18·54	3	cF; cS; R; pmbM; r; am B st	5
2149	……	II. 46	…………	10 29 13·4	3·285	1	67 15 27·2	18·54	1	pF; S; r; Δ pB st n…………	1
2150	728	II. 46??	…………	10 29 18·1	3·283	2	67 23 45·2	18·54	2	cB; S; lE; psbM; r ………	3
2151	3274	……	…………	10 29 20·3	+2·757	1	121 37 4·2	+18·54	1	eF; S; R…………	1

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	h.	H.		h m s	s		° ′ ″	″			
2152	3275	......	....	10 29 22·6	+2·756	1	121 38 41·2	+18·54	1	vF; S; R	1.
2153	730	III. 66	....	10 29 27·1	3·245	2	71 8 48·2	18·54	2	vF; vS; vlE; glbM; r.........	3
2154	729	III. 615	....	10 29 33·9	3·470	2	51 49 25·5	18·55	2	vF; cS; psbM; er	4
2155	3277	......	....	10 29 38·5	2·818	1	116 26 17·5	18·55	1	vF; S; R; 2nd of 9	1
2156	3278	......	....	10 29 41·3	2·813	1	116 52 38·5	18·55	1	eeF; 3rd of 9	1
2157	3279	......	....	10 29 48·4	2·816	4	116 42 50·5	18·55	4	F; S; R; 4th of 9	4
2158	731	IV. 60	....	10 29 59·7	3·770	1	35 45 49·1	18·55	1	○? cB; pL; R; vg, vsmbMN 15″.	3†
2159	3280	......	....	10 30 1·1	2·815	2	116 48 5·8	18·56	2	B; L; R; p of D neb; 5th of 9	3
2160	3281	......	....	10 30 9·5	2·815	1::	116 49 17·1	18·57	1::	B; L; R; f of D neb; 6th of 9	1
2161	3282	......	....	10 30 28·7	2·815	2	116 53 42·1	18·57	2	cF; E; gbM; 7th of 9	2
2162	3283	......	....	10 30 39·9	2·815	1	116 56 48·4	18·58	1	8th of 9	1
2163	3284	......	....	10 30 59·8	2·817	3	116 52 45·7	18·59	3	F; S; R; bM; 9th of 9	3
2164	3285	......	....	10 31 6·1	2·636	2	130 54 24·7	18·59	2	cF; pL; pmE; lbM	2
2165	......	III. 700	....	10 31 7·4	3·526	1	47 36 30·3	18·61	1	cF; L; iE; mb, s of M	1
2166	732	II. 745	....	10 31 12·8	3·626	2	41 51 29·3	18·61	2	F; pS; mE 0...90°; ✱ 10 nf.	4
2167	3286	......	Δ. 322?	10 32 2·9	2·277	1	147 53 48·6	18·62	1	pB; vvL; iF; ⁑ inv	1
2168	734	II. 348	....	10 32 17·5	3·299	2	65 11 2·2	18·64	2	vF; S; R; gbM; vS ✱ att ...	3
2169	733	......	....	10 32 34·7	5·264	3	12 26 32·4	18·68	3	pB; S; lE; psmbM............	3
2170	......	I. 272	....	10 32 34·7	3·158	1::	79 59 19·5	18·65	1::	B; S; iR; mbMBN	2
2171	3287	......	Δ. 355	10 33 1·6	2·409	1	143 24 4·5	18·65	1	Cl; P; st 9	1
2172	3288	......	....	10 33 26·3	2·729	1	125 19 17·0	18·70	1	eF; vS; mE; ✱ 15 att.........	1
2173	735	II. 641	....	10 33 28·5	3·451	1	51 58 16·4	18·68	1	cF; vS; R; bM	4
2174	3289	......	....	10 33 43·6	2·823	1	117 1 41·4	18·68	1	vF; pL; lE; glbM	1
2175	737	II. 77	....	10 34 42·7	3·195	2	75 31 44·6	18·72	2	F; cL; E; vgbM; r; ✱7p10ˢ	4
2176	736	III. 317	....	10 35 17·6	4·724	1	15 54 50·5	18·75	1	pF; S; R; gbM	2
2177	......	III. 5	....	10 35 25·5	3·157	1	79 49 40·2	18·74	1	eF; eS	1
2178	739	I. 81	....	10 35 49·5	3·298	1	64 20 20·5	18·75	1	cB; L; gbM; ✱inv; 2st f ...	3
2179	740	I. 26	....	10 36 8·6	3·178	1	77 16 41·8	18·76	1	cB; pL; E; mbM	2
2180	3290	V. 7	....	10 36 10·9	3·203	1	74 23 11·8	18·76	1	cF; vL; R; vgvlbM; er......	2
2181	3291	......	....	10 36 23·8	2·732	4	125 37 46·8	18·76	3	pF; S; mE 0°±; vsvmbM; 1st of 3.	4
2182	738	I. 80	....	10 36 29·0	4·648	1	16 25 22·7	18·79	1	B; S; ilE; psbM; ✱11, 281°·8, 20ˢ·0.	2
2183	742	......	....	10 36 33·7	3·358	1	58 32 21·1	18·77	1	eF; vS; 2st 9·10, s	1
2184	743	......	M. 95	10 36 36·7	3·175	2	77 34 22·1	18·77	4	B; L; R; pgmbMrN	8
2185	741	III. 842	....	10 36 39·2	3·782	1	33 18 30·6	18·78	1	F; cS; R; pgbM; ✱s 90″ ...	2
2186	3292	......	....	10 36 41·4	2·734	4	125 38 53·1	18·77	3	F; S; vlE; psbM; 2nd of 3...	4
2187	744	III. 107	....	10 36 53·2	3·133	2	82 30 48·4	18·78	2	vF; pS; R; bM; ✱9, 150″ ...	4
2188	3293	......	....	10 37 11·7	2·735	...	125 38 59·7	18·79	...	cF; vS; vlE; vS✱att; 3rd of 3	2
2189	745	V. 52	....	10 37 21·0	4·018	1	26 2 25·3	18·81	1	pB; L; E 0°; glbM............	2*
2190	746	III. 318	....	10 37 57·1	4·583	1	16 49 37·9	18·83	1	vF; L; R; vgbM; r; ⁑ sf ...	2
2191	747	......	....	10 38 59·5	3·091	1	87 28 32·5	18·85	1	eF; L; eE; vgvlbM; a ray...	1
2192	3294	......	....	10 39 4·1	2·644	1?	132 58 43·5	18·85	1	F; E; gbM; ✱6, 7 v nr	1*
2193	748	II. 78	....	10 39 10·5	3·189	2	75 31 1·5	18·85	2	pB; cL; iR; vglbM; r; 1st of 3	5
2194	749	......	M. 96	10 39 20·4	3·173	4	77 26 55·8	18·86	4	vB; vL; lE; vsvmbM; r......	8
2195	750	II. 81	....	10 39 34·8	3·219	2	71 59 40·1	18·87	2	cB; pL; vlE; gbM; r.........	3
2196	751	......	....	10 39 37·3	3·188	1	75 28 25·1	18·87	1	F; R; 2nd of 3	1
2197	3295	......	Δ. 309	10 39 36·8	2·313	3	148 56 44·8	18·86	3	η Argûs. The great neb ...	Mon.†
2198	753	......	....	10 39 49·3	3·187	1	75 35 15·1	18·87	1	F; R; 3rd of 3	1
2199	752	III. 701	....	10 39 49·4	3·506	1	46 4 18·4	18·88	1	vF; cS; iR	2
2200	......	......	D'Arrest, 75	10 40 12	3·12	[1]	83 12 24	18·88	[1]	vF; S	0
2201	754	II. 99	....	10 40 16·7	3·189	2	75 16 24·7	18·89	2	vB; cL; R; svmbMBN	5*
2202	3296	......	....	10 40 22·4	2·702	3	129 17 2·7	18·89	3	cF; S; R; glbM	4
2203	757	I. 17	Mechain.	10 40 25·7	3·177	3	76 41 10·7	18·89	3	vB; cL; R; psbM; r	8
2204	755	II. 360	....	10 40 27·4	3·323	5	60 39 47·7	18·89	5	pB; pS; R; sbM................	6
2205	756	II. 565	....	10 40 33·2	3·390	2	54 33 18·7	18·89	2	pF; cL; iR; vglbM; 1st of 3	3
2206	3297	......	....	10 40 39·5	2·874	2	113 41 36·7	18·89	2	F; pL; iR; glbM	2
2207	758	I. 18	....	10 40 52·2	3·177	3	76 38 27·0	18·90	3	vB; L; R; psmbM; 2nd of 3	8
2208	759	......	....	10 40 54·9	3·116	1	84 15 58·0	18·90	1	vF; R	1
2209	760	......	....	10 40 55·7	+ 3·116	2	84 20 22·0	+18·90	2	F; S; lE; bM	2

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	h.	H.		h m s	s		° ′ ″	″			
2210	762	......	....	10 41 2·5	+3·116	1	84 18 1·3	+18·91	1	Suspected; ✱nr	1
2211	761	II. 41	....	10 41 3·8	3·176	2	76 43 38·3	18·91	2	F; L; E 90°±; vglbM; 3rd of 3.	9
2212	3298	......	....	10 41 32·5	2·807	1	120 48 44·6	18·92	1	F; S; pmE 0°	1
2213	763	III. 881	....	10 41 36·0	4·082	1	23 28 40·9	18·93	1	vF; S; psbM; st nr	2
2214	3299	......	....	10 41 41·8	2·870	3	114 25 38·6	18·92	3	F; S; R; psbM; 2st 10f	3
2215	764	II. 872	....	10 41 59·8	4·075	1	23 30 51·2	18·94	1	cF; S; lE; vgbM	2
2216	765	I. 116	....	10 42 2·4	3·364	2	56 16 38·2	18·94	2	cB; pS; iIE; 1st of 2	4†
2217	766	I. 117	....	10 42 7·5	3·364	2	56 15 59·2	18·94	2	pB; pS; iIE; 2nd of 2.........	4†
2218	......	I. 284	....	10 42 34·7	5·103	1	11 57 49·1	18·97	1	cB; vS; iF	1
2219	......	III. 792	....	10 42 57·4	3·717	1	33 49 54·1	18·97	1	vF; S; E; er	1
2220	768	II. 361	....	10 43 2·3	3·314	3	60 47 45·1	18·97	3	pF; S; R; bM	4
2221	767	II. 335	....	10 43 15·4	4·609	1	15 34 35·7	18·97	1	pF; L; iE; vgbM	2
2222	771	......	....	10 43 17·6	3·619	1	38 13 52·4	18·98	1	pB; R; pgbM..................	1
2223	769	III. 919	....	10 43 20·3	3·884	1	27 53 7·4	18·98	1	vF; vS; R; vS✱ nr	2
2224	770	III. 913	....	10 43 21·3	3·792	1	30 49 46·4	18·98	1	vF; cS; R; 2pB st s	2
2225	772	II. 718	....	10 43 33·3	3·496	1	45 32 51·4	18·98	1	pB; S; vlE; stellar; 3S st nr.	3
2226	772, a	......	R. nova	10 43 ±	......	...	45 ±	......	...	3′ dist. from h. 772	0
2227	773	II. 362	....	10 43 33·5	3·307	4	61 17 10·4	18·98	4	B; pL; R; mbM..............	5
2228	776	III. 522	....	10 43 28·5	2·982	(3)	102 6 30·0	19·00	1: :	F; S; R; lbM..................	4
2229	774	I. 27	....	10 43 29·8	3·180	2	75 50 51·4	18·98	2	B; S; lE 135°±; smbMN...	5
2230	775	II. 363	....	10 43 41·7	3·308	2	61 9 14·7	18·99	4	cF; S; R; bM................	5
2231	......	{ IV. 6 = II. 131 }	....	10 43 49·0	3·117	4	83 26 44·4	18·99	4	B; vL; R; bM; r	5✱
2232	......	II. 493	....	10 43 52·8	3·355	1	56 29 54·7	18·99	1	F; S..............................	1
2233	......	I. 118	....	10 43 54·7	3·327	1	57 16 54·7	18·99	1	cB; cL; iR; mbM (?58°P.D.).	1✱
2234	777	III. 88	....	10 43 57·2	3·121	3	83 25 0·7	18·99	3	F; vL; R; vgbM; rr	5✱
2235	778	II. 494	....	10 44 1·8	3·356	1	56 21 52·0	19·00	1	pF; pL; lE; sp of 3	3
2236	779	I. 118?	....	10 44 27·8	3·355	1	56 18 10·3	19·01	1	pB; L; iE; gbM; 2nd of 3...	2✱
2237	......	III. 108	....	10 44 33·7	3·141	1	80 44 57·3	19·01	1	eF; eS; R	1
2238	780	I. 172	....	10 44 35·5	3·395	2	52 38 29·3	19·01	2	pB; pL; vmE 42°·5; ✱✱ inv?	3✱
2239	782	......	....	10 44 40·7	3·356	1: :	56 10 30·3	19·01	1: :	pB; nf of 3 in a line............	1
2240	783	III. 20	....	10 44 40·7	3·156	1	79 6 47·3	19·01	1	vF; vL; R; vgbM	2
2241	784	III. 497	....	10 44 43·3	3·105	1	85 28 16·3	19·01	1	F; pS; R; vglbM	3
2242	781	II. 887	....	10 44 54·9	3·865	1	27 58 21·6	19·02	1	cF; pS; lE; vgbM	2
2243	786	II. 47	....	10 44 59·2	3·257	2	66 19 33·6	19·02	3	pB; pL; lE 120°; gbM	5
2244	785	III. 914	....	10 45 10·9	3·740	1	32 8 27·9	19·03	1	vF; S; lE	2
2245	787	I. 267	....	10 45 58·4	3·730	1	32 16 18·5	19·05	1	cB; pL; iR; vglbM; ✱10nf2′	2
2246	3301	......	....	10 45 58·5	2·660	2	134 24 21·2	19·04	2	Cl; pL; P; lC; iF; st 9...13	2
2247	3300	......	....	10 45 59·6	3·203	1	72 29 30·5	19·05	1	eF; vL; vgvlbM; B ✱✱ sp ...	1
2248	788	I. 233	....	10 46 5·2	3·666	1	34 56 48·5	19·05	1	B; pL; mE 67°·0; gbM	3
2249	3302	......	....	10 46 20·2	2·809	1	122 10 58·5	19·05	1	F; S; R; ✱6·7 sf.............	1
2250	3303	......	....	10 46 25·7	2·919	1	110 6 14·5	19·05	1	vF; L; R; vglbM; r	1
2251	789	II. 364	....	10 46 40·2	3·291	4	62 1 7·1	19·07	4	F; pL; vlE; vlbM	5
2252	3304	......	....	10 46 52·3	2·912	1	111 2 13·1	19·07	1	F; S; R; bM	1
2253	790	......	....	10 47 1·0	3·206	1	71 54 33·4	19·08	1	pF; lE; np of 2	1
2254	791	II. 82	....	10 47 3·0	3·205	1	71 58 7·4	19·08	2	pF; S; E; gbM; r; sf of 2...	3
2255	792	IV. 29	....	10 47 11·5	2·958	1	105 16 57·4	19·08	1	eF; att to ✱12f	2
2256	793	......	....	10 47 21·7	3·207	1	71 38 54·7	19·09	1	2 or 3 S st & neb	1
2257	......	I. 268	....	10 47 23·7	3·721	1	32 8 1·7	19·09	1	vB; vS; R; stellar	1
2258	794	II. 16	....	10 48 2·0	3·132	1	81 33 42·0	19·10	1	vF; vS; vlE; psbM	5
2259	3305	......	....	10 48 29·5	2·879	1	115 23 58·3	19·11	1	F; S; R; glbM	1
2260	795	......	....	10 48 57·1	4·652	2	14 3 47·2	19·14	2	eF; pL; R; vglbM; ✱✱ nf ...	2
2261	796	......	....	10 48 57·6	3·146	1	79 30 48·9	19·13	1	vF; ✱9, 90°; p of 2...........	1
2262	798	......	....	10 49 26·2	3·145	1	79 29 49·2	19·14	1	vF; R; vsmbM✱12; f of 2...	1
2263	797	III. 632	....	10 49 33·9	3·425	2	48 18 8·2	19·14	2	F; eS; R; bM..................	4
2264	3306	......	....	10 49 58·9	2·976	1	103 33 21·5	19·15	1	eeF; S	1
2265	799	II. 888	....	10 50 5·6	3·759	1	29 44 47·8	19·16	1	vF; S; R; vgbM...............	2
2266	......	III. 972	....	10 50 15·1	3·819	1	27 39 6·1	19·17	1	vF; S; R; bM.................	1
2267	......	III. 67	....	10 50 35·3	+3·197	1	72 9 7·1	+19·17	1	vF; E; bet 2 st	1

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	h.	H.		h m s	s		° ′ ″	″			
2268	800	III. 332	....	10 50 49·8	+3·253	1	65 1 18·4	+19·18	1	vF; R; gbM; ✱13 {H. 1'n / h. 2's}	2
2269	801	III. 705	....	10 51 18·6	3·485	1	43 8 17·7	19·19	1	cF; S; R........................	2
2270	3308	......	....	10 52 16·3	2·675	1	135 49 22·3	19·21	1	eF; S; R; gbM	1
2271	3307	......	....	10 52 17·7	2·869	1	117 43 48·3	19·21	1	pF; S; R; bM; am st.........	1
2272	802	......	....	10 52 29·0	4·670	1	13 25 47·9	19·23	1	Very doubtful object	1
2273	{804 = 3309}	II. 100	....	10 52 37·6	3·178	4	74 24 56·6	19·22	4	F; L; R; glbM; r	5
2274	805	I. 87	....	10 52 44·4	3·288	10	60 16 20·6	19·22	10	cB; cL; R; gmbM............	13
2275	803	I. 269	....	10 52 50·3	3·686	1	31 34 53·9	19·23	1	{H. eF; / h. cB;} vlE; pS; ✱13s att	2
2276	806	II. 101	....	10 52 56·1	3·170	2	75 20 58·9	19·23	2	vB; pL; lE 80°±; smbMN.	4*
2277	807	III. 21	....	10 53 14·5	3·158	1	77 5 16·2	19·24	1	eF; cS; R; bMN	3
2278	808	......	....	10 53 49·7	3·274	2	61 32 11·5	19·25	2	vF; R; bM; ✱sp...............	2
2279	809	III. 498	....	10 54 0·3	3·100	3	85 37 30·5	19·25	3	vF; pL; mE	4
2280	3310	......	....	10 54 11·7	2·432	1	149 34 46·5	19·25	1	Cl; pL; pRi; lC; st13	1
2281	......	III. 824	....	10 54 29·4	2·945	1	108 43 14·1	19·27	1	vF; vS; iR; glbM............	1
2282	......	III. 75	....	10 54 30·4	3·171	1	74 52 13·1	19·27	1	eF; pL..........................	1
2283	......	III. 793	....	10 54 45·6	3·636	1	33 2 13·4	19·28	1	vF; vS; stellar	1
2284 2285	......	III. {967 / 968}	....	10 55 9·9	4·582	1	13 27 12·0	19·30	1	{vF / eF}; D neb; v near	1
2286	3311	......	....	10 55 32·8	2·457	1	149 5 48·7	19·29	1	3S st10 m in vF neb...........	1
2287	810	I. 88	....	10 55 33·8	3·271	5	61 16 22·7	19·29	6	B; L; E; mbMN; rr; p of 2	8
2288	3312	......	....	10 55 50·2	2·976	1	104 44 26·0	19·30	1	pF; S; R; glbM; ✱14 nr ...	1
2289	811	III. 22	....	10 55 53·7	3·147	3	78 10 23·0	19·30	3	vF; cS; R; vgvlbM	4
2290	812	IV. 7	....	10 55 59·3	3·195	2	71 6 42·0	19·30	2	cF; pL; R; sbMS✱; ✱9 att 25°.	4
2291	814	II. 507	....	10 56 3·9	2·970	1	105 32 3·0	19·30	1	F; {H. S / h. vL}; bM; ✱nf inv...	2
2292	......	III. 598	....	10 56 5·4	3·106	1	84 28 16·3	19·31	1	eF; S; lE; ?	1
2293	813	II. 365	....	10 56 6·3	3·277	2	60 21 55·3	19·31	2	F; L; cE; ✱7, 310° 8′	5
2294	......	V. 39	....	10 56 27·4	2·923	2	112 21 17·3	19·31	2	vF; vL; mE	2
2295	815	II. 366	....	10 56 28·0	3·269	1	61 11 48·3	19·31	1	F; pS; R; pgbM, f of 2......	2
2296	......	V. 40	....	10 56 43·8	2·923	2	112 29 17·6	19·32	2	vF; vL; mE	2
2297	3313	......	....	10 56 50·2	2·955	1	108 1 41·6	19·32	1	vF; pL; R; vgvlbM	1
2298	816	II. 336	....	10 57 1·2	4·248	1	16 40 22·2	19·34	1	pB; vS; iR; psmbM✱	2
2299	817	II. 884	....	10 57 13·1	3·622	1	32 43 27·2	19·34	1	cF; S; R; vgbM...............	2
2300	3314	......	....	10 58 19·3	2·446	1	150 36 38·5	19·35	1	Cl; pRi; pC	1
2301	818	I. 13	....	10 58 38·7	3·076	1	89 16 57·8	19·36	1	cB; cL; mE 140°±; vsmbMN	4
2302	......	II. 904	....	10 59 4·5	4·481	1	13 33 18·7	19·39	1	F; pL; lbM....................	1
2303	819	III. 23	....	10 59 13·5	3·145	2	77 51 22·4	19·38	3	F; S; lE; psbM; 2st np in line.	5
2304	820	III. 350	....	10 59 41·3	3·263	3	60 43 33·7	19·39	3	eF; S; ✱10 p 60″	5
2305	3316	......	....	11 0 17·4	2·956	1	108 43 4·0	19·40	1	F; S; R; pslbM; p of 2......	1
2306	3317	......	....	11 0 19·9	2·956	1	108 47 19·0	19·40	1	eF; S; R; vlbM; f of 2	1
2307	821	III. 915	....	11 0 20·6	3·609	1	32 1 29·0	19·40	1	vF; S; R; pgbM...............	2
2308	3315	......	Δ. 323	11 0 33·8	2·530	2	147 55 1·0	19·40	2	!!; Cl; eL; R; lC; st8...12...	4
2309	3318	......	....	11 0 45·4	2·823	1	126 25 0·3	19·41	1	eeF; vS✱att....................	1
2310	823	III. 111	....	11 1 17·1	3·104	1	84 24 45·6	19·42	1	cF; vS; R; bM; r	2*
2311	822	......	....	11 1 18·9	3·260	1	60 47 14·0	19·40	1	F; S; R; bM	1
2312	825	......	....	11 1 31·1	3·258	1	60 34 29·9	19·43	1	eF	1
2313	824	......	....	11 1 32·2	3·319	1	53 12 44·9	19·43	1	vF; R; psbM; ✱7 p 7′	1
2314	826	III. 920	....	11 2 29·3	3·683	1	27 54 1·5	19·45	1	eF; vS; E 0°±; r	2
2315	828	II. 42	....	11 2 38·9	3·137	1	78 30 58·5	19·45	1	F; S; lE; vlbM	2*
2316	827	......	....	11 2 40·8	3·314	1	53 12 35·5	19·45	1	eF; S; ✱8, p	1
2317	......	I. 220	....	11 2 44·9	3·518	2	35 51 45·8	19·46	2	cB; cL; cE 160°...............	2
2318	831	V. 46	....	11 3 3·0	3·554	1	33 34 44·1	19·47	1	cB; vL; vmE 79°·0; pbM; r.	3
2319	829	III. 351	....	11 3 3·7	3·254	5	60 28 50·8	19·46	6	!; F(? var); S; R; bM; ✱9 f 1′; 1st of 4.	8*
2320	832	III. 352	....	11 3 9·6	+3·253	3	60 33 4·1	+19·47	3	eF; vS; 2nd of 4	5

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	h.	H.		h m s	s		° ′ ″	″			
2321	833	......	....	11 3 12·0	+3·253	1	60 36 21·1	+19·47	2	vF; pS; R; bM; 3rd of 4 ...	2
2322	3319	......	....	11 3 19·8	2·830	2	126 46 48·1	19·47	2	B; S; R; pgmbM; 1st of 3...	2
2323	......	III. 79	....	11 3 25·7	3·144	1	77 19 26·1	19·47	1	eF; pS; lE; r	1
2324	834	......	....	11 3 26·1	3·139	1	78 3 34·1	19·47	1	F; S; R; gbM................	1
2325	830	II. 337	....	11 3 30·4	4·167	1	16 22 0·4	19·48	1	pF; pS; lE; gbM; *15, 22°·1, 70″.	2
2326	835	......	....	11 3 36·1	3·252	3	60 32 50·1	19·47	3	vF; pL; 4th of 4	3
2327	3320	......	....	11 3 56·5	2·832	2	126 46 43·4	19·48	2	pF; S; R; bM; 2nd of 3......	2
2328	836	III. 89	....	11 4 2·4	3·108	2	83 25 3·4	19·48	2	eF; R; sbM; r	3
2329	3321	......	....	11 4 9·7	2·835	1	126 42 10·4	19·48	2	vF; pL; R; * inv; 3B st nr...	2
2330	......	II. 819	....	11 4 32·1	2·972	1	107 31 27·7	19·49	1	pF; pL; iF; bM	1
2331	3323	......	....	11 4 32·5	2·534	1	149 28 52·7	19·49	1	Cl; pRi; lC	1
2332	3322	......	....	11 4 39·9	2·842	1	126 5 31·7	19·49	1	eF; S; R; glbM; 3 st 11 f ...	1
2333	3324	......	....	11 5 52·5	2·522	1	150 36 57·3	19·51	1	F; lE; 1st of 6..............	1†
2334	......	III. 723	....	11 5 53·4	3·424	1	40 53 27·6	19·52	1	eF; vS; p of 2	1
2335	837	......	....	11 5 54·4	2·988	1	105 11 45·6	19·52	1::	Neb (?)	1
2336	3325	......	....	11 5 56·2	2·526	2	150 28 3·6	19·52	2	F; lE; sbM; 2nd of 6.........	2†
2337	3326	......	....	11 6 3·2	2·526	2	150 32 58·6	19·52	2	*12 with fan-shaped neb att; 3rd of 6.	2†
2338	3327	......	....	11 6 14·4	2·528	2	150 30 34·6	19·52	2	B; bM*; 4th of 6	2†
2339	......	II. 728	....	11 6 18·3	3·422	2	40 51 28·9	19·53	2	pB; pL; R; vgmbM	2
2340	3329	......	....	11 6 21·4	2·541	2	150 26 42·6	19·52	2	F; L; E 0°; bM; 5th of 6 ...	2†
2341	3328	II. 269	....	11 6 28·1	2·923	2	115 59 51·9	19·53	2	B; pL; E; vsmbMN; 2 B st △	4
2342	3330	......	....	11 6 31·0	2·540	2	150 35 29·9	19·53	2	eF; S; E 160°±; 6th of 6 ...	2†
2343	838	......	M. 97	11 6 34·8	3·514	1	34 13 38·2	19·54	1	!!; ○; vB; vL; R; vvg, vsbM○; 19″·0 d.	4†
2344	839	III. 921	....	11 6 51·9	3·621	1	28 32 29·2	19·54	1	vF; L; E; vgbM; in △ of L st	2
2345	3332	......	....	11 6 59·1	2·546	1	150 2 19·2	19·54	1	Cl; pRi; C; E................	1
2346	3331	III. 529	....	11 7 4·5	3·000	1	103 19 48·2	19·54	1	vF; S; iR; lbM	3
2347	840	I. 29	....	11 7 16·1	3·144	3	76 25 29·5	19·55	3	B; cL; E 90°±; psmbM......	4
2348	......	III. 770	....	11 7 26·0	3·520	1	33 29 29·5	19·55	1	vF; vS; stellar..............	1
2349	......	III. 706	....	11 7 30·1	3·404	2	41 45 30·5	19·55	2	vF; vS; vlE; stellar; cB* n...	2
2350	841	II. 102	....	11 7 43·8	3·154	1	74 26 53·8	19·56	2	pF; L; R; glbM	2
2351	3333	......	....	11 7 49·1	2·946	1	112 57 42·8	19·56	1	vF; pS; R; bM	1
2352	843	II. 49	....	11 8 4·2	3·172	1	71 7 42·8	19·56	1	B; pS; R; pgmbM............	2
2353	842	II. 709	....	11 8 6·6	3·339	1	47 38 28·8	19·56	1	pF; S; lE 0°±; vgbM	4
2354	3334	......	....	11 9 7·0	2·558	1	150 29 39·4	19·58	1	⊕ and neb; st 15...18	2
2355	......	II. 626	....	11 9 7·9	3·098	1	84 42 34·4	19·58	1	pB; S; lE; mbM..............	1
2356	844	III. 27	....	11 9 21·8	3·169	2	71 14 8·7	19·59	2	F; S; R; sp of 3	3
2357	3335	......	....	11 9 29·9	2·885	1	123 3 54·7	19·59	1	eF; S; R; gbM	1
2358	845	II. 50	....	11 9 30·6	3·169	2	71 11 38·7	19·59	2	vB; L; R; vmbM; 2nd of 3..	3
2359	846	II. 51	....	11 9 35·3	3·169	2	71 5 49·7	19·59	2	B; pL; R; psbM; 3rd of 3...	3
2360	847	I. 270	....	11 10 11·5	3·548	3	30 27 29·0	19·60	3	vB; pS; lE 90°±; vsvmbMSN.	5
2361	849	II. 521	....	11 10 16·1	3·098	2	84 40 50·0	19·60	2	pF; cS; iR; psmbM; *10, 330°, 3′.	5
2362	848	I. 271	....	11 10 20·9	3·533	1	31 14 40·3	19·61	1	vB; cL; mE 305°·0; smbMN	2
2363	850	II. 729	....	11 10 29·9	3·366	1	43 29 5·3	19·61	1	F; pL; lE 90°±; glbM; r ...	3
2364	851	III. 333	....	11 10 39·5	3·197	1	65 49 46·3	19·61	1	cF; vS; smbM; stellar; p of 2	2
2365	851, a	......	R. nova	11 10 ±	......	...	65 49 ±	......	...	F; S; bM; place from MS ...	0
2366	......	III. 76	....	11 10 53·0	3·149	1	74 29 36·6	19·62	2	eF; pL........................	1
2367	3336	......	....	11 10 59·5	2·939	1	115 21 55·6	19·62	1	F; S; R; gbM.................	1
2368	......	III. 334	....	11 11 0·0	3·197	1	65 42 34·6	19·62	1	vF; S; f of 2	1
2369	852	I. 244	....	11 11 9·6	3·520	1	31 30 10·6	19·62	1	cB; cL; R; vgmbM	3
2370	3338	......	....	11 11 21·4	1·992	1	165 27 17·6	19·62	1	F; pS; pmE; gbM	1
2371	3337	I. 241	Δ. 617	11 11 29·6	2·899	1	122 2 45·9	19·63	1	cB; vL; E 160°±; am 4 st...	2
2372	853	II. 879	....	11 11 32·8	3·745	1	21 59 31·9	19·63	1	pB; S; R; gbM	2
2373	854	......	M. 65	11 11 36·7	3·139	3	76 8 46·9	19·63	3	B; vL; mE 165°±; gbMBN.	8*†
2374	855	......	....	11 11 40·5	3·111	1	81 42 52·9	19·63	1	eF	1
2375	......	II. 885	....	11 12 17·2	3·512	1	31 22 36·2	19·64	1	F; S; lE; 135°±	1
2376	856	II. 52	....	11 12 43·1	+3·164	1	70 52 55·5	+19·65	1	B; S; vlE; sbM	2

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	h.	H.		h m s	s		° ′ ″	″			
2377	857 = 875?	......	M. 66	11 12 48·3	+3·137	2	76 15 17·5	+19·65	3	B; vL; mE150°; mbM; 2st np	9*†
2378	859	V. 8	....	11 12 57·4	3·140	4	75 38 28·5	19·65	4	pB; vL; vmE102°·0	8†
2379	858	I. 226	....	11 13 1·7	3·435	1	36 4 6·5	19·65	1	pB; L; R; svmbMrN........	2†
2380	860	II. 338	....	11 13 3·3	3·211	7	62 16 12·5	19·65	7	cF; L; R; vgvlbM...........	9
2381	861	......	....	11 13 5·2	3·089	2	86 16 11·5	19·65	2	pB; S; R; smbMN............	3
2382	......	II. 30	....	11 13 6·1	3·163	1	71 4 37·5	19·65	1	pB; * inv.........................	1*
2383	862	II. 550	....	11 13 21·8	3·027	2	99 30 44·8	19·66	2	F; vS; R; lbM; *7 f; p of 2.	4
2384	863	II. 551	....	11 13 36·0	3·028	2	99 28 56·8	19·66	2	F; vS; R; psbM; *7p; f of 2	4
2385	856, a	......	R. nova	11 13 46·1	3·164	::	70 52 55·5	19·66	::	pF; S; R; vlbM; foll h. 856, 15′.	0
2386	864	II. 33	....	11 13 53·9	3·090	1	85 59 53·1	19·67	1	B; pL; R; psbM	3
2387	865	I. 245	....	11 14 10·0	3·515	4	30 9 43·1	19·67	4	pB; pL; R; vgbM	7
2388	867 = 861?	II. 32	....	11 14 20·6	3·088	1?	86 16 50·1	19·67	1?	pB; S; E; bM	1*
2389	866	III. 15	....	11 14 20·9	3·170	2	69 4 20·4	19·68	2	cF; cL; lE; gbM; sp of 2 ...	3
2390	868	......	....	11 14 51·7	3·290	1	49 22 36·7	19·69	1	pB; S; pmE; bMN=close *?	1
2391	869	III. 16	....	11 14 52·5	3·169	2	69 1 43·7	19·69	2	vF; pS; R; gbM; nf of 2 ...	3
2392	870	III. 335	....	11 15 1·4	3·190	1	64 56 3·7	19·69	1	cF; vS; R; bM; np of 2	2
2393	872	III. 336	....	11 15 6·1	3·190	1	64 57 48·7	19·69	1	vF; vS; sf of 2	2
2394	871	II. 775	....	11 15 6·6	3·274	1	51 27 49·7	19·69	1	pF; cL; lE; vgbM	2
2395	......	II. 880	....	11 15 24·0	3·766	2	19 47 39·0	19·70	2	F; S; lE 15°±	2
2396	873	I. 5	....	11 15 33·5	3·150	3	72 38 30·0	19·70	3	pB; pS; iR; bM; r	5
2397	874	II. 782	....	11 15 40·7	3·423	1	35 23 23·0	19·70	1	pB; S; R; vgbM; *12 p ...	2
2398	876	III. 768	....	11 15 58·6	3·409	1	36 18 44·3	19·71	1	cF; vS; R; stellar	3
2399	878	II. 53	....	11 16 22·1	3·154	1	71 25 28·3	19·71	1	cF; S; lE; r	3
2400	877	IV. 59	....	11 16 23·3	3·273	1:	50 42 2·3	19·71	1:	H. cB; S; R; svmbMN h. F; R; *17 M.	3
2401	......	II. 635	....	11 16 27·0	3·037	1	97 52 41·3	19·71	1	F; pL; iR; vgbM	1
2402	3339	III. 530	....	11 16 37·0	3·014	1	103 3 31·3	19·71	1	F; S; R; stellar; p of 2	2
2403	879	IV. 4	....	11 16 39·9	3·070	1	90 19 53·3	19·71	1	vF; S; att to *13 m	3
2404	881	I. 219	....	11 17 10·2	3·271	2	50 28 40·6	19·72	2	cB; cL; iR; pgmbM	3
2405	882	I. 20	....	11 17 11·3	3·123	3	77 53 21·6	19·72	3	F; E 90°±; B* f 34s	5*
2406	3340	III. 531	....	11 17 11·8	3·015	2	103 4 29·6	19·72	1	pF; pL; iR; vlbM	4
2407	880	II. 845	....	11 17 14·0	3·569	1	25 47 2·9	19·73	1	F; pS; iR; gbM; *9 np......	4
2408	883	II. 829	....	11 17 24·4	3·462	1	31 30 53·9	19·73	1	vF; pL; pmE 135°±; er ...	3
2409	884	III. 337	....	11 17 25·2	3·182	1	65 16 34·9	19·73	1	vF; vS; R	2
2410	885	III. 922	....	11 17 54·2	3·503	1	28 45 10·2	19·74	1	vF; vS; 2 vS st inv	2
2411	886	I. 131	....	11 17 58·3	3·034	1	99 1 48·9	19·73	2	pB; L; E 0°±; gbM	3*
2412	3341	......	....	11 18 20·4	2·956	1	115 58 29·2	19·74	1	F; vL; gvlbM; *7 s 6′	1
2413	887	I. 194	....	11 18 26·4	3·301	2	45 38 32·2	19·74	2	vB; cL; vmE 0°±; vsmbMN; st p.	4
2414	888	......	....	11 18 34·5	3·328	2	42 14 27·2	19·74	2	eF; S; R; vsbM*; 2 st 11 nf.	2
2415	......	II. 886	....	11 18 36·9	3·442	1	32 6 42·5	19·75	1	pF; iF	1
2416	889	......	....	11 18 47·7	3·199	1	61 21 44·5	19·75	1	vF; S; R; psbM; *12 nf ...	1
2417	......	III. 112	....	11 18 51·5	3·051	2?	95 4 52·5	19·75	2?	eF; cL; R; r (v near vB*)...	3*
2418	3342	......	Δ. 481	11 18 56·6	2·858	2	132 27 50·5	19·75	2	Cl; cL; pRi; lC; st 10...14...	2
2419	891 = 3343	II. 159	....	11 19 10·3	3·145	4	72 22 2·5	19·75	4	B; pS; R; bM	5
2420	890	I. 262	....	11 19 19·3	3·618	1	22 38 33·8	19·76	1	cB; S; iR; spmbMN	2
2421	892	I. 246	....	11 19 32·8	3·431	1	32 20 57·8	19·76	1	cB; pL; E	3
2422	893	......	....	11 19 52·5	3·144	2	72 12 19·8	19·76	3	pB; pL; E; vgbM	3
2423	894	II. 160 = III. 28	....	11 20 23·8	3·144	2	72 0 27·1	19·77	2	pB; L; vlE; vgbM; r.........	5
2424	895	II. 770	....	11 20 32·3	3·202	5	59 42 52·1	19·77	5	pB; pS; R; lbM; r...........	6
2425	896	I. 247	....	11 20 44·8	+3·441	1	30 40 47·4	+19·78	2	pB; pS; vlE 80°±; pgbM; S st sf nr.	4

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	h.	H.		h m s	s		° ′ ″	″			
2426	897	II. 339	....	11 20 45·7	+3·182	1	63 34 36·4	+19·78	1	pB; pL; lE; bM	2
2427	898	II. 54	....	11 20 49·6	3·142	2	72 18 40·4	19·78	2	F; pS; lE; r	4
2428	......	II. 152	....	11 20 59·6	3·111	1	79 41 44·4	19·78	1	F; mE; r..	1
2429	3334	III. 532	....	11 21 9·0	3·023	1	102 24 55·4	19·78	1	cF; S; E; gbM	2
2430	899	......	....	11 21 22·8	3·231	1	53 48 40·7	19·79	1	cF; S; R; sbM*?	1
2431	900	......	....	11 21 30·0	3·158	1	68 25 51·7	19·79	1	eF; vS; E90°	1
2432	3345	......	....	11 21 33·5	2·706	1	149 11 3·7	19·79	1	B; pL; iR; pgpmbM	1
2433	901	II. 349	....	11 22 5·3	3·171	1	65 7 52·0	19·80	1	pF; pL; lE	2
2434	902	II. 13	....	11 22 53·7	3·109	1	79 57 4·3	19·81	1	pF; pL; R; vsmbM; r	4
2435	3346	......	....	11 22 53·9	2·920	3	125 37 34·3	19·81	3	pB; cS; R; psmbM	3
2436	903	......	....	11 23 0·3	3·109	1::	79 52 19·3	19·81	1	pB; cS; E90°	1
2437	904	II. 350	....	11 23 43·7	3·161	1	66 27 39·6	19·82	1	F; S; *7·8 nf 5′	2
2438	905	......	....	11 24 15·2	3·185	1	60 44 5·6	19·82	1	F; vS; R; smbM	2
2439	906	II. 367	....	11 24 17·1	3·183	3	61 4 23·6	19·82	3	F; cS; R; sbMN	4
2440	907	III. 353	....	11 24 28·7	3·183	3	60 52 0·9	19·83	3	F; S; R; psbM	5*
2441	3347	II. 562	....	11 24 29·0	3·024	2	103 27 38·9	19·83	2	pF; S; R; vgvlbM	5*
2442	3348	......	....	11 24 36·6	2·958	2	119 29 4·9	19·83	2	pB; S; mE; * 13 att	2
2443	908	I. 221	....	11 24 46·4	3·342	2	36 9 21·9	19·83	2	pB; vL; R; vglbM	4
2444	909	II. 836	....	11 25 43·2	3·442	2	27 21 32·2	19·84	2	cF; S; R; gvlbM; r	3
2445	910	II. 730	....	11 25 44·5	3·284	2	42 10 57·2	19·84	2	pB; vL; lE0°; vsmbM*15; *11 n.	3†
2446	912	II. 351	....	11 26 2·1	3·162	1	64 46 58·5	19·85	1	F; S; R; bM	2
2447	911	I. 222	....	11 26 2·3	3·333	1	36 5 47·5	19·85	1	pB; pL; lE0°±; gbM; *12nr	3
2448	......	III. 80	....	11 27 5·9	3·115	1	76 43 50·8	19·86	1	vF; vS; R	1
2449	913	II. 552	....	11 27 8·0	3·042	2	99 3 46·8	19·86	2	F; S; R; psbM; *14 sp 225°	3
2450	......	III. 771	....	11 27 10·4	3·340	1	34 23 50·8	19·86	1	eF; S; iR; L * in field	1
2451	3349	III 935	....	11 27 37·1	3·038	1	103 19 8·1	19·87	1	cF; S; R; gbM	2
2452	914	I. 287	....	11 27 44·4	3·602	1	18 41 33·1	19·87	1	pB; L; mE130°·4; mbM ...	2
2453	......	III 772	....	11 27 44·9	3·340	1	34 15 51·1	19·87	1	vF; stellar	1
2454	......	II. 783	....	11 28 6·8	3·331	1	34 41 52·1	19·87	1	pB; pL; bM	1
2455	915	III. 847	....	11 28 23·8	3·387	1	29 15 14·4	19·88	1	vF; vS; R; vgbM	2
2456	916	......	....	11 28 33·7	3·255	1	43 56 23·4	19·88	1	vF; S; R; vgbM	1
2457	3350	......	....	11 28 37·1	2·938	2	127 10 35·4	19·88	2	pF; pL; vlE; glbM	2
2458	......	III. 969	....	11 28 56·6	3·748	1	14 14 50·7	19·89	1	eF; S	1
2459	3351	......	....	11 28 59·4	2·939	2	127 12 45·4	19·88	2	F; cS; lE; gvlbM	2
2460	917	II. 905	....	11 29 3·2	3·784	1	13 56 57·4	19·88	1	pB; pL	2
2461	918	II. 784	....	11 29 4·5	3·319	1	34 55 43·7	19·89	1	pF; L; lE	2*
2462	920	......	....	11 29 4·7	3·203	1	52 49 3·7	19·89	1	eF; pL; pmE; gbM	1
2463	919	III. 843	....	11 29 8·5	3·360	1	30 49 18·7	19·89	1	vF; R; stellar; vS* 1 d sf ...	1
2464	......	......	D'Arrest, 78	11 29 16	3·14	[1]	67 24 12	19·88	[1]	B; pS; mbMN=*13; *11 p 4ˢ, s 175″.	0
2465	921	II. 837	....	11 29 29·6	3·398	1	27 29 1·7	19·89	1	F; vlE; gbM	2
2466	......	......	D'Arrest, 79	11 29 39	3·13	[2]	71 20 42	19·89	[2]	F; S; R	0
2467	922	......	....	11 29 41·9	3·150	1	65 7 26·7	19·89	1	vF; S; R	1
2468	3352	......	Δ. 289	11 29 42·0	2·764	2	150 49 29·7	19·89	2	Cl; pL; pRi; pC; st 8...13 ..	2
2469	923	III. 29	....	11 29 54·8	3·128	1	71 23 48·7	19·89	1	vF; eS; stellar...............	2
2470	924	......	....	11 29 55·8	3·125	1	72 20 37·7	19·89	1	vF; S; bM	1
2471	925	II. 731	....	11 30 10·0	3·262	2	41 19 6·0	19·90	2	pB; S; pmE	4
2472	926	II. 838	....	11 30 13·1	3·364	2	29 36 37·0	19·90	2	pF; S; R; gbM; r	3
2473	927	II. 352	....	11 30 27·9	3·143	2	66 32 34·0	19·90	2	vF; S; E; r	4
2474	928	III. 81	....	11 30 58·4	3·109	1	77 6 36·3	19·91	1	cF; cS; R; psbM	4
2475	3353	......	....	11 31 34·0	2·881	1	139 55 51·3	19·91	1	eF; S; R; am 50 S st	1
2476	929	I. 227	....	11 31 41·7	3·315	2	32 57 27·3	19·91	2	pF; L; vlE; vgbM; r	4
2477	930	II. 732	....	11 32 1·9	3·241	1	42 45 34·6	19·92	1	F; S; att to *15; another * cont.	2
2478	3354	......	....	11 32 6·8	2·954	1	126 57 46·6	19·92	2	cB; R; sbMN*; *9 sf	2
2479	931	......	....	11 32 18·7	3·171	3	57 18 46·6	19·92	3	pB; pL; pmE; gbM; p of 2...	3
2480	932	......	....	11 32 21·7	3·171	3	57 17 20·6	19·92	3	cB; pL; pmE0°; pgbM	3
2481	933	III. 109	....	11 32 29·4	3·123	2	71 31 5·6	19·92	2	cF; vS; pmE; sbM; 2 S st f; 1st of 3.	3
2482	935	III. 609	....	11 32 35·2	+3·048	1	98 35 14·6	+19·92	1	vF; vS; R; gbM; *8 s 6′ ...	2

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	h.	H.		h m s	s		° ′ ″	″			
2483	......	III. 773	....	11 32 45·1	+3·305	1	32 59 54·6	+19·92	1	cF; pS; vS*v nr	1
2484	......	III. 844	....	11 32 47·4	3·327	1	30 36 24·9	19·93	1	vF; S; mE	1
2485	937	II. 839	....	11 32 47·9	3·345	2	28 56 14·9	19·93	2	F; cS; R; mbM	3
2486	934 = 3355	......	....	11 32 54·0	3·116	2	73 53 41·9	19·93	3	cF; R; p of 2	3†
2487	938	II. 340	....	11 32 54·5	3·144	1	64 31 35·9	19·93	1	F; cS; lE; stellar; r	3
2488	936 = 3356	II. 103	....	11 32 57·8	3·116	4	73 52 36·9	19·93	6	F; pS; E; pglbM; r; f of 2...	8†
2489	936, a	......	R. nova	11 33 ±	......	...	73 52 ±	......	...	Smaller than h. 936	0
2490	939	II. 161	....	11 33 0·4	3·122	3	71 29 46·9	19·93	3	pF; pL; R; bM; r; 2nd of 3	4
2491 2492 5069	939, a 939, b	......	R. 3 novæ	11 33 ±	......	...	71 29 ±	......	...	No description (for 939, c, see No. 5069).	0
2493	940	III 30	....	11 33 5·0	3·122	3	71 27 36·9	19·93	2	vF; pS; r; 2 vB st p; 3rd of 3	4
2494	......	II. 830	....	11 33 23·7	3·300	1	32 58 24·9	19·93	1	pB; E	1
2495	941	III. 375	....	11 33 26·1	3·129	1	68 53 9·9	19·93	1	cB; cS; R; bM; r	3
2496	......	......	D'Arrest, 80	11 33 29	3·12	[1]	71 25 42	19·93	[1]	F; pL; *9·10, s 5′	0
2497	......	III. 338	....	11 33 35·4	3·135	1	66 46 41·9	19·93	1	vF; vS............	1
2498	942	II. 737	....	11 33 45·6	3·237	2	41 30 46·9	19·93	2	F; S; vlE; glbM	4
2499	943	I. 21	....	11 33 45·7	3·104	3	77 45 10·9	19·93	4	B; L; vlE	8
2500	944	III. 320	....	11 33 48·3	3·142	1	64 24 10·9	19·93	1	cF; vS; R; p of 2; * 6 sf 3′..	3
2501	945	I. 94	....	11 33 53·4	3·183	3	52 40 24·2	19·94	3	cB; pL; pmE 90° ±; bM ...	5*
2502	946	III. 339	....	11 34 20·2	3·140	1	64 25 31·2	19·94	1	cF; S; f of 2	2
2503	947	......	....	11 34 40·4	3·099	1	78 55 43·2	19·94	1	F; 1st of 4	1
2504	948	III. 284	....	11 34 47·6	3·058	1	95 23 14·2	19·94	1	F; pS; R; psbM............	3
2505	950	......	....	11 34 51·9	3·099	1	78 54 43·2	19·94	1	vF; 2nd of 4	1
2506	949	III. 376	....	11 34 52·5	3·126	2	68 54 26·5	19·95	2	vF; cS; R; bM; bet 2 st......	5
2507	951	II. 153	....	11 34 58·4	3·099	1	78 57 13·5	19·95	1	pF; pS; 3rd of 4............	2
2508	3357	......	....	11 35 9·4	3·039	1	103 4 35·5	19·95	1	F; cS; lE; pslbM	1
2509	952	III. 774	....	11 35 10·2	3·258	1	36 26 51·5	19·95	1	vF; cS; pmE	3
2510	953	II. 154	....	11 35 11·4	3·099	1	78 58 3·5	19·95	1	pF; pS; 4th of 4............	2
2511	954	II. 341	....	11 35 14·4	3·143	3	62 44 1·5	19·95	3	pF; S; R; psbM; stellar ...	4
2512	......	......	D'Arrest, 81	11 35 21	3·12	[1]	70 22 42	19·94	[1]	F; S; lbM	0
2513	955	III. 775	....	11 35 51·3	3·253	1	36 29 51·5	19·95	1 :	vF; vS............	2
2514	956	......	....	11 35 56·5	3·141	1	62 43 32·5	19·95	1	eF	1
2515	957	......	....	11 36 11·6	3·043	1	102 5 22·8	19·96	1	F; vS; R; bM	1
2516	......	III. 340	....	11 36 15·0	3·130	1	66 24 56·8	19·96	1	vF; pL; 2 suspected neb nr ..	1
2517	......	III. 102	....	11 36 15·9	3·097	1	79 6 56·8	19·96	1	eF; pS............	1
2518	......	......	D'Arrest, 82	11 36 21	3·12	[1]	70 7 42	19·95	[1]	vF; vS; slbMN*13 m	0
2519	958	......	....	11 36 24·4	3·306	2	29 6 47·8	19·96	2	pB; E; gbM; *8 nf 5′	1
2520	959	II. 831	....	11 36 29·2	3·286	2	31 16 21·8	19·96	2	pB; cS; E; psbM*12.........	3
2521	960	......	....	11 36 32·6	3·121	2	69 15 52·8	19·96	2	cF; S; R; 1st of 5	1
2522 2523 2524 2525	960, a	......	R. 4 novæ	11 36 ±	......	...	69 15 ±	......	...	8 "knots" (vide h. 960, 1, 2, 3)	0
2526	961	......	....	11 36 39·1	3·120	2	69 20 12·8	19·96	2	cF; S; R; 2nd of 5............	1
2527	962	III. 377	....	11 36 44·2	3·120	3	69 17 13·8	19·96	3	F; S; R; vglbM; 3rd of 5 ...	5
2528	963	......	....	11 36 46·7	3·120	1	69 14 9·8	19·96	1	vF; pS; 4th of 5............	1
2529	964	......	....	11 36 53·3	3·264	1	33 34 40·8	19·96	1	F; pL; R; vgbM	1
2530	965	......	....	11 36 53·6	3·159	1	55 42 18·8	19·96	1	F; S; R; psbM	1
2531	......	III. 35	....	11 36 57·9	3·098	1	78 55 57·8	19·96	1	eF; vS	1
2532	......	III. 776	....	11 37 2·1	3·265	1	33 19 56·8	19·96	1	eF; pL; lE	1
2533	966	III. 378	....	11 37 6·3	3·120	1	69 15 33·8	19·96	1	eF; vS; R; 5th of 5	3
2534	......	III. 36	....	11 37 11·9	3·098	1	78 55 57·8	19·96	1	eF; vS............	1
2535	......	III. 386	....	11 37 37·2	3·118	1	69 27 57·1	19·97	1	vF; vS; r	1
2536	970	......	....	11 37 45·7	3·118	1	69 14 20·1	19·97	1	F; S; R; bM (?)............	1
2537	......	III. 385	....	11 37 53·2	3·117	1	69 35 57·1	19·97	1	vF; vS; r............	1
2538	971	......	....	11 38 30·2	+3·096	2	78 23 56·4	+19·98	2	F; S; iR; psbM	1

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	h.	H.		h m s	s		° ′ ″	″			
2539	972	III. 833	....	11 38 34·3	+3·218	3	39 1 19·4	+19·98	3	cF; cS; R; psbM	4
2540	967	......	....	11 38 34·6	3·152	1?	56 6 59·4	19·98	1?	eF; R; gbM; 1st of 4.........	1*
2541	973	II. 104	....	11 38 35·6	3·102	3	75 27 32·4	19·98	3	B; S; R; smbM*	5
2542	......	III. 104	....	11 38 36·0	3·091	1	80 40 29·4	19·98	1	vF; vS; suspected	1
2543	......	III. 387	....	11 38 37·2	3·116	1	69 27 58·4	19·98	1	vF; vS; r	1
2544	......	III. 103	....	11 38 37·6	3·093	1	80 1 58·4	19·98	1	vF; r	1
2545	......	I. 201	....	11 38 55·9	3·202	2	41 43 58·4	19·98	2	B; L; mE 25°±	2
2546	974	......	....	11 38 56·4	3·150	1	56 1 20·4	19·98	1	vF; R; 2nd of 4	1
2547	......	II. 881	....	11 39 0·8	3·391	1	19 49 57·4	19·98	1	F; pL; mE 105°±	1
2548	968 = 975	......	....	11 39 1·4	3·150	2	56 3 20·4	19·98	2	vF; R; gbM; 3rd of 4	2
2549	969 = 976	......	....	11 39 14·9	3·149	2	56 7 5·4	19·98	2	vF; R; gbM; 4th of 4	2
2550	3358	......	....	11 39 16·6	2·903	2	145 36 20·4	19·98	2	vF; lE; 2 st inv	1
2551	......	III. 372	....	11 39 21·8	3·112	1	68 35 58·4	19·98	1	vF; cL..........	1
2552	977	III. 388	....	11 39 30·9	3·116	1	68 49 39·4	19·98	1	cF; S; iR; gbM; r; *7 sp 6′	4
2553	3359	III. 828	....	11 39 45·1	3·014	2	117 8 33·7	19·99	2	cF; vS; vlE; bM; vF* sf ...	2
2554	979 = 3360	I. 120	....	11 39 56·8	3·040	2	106 4 52·7	19·99	2	pB; L; iR; vgpmbM	3
2555	978	II. 785	....	11 40 6·3	3·238	2	33 15 24·7	19·99	2	pB; S; lE; pgbM	3
2556	980	II. 723	....	11 40 45·5	3·136	3	58 51 30·7	19·99	3	pB; S; bM	4
2557	981 = 3361	II. 553	....	11 40 53·3	3·053	2	100 10 27·7	19·99	2	pB; pL; R; gbM; r	4
2558	......	III. 940	....	11 40 54·2	3·462	1	14 52 59·0	20·00	1	vF; S; R; bM	1
2559	982	II. 738	....	11 41 12·9	3·192	1	40 30 26·0	20·00	1	B; pL; R; mbM...............	2†
2560	983	I. 248	....	11 41 18·7	3·250	2	29 48 43·0	20·00	2	B; pL; iR; pgmbM; p of 2 ..	4
2561	984	II. 832	....	11 41 29·7	3·248	1	29 47 51·0	20·00	1	pF; pL; vlE; gbM; f of 2...	3
2562	......	II. 739	....	11 41 36·1	3·189	1	40 30 59·0	20·00	1	F; vS	1
2563	986	II. 408	....	11 41 41·5	3·144	3	54 11 35·0	20·00	3	F; S; R; bM	5
2564	985	I. 228	....	11 41 45·6	3·224	4	33 8 26·0	20·00	4	B; pL; lE; svmbM............	6
2565	987	......	....	11 41 51·5	3·123	1	62 46 31·0	20·00	1	pB; R; smbM.................	1
2566	988	I. 82	....	11 41 53·3	3·124	5	62 11 43·0	20·00	5	B; pL; vlE 0°±; bMN	8
2567	......	III. 970	....	11 41 58·8	3·564	1	11 7 57·0	20·00	1	pF; pL; r	1
2568	989	III. 321	....	11 42 1·2	3·122	2	63 5 50·0	20·00	2	F; pS; lE; vglbM	3
2569	3362	......	....	11 42 1·4	2·998	2	126 44 21·0	20·00	2	pB; cS; vlE; lbM	2
2570	3363	II. 864	....	11 42 9·2	3·019	1	118 32 27·0	20·00	1	pB; S; R; mbM...............	2
2571	......	III. 715	....	11 42 11·9	3·185	1	40 48 0·0	20·00	1	eF; pL........................	1
2572	990	......	....	11 42 19·8	3·065	1	90 19 7·0	20·00	1	eF; S; psbM	1
2573	3364	......	....	11 42 34·3	2·967	1	137 29 23·0	20·00	1	Cl; vL; lC; st 9...14	2
2574	991	III. 341	....	11 42 42·2	3·117	(1)	64 17 48·0	20·00	1:	vF; S; p of 2	2
2575	992	II. 342	....	11 42 49·0	3·120	3	62 44 0·3	20·01	3	F; pL; R; pgbM	4
2576	......	II. 786	....	11 43 0·7	3·209	1	33 53 0·3	20·01	1	F; E	1
2577	......	III. 113	....	11 43 4·8	3·066	1	94 21 0·3	20·01	1	eF; eS; bet 2 st	1*
2578	993	II. 787	....	11 43 20·3	3·206	1	34 4 32·3	20·01	1	eF; R; gbM	2
2579	994	......	....	11 43 20·4	3·190	1	37 23 43·3	20·01	1	F; L; vmE; vgbM............	1
2580	995	III. 90	....	11 43 20·5	3·084	2	82 38 55·3	20·01	3	F; vS; R; lbM; *13 np 80″.	4
2581	3365	......	....	11 43 23·9	2·934	4	146 24 8·3	20·01	5	○; !; S; R; blue; = *7m; 1″·5=d.	6
2582	996	......	....	11 43 33·5	3·115	1::	64 17 48·3	20·01	1	Neb; f of 2	1
2583	......	II. 824	....	11 43 36·9	3·193	1	36 21 30·3	20·01	1	pB; L; mE	1
2584	997	II. 788	....	11 43 37·1	3·203	1	34 8 32·3	20·01	1	pF; S; R; pspmbM	2
2585	......	III. 716	....	11 43 47·7	3·180	1	39 1 6·3	20·01	1	vF; vS.........................	1
2586	3366	I. 259	....	11 43 57·8	3·025	2	118 3 5·3	20·01	2	B; pL; lE; gmbM; r; vS*sp inv.	3
2587	......	II. 825	....	11 43 59·4	3·189	1	39 8 0·3	20·01	1	pB; S; iF; bM	1
2588	......	......	D'Arrest, 83	11 44 6	3·11	[1]	67 19 48	20·01	[1]	vF; vS.........................	0
2589	998	III. 379	....	11 44 11·2	+3·108	1	67 12 20·3	+20·01	1	eF; eS; vlE; er; st nr.........	4

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	h.	H.		h m s	s		° ′ ″	″			
2590	999	II. 740	....	11 44 24·7	+3·170	1	40 31 56·6	+20·02	1	pF; S; R; pspmbM	2
2591	1000	III. 616	....	11 44 29·7	3·139	2	51 18 20·6	20·02	2	eF; cL; iF; glbM; *6n5′; *7f	4*
2592	......	III. 769	....	11 44 39·8	3·182	1	37 16 0·6	20·02	1	eF; S	1
2593	......	......	D'Arrest, 84	11 44 49	3·17	[1]	40 35 48	20·01	[1]	vF; v diffic; H. II. 740 np ...	0
2594	1001	......	....	11 45 8·9	3·124	3	56 48 50·6	20·02	3	pF; S; lE; psbM	3
2595	3367	......	....	11 45 19·8	3·033	1	116 7 21·6	20·02	1	vF; cL; vmE 59°·3	1
2596	1003	III. 389	....	11 45 28·9	3·103	1	68 35 6·6	20·02	1	vF; cS; R	2
2597	1002	I. 203	....	11 45 30·5	3·150	2	45 5 35·6	20·02	2	B; vL; R; bMpBN; er	3†*
2598	......	III. 971	....	11 45 31·1	3·378	1	14 7 0·6	20·02	1	eF; vS; R	1
2599	1004	III. 380	....	11 45 34·2	3·103	1	68 14 0·6	20·02	1	vF; cS; R	3
2600	1005	I. 173	....	11 45 38·3	3·132	3	52 13 50·6	20·02	3	vB; pL; R; smbM*9	4
2601	1007	III. 322	....	11 45 50·3	3·111	3	63 0 39·6	20·02	3	pF; pS; R; psbM	4
2602	1006	I. 251	....	11 45 50·5	3·211	2	28 33 14·6	20·02	2	B; pL; R; gmbM; r; *f	3
2603	1008	II. 403	....	11 46 7·9	3·101	1	68 28 0·6	20·02	2	F; pS; iE; lbM; ** p	5
2604	1009	I. 202	....	11 46 21·7	3·155	2	41 21 59·9	20·03	2	cB; pL; pmE; vgbM	4*
2605	1010	III. 342	....	11 46 22·4	3·105	1	65 49 28·6	20·02	1	vF; cS; vlE	2
2606	1011	V. 45	....	11 46 27·4	3·169	1	36 53 3·9	20·03	1	cB; L; E 0°±; vsbMLrN ...	3†
2607	1012	III. 612	....	11 46 28·1	3·068	2	93 13 23·9	20·03	2	cF; cS; lE 90°±; bM; r ...	4
2608	1013	III. 381	....	11 46 30·0	3·101	1	68 20 44·9	20·03	1	eF; R	2*
2609	......	II. 623	....	11 46 53·7	3·043	2	112 24 0·9	20·03	2	cF; S; E 170°±; lbs	2
2610	3368	III. 290	....	11 46 56·8	3·047	1	109 47 29·9	20·03	1	cF; pL; pmE 56°·8	2
2611	......	II. 294	....	11 46 58·4	3·048	1	108 47 0·9	20·03	1	F; S; E; r	1
2612	1014	II. 833	....	11 47 1·8	3·188	2	30 51 20·9	20·03	2	pF; pS; pmE; vgbM	3
2613	1015	IV. 67	....	11 47 27·9	3·184	2	30 43 51·9	20·03	2	pF; cL; R; vg, sbM	3
2614	3369	......	Δ. 349	11 47 29·3	2·976	1::	144 56 21·9	20·03	1	Cl; pL; pRi; gplmbM; st 13	3
2615	......	III. 905	....	11 47 31·4	3·254	1	19 54 0·9	20·03	1	eF; vS	1
2616	3370	I. 67	....	11 47 32·4	3·056	2	103 11 30·9	20·03	2	cB; pL; iR; gmbM; Δ 2 st...	6
2617	1016	......	....	11 47 39·1	3·108	2	60 57 8·9	20·03	2	vF; S; E; *10nf att	3
2618	......	II. 789	....	11 48 4·2	3·166	1	33 54 0·9	20·03	1	pB; E	1
2619	......	II. 790	....	11 48 4·2	3·166	1	33 54 0·9	20·03	1	F; S	1
2620	1017	IV. 62	....	11 48 6·5	3·165	1	34 6 0·9	20·03	1	B; pL; R; g, sbM disc	2
2621	1018	II. 162	....	11 48 16·5	3·086	2	77 15 16·9	20·03	2	pB; L; iR; bM; *10, 25°, 5′	4
2622	1018, a	......	R. nova	11 48 +	......	...	77 15 −	......	...	nf h. 1018	0
2623	1020	......	....	11 48 18·9	3·060	1	101 15 54·9	20·03	1	F; S; R; psbM; p of 2	1
2624	1019	II. 724	....	11 48 20·0	3·109	1	59 13 39·9	20·03	1	pF; vS; R; bM	2
2625	1021	......	....	11 48 32·4	3·060	1	101 12 44·9	20·03	1	vF; S; R; bM; f of 2	1
2626	1022	II. 132	....	11 48 45·9	3·079	3	82 28 4·2	20·04	3	B; pL; cE 30°; vsmbMN ...	4
2627	1023	II. 840	....	11 48 54·9	3·178	2	28 42 30·2	20·04	2	cF; S; lE; bM; *8, 90°, 6′...	3
2628	......	III. 274	....	11 49 3·7	3·052	1	109 7 1·2	20·04	1	vF; pL; iF	1
2629	1024	III. 343	....	11 49 10·1	3·098	1	65 20 55·2	20·04	1	cF; cS; R; psbM	2
2630	1026	......	....	11 49 23·3	3·104	1	60 13 26·2	20·04	1	eF; S; R; bM	1
2631	1025	III. 707	....	11 49 23·7	3·136	1	40 52 57·2	20·04	1	vF; cS; another suspected ...	3
2632	1027	......	....	11 49 30·3	3·107	3	57 11 59·2	20·04	3	pF; S; pmE 90°±; *11 nr...	1
2633	1028	......	....	11 50 9·6	3·100	1	61 20 43·2	20 04	1	vF; S; R; bM*; p of 2	2
2634	1029	II. 791	....	11 50 14·3	3·147	1	33 46 31·2	20·04	1	pF; S; lE; pslbM	2
2635	1030	IV. 61	....	11 50 15·6	3·142	1	65 50 34·2	20·04	1	cB; vL; pmE; sbMBrN	3
2636	1032	......	....	11 50 34·2	3·085	3	74 55 29·2	20·04	2	vF; pL; R; 2 st f	3
2637	1031	I. 229	....	11 50 34·3	3·145	1	33 46 21·2	20·04	1	cB; pS; R; vg, smbM	2
2638 2639 2640	1033, a	......	R. 3 novæ	11 50 ±	......	...	63 57 ±	......	...	one S, R; the other two E ...	0
2641	1033	III. 323	....	11 50 36·1	3·095	1	63 57 34·2	20·04	1	pF; vS; E 25° bet 2 st	2
2642	1040, a	......	R. nova	11 50 44·7	3·121	::	41 54 13·2	20·04	::	S; R; 7′ np h. 1040	0
2643	1034	III. 344	....	11 50 45·6	3·093	1	66 0 39·2	20·04	1	vF; vS; R; n of 2	2
2644	1035	III. 345	....	11 50 45·6	3·093	1	66 4 59·2	20·04	1	vF; vS; R; s of 2	2
2645	1036	III. 354?	....	11 50 52·9	3·097	2	61 20 22·2	20·04	2	F; vS; R; *12 near	3
2646	......	III. 324	....	11 50 53·4	3·095	1	63 53 51·2	20·04	1	eF; suspected	1
2647	1037	......	....	11 50 56·3	3·070	1	91 21 8·2	20·04	1	F; S; R; bM; *11 nf	1
2648	......	III. 325	....	11 50 57·6	3·087	1	66 6 2·2	20·04	1	eF; vS	1
2649	1038	II. 368	....	11 51 3·0	3·097	4	61 1 33·2	20·04	3	pB; pS; R; psbM; r	5
2650	1039	......	....	11 51 4·6	+3·097	1	61 10 ±?	+20·04	0	vB; mE; mbM (?P.D.)	1*

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	h.	H.		h m s	s		° ′ ″	″			
2651	1040	......	....	11 51 14·4	+3·121	1	41 59 10·2	+20·04	1	F; pL; mE; vglbM	1
2652	1041	II. 733	....	11 51 19·2	3·116	2	45 17 9·2	20·04	2	B; cL; mE 62°·3; vsvmbM * 10	3*†
2653	1042	......	....	11 51 24·9	3·086	2	73 2 32·2	20·04	2	pB; pS; R; psbM	2*
2654	1043, a	......	R. nova	11 51 33	3·095	:	61 39 50	20·04	:	vF	0
2655	1043	II. 369	....	11 51 33·0	3·095	(1)	61 44 50·2	20·04	2	F; L; E; gbfM	4
2656	1044	......	....	11 51 42·8	3·083	1	75 0 46·2	20·04	1	eF; * 9 sf 5′	1
2657	1045	II. 725	....	11 51 43·9	3·097	1	58 47 57·5	20·05	1	pB; pL; E 19°·5; biN	2
2658	......	II. 295	....	11 51 56·2	3·059	1	107 35 2·5	20·05	1	F; vS; iF; bM	1
2659	1046	III. 617	....	11 51 57·7	3·104	1	51 24 39·5	20·05	1	eF; pL; R	3
2660	1047	I. 223	....	11 52 6·8	3·122	1	38 15 44·5	20·05	1	vB; cL; mE 160° ±; vsvmbMBN.	3
2661	3371	II. 296	....	11 52 23·1	3·059	1	108 29 25·5	20·05	1	⊕; pF; pL; R; rr; st 16 ...	2
2662	......	III. 3	....	11 52 45·0	3·083	2	73 0 1·5	20·05	2	vF; vS; vlE; r	2
2663	1048	I. 121	....	11 53 12·3	3·072	2	90 19 7·5	20·05	2	cB; L; vlE; psmbM; B st nr	4
2664	1049	II. 404	....	11 53 21·0	3·084	6	69 8 34·5	20·05	6	pF; pL; R; gbM; * 12 nf...	7
2665	......	II. 508	....	11 53 21·4	3·062	1	107 3 2·5	20·05	1	pB; S; lE; bM	1
2666	......	III. 903	....	11 53 45·4	3·155	1	19 52 1·5	20·05	1	eF; S; iF; gvlbM	1
2667	3372	III. 279	....	11 53 50·5	3·064	1	105 10 13·5	20·05	1	eF; pL; * 9 45° ±	2
2668	1050	I. 253	....	11 54 12·4	3·125	1	27 19 59·5	20·05	1	{ H. vb; vL; E } { h. pB; 25″; R } *	2*
2669	1051	III. 77	....	11 54 13·1	3·079	2	75 49 13·5	20·05	2	eF; pL; R; r	3
2670	1052	IV. 28·1	....	11 54 43·7	3·064	1	108 5 11·5	20·05	1	pB; cL; R; vgbM	2†
2671	1053	IV. 28·2	....	11 54 43·7	3·064	1	108 7 11·5	20·05	1	pF; pL	2†
2672	1054	I. 252	....	11 54 58·6	3·117	2	27 5 10·5	20·05	2	B; cL; R; g, psvmbMrN ...	3
2673	1055	......	....	11 55 10·9	3·074	1	84 52 38·5	20·05	2	pF; S; R; psbM; ** f 30ˢ ...	2
2674	1056	III. 491	....	11 55 18·9	3·072	1	89 25 53·5	20·05	1	cF; cS; R; bM	3
2675	1057	II. 276	....	11 55 32·9	3·072	1	87 14 48·5	20·05	1	pF; L; R; sbM; * sf	4
2676	1058	II. 741	....	11 55 36·5	3·095	1	40 35 7·5	20·05	1	pB; pS; R	2
2677	1059	......	....	11 55 38·9	3·079	2	71 12 12·5	20·05	2	vF; vS; R; psbM	2
2678	1060	III. 390	....	11 55 41·8	3·079	1	70 28 21·5	20·05	3	eF; pS; R; glbM	4
2679	......	II 509	....	11 55 47·7	3·066	1	105 36 2·5	20·05	1	F; cL; iR; lbM	1
2680	1061	IV. 56	....	11 55 58·3	3·089	4	44 41 3·5	20·05	4	B; vL; E; vg, vsmbM * 11...	5†
2681	3373	......	....	11 55 59·6	3·039	1	152 24 13·5	20·05	1	Cl; pRi; lC	1
2682	......	III. 794	....	11 56 8·6	3·099	1	31 18 2·5	20·05	1	eF; S	1
2683	1062	......	....	11 56 32·1	3·078	1	68 10 1·5	20·05	1?	pB. P.D. very doubtful ...	1*
2684	1063	......	....	11 56 36·6	3·078	1::	68 8 1·5	20·05	1?	pB. P.D. very doubtful ...	1*
2685	1064	......	....	11 56 40·6	3·078	1::	67 55 1·5	20·05	1?	pB. P.D. very doubtful ...	1*
2686	1065	......	....	11 56 51·4	3·077	3	68 59 39·5	20·05	3	vF; S; R; D neb pos 70° ...	3*
2687	1066	I. 174	....	11 56 53·2	3·080	4	57 19 32·5	20·05	4	pB; vL; mE 97°; vgbM......	5
2688	......	......	D'Arrest, 85	11 56 57	3·07	[1]	70 46 42	20·06	[1]	B; E; gbM * 17 p, 82″ dist..	0
2689	1067	......	....	11 56 57·5	3·077	2	68 59 58·5	20·05	2	pF; R	2*
2690	1068	......	....	11 56 59·7	3·076	4	68 51 54·5	20·05	4	pB	4*
2691	1069	III. 37	....	11 57 1·0	3·074	5	78 21 53·5	20·05	5	F; pS; R; gbM	6
2692	......	II. 781	....	11 57 1·6	3·087	2	36 40 2·5	20·05	2	pF; S; stellar	2
2693	1070	III. 392	....	11 57 1·7	3·076	1	69 53 38·5	20·05	1	vF; vS..........................	2*
2694	1071	III. 391	....	11 57 2·0	3·076	2	68 49 13·5	20·05	2	F; vS	2*
2695	3374	......	....	11 57 4·9	3·046	1	156 31 32·5	20·05	1	vF; vS; R; bM*; am st ...	1
2696	1072	II. 277	....	11 57 17·2	3·072	4	87 19 26·5	20·05	4	F; pS; R; pgbM; np of 2 ...	7
2697	1073	III. 393	....	11 57 20·8	3·076	1	68 54 1·5	20·05	1	eF; vS..........................	2*
2698	1074	......	....	11 57 20·9	3·072	1	87 9 8·5	20·05	1	F; S; R	1
2699	1075	III. 394	....	11 57 24·7	3·076	1	69 3 31·8	20·06	1	vF; vS..........................	2*
2700	1076	III. 258	....	11 57 29·5	3·072	1	87 26 4·8	20·06	1	cF; cS; vlE; bM; sf of 2 ...	3
2701	......	III. 395	....	11 57 32·3	3·075	2?	69 7 1·8	20·06	2?	vF; vS..........................	1*
2702	......	III. 396	....	11 57 32·3	3·075	2?	69 7 1·8	20·06	2?	vF; vS..........................	1*
2703	1077	......	....	11 57 38·1	3·072	1	91 36 8·8	20·06	1	F; L; R; * 10 n 60″	1
2704	1078	III. 355	....	11 57 42·0	3·076	4	62 13 27·8	20·06	4	cF; pS; E; gbM...............	5
2705	......	......	D'Arrest, 86	11 58 13	3·08	[1]	38 54 7	20·06	[1]	F; lE; I. 206 nr	0
2706	3375	III. 754	....	11 58 24·4	3·070	1	115 44 43·8	20·06	1	pB; S; R; bM	2
2707	......	I. 224	....	11 58 40·3	3·074	2	38 56 2·8	20·06	2	B; pL; pmE; vsbM	2
2708	......	I. 206	....	11 58 42·2	3·074	3	38 42 22·8	20·06	3	B; cL; pmE 135° ±; lbM ...	3
2709	3376	......	....	11 58 44·2	+3·072	1	103 45 28·8	+20·06	1	eF; L; pmE; vgbM; 2 st 11 nr	1

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	h.	H.		h m s	s		° ′ ″	″			
2710	1079	III. 382	…………	11 58 45·3	+3·072	1	68 38 1·8	+20·06	2	vF; vS…………………………	4
2711	1081	I. 207	…………	11 58 51·9	3·072	1	41 44 34·8	20·06	1	pB; vL; mE 32°·0 …………	4
2712	1080	III. 400	…………	11 58 53·1	3·072	2	52 21 4·8	20·06	2	eF; vS; R; stellar; * 10 sp 2′	6
2713	1082	III. 383	…………	11 58 54·0	3·072	2	68 36 5·8	20·06	3	eF; eS; R; bM …………	5
2714	……	III. 384	…………	11 58 57·0	3·072	1	68 35 2·8	20·06	1	eF; eS…………………………	2
2715	1084	III. 717	…………	11 58 59·8	3·072	1	39 38 47·8	20·06	1	pB; vL; vmE 166°·5; vgvlbM	8
2716	1083	III. 326	…………	11 59 0·1	3·072	2	63 39 44·8	20·06	2	eF; vS; R; vgbM …………	3
2717	1085	I. 225	…………	11 59 15·4	3·070	1	36 30 34·8	20·06	1	B; pS; R; bMBrN; * 12 sp, v, nr	3
2718	3377	……	Δ. 291	11 59 28·0	3·077	3	150 27 56·8	20·06	3	Cl; pL; pC; iR; st 10…14…	3
2719	1086	II. 370	…………	11 59 28·8	3·070	6	61 2 37·8	20·06	6	pB; pS; lE; bM …………	7
2720	3378	II. 865	…………	11 59 29·4	3·074	2	119 0 26·8	20·06	2	pF; pS; R; psbM; r; p of 2	3
2721	3379	II. 866	…………	11 59 34·4	3·074	2	119 0 41·8	20·06	2	pF; pS; R; pgbM; f of 2 …	3
2722	1087	……	…………	11 59 39·8	3·062	1	22 3 42·8	20·06	1	B; S; R; gbM…………	1
2723	1088	I. 195	…………	11 59 52·9	3·067	2	46 9 26·8	20·06	2	vB; pS; mE 151°·0…………	4
2724	3380	……	…………	11 59 55·8	3·077	2	129 25 18·8	20·06	2	F; S; vlE; glbM; 3B st nr…	2
2725	1089	……	…………	12 0 0·5	3·068	1	55 13 46·8	20·06	1	eF …………………………	1
2726	3381	III. 533	…………	12 0 0·9	3·074	1	103 24 23·8	20·06	1	cF; S; iR; gbM…………	2
2727	1090	……	…………	12 0 1·7	3·070	1	74 49 13·8	20·06	1	eF; suspected …………	1
2728	1092, a	……	R. nova	12 0 19·8	3·072	…	86 30 33·5	20·06	…	Hook-shaped; h. 1092 is nf 45°; 14′ dist.	0
2729	1091	III. 708	…………	12 0 26·9	3·064	1::	46 12 1·8	20·06	1::	vF; vS…………………………	2
2730	……	II. 14	…………	12 0 42·1	3·070	1	79 41 2·5	20·05	1	lE …………………………	1*
2731	……	III 904	…………	12 0 56·3	3·040	1	19 38 2·5	20·05	1	eF; vS; E …………	1
2732	1093	……	…………	12 0 59·2	3·064	1	56 13 7·5	20·05	1	eF; vS; R; mbM …………	1
2733	1092	V. 4	…………	12 0 59·4	3·072	2	86 20 39·5	20·05	2	cF; vL; E 90°±; bM * 16…	4†
2734	1094	{I. 33 = II. 60}	…………	12 1 1·2	3·070	2	78 50 36·5	20·05	3	pB; pL; mE 120°; bM; r …	5
2735	……	……	Auw. N. 28	12 1 3·1	3·045	…	24 2 50·6	20·05	…	pB; pL; cE; mbMN (Hind, Jan. 5, 1850).	0
2736	1095	III. 68	…………	12 1 28·2	3·067	1	73 5 11·5	20·05	1	vF; S; R; pslbM; bet 2 vS st	2
2737	1096	I. 279	…………	12 1 29·2	3·005	2	12 25 10·5	20·05	2	F; pL; vlE; glbM …………	4
2738	……	I. 263	…………	12 1 33·2	3·032	1	20 25 2·5	20·05	1	cB; lE; bM…………………	1
2739	{1097 = 3382}	II. 548	…………	12 1 42·3	3·074	3	98 15 19·5	20·05	3	F; pL; pmE 95°±; vglbM	4
2740	1098	III. 356	…………	12 1 47·9	3·063	2	59 56 7·5	20·05	2	cF; S; R; 1st of 3 …………	3
2741	1099	III. 357	…………	12 1 52·9	3·063	3	59 58 57·5	20·05	3	cF; S; iR; 2nd of 3 …………	4
2742	1100	I. 278	…………	12 1 59·4	3·003	2	14 19 20·5	20·05	2	pB; cL; R; gmbM…………	3
2743	1101	II. 371	…………	12 2 1·4	3·062	2	60 3 7·5	20·05	2	pF; pL; lE; 3rd of 3 …………	4
2744	1108	II. 321	…………	12 2 6·7	3·061	1	59 18 5·5	20·05	1	F; vL; vgmbM …………	3
2745	……	I. 196	…………	12 2 23·2	3·053	2	45 32 32·5	20·05	2	B; pL; lE; vgbM; * np……	2
2746	1102	III. 795	…………	12 2 29·8	3·037	1	30 22 0·5	20·05	1	vF; pS; lE; gbM; r …………	3
2747	1103	III. 814	…………	12 2 31·4	3·044	1	36 6 6·5	20·05	1	vF; S; iF; vglbM; er…………	2*
2748	1104	IV. 54	…………	12 2 43·5	3·051	1::	46 44 3·5	20·05	1	cB; R; vg, vsbMN…………	2
2749	1107	II. 747	…………	12 2 54·1	3·047	2	42 46 19·5	20·05	2	pF; cL; vmE 109°·0; vgbM..	3
2750	1105	I. 169	…………	12 2 54·5	3·053	1	49 20 37·5	20·05	1	B; vL; vglbM…………………	2
2751	1106, a	……	R. nova	12 2 ±	……	…	70 40 ±	……	…	S; prec h. 1106 …………	0
2752	1106	I. 19	…………	12 2 55·5	3·064	3	70 40 17·5	20·05	3	⊕; vB; pL; R; gbM; rrr …	4
2753	……	III. 327	…………	12 2 56·8	3·060	1	62 48 2·5	20·05	1	vF; pS…………………………	1
2754	1109	II. 802	…………	12 3 22·6	3·030	1	30 56 13·5	20·05	1	F; S; E …………………	2
2755	1110	I. 73	…………	12 3 26·7	3·057	1	58 49 5·5	20·05	1	B; S; R; pgmbM …………	3
2756	1111	I. 165	…………	12 3 27·0	3·050	2	49 49 31·5	20·05	2	vB; S; R; vsmbMBN; p of 2	4†
2757	1112	II. 83	…………	12 3 28·7	3·064	3	73 11 28·5	20·05	3	pB; pL; R; pgmbM; r …………	4
2758	……	I. 11	…………	12 3 36·9	3·063	1	70 52 2·5	20·05	1	B; pL; E; bM …………	1
2759	……	III. 845	…………	12 3 42·6	3·026	1	30 53 2·5	20·05	1	vF; S; E 90°± …………	1
2760	1113	II. 642	…………	12 3 46·5	3·049	2:	49 45 45·5	20·05	2:	pF; S; E; vgbM; f of 2……	4†
2761	1114	I. 208	…………	12 4 0·9	3·036	1	38 43 54·5	20·05	1	pF; cL; vmE 60°± …………	4
2762	1115	II. 405	…………	12 4 4·2	3·061	1	69 2 58·5	20·05	1	F; pS; lE; bM; pB * nf……	3
2763	1116	III. 941	…………	12 4 4·7	2·945	1	13 5 48·5	20·05	1	eF; pS; R; Δ 2 st …………	2
2764	……	II. 803	…………	12 4 39·4	+3·018	2	31 27 2·5	+20·05	2	F; S; R …………………	2

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	h.	H.		h m s	s		° ′ ″	″			
2765	1117	II. 353	....	12 4 46·9	+3·057	1	65 5 49·5	+20·05	1	B; L; iE; bM..................	2
2766	......	III. 399	....	12 4 56·4	3·046	2	53 2 2·5	20·05	2	vF; pL; vlE; er	2
2767	1118	......	....	12 5 4·5	3·045	1	52 43 3·5	20·05	1	F; pL; R; vgbM; ** sp 10′..	1
2768	1119	II. 105	....	12 5 8·6	3·063	1	76 1 5·5	20·05	1	pB; pL; iF; psbM; r; * inv	3
2769	1120	III. 358	....	12 5 14·5	3·051	2	60 3 17·5	20·05	2	F; S; 1st of 4	4
2770	1123	II. 792	....	12 5 16·6	3·016	1	33 2 58·5	20·05	1	F; S; lE; gbM	2
2771	1121	II. 372	....	12 5 17·2	3·051	2	60 0 1·5	20·05	1::	F; S; 2nd of 4..................	4*
2772	1122	III. 359	....	12 5 18·7	3·051	2	60 5 31·5	20·05	2	F; S; 3rd of 4..................	4
2773	1124	III. 360	....	12 5 27·2	3·050	2	60 3 31·5	20·05	2	F; eS; 4th of 4	4*
2774	3383	III. 534	....	12 5 30·9	3·081	1	103 14 38·5	20·05	1	vF; pL; R; vgbM	2
2775	1125	......	....	12 5 35·2	2·993	1	78 22 7·5	20·05	2	vF; vL; E 45°±; *7 f	2
2776	1126	I. 9	....	12 5 42·9	3·070	1	87 55 28·5	20·05	1	pB; pS; pmE 135°±; bMN	5
2777	1127	II. 133	....	12 5 53·0	3·067	3	82 11 3·5	20·05	3	pF; S; lE 0°±; r	5
2778	......	III. 777	....	12 6 4·8	3·016	1	36 20 2·5	20·05	1	eF; S; stellar	1
2779	1128	III 697	....	12 6 13·1	3·031	2	45 32 36·5	20·05	2	vF; cL; mE 170°±	5
2780	3384	......	....	12 6 14·4	3·152	1	151 56 12·2	20·04	1	Cl; mC; st eS.................	1
2781	1129	II. 373	....	12 6 15·1	3·048	3	60 42 57·2	20·04	3	cF; L; R; gbM	5
2782	......	II. 813	....	12 6 28·8	3·018	1	38 30 2·2	20·04	1	pB; S; lE	1
2783	1131	II. 106	....	12 6 38·7	3·061	1	75 48 11·2	20·04	1	F; L; lE; vglbM; r	3
2784	1133	II. 409	....	12 6 39·4	3·038	1	52 35 28·2	20·04	1	cF; pS; R; vglbM; r	3
2785	1130	......	....	12 6 39·6	3·066	4	82 1 3·2	20·04	4	cF; R; bM; near S*	4
2786	1132	......	M. 98	12 6 40·0	3·060	3	74 19 1·2	20·04	4	B; vL; vmE 152°·1; vsvmbM	7
2787	1134	II. 163	....	12 6 44·6	3·061	1	76 3 5·2	20·04	1	vF; pL; E; vgbM	2
2788	1135	II. 867	....	12 7 6·6	3·004	1	34 40 46·2	20·04	1	pB; vS; vsbM *12............	2
2789	......	III. 796	....	12 7 8·8	2·988	1	29 34 2·2	20·04	1	eF	1
2790	1136	II. 374	....	12 7 26·4	3·045	4	60 48 15·2	20·04	4	pB; S; R; vsmbM *	5
2791	1137	II. 134	....	12 7 28·5	3·066	1	83 24 54·2	20·04	1	pF; pmE; vgbM...............	2
2792	1139	II. 793	....	12 7 29·7	2·997	1	33 12 18·2	20·04	1	pF; pS; lE; gbM	3
2793	......	III. 797	....	12 7 32·0	2·983	2	29 16 2·2	20·04	2	vF; S	2
2794	1138	II. 164	....	12 7 32·7	3·061	2	77 3 13·2	20·04	2	cF; { H. vmE / h. R, 2 obs. } lbM ...	3
2795	......	II. 165	....	12 7 44·7	3·060	1	76 4 2·2	20·04	1	F; vmE	1
2796	1140	I. 175	....	12 8 1·5	3·037	2	56 1 26·2	20·04	2	vB; S; R; psmbM	3
2797	1141	III. 397	....	12 8 8·0	3·051	1	68 33 36·2	20·04	2	vF; cL; iR; vgbM	4
2798	......	......	D'Arrest, 87	12 8 9	2·96	[1]	25 25 42	20·04	[1]	pB; pS; R; *12 f; ln.........	0
2799	1142	II. 107	....	12 8 19·4	3·058	1	75 19 25·2	20·04	1	vF; pL; R; gbM	2
2800	......	II. 375	....	12 8 21·7	3·041	1	60 43 22·2	20·04	1	F; pS	1
2801	1143	III. 850	....	12 8 23·4	2·945	1	23 14 29·2	20·04	1	pF; pS; R; vgbM	2
2802	1144	II. 108	....	12 8 31·3	3·057	2	75 19 1·2	20·04	2	B; L; E 90°±; g, sbM; r ...	3
2803	1145	II. 354	....	12 8 31·6	3·047	1	65 13 59·2	20·04	1	cF; vS; R	2
2804	1146	I. 95	....	12 8 36·0	3·030	1	52 54 3·2	20·04	1	cB; cL; iE; biN	3†
2805	1147	II. 135	....	12 8 44·8	3·065	3	82 49 9·2	20·04	3	B; pS; E; sbM *11	4
2806	1148	I. 35	....	12 8 46·7	3·058	2	76 4 2·2	20·04	3	vB; vL; vmE 17°±; sbMN	5†
2807	1149	II. 748	....	12 8 49·1	3·009	4	42 8 55·2	20·04	5	pF; L; mE 45°·0; ** n, p of 2	6†
2808	......	III. 718	....	12 8 55·0	3·006	1	41 5 2·2	20·04	1	vF; vS..........................	1
2809	3385	......	....	12 9 7·3	3·126	4	132 32 44·2	20·04	4	pF; pL; pmE; vglbM.........	4
2810	1150	......	....	12 9 9·1	2·931	1	22 59 24·2	20·04	1	pB; S; R; psbM...............	1
2811	1151	I. 209	....	12 9 12·9	3·005	1	41 20 34·2	20·04	1	cB; pL; pmE 134°·4; psbM..	3
2812	1152	II. 137	....	12 9 17·5	3·065	1::	82 31 31·9	20·03	1	pF; pL; R; r (? R.A. 10m)...	3
2813	1153	II. 136	....	12 9 21·7	3·063	2	81 45 38·9	20·03	2	pB; pS; lE; gb, *not* M; r ...	4
2814	......	II. 109	....	12 9 23·0	3·057	1	76 8 1·9	20·03	1	r	1*
2815	1154	......	....	12 9 25·1	3·084	1	101 31 49·9	20·03	1	F; eS; R; *170°, 60″.........	1
2816	1155	......	....	12 9 28·9	3·005	1	42 12 32·9	20·03	1	F; S; lE; f of 2	1
2817	1156	II. 518	....	12 9 31·4	3·030	2	55 42 6·9	20·03	2	F; vS; vlE; psbM; sp of 2...	4
2818	1157	......	....	12 9 32·3	3·026	1	52 53 41·9	20·03	1	vF; L; R; gbM	1
2819	1158	II. 519	....	12 9 36·9	3·030	2	55 39 37·9	20·03	2	cF; vS; lE; psbM; nf of 2...	4
2820	3386	......	....	12 9 41·0	3·160	1	144 31 20·9	20·03	1	Cl; F; pL; iF; st 13...15 ...	1
2821	1159	II. 17	....	12 9 58·6	3·063	2	82 1 40·9	20·03	2	pB; pL; pmE; lM; p of 2 ...	5
2822	1160	......	....	12 9 59·5	3·067	1	85 32 30·9	20·03	1	pB; L; R; gbM	1
2823	1161	II. 496	....	12 9 59·5	3·063	1	81 35 59·9	20·03	1	pF; R; vsbMSN...............	2
2824	1162	II. 11	....	12 10 3·8	+3·054	2	73 54 23·9	+20·03	2	pB; pL; lE; vgbM; r.........	5

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	h.	H.		h m s	s		° ′ ″	″			
2825	1163	V. 51	....	12 10 4·1	+2·894	1	19 48 0·9	+20·03	1	vF; eL; mE 160°±; vgbM..	3
2826	1164	III. 851	....	12 10 5·7	2·939	1	25 48 56·9	20·03	1	vF; pS; iR; vglbM...........	2
2827	......	III. 719	....	12 10 13·6	2·999	1	41 44 31·9	20·03	1	vF; vS; n of D neb	1
2828	......	III. 720	....	12 10 13·6	2·999	1	41 45 31·9	20·03	1	vF; vS; s of D neb...........	1
2829	1165	III. 480	....	12 10 17·5	3·063	2	82 32 50·9	20·03	2	vF; L; vgbM; * 7 s	3
2830	1166	III. 725	....	12 10 20·5	3·003	2	43 35 49·9	20·03	2	vF; cL; iR; vgbM; r.........	4
2831	1167	V. 41	....	12 10 27·9	3·019	2	51 24 39·9	20·03	2	pB; vL; eE 43°·2; vgbM ...	3
2832	1168	I. 74	....	12 10 32·5	3·033	3	59 36 54·9	20·03	3	cB; pL; vlE; smbM; r	5
2833	......	III. 91	....	12 10 43·7	3·063	1	82 6 1·9	20·03	1	eF	1
2834	1169	II. 742	....	12 10 48·8	2·995	2	41 49 7·9	20·03	2	vF; S; pmE; psbM...........	4
2835	1170	I. 264	....	12 10 57·2	2·864	1	18 25 17·9	20·03	1	pB; S; R; pgbM	2
2836	1171	I. 89	....	12 11 3·5	3·034	2	61 2 42·9	20·03	3	vB; S; E; vsvmbMN; * 6·7 f 90ˢ.	4
2837	1172	III. 702	....	12 11 30·9	3·029	1::	59 23 0·9	20·03	1::	vF; vS; R	2
2838	1173	......	M. 99	12 11 41·8	3·052	3	74 48 7·6	20·02	4	!!; {(H. h.)B; L; R; gbM; r / (L) 3-branched spiral}	6†
2839	1174	II. 846	....	12 11 56·0	2·898	1	23 19 23·6	20·02	1	pB; L; cE 38°·2; bMBN ...	2
2840	......	......	D'Arrest, 88	12 12 1	3·06	[1]	83 29 42	20·02	[1]	vF pS; R; *18 s 2′	0
2841	1175	V. 43	....	12 12 1·7	2·988	3	41 55 40·6	20·02	4	vB; vL; vmE 0°; sbMBN...	8†
2842	1176	II. 139	....	12 12 4·6	3·063	1	83 23 51·6	20·02	1	F; pS; R; gbM	4
2843	1177	II. 138	....	12 12 12·9	3·063	2	83 9 22·6	20·02	2	pB; E; psbM	5
2844	1178	......	....	12 12 13·0	3·064	1	83 53 0·6	20·02	1	neb; "1st of 5"	1
2845	1179	II. 110	....	12 12 22·0	3·051	1	74 20 51·6	20·02		B; S; R; r	3
2846	......	III. 535	....	12 12 26·0	3·090	1	101 28 31·6	20·02	1	vF; pL; iF	1*
2847	1180	II. 140	....	12 12 26·3	3·063	1	83 22 30·6	20·02	1	F; pS; R; gbM	4
2848	1181	II. 166	....	12 12 38·0	3·053	2	76 26 0·6	20·02	2	pB; vS; R; vsmbM...........	3
2849	......	......	D'Arrest, 89	12 12 41	3·06	[2]	83 12 42	20·02	[2]	pF; S; R; *9 f 1ˢ·7, n 85″...	0*
5070	......	......	....	12 12 41	......	...	83 46 42	......	...	See No. 5070	0
2850	1182	III. 299	....	12 12 45·0	3·024	1	58 52 49·6	20·02	1	cF; S; iR; gmbM	3
2851	1185	I. 75	....	12 12 47·4	3·025	1	59 36 15·6	20·02	1	vB; vL; E 90°±; mbMN ...	3
2852	1183	II. 568?	....	12 12 48·2	3·064	1	83 53 1·6	20·02	1	B; L; E; gbM	1*
2853	......	II 804	....	12 12 48·3	2·946	1	32 29 1·6	20·02	1	pB; pL; iF	1
2854	1184	II. 376	....	12 12 49·1	3·029	1	61 36 0·6	20·02	2	F; S; vlE; gbM; * 15 nr ...	3
2855	1186	{I. 90 = II. 322}	....	12 13 3·4	3·025	2	59 56 25·6	20·02	2	vB; pL; R; mbM; r; p of 2	5*
2856	......	II. 571??	....	12 13 8·7	3·062	1	82 51 1·6	20·02	1	4 neb sc about. Place of the last (see note).	1*
2857	1187	{II. 573 = II. 569?}	....	12 13 12·5	3·063	1	83 50 34·6	20·02	1	vB; vL; R; pgbM; "3 more seen."	3*
2858	1188	II. 323	....	12 13 18·6	3·024	2	59 54 20·6	20·02	2	B; S; R; bM; 2nd of 3	4
2859	......	II. 377	....	12 13 23·0	3·025	1	60 0 39·6	20·02	1	....	1*
2860	......	III. 798	....	12 13 25·7	2·933	1	31 6 1·6	20·02	1	cF; lE; p of 2................	1
2861	......	III. 300	....	12 13 35·7	3·023	2	59 51 31·3	20·01	2	vF	2
2862	1189	II. 570?	....	12 13 35·9	3·063	1::	83 54 0·6	20·02	1::	vF; S	1*
2863	1188, a	......	R. nova?	12 13 38·6	3·024	...	59 51 20·6	20·02	...	Most probably = H. III. 300	0?
2864	1191	III. 726	....	12 13 41·0	2·980	3	42 56 17·3	20·01	3	vF; pS; R; vgbM; r	5
2865	1190	II. 571?	....	12 13 46·0	3·063	1	83 52 34·3	20·01	1	vB; R; central of 4............	1*
2866	1193	II. 805	....	12 13 48·1	2·930	1	31 7 37·3	20·01	1	pB; L; R; gmbM	2
2867	1195	V. 5	....	12 14 4·2	3·042	2	70 50 17·3	20·01	2	F; vL; E; lbM; r	3
2868	1192	I. 275	....	12 14 4·8	2·720	3	13 51 7·3	20·01	4	pB; vS; R; lbM; 3 st f	6
2869	1194	......	....	12 14 6·1	3·063	2	83 49 57·3	20·01	2	B; pL; lE; bM; 4th of 4......	3*
2870	1196	......	....	12 14 6·1	3·064	1	84 38 4·3	20·01	2	F; S; R; vglbM; B*340°, 60″	2†
2871	1197	II. 61	....	12 14 8·4	3·053	4	77 42 56·3	20·01	4	F; L; mE 135°±; bi-N; p of 2	6
2872	......	III. 92	....	12 14 23·6	3·061	2	82 34 1·3	20·01	2	vF; vS..........................	2
2873	......	III. 93	....	12 14 23·6	3·061	2	82 34 1·3	20·01	2	eF; eS	2
2874	1198	II. 111	....	12 14 24·9	3·048	2	74 36 52·3	20·01	2	F; L; E 0°±; vgbM, p of 2	4
2875	1200	II. 62	....	12 14 32·7	3·053	2	77 43 1·3	20·01	2	F; L; lE; vgbM; f of 2	4
2876	1201	II. 572	....	12 14 33·1	3·063	1	83 50 23·3	20·01	1	F; lE; vgbM	2
2877	1199	II. 112	....	12 14 33·2	+3·047	2	74 36 53·3	+20·01	2	L; vmE 0°±; f of 2	4

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	h.	H.		h m s	s		° ′ ″	″			
2878	1202	I. 139	M. 61	12 14 45·1	+3·064	2	84 44 55·3	+20·01	2	vB; vL; vsbM*; biN.........	5*†
2879	3387	......	....	12 14 51·8	3·132	1	122 41 54·3	20·01	1	vF; vL; R; vgvlbM; r	1
2880	1203	......	....	12 14 56·7	3·050	1	76 29 35·3	20·01	1	vF; R	1
2881	1204	I. 76	....	12 15 23·5	3·016	1	59 19 46·0	20·00	1	cB; L; E 150°±; sbM; * np	3
2882	1205	II. 378	....	12 15 24·5	3·017	1::	60 0 34·0	20·00	2	B; cL; lE; np of 2	3
2883	1206	......	....	12 15 24·5	3·017	1	60 1 8·0	20·00	1	F; sf of 2........................	1
2884	1202, *a*	......	R. nova?	12 15 30	3·06	...	84 35 40	20·00	...	F; E; 10′ nf h. 1202	0†*
2885	1207	II. 63	....	12 15 30·2	3·051	2	77 26 25·0	20·00	2	vF; L; E 135°±; r...........	4
2886	1209	II. 628	....	12 15 31·6	3·044	(1)	73 40 57·0	20·00	1	pB; cL; E; gbM...............	2
2887	......	II. 324	....	12 15 34·9	3·011	1	58 11 0·0	20·00	1	F; S..............................	1
2888	1210	I. 276	....	12 15 36·3	2·686	3	13 53 59·0	20·00	3	pB; pS; vlE; sbM	5
2889	1208	......	....	12 15 37·4	3·057	1	81 1 20·0	20·00	1	eF; *8 n 5′	1
2890	1211	......	M. 100	12 15 50·6	3·043	3	73 23 54·0	20·00	4	!!; {(H, h) pF; vL; R; vg, psbMrN (L) 2-branched spiral}	5
2891	1212	II. 85	....	12 16 1·2	3·041	1	72 30 5·0	20·00	1	pB; S; R; psbM...............	2
2892	......	......	D'Arrest, 90	12 16 2	3·06	[4]	83 58 42	19·99	[4]	pB; R or lE; bM	0*
2893	1213	II. 141	....	12 16 2·0	3·060	1	83 8 51·0	20·00	1	vF; S; R; bM; 1st of 3	2
2894	......	II. 84	....	12 16 4·6	3·043	1	73 25 0·0	20·00	1	F; S; R; r	1
2895	1216	II. 847	....	12 16 6·6	2·843	1	23 22 43·0	20·00	1	pF; S; vlE; vgbM	2
2896	1214	......	....	12 16 8·6	3·093	1	101 45 29·0	20·00	1	vF; vS; R; bMN...............	1
2897	1217	II. 806	....	12 16 8·9	2·905	2	30 47 2·0	20·00	2	pB; S; E; gbM	3
2898	1220	III. 942	....	12 16 11·2	2·637	1	13 3 7·0	20·00	1	eF; E 0°±	2
2899	1215	II. 142	....	12 16 12·5	3·060	1	83 11 1·0	20·00	1	F; pS; R; bM; 2nd of 3......	2
2900	1218	......	....	12 16 16·0	3·058	1	81 45 7·0	20·00	1	pF; S; R; *v nr...............	1
2901	1219	II. 406	....	12 16 21·1	3·035	2	69 48 9·0	20·00	2	vF; pL; iR; biN?	4
2902	3388	......	....	12 16 23·3	3·231	1	147 20 30·0	20·00	1	Cl; pRi; lC; st 12...14	1
2903	1221	II. 86	....	12 16 23·7	3·040	1	72 31 5·0	20·00	1	cB; vS; mE; vsbM............	2
2904	1222	II. 143	....	12 16 26·6	3·059	3	83 8 10·0	20·00	3	B; pL; R; bM; 3rd of 3......	4
2905	......	III. 95	....	12 16 29·8	3·058	2	82 14 31·0	20·00	2	eF; vS; R	2
2906	......	III. 96	....	12 16 29·8	3·058	2	82 14 31·0	20·00	2	eF; vS; R	2
2907	1223	III. 94	....	12 16 30·1	3·060	1	82 16 59·0	20·00	1	pF; S; E; ?D..................	3
2908	1224	III. 31	....	12 16 32·7	3·038	1	71 41 1·0	20·00	1	vF; pS; R; vglbM; Δ 2 st ...	2
2909	1200, *a*	......	R. nova	12 16 34·7	3·053	::	77 43 1·3	20·00	::	vF; vmE	0
2910	1225	I. 210	....	12 16 34·8	2·960	5	42 14 8·0	20·00	6	vF; S; mE 100°±; vsmbMBN	8†
2911	1226	II. 625	....	12 16 43·1	3·077	1	92 40 18·0	20·00	1	F; pL; E 70°±; vlbM	4
2912	3389	......	Δ. 292	12 16 49·2	3·262	3	151 7 11·0	20·00	3	Cl; vB; vL; lC; st 12...14...	3
2913	......	III. 481	....	12 17 3·3	3·055	1	80 42 0·7	19·99	1	vF	1
2914	1230	III. 799	....	12 17 9·0	2·896	1	30 50 31·7	19·99	1	cF; cS; lE (? 18ᵐ R.A.)......	2
2915	1228	I. 123	....	12 17 9·3	3·061	1	84 17 31·7	19·99	1	B; S; * 8·9 sf 3′	3
2916	1229	III. 648	....	12 17 10·3	3·006	1	57 42 34·7	19·99	1	cF; pmE 90°; vlbM	2
2917	1231	I. 65	....	12 17 16·6	3·107	1	108 0 3·7	19·99	1	vB; L; R; vsmbMn; r	2
2918	1233	III. 800	....	12 17 17·0	2·895	1	30 51 31·7	19·99	1	vF; cS; R; r	2
2919	......	III. 938	....	12 17 17·6	2·659	1	14 17 0·7	19·99	1	eF; pL; iF	1
2920	......	III. 801	....	12 17 18·5	2·894	1	30 48 59·7	19·99	1	cF; cS; R	1
2921	1232	I. 30	....	12 17 21·3	3·057	2	81 54 17·7	19·99	3	cB; pL; vlE; gl, smbM	5
2922	......	III. 97	....	12 17 23·3	3·057	1	81 50 0·7	19·99	...	eF	2
2923	......	III. 38	....	12 17 35·0	3·050	1	78 37 59·7	19·99	1	vF; vS............................	1
2924	1234	I. 166	....	12 17 40·3	2·981	2	49 51 4·7	19·99	2	cB; S; R; mbMN; r	4
2925	1235	I. 22	....	12 17 45·8	3·048	3	77 31 34·7	19·99	3	B; pS; R; gbM	6
2926	1236	II. 144	....	12 17 46·2	3·057	2	81 46 52·7	19·99	2	pF; pS; lE; bM	3
2927	3390	......	Δ. 67??	12 17 50·1	3·412	1	161 53 19·7	19·99	1	⊕; pF; L; R; st 12...16 ...	2
2928	3391	......	....	12 17 55·9	3·162	1	128 58 12·7	19·99	1	pB; S; R; pgvmbM	1
2929	1227	II. 64	....	12 17 56·9	3·050	1	77 59 51·7	19·99	2	cF; cS; lE	3
2930	1237	......	M. 84	12 17 57·6	3·045	1	76 20 8·7	19·99	1	vB; pL; R; psbM; r	2
2931	1238	II. 379	....	12 17 59·0	2·994	2	60 40 3·7	19·99	2	F; S; R; bM; * nf 90″	3
2932 2933 2934 2935 2936	1237, *a*	......	R. 9 novæ	12 18 ±	+3·045	...	76 20 ±	+19·99	...	"Twelve knots exam." (see h. 1237, 1244, 1250).	0

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	h.	H.		h m s	s		° ′ ″	″			
2937 2938 2939 2940	1237, a	......	R. 9 novæ (continued)	12 18 ±	+3·045	...	76 20 ±	+19·99	...	"Twelve knots exam." (see h. 1237, 1244, 1250).	0
2941	......	II. 530	....	12 18 2·6	3·060	1	83 29 59·7	19·99	1	F; S	1
2942	1239	I. 12	....	12 18 7·3	3·041	1	74 27 43·7	19·99	1	B; S; R; smbM	4
2943	1240	II. 87??	....	12 18 13·0	3·039	1	73 35 30·7	19·99	1	pS; R; psbMN	2
2944	1241	......	....	12 18 17·0	3·051	1	79 13 0·4	19·98	1	vF; pL; R; lbM	1
2945	......	II. 743	....	12 18 19·0	2·940	1	40 23 59·4	19·98	1	F; S	1
2946	1242	......	M. 85	12 18 19·5	3·033	2	71 2 10·4	19·98	2	vB; pL; R; bM; * np	3
2947	1243	III. 879	....	12 18 22·3	2·910	2	34 43 1·4	19·98	2	cF; S; iR	3
2948	1247	I. 277	....	12 18 36·7	2·609	2	13 42 3·4	19·98	2	B; cL; lC; psmbM	4
2949	1244	......	....	12 18 41·2	3·045	1	76 34 33·4	19·98	1	vF; E; p of 2	1
2950	1245	II. 749	....	12 18 41·2	2·950	3	43 32 16·4	19·98	3	pB; pL; iE; vglbM	5†
2951	......	II. 87	....	12 18 43·3	3·039	1	73 41 59·4	19·98	1	S; bM; r	1*
2952	1248	III. 852	....	12 18 44·3	2·819	2	24 17 14·4	19·98	2	cF; S; R; sbM; ** sp	3
2953	1249	III. 729	....	12 18 48·4	2·951	1	43 25 20·4	19·98	1	cF; S; R; vgbM	2
2954	1246	III. 361	....	12 18 48·5	3·010	1:	61 39 49·4	19·98	1	vF; vL; iF; B * p	2
2955	1250	II. 167	....	12 18 51·3	3·044	1::	76 29 1·4	19·98	1	Northern of 2; no description	2
2956	1250	II. 168	....	12 18 51·3	3·044	1::	76 29 1·4	19·98	1	Southern of 2; E	2
2957	1251	II. 55	....	12 18 52·0	3·032	2	71 0 52·4	19·98	2	pB; lE; bM	3
2958	1252	V. 29·1	....	12 18 52·0	2·993	1	55 40 35·4	19·98	1	eF; vL; np of D neb	3†
2959 2960	1252, a	......	R. 2 novæ	12 18 ±	2·993	...	55 40 ±	19·98	...	....	0
2961	1253	......	M. 86	12 19 0·2	3·044	1	76 17 9·4	19·98	1	vB; L; R; gbMN; r	5*
2962	1252	V. 29·2	....	12 19 1·1	2·993	2	55 42 28·4	19·98	2	vF; vL; pvlbM; sf of D neb	3†
2963	......	III. 755	....	12 19 2·5	3·086	1	96 54 29·4	19·98	1	vF; vS; E	1
2964	......	III. 756	....	12 19 2·5	3·086	1	96 54 29·4	19·98	1	vF; vS; E	1
2965	......	......	Auw. N. 30	12 19 2·6	3·041	A	76 6 32·4	19·98	A	F; L; mE 90° (Auwers, Mar. 5, 1862).	0
2966	1254	II. 88	....	12 19 3·4	3·037	1	73 3 10·4	19·98	1	pF; S; R; vsbM; r	2
2967	......	III. 39	....	12 19 5·0	3·050	1	78 42 59·7	19·98	1	vF; B * nr	1
2968	1255	......	....	12 19 11·1	3·044	1	76 34 59·4	19·98	1::	f of 2 neb	1
2969	1256	......	....	12 19 14·6	3·052	1	80 12 44·4	19·98	1	eF; vL; R; gbM	1
2970	......	III. 17	....	12 19 17·0	3·065	1	86 43 59·4	19·98	1	vF; pS; r	1
2971	1257	II. 34	....	12 19 26·2	3·063	2	85 15 53·4	19·98	2	F; pL; R; gbM; r	6
2972	1258	I. 77	....	12 19 29·4	2·997	1	57 30 19·4	19·98	2	vB; L; E; g, vsmbM*	5†
2973	......	III. 482	....	12 19 30·1	3·053	1	80 47 0·4	19·98	1	eF	1
2974	1259	II. 169	....	12 19 32·4	3·044	1	76 39 31·4	19·98	1	cF; S; gbM; 2 st n, np	2
2975	1260	......	....	12 19 37·2	3·054	1	81 18 20·1	19·97	1	vF; L; R; * sp 5′	1
2976	1261	III. 492	....	12 19 44·6	3·072	1	90 6 39·1	19·97	2	{H. vF; cL; mE / h. F; S; R; * nr}	3*
2977	1262	II. 113	....	12 19 49·3	3·038	1	74 10 51·1	19·97	1	B; pmE 135°±; sbM	3
2978	1263	II. 23	....	12 19 50·7	3·065	2	86 43 44·1	19·97	2	F; pL; lE; r: (?=III. 17)...	4
2979	......	II. 155	....	12 19 56·1	3·052	1	79 37 0·7	19·97	1	F; pL; E; lbp	1
2980	1265	III. 114	....	12 19 59·7	3·083	2	95 3 10·1	19·97	2	F; vS; R; psbM; 2S st nr ...	5
2981	1264	II. 89	....	12 19 59·8	3·037	2	73 45 10·1	19·97	2	pB; pL; pgbM; B * np	5
2982	1266	II. 145	....	12 20 1·6	3·058	1	83 20 42·1	19·97	1	vF; vS; E	3
2983	1267	II. 170	....	12 20 5·0	3·043	2	76 30 10·1	19·97	2	pF; S; R; bM	3
2984	1268	II. 171	....	12 20 8·2	3·044	1	76 55 44·1	19·97	1	vF; vS; cE; gbM	2
2985	1269	......	....	12 20 16·0	3·088	1	97 24 14·1	19·97	1	vF; pL	1
2986	1270	II. 146	....	12 20 19·4	3·057	2	82 58 0·1	19·97	2	cF; L; R; gbM	3
2987	1271	II. 65	....	12 20 20·7	3·046	2	78 7 49·1	19·97	2	B; L; cE; psbM; * 10 nf ...	3
2988	1272	II. 172	....	12 20 22·2	3·043	1	76 53 54·1	19·97	1	cF; S; gbM	2
2989	......	II. 497	....	12 20 25·0	3·053	1	81 0 59·1	19·97	1	pF; vS	1
2990	1273	......	....	12 20 26·1	3·089	1	97 30 49·1	19·97	1	pF; pL; lE	1
2991	1274	I. 28, 1	....	12 20 33·7	3·041	2	76 9 14·1	19·97	3	vB; cL; R; p of 2	2
2992	1274, a	......	R. nova	12 20 ±	3·041	...	76 9 ±	19·97	...	See note	0*
2993	1276	II. 173	....	12 20 35·2	3·043	1	76 55 49·1	19·97	1	B; pS; R; bM; r	2
2994	1275	I. 28, 2	....	12 20 36·7	3·041	3	76 13 17·1	19·97	3	B; cL; vlE; r; f of 2	5
2995	1275, a	......	R. nova	12 20 ±	+3·041	...	76 13 ±	+19·97	...	See note	0*

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	h.	H.		h m s	s		° ′ ″	″			
2996	1277	......	....	12 20 39·3	+3·070	1	89 5 41·1	+19·97	1	F; eE 75°; ✱10nf; place... that of ✱).	1
2997	3392	......	Δ. 300 ?	12 20 45·1	3·287	1	149 19 6·8	19·96	1	Cl; S; st 11...12	1
2998	......	II. 848	....	12 20 48·0	2·795	1	24 25 19·8	19·96	1	F; S; iR; bM	2
2999	1279	II. 156	....	12 20 58·0	3·048	1	79 24 53·8	19·96	1	vb; pL; R; smbM	2*
3000	3393	......	....	12 21 7·5	3·191	1	132 29 6·8	19·96	1	eF; L; R; vgbM..............	1
3001	1280	I. 91	....	12 21 15·0	2·999	3	60 36 32·8	19·96	4	B; L; E 90°; sbM	6
3002	1281	I. 213	....	12 21 17·5	2·944	4	45 8 8·8	19·96	4	vB; cL; mE 15°; rrr; ✱ 9, 5′	5
3003	1282	II. 56 = II. 90	....	12 21 24·9	3·030	6	72 8 33·8	19·96	7	B; L; R; gvmbM✱; r; B✱ nr	10*
3004	1283	II. 26	....	12 21 39·0	3·055	1	82 42 49·8	19·96	1	F; pS; bM; r	2
3005	1284	II. 180	....	12 21 40·2	3·075	1	91 10 10·8	19·96	1	F; L; R; gbM; er	5
3006	1285	II. 355	....	12 21 41·6	3·014	2	66 24 25·8	19·96	2	F; L; E; gbM; 2 B st nf......	3
3007	3394	......	....	12 21 43·2	3·147	1	119 19 41·8	19·96	1	eeF; vS; ✱ 13 att	1
3008	......	I. 23	....	12 21 49·1	3·043	2	77 32 29·8	19·96	2	pB; S; vmE	2*
3009	1286	II. 35	....	12 21 50·2	3·063	3	85 39 10·8	19·96	3	cB; pS; R; smbMN	5
3010	1287	II. 121	....	12 21 51·6	3·039	1	75 59 6·8	19·96	1	pB; S; R; bM; p of 2	2
3011	1289	I. 212 = II. 750	....	12 21 52·8	2·936	1	44 20 29·8	19·96	1	B; pL; E 45°±; psbM	4*
3012	1288	I. 161	....	12 21 54·1	3·037	2	75 14 55·5	19·95	2	pB; pL; iR; bM; r; ✱ 8 sf 2′	3
3013	1290	II. 122 = II. 174	....	12 21 57·1	3·039	1	76 1 41·5	19·95	2	pF; S; R; bM; f of 2.........	4*
3014	3396	III. 764	....	12 22 2·0	3·128	2	112 23 40·5	19·95	2	pB; pS; E 130°; vbM	3
3015	3395	......	....	12 22 3·2	3·350	1	154 1 1·5	19·95	1	Cl; P; vlC	1
3016	1291	......	....	12 22 4·3	2·778	1	24 25 41·5	19·95	1	pB; R; gbM	1
3017	......	II. 630	....	12 22 15·1	3·037	1	75 15 58·5	19·95	1	cL	1
3018	1292	III. 483	....	12 22 15·4	3·051	1	81 4 3·5	19·95	1	F; vS; R; pgbM..............	2
3019	......	II. 157	....	12 22 25·8	3·049	1	80 23 58·5	19·95	1	pF; pL; mE; bM r	1
3020	1293	II. 18 = II. 498	....	12 22 30·4	3·049	2	81 24 5·5	19·95	2	F; pL; iR; bM	6
3021	1294	......	M. 49	12 22 39·3	3·051	4	81 13 44·5	19·95	5	vB; L; {H. E / h. R}; mbM......	6*
3022 3023 3024	1294, a	......	R. 3 novæ	12 22 ±	3·051	...	81 13 ±	19·95	...	"Four found" (one being h. 1294).	0
3025	......	II. 115	....	12 22 43·3	3·016	2	75 36 0·5	19·95	2	vB; cL..........................	2
3026	1295	II. 117 = II. 629	....	12 22 48·2	3·036	1	75 9 20·5	19·95	1	pF; R; r	3*
3027	1297	III. 362	....	12 22 48·7	2·998	1	61 58 28·5	19·95	2	eF; pL; R	5
3028	1296	II. 123	....	12 22 51·8	3·040	2	76 54 3·5	19·95	2	F; S; R; bM; 1st of 3	3
3029	......	II. 116	....	12 22 58·9	3·037	2	75 39 0·5	19·95	2	pB; pL..........................	2*
3030	......	II. 114	....	12 23 1·6	3·037	2	75 48 33·5	19·95	2	vF; r	2
3031	1298	II. 124	....	12 23 11·1	3·040	4	76 54 41·2	19·94	5	pB; S; R; psbM; 2nd of 3...	7
3032	1299	II. 531	....	12 23 17·7	3·060	1	84 58 43·2	19·94	1	pF; pS; E; bs	3
3033	......	III. 40	....	12 23 33·4	3·043	3	78 28 58·2	19·94	3	eF; pL..........................	1
3034	1300	......	....	12 23 37·1	3·100	1	100 51 52·2	19·94	1	pF; S; R; gbM	1
3035	1301	......	M. 87	12 23 44·0	3·039	4	76 50 39·2	19·94	4	vB; vL; R; mbM	7
3036	......	II. 776	....	12 23 44·2	3·091	1	97 18 28·2	19·94	1	F; vL; er......................	1
3037	1302	III. 484	....	12 23 44·6	3·049	1	80 51 38·2	19·94	1	vF; vS; lE	2
3038	1303	II. 91	....	12 23 48·1	3·027	3	72 27 55·2	19·94	3	pF; cS; R; gbM..............	4
3039	1304	III. 41	....	12 23 50·0	3·041	1	77 45 3·2	19·94	1	F; L; R	2
3040	1305	II. 499	....	12 23 53·2	3·050	1	81 9 14·2	19·94	1	pF; pL; vglbM; 2 st nr	2
3041	1306	I. 197	....	12 24 17·8	2·938	1	47 36 37·9	19·93	1	B; pS; iR; p of 2	2†
3042	1308	I. 198	....	12 24 23·6	2·938	1	47 39 25·9	19·93	1	vB; vL; mE 130°; rr	2†
3043	1307	I. 83	....	12 24 24·3	2·998	1	63 26 55·9	19·93	1	vB; pL; R; vsmbMN.........	2*
3044	1310	III. 301	....	12 24 26·4	+2·987	1	60 5 7·9	+19·93	2	pF; cS; R; pslbM	3

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	h.	H.		h m s	s		° ′ ″	″			
3045	1309	II. 36	....	12 24 29·8	+3·060	3	85 17 47·9	+19·93	3	F; cL; biN or D neb	6
3046	......	III. 42	....	12 24 33·4	3·040	1	77 37 57·9	19·93	1	vF	1
3047	3397	......	....	12 24 40·2	3·195	2	129 12 18·9	19·93	2	vF; L; R; vglbM	2
3048	1311	I. 234	....	12 24 41·5	2·827	1	31 16 14·9	19·93	1	B; cS; E; pgbM; * 9 f 30″	3
3049	1312	......	M. 88	12 24 54·5	3·031	3	74 48 26·9	19·93	3	B; vL; vmE; p of D neb	8
3050	......	II. 118	....	12 24 +	3·031	...	74 48 —	19·93	...	F; S; f of D neb (not obs by h.).	
3051	......	III. 69	....	12 24 57·5	3·024	1	72 25 57·9	19·93	1	vF; vS	1
3052	1313	II. 66	....	12 24 58·5	3·040	2	78 3 15·9	19·93	2	pB; S; R; gbM	3
3053	1314	II. 92	....	12 24 59·4	3·025	1	72 32 23·9	19·93	1	vF; S	2
3054	3398	II. 771	....	12 25 4·1	3·090	1	96 46 40·6	19·92	1	pB; cL; iE; gvlbM; er	3
3055	1315	III. 18	....	12 25 6·8	3·060	1::	85 14 53·6	19·92	1	vF; cL; r; f of 2	2
3056	1316	II. 631	....	12 25 6·8	3·034	1	75 48 23·6	19·92	1	cF; pmE 90°±; gbM; *9 f 8s	2
3057	3399	......	....	12 25 9·9	3·197	2	129 8 6·6	19·92	2	pB; S; R; psmbM *16	2
3058	1317	......	....	12 25 9·9	3·055	1	83 24 1·6	19·92	1	vS; R; sbM *13	1
3059	1318	......	....	12 25 14·6	2·974	1	57 7 59·6	19·92	1	vF; S; R; lbM	1
3060	1196, a	......	R. nova	12 25 30	3·058	???	84 47	19·92	???	Query R.A.; vF; 10′s of scarlet *.	0
3061	1319	III. 834	....	12 25 33·9	2·833	1	32 45 54·6	19·92	1	pF; vS; iR; vgbM	2
3062	1321	......	....	12 25 45·5	2·748	1	25 29 50·6	19·92	1	pB; S; R; psbM	1
3063	1320	III. 302	....	12 25 50·1	2·980	2	59 30 52·6	19·92	2	eF; vS; R; bM	3
3064	1322	......	....	12 26 3·1	3·049	1	81 22 31·6	19·92	1	F; S; R; bM	1
3065	1323	III. 78	....	12 26 3·1	3·029	1	74 38 42·6	19·92	1	F; pS; R; r	3
3066	......	IV. 5	....	12 26 3·3	3·070	3	89 8 56·6	19·92	3	cB; vL; vmE 95°±; B * in cont.	3
3067	1324	II. 93	....	12 26 4·0	3·024	1	72 56 23·6	19·92	1	F; vS; bM*	2
3068	......	II. 158	....	12 26 20·2	3·046	3	80 33 36·3	19·91	3	F; pL; R; bM; r	3
3069	......	II. 849	....	12 26 23·0	2·732	1	25 12 55·5	19·91	1	pB; vS; lE; sbMSN	1
3070	......	II. 757	....	12 26 24·2	3·091	2	96 36 57·3	19·91	2	vF; S; 2 vS st inv	2
3071	1326	......	....	12 26 26·9	2·741	1	25 37 18·3	19·91	1	pB; S; pmE; pgbM; * 9 inv	1
3072	1325	......	....	12 26 32·9	3·044	1	80 2 47·3	19·91	1	eF; pL; lE; vlbM	1
3073	1327	......	....	12 26 38·0	3·104	1	101 14 26·3	19·91	1	vF; iF; bM	1
3074	1328	II. 325	....	12 26 53·3	2·975	1::	58 57 57·3	19·91	1::	F; pL; iR; bM	3
3075	1329	I. 31 = I. 38	....	12 26 56·3	3·048	3	81 31 57·3	19·91	3	vB; vL; mE 120°±; psmbM; L * f; * 9 p.	5*
3076	1330	II. 37	....	12 27 0·4	3·063	2	86 34 26·3	19·91	2	pB; L; pmE 60°±; mbM	4
3077	1331	II. 67	....	12 27 2·1	3·037	3	77 54 52·3	19·91	4	pF; cS; R; bM; * 9 f 30s	5
3078	......	III. 26	....	12 27 2·2	3·009	1	68 41 56·3	19·91	1	eF; L	2*
3079	1332	......	8 Canum	12 27 7·7	2·925	4	47 52 34·0	19·90	4	Nebulous *	4*
3080	......	II. 500	....	12 27 8·5	3·047	1	81 1 56·0	19·90	1	vL; er	1
3081	1333	II. 175	....	12 27 10·6	3·032	2	76 9 20·0	19·90	2	F; pL; R; vgbM	3
3082	1334	II. 147	....	12 27 11·7	3·051	1	82 46 13·0	19·90	1	pB; pL; pmE; vgbM; r	2
3083	1336	II. 410	....	12 27 13·1	2·952	2	53 49 9·0	19·90	2	cF; L; lE; vglbM; r	4
3084	1335	II. 94 = II. 119	....	12 27 14·9	3·024	2	73 40 25·0	19·90	2	F; pS; bM; r	5
3085	1337	V. 2	....	12 27 19·6	3·064	1	87 2 47·0	19·90	1	B; vL; mE 110°; sbM; er	5†
3086	1338	......	....	12 27 32·8	3·015	1	71 1 23·0	19·90	1	pB; pmE	1
3087	1341	......	....	12 28 6·8	2·860	1	38 25 19·7	19·89	1	eF; pL; R	1
3088	1340	......	....	12 28 12·2	3·052	1	83 6 37·7	19·89	1	pF; cS; R; bM	1
3089	1346	II. 850	....	12 28 13·8	2·721	1	25 42 2·7	19·89	1	F; L; iR; vgbM; S * nf	2
3090	1342	III. 493	....	12 28 15·1	3·071	1::	89 28 3·7	19·89	1:	eF; S; R; gbM	3
3091	1344	III. 802	....	12 28 15·7	2·783	2	30 19 21·7	19·89	2	vF; pS; E; vgbM; * 9 f 2′; p of 2.	4
3092	1339	I. 160	....	12 28 17·8	3·081	4	93 1 7·7	19·89	4	vB; cL; pmE 63°±; vsmbMN.	6
3093	1345	II. 120	....	12 28 22·7	3·026	3	74 43 45·7	19·89	3	B; L; lE; lbM	5
3094	1347	III. 807	....	12 28 23·5	2·781	2	30 17 43·7	19·89	2	eF; pS; E; f of 2	3
3095	1343	I. 36	....	12 28 24·2	3·033	2	77 0 46·7	19·89	3	pB; S; vlE; sp of 2	4
3096	1349	I. 37	....	12 28 32·3	+3·033	2	76 58 38·7	+19·89	2	pB; S; R; bM; nf of 2	3

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	h.	H.		h m s	s		° ′ ″	″			
3097	1348	......	M. 89	12 28 35·2	+3·032	1	76 40 32·7	+19·89	4	pB; pS; R; gmbM	5*
3098	3400	......	....	12 28 36·9	3·211	1	128 39 49·7	19·89	1	F; vlE; glbM	1
3099	1350	II. 343	....	12 28 44·3	2·983	2	62 42 18·7	19·89	2	B; pS; iR; vsmbM * 12	3
3100	1351	II. 380	....	12 28 50·3	2·981	1	62 18 47·7	19·89	2	F; pL	3
3101	1352	I. 92	....	12 28 59·6	2·980	3	61 16 7·4	19·88	3	vB; vL; mE 150°; gbM; 3 st f	4†
3102	1354	......	....	12 29 0·3	2·980	1	62 16 6·4	19·88	1	vF; nf of 2 or ?3	1
3103	1353	I. 119	....	12 29 0·4	3·046	1::	81 33 21·4	19·88	1 :	cB; pL; R; gbM	2*
3104	1355	II. 407	....	12 29 5·7	3·008	2	69 54 20·4	19·88	2	pB; pL; vlE; lbM; r	4
3105	1356	II. 68	....	12 29 21·6	3·034	2	77 47 21·4	19·88	2	pB; S; lE; psbM	4
3106	1357	V. 24	....	12 29 22·9	2·993	4	63 14 30·4	19·88	4	B; cL; eE 136°·1; vsbMN =*10, 11.	5†
3107	1360	III. 880	....	12 29 26·1	2·821	1	35 0 1·4	19·88	1	pF; S; iR; gbM	2
3108	1358 = 1363	IV. 8	....	12 29 26·5	3·035	4	77 59 26·4	19·88	4	vF; L; np of D neb .... pos 160° ±	6*†
3109	1359 = 1363	IV. 9	....	12 29 28·0	3·035	2	78 0 26·4	19·88	2	vF; L; sf of D neb	6*†
3110	1361	I. 32	....	12 29 45·6	3·047	5	81 59 3·1	19·87	5	cB; pS; mE 0° ±; sbMrN ...	9
3111	......	......	M. 90	12 29 52·8	3·028	2	76 4 18·1	19·87	2	pL; bMN	2*
3112	1364	III. 939	....	12 29 55·6	2·392	1	14 59 52·8	19·86	1	eF; S	2
3113	1362	III. 602	....	12 29 59·1	3·024	1	74 58 8·1	19·87	1	vF; L; E; vgbM; cB * att...	2†
3114	3401	......	....	12 30 6·6	3·241	1	132 51 14·1	19·87	1	vF; S; * 10 n 30″	1
3115	3402	......	....	12 30 17·6	3·199	1	124 44 3·1	19·87	1	vF; L; lE; vglbM	1
3116	3403	......	....	12 30 19·4	3·224	1	129 45 59·1	19·87	1	F; S; pmE; 2 st p	1
3117	......	III. 13	....	12 30 23·9	3·051	1	83 9 55·8	19·86	1??	vF; vS	1
3118	1365	II. 15	....	12 30 26·2	3·039	1	79 40 5·8	19·86	1	pF; pS; R; sbMN; * np ...	3
3119	1366	......	....	12 30 26·7	3·039	1	79 45 21·8	19·86	1	F; R; bM (?=II.15+5′P.D.)	1
3120	1367	......	M. 91 ??	12 30 30·8	3·025	1?	75 26 55·8	19·86	1?	np this place is a F neb; *not* M. 91, whose existence ?.	1
3121	1368	......	M. 58	12 30 36·6	3·031	3	77 24 52·8	19·86	3	B; L; iR; vmbM; r	6
3122	1369	I. 124	....	12 30 40·5	3·053	1	83 51 55·8	19·86	1	pB; L; vgbM	3
5071	......	......	....	12 31 0·5	......	...	89 2 46·5	......	...	See No. 5071.	
3123	1370	III. 495	....	12 31 14·1	2·945	2	55 46 15·8	19·86	2	cF; S; lE; bM	3
3124	......	......	D'Arrest, 91	12 31 17	3·02	[3]	76 7 12	19·85	[3]	vF; S; R	0
3125	1371	I. 125	....	12 31 18·5	3·056	2	84 54 40·5	19·85	2	pB; L; E; psbM	4
3126	......	III. 98	....	12 31 45·7	3·047	1::	82 25 53·5	19·85	1::	vF; eS	1
3127	1374	I. 273	....	12 31 53·3	2·369	3	15 2 24·5	19·85	4	cB; L; lE; pgmbM............	7*
3128	3404	......	M. 68	12 32 5·1	3·166	1	115 58 45·2	19·84	1	⊕; L; eRi; vC; iR; rrr; st 12, red.	4
3129	1372	III. 504	....	12 32 6·7	3·050	1	83 12 29·2	19·84	1	vF; cS	3
3130	1373	II. 31	....	12 32 8·0	3·072	1	89 16 17·2	19·84	1	F; L; E 90° ±; vgbM.........	4
3131	1375	II. 183	....	12 32 26·6	3·088	2	94 34 23·2	19·84	2	pB; cL; E; sbMN=*	4
3132	1376	I. 43	....	12 32 44·2	3·110	1	100 50 14·2	19·84	1	!; vB; vL; eE 92°; vsmbMN.	3†
3133	1377	II. 632	....	12 32 49·6	3·016	3	73 56 8·2	19·84	4	pF; pL; R; gbM	5
3134	1378	I. 24	....	12 32 50·8	3·034	5	79 2 56·2	19·84	5	B; pS; R; gmbM; r; 3 st f...	7
3135	......	II. 636	....	12 32 53·7	3·089	1	95 2 22·9	19·83	1	F; vL; bM	1
3136	......	III. 105	....	12 33 11·7	3·040	2	80 51 21·9	19·83	2	eF; L; R; vlbM	2
3137	......	III. 509	....	12 33 13·9	3·065	1	88 0 51·9	19·83	1	vF; vS............	1
3138	1379	II. 577	....	12 33 14·1	3·059	2	86 6 33·9	19·83	2	F; S; R; 2 st 8 f............	3*
3139	3405	......	....	12 33 23·5	3·242	1	130 8 57·9	19·83	1	eF; L; R; pslbM; p of 2......	1
3140	1380	II. 184	....	12 33 26·5	3·088	1	94 21 50·9	19·83	1	F; L; E; vglbM	3
3141	3406	......	....	12 33 31·0	3·242	1	130 12 2·6	19·82	1	F; L; R; vgbM; r	1
3142	1381	I. 254	....	12 33 42·8	2·689	1	27 36 49·6	19·82	1	B; L; vmE 118°·6; glbM ...	2
3143	1382	III. 43	....	12 33 53·3	3·027	2	77 20 9·6	19·82	3	vF; pS; E; 2 or 3 vS st inv ..	5
3144	1383	II. 69	....	12 34 10·3	3·033	2	79 3 58·6	19·82	2	pB; pL; R; psbM; r; * 12 np 1′.	4
3145	3407	......	Δ. 272	12 34 13·2	3·463	1	152 12 8·6	19·82	1	Cl; pL; pC; cE; st 10... ...	1
3146	......	I. 7	....	12 34 16·3	3·042	1::	81 24 51·6	19·82	1::	vB; vL (no doubt a comet) ..	1
3147	......	II. 19	....	12 34 20·3	3·042	1	81 30 51·6	19·82	1	F; vL	2
3148	1384	II. 146	....	12 34 23·9	+3·043	3	81 55 10·3	+19·81	3	pB; S; R; psmbM	6*

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	h.	II.		h m s	s		° ′ ″	″			
3149	3408	......	....	12 34 44·6	+3·247	1	129 54 10·3	+19·81	1	eF; vS; R; * att nf; p of 2...	1
3150	......	II. 744	....	12 34 46·0	2·815	2	38 48 50·3	19·81	2	pF; S; iR; er	2
3151 3152	1385	I. 178 I. 179	....	12 34 48·0	2·886	4	48 4 47·3	19·81	4	B; L; R; mbM D neb; pb .... pos 160°	5†
3153	1387	......	....	12 34 56·3	3·021	1	76 17 9·3	19·81	1	vF; S; R; vgbM............	1
3154	1388	II. 411	....	12 34 56·5	2·922	3	54 9 54·3	19·81	3	F; pS; R; lbM; * 8·9 f	5
3155	1386	......	M. 59	12 34 56·9	3·026	3	77 34 1·3	19·81	3	B; pL; lE; vsvmbM; 2 st p..	5
3156	3409	......	....	12 35 3·9	3·249	1	129 58 30·3	19·81	1	pF; S; R; pslbM; f of 2......	1
3157	1389	II. 149	....	12 35 3·9	3·041	1	81 33 55·3	19·81	1	cF; pL; E; pslbM; r	5
3158	1390	......	....	12 35 7·3	3·059	1	86 10 20·3	19·81	1	B; E	1
3159	1391	II. 659	....	12 35 8·7	2·935	1	56 39 40·3	19·81	1	F; S; R; np of 2..............	2
3160	1392	II. 660	....	12 35 9·8	2·883	2	47 56 36·3	19·81	2	pF; S; R......................	3
3161	1393 = 3410	II. 772	....	12 35 10·4	3·095	2	96 16 10·3	19·81	2	vF; cS; lE; glbM	4
3162	1394 = 3411	II. 773	....	12 35 11·3	3·095	2	96 11 25·3	19·81	2	cF; S; E; gbM	4
3163	......	......	D'Arrest, 92	12 35 21	3·07	[1]	91 2 12	19·80	[1]	pB; pL; E; lbM; ? biN	0
3164	1395	II. 532	....	12 35 21·7	3·055	1	85 16 24·0	19·80	1	cF; S; R; lbM	3
3165	1397	V. 42	....	12 35 22·0	2·934	2	56 41 21·0	19·80	2	!; vB; vL; eE 70° ±; bMN; B * nr.	3†
3166	1396	I. 14	....	12 35 23·8	3·070	1	89 18 56·0	19·80	1	pB; L; E 45° ±	4
3167	1398	III. 603	....	12 35 38·1	3·015	2	74 55 47·0	19·80	2	vF; L; mE 135° ±; vgbM ...	3
3168	1400	......	....	12 35 39·8	2·992	2	69 17 27·0	19·80	2	vF; L; vglbM................	2
3169	1399	II. 38	....	12 35 41·1	3·060	2	86 32 30·0	19·80	2	B; L; iR; vgvmbM; r	4
3170	1401	......	....	12 35 41·9	3·055	1	85 32 15·0	19·80	1	vB; cS; R; smbM	1*
3171	1402	II. 70 = II. 176	....	12 35 43·8	3·026	1::	77 49 6·0	19·80	1	F; R; gbM	3
3172	1402, a	......	R. nova	12 35 43±	......	...	77 49 ±	......	...	Makes a D or biN neb with h. 1402.	
3173	1403	II. 125	....	12 35 49·4	3·019	1	75 58 44·0	19·80	1	pB; S; E; r; * 12 sf 1′	2
3174	......	II. 20	....	12 35 56·9	3·043	1	81 53 10·3	19·79	1	vS	1*
3175	......	III. 494	....	12 36 3·7	3·072	2	89 53 20·7	19·79	2	vF; cS; E	2
3176	1402	I. 10	....	12 36 12·1	3·062	1	87 14 46·7	19·79	1	cB; pS; lE; mbM	5
3177	1406	II. 794 No. 1	....	12 36 15·9	2·754	1	34 4 23·7	19·79	1	vF; S; R; gbM	1*
3178	3412	......	....	12 36 21·6	3·262	1	130 58 46·7	19·79	1	pB; S; psbM	1
3179	1407	II. 794 No. 2	....	12 36 23·0	2·756	1	34 22 23·7	19·79	1	F; S; 4 vS st sp	2*
3180	1405	III. 44	....	12 36 25·5	3·025	4	77 39 25·7	19·79	3	vF; pL; lE 115° ±; np of D neb	6*†
3181	1410	I. 274	....	12 36 33·0	2·258	4	14 48 37·7	19·79	5	pB; cS; R; gbM; ** p	8
3182	1408	......	M. 60	12 36 34·8	3·025	5	77 40 38·4	19·78	6	vB; pL; R; f of D neb	8†
3183	3413	......	....	12 36 41·2	3·256	1	129 57 35·4	19·78	1	vF; R; bM; r..................	1
3184	1409	II. 12	....	12 36 42·3	3·005	3	72 50 28·4	19·78	4	cB; L; E 90°; gbM; r	6
3185	1413	......	....	12 36 48·8	2·698	1	30 16 19·4	19·78	1	pF; pL; gbM; 2 B st f	1
3186	......	III. 662	....	12 36 50·2	3·072	1	89 47 49·4	19·78	1	vF; pL	1
3187	1411	II. 126	....	12 36 54·0	3·018	2	76 7 8·4	19·78	2	F; vL; pmE; ?D; 3 st nr ...	4
3188	1412	II. 661	....	12 36 54·5	2·876	2	48 12 25·4	19·78	3	vF; vS; stellar; * 15, 16 f ...	3
3189	1414	I. 176	....	12 37 7·8	2·929	4	57 3 48·4	19·78	5	!; pB; L; vmE 34°·3; sp of 2	6*†
3190	1415	I. 177	....	12 37 16·6	2·929	3	57 1 24·1	19·77	3	!; pF; L; E 90° ±; nf of 2 ...	6*†
3191	3414	II. 558	....	12 37 23·1	3·109	1	99 18 53·1	19·77	1	vF; L; E; * 16 att; * 9 p	2
3192	1416	II. 127	....	12 37 27·8	3·016	1	75 43 35·1	19·77	1	F; cS; R; bM; r	3
3193	1417	II. 71	....	12 37 29·4	3·025	1	78 2 34·1	19·77	1	vB; S; vsvmbMN	2
3194	3415	......	....	12 37 37·9	3·264	1::	130 19 53·1	19·77	1::	F; pL; R; gbM	1
3195	1418	II. 643	....	12 37 42·5	2·898	1	52 5 58·1	19·77	1	pF; pL; R; gbM; r	2
3196	......	II. 39	....	12 37 46·2	3·057	1	86 0 49·1	19·77	1	pB; 2 S st in M; S * p	1
3197	1419	I. 142	....	12 37 58·3	3·057	2	86 11 14·8	19·76	2	B; pL; iR; mbM; * 10 sp ...	3
3198	1420	I. 15	....	12 37 59·0	3·072	1	89 41 29·8	19·76	1	B; vL; mE 45° ±; psbM ...	4
3199	1421	......	....	12 38 12·3	+3·023	1	77 47 31·8	+19·76	1	B; S; R; psbM	1

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	h.	H.		h m s	s		° ′ ″	″			
3200	......	III. 663	....	12 38 15·2	+3·072	1	89 46 47·8	+19·76	1	vF; S; iF	1
3201	1422	III. 328	....	12 38 23·9	2·952	3	62 6 37·8	19·76	4	pF; cS; R; bM; r; p of 2 ...	6
3202	1423	II. 774	....	12 38 32·4	3·098	1	96 18 3·8	19·76	1	pF; S; R; psmbM	3
3203	3416	......	....	12 38 36·0	3·273	1	130 56 55·5	19·75	1	cF; S; R; vgbM	1
3204	1424	III. 329	....	12 38 43·7	2·951	1	62 10 24·5	19·75	2	F; vS; R; sbM*10; f of 2 ...	4
3205	3417	......	....	12 38 47·3	3·104	1	97 52 44·5	19·75	1	vF; cS; R; glbM	1
3206	......	III. 778	....	12 39 11·1	2·733	2	34 28 43·5	19·75	2	cF; S; lE	2*
3207	1425	II. 326	....	12 39 17·7	2·928	1	58 30 6·2	19·74	1	vF; pmE; ?biN	3
3208	3418	......	....	12 39 18·0	3·274	1	130 49 55·2	19·74	1	eF; lE; vgbM	1
3209	3419	......	....	12 39 36·0	3·263	1	128 48 5·2	19·74	1	eeF; pL; R	1
3210	3420	......	....	12 39 36·4	3·118	1	100 52 34·2	19·74	1	eF; S; 1 or 2 st inv	1
3211	3421	......	....	12 39 45·5	3·291	1	132 34 50·2	19·74	1	pF; S; R; gbM	1
3212	3423	III. 523	....	12 39 58·2	3·111	1	99 17 29·9	19·73	1	cF; L; E 45°±; gvlbM	2
3213	3422	......	....	12 40 2·9	3·279	1	130 47 24·9	19·73	1	eF; pS; R; vgbM; S * sp ...	1
3214	1426	II. 181	....	12 40 6·3	3·081	1	91 58 8·9	19·73	1	B; pL; pmE 25°	5*
3215	1427	III. 398	....	12 40 13·3	2·984	3	69 46 52·9	19·73	3	F; S; R; sbM*; rr	4
3216	1428	II. 795	....	12 40 16·6	2·728	1	34 41 11·9	19·73	1	pF; vS; vmE; vsmbM	3*
3217	1430	......	....	12 40 39·3	2·897	1	53 52 53·6	19·72	1	vF; vS; R; psbM	1
3218	1429	III. 543	....	12 40 43·4	3·051	1	84 53 57·6	19·72	2	eF; pL; *9·10 p 10s	3
3219	1431	II. 128	....	12 40 44·5	3·010	3	75 28 34·6	19·72	3	pB; vL; E; vglbM; r	5
3220	......	III. 664	....	12 40 48·3	3·076	1	90 55 44·6	19·72	1	vF; S	1
3221	1432	II. 182	....	12 40 58·2	3·081	1	92 33 48·6	19·72	1	pB; pL; E 90°±; mbM	5
3222	1433	II. 381	....	12 41 2·8	2·943	2	62 0 41·6	19·72	3	F; cS; R; bM	4
3223	......	III. 906	....	12 41 5·5	2·332	1	18 3 44·6	19·72	1	vF; pL; E	1
3224	1435	II. 796	....	12 41 9·5	2·731	1	34 51 21·6	19·72	1	cF; pS; vlE; mbMN	3*
3225	1434	II. 72	....	12 41 11·2	3·021	6	78 14 47·6	19·72	6	pF; S; vlE	7
3226	3424	......	Δ. 510 ?	12 41 12·2	3·282	1	130 32 29·3	19·71	1	pB; L; R; gbM; r	1
3227	1436	I. 39	....	12 41 21·8	3·094	2	95 2 2·3	19·71	2	vB; L; lE 45°±; smbMrN ..	5
3228	......	I. 8 = III. 6	....	12 41 28·1	3·032	6	80 44 53·3	19·71	6	cB; pL; iR; bM; r	6*
3229	1437 = 3425	I. 129	....	12 41 46·4	3·107	2	97 54 3·3	19·71	2	vB; R; vmbM rN; r	3
3230	1438 = 3426	III. 524	....	12 41 49·6	3·119	2	100 38 16·3	19·71	2	F; L; mE40°; vlbM; B*p ...	4
3231	......	II. 578	....	12 42 2·1	3·054	1	85 50 43·0	19·70	1	F; S	1
3232	......	III. 514	....	12 42 5·8	3·109	2	98 21 44·0	19·70	2	eF; cS; pmE	2
3233	1439	II. 662	....	12 42 8·9	2·842	2	47 18 39·0	19·70	2	cF; S; R; gbM	3
3234	3427	......	....	12 42 15·5	3·288	1	130 31 49·0	19·70	1	vF; vS; R; psbM	1
3235	......	III. 815	....	12 42 19·7	2·753	1	38 2 43·0	19·70	1	S; stellar	1
3236	......	III. 722	....	12 42 22·8	3·118	1	100 20 43·7	19·69	2	eF; S	1
3237	3428	......	Δ. 511	12 42 26·6	3·289	1	130 36 13·7	19·69	1	pB; cS; R; gbM	1
3238	1440	III. 610	....	12 42 29·3	3·092	1	94 26 6·7	19·69	1	cF; pL; lE	2
3239	1451, *a*	......	R. nova	12 42 35·9	2·95	: :	63 45 13·4	19·69	: :	E	0
3240	1441	II. 95	....	12 42 39·1	3·000	2	74 4 17·7	19·69	2	cB; pL; vmE 28°·5; sbMN ..	4†
3241	1442	......	....	12 42 39·4	2·948	1	63 45 43·7	19·69	1	vF; pL	1
3242	1443	II. 412	....	12 42 40·1	2·888	1	53 54 17·7	19·69	1	F; S; E; glbM; er	4
3243	1444	I. 140	....	12 42 51·5	3·045	4	83 55 22·7	19·69	4	pB; L; vlE; glbM	6
3244	1445	III. 536	....	12 43 1·6	3·130	2	102 33 51·7	19·69	2	F; pS; R; gbM	3
3245	1446	......	....	12 43 19·9	3·093	1	94 30 56·4	19·68	1	eF; vS; bet 2 st	1
3246	1448	III. 424	....	12 43 22·9	2·900	1	56 4 35·4	19·68	1	vF; stellar	2
3247	1475, *a*	......	R. nova	12 43 26	2·92	: :	60 23 ±	19·68	: :	E; bMN	0
3248	1447	III. 611	....	12 43 26·4	3·088	1	93 23 9·4	19·68	1	eF; S; bM	2
3249	1451	I. 84	....	12 43 32·0	2·945	1	63 44 13·4	19·68	1	vB; vL; E; vg, vsvmbMeBN	2†
3250	1449	III. 280	....	12 43 34·1	3·135	1	103 34 27·1	19·67	1	F; vS; R; stellar; np of 2 ...	2
3251	1450	II. 298	....	12 43 37·5	3·135	1	103 34 57·1	19·67	1	F; pL; R; lbM; sf of 2	2
3252	3430	......	....	12 43 38·8	3·293	1::	130 19 50·1	19·67	1::	neb; 1st of 3	1
3253	3431	......	....	12 43 38·8	3·293	1::	130 19 50·1	19·67	1::	2nd of 3	1
3254	1452	I. 41	....	12 43 45·4	+3·098	1	95 38 6·1	+19·67	1	vF; pL; E	3

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	h.	H.		h m s	s		° ′ ″	″			
3255	......	II. 814	....	12 43 53·8	+2·720	1	36 20 41·1	+19·67	1	F; S; vsmbM	1
3256	1453	II. 73	....	12 44 3·4	3·019	4	78 19 45·1	19·67	5	cF; pL; {R mE90°} r; *12p.	6*
3257	1454	......	....	12 44 5·1	3·047	1	84 23 15·1	19·67	1	vF; vS; R	1
3258	1456	......	M. 94	12 44 17·2	2·838	5	48 6 31·8	19·66	5	vB; L; iR; vsvmbMBN; r...	10†
3259	1457	III. 496	....	12 44 18·7	2·890	1::	55 5 14·8	19·66	1	eF; vS; pmE	2
3260	1455	III. 515	....	12 44 20·7	3·108	1	97 39 9·8	19·66	1	F; pL; lE; pglbM	2
3261	1458	III. 721	....	12 44 30·1	2·776	1	41 33 55·8	19·66	1	vF; S; R; psbM	2
3262	3432	I. 133	....	12 44 30·5	3·118	1	99 41 33·8	19·66	1	cB; vS; vbMN=*9; *10 sf	2
3263	3429	......	....	12 44 31·3	3·300	1	130 37 52·8	19·66	1	F; R; gbM	1
3264	3433	......	....	12 44 37·2	3·297	1	130 18 42·8	19·66	1	F; L; E; gbM; 3rd of 3	1
3265	1460	......	....	12 44 52·3	3·012	1	77 9 54·5	19·65	1	pB; mE; r	1
3266	......	II. 344	....	12 44 54·5	2·940	1	63 28 40·5	19·65	1	F; pL; lE	1
3267	1459	III. 537	....	12 44 55·1	3·132	1	102 38 47·5	19·65	1	F; vS; iR; gbM	2
3268	......	III. 907	....	12 44 55·5	2·244	1	17 36 39·8	19·66	1	vF; cL; E 135°±	1
3269	1451, b	......	R. nova	12 45 0·0	2·95	::	63 23 0·4	19·65	::	E 0°	0
3270	1463	IV. 78	....	12 45 0·2	2·175	1	16 21 38·5	19·65	1	pB; L; R; vg, vsbM	2
3271	3434	......	....	12 45 6·7	3·313	1	131 54 27·5	19·65	1	B; pS; R; vg, vsmbM	1
3272	......	III. 82	....	12 45 10·2	3·005	1	75 44 40·5	19·65	1	vF; S; E; r	1
3273	1461	I. 16	....	12 45 11·6	3·075	1	90 26 26·5	19·65	1	cB; L; vlE; vglbM............	5
3274	1462	{I. 25 = II. 74}	....	12 45 15·0	3·015	5	77 55 36·5	19·65	5	B; pL; R; psbM; p of 2......	7
3275	3435	......	Δ. 301	12 45 22·2	3·533	2	149 35 20·2	19·64	3	Cl; vL; st vB (κ Crucis)......	3†
3276	1464	III. 281	....	12 45 35·3	3·143	1::	104 38 49·2	19·64	1::	vF; pS; r..........................	2
3277	1465	III. 70	....	12 45 46·1	2·992	3	73 23 18·2	19·64	3	vF; pL; E?........................	4
3278	1466	II. 75	....	12 45 53·4	3·015	4	78 0 38·2	19·64	4	pB; vmE 34°·0; 3B st s; f of 2	6†
3279	......	III. 489	....	12 45 55·7	3·152	1	106 13 39·9	19·63	1	vF; S; lbM	1
3280	1467	III. 544	....	12 46 6·4	3·048	1	84 46 58·9	19·63	1	F; cS; R; gbM	2
3281	3436	......	....	12 46 11·3	3·295	2	128 58 3·9	19·63	2	B; pS; lE; mbM................	3
3282	......	III. 525	....	12 46 12·7	3·115	1	98 45 39·9	19·63	1	vF; vS..............................	1
3283	1468	II. 535	....	12 46 13·2	3·063	1	87 58 25·9	19·63	1	F; pL; mE; *9 p 90°.........	2
3284	......	III. 516	....	12 46 22·4	3·111	1	97 54 38·9	19·63	1	vF; S	1
3285	1469	II. 24	....	12 46 22·4	3·058	2	87 3 50·9	19·63	2	pF; pS; R; mbM	5
3286	1471	III. 618	....	12 46 26·5	2·862	1	52 25 5·9	19·63	1	eF; cS; R; bM	2
3287	1470	II. 186	....	12 46 29·7	3·101	1	95 51 34·9	19·63	1	F; cL; R; vglbM; r	3
3288	3437	II. 559	....	12 46 46·5	3·114	1	98 26 38·6	19·62	1	F; S; R; vlbM; p of D neb...	2
3289	......	III. 517	....	12 46 47·5	3·112	1	98 1 38·6	19·62	1	vF; S	1
3290	3438	......	....	12 46 47·5	3·114	1	98 26 17·6	19·62	1	vF; S; R; vlbM; f of D neb	1
3291	1472	III. 106	....	12 46 48·1	3·020	2	79 31 37·0	19·60	2	vF; pL; R; r	3
3292	......	I. 134	....	12 47 4·9	3·120	1	99 46 38·3	19·61	1	cB; vL; mE	1
3293	......	I. 135	....	12 47 15·1	3·131	2	101 49 8·3	19·61	2	pF; pS; R; mbM; p of D neb	2
3294	......	I. 136	....	12 47 16·1	3·131	2	101 50 8·3	19·61	2	pF; pS; R; mbM; f of D neb	2
3295	......	III. 526	....	12 47 16·9	3·121	1	99 51 38·3	19·61	1	eF; eS	1
3296	3439	......	....	12 47 17·2	3·384	1	137 58 54·3	19·61	1	vF; S; R; glbM	1
3297	......	II. 187	....	12 47 21·4	3·102	2	96 6 8·3	19·61	2	pB; pS; mbM; r................	2
3298	1473	II. 345	....	12 47 29·9	2·925	3	62 9 58·3	19·61	2	F; R; *9 att l'n	4
3299	......	II. 560	....	12 47 31·8	3·120	1	99 28 38·3	19·61	1	pF; pS; iR	1
3300	1475	I. 93	....	12 47 54·0	2·911	2	60 17 53·0	19·60	2	pB; pS; lE; *8nf 1′	4
3301	......	II. 538	....	12 47 55·2	3·132	2	101 52 8·0	19·60	2	vF; S; 2 or 3 st near	2
3302	1474	II. 21	....	12 47 59·1	3·029	3	81 10 55·0	19·60	3	pF; pL; R; bM; r	7
3303	1477	II. 382	....	12 48 6·7	2·921	2	61 49 20·0	19·60	2	pF; pS; gbM	3
3304	1476	III. 548	....	12 48 7·8	3·054	1	86 20 18·0	19·60	1	cF; S; vS* att	2
3305	1478	I. 211	....	12 48 9·1	2·765	1	42 43 7·0	19·60	1	pB; cS; R; psbM; *14 p ...	5
3306	1479	III. 816	....	12 48 26·6	2·683	1	36 8 27·7	19·59	1	eF; S; lE..........................	2
3307	......	IV. 40	....	12 48 33·9	3·136	1	102 17 37·7	19·59	1	S; att to pB*	1
3308	3440	......	....	12 48 39·2	3·230	1	118 45 0·7	19·59	1	F; cS; R; gvlbM	1
3309, 3310	} 1509, a	......	R. 2 novæ	12 48 ±45	3·05	::	86 42 ±	19·58	::	{F; D neb; E at right angles to each other.	0
3311	1480	I. 141	....	12 48 53·4	+3·047	1::	84 56 47·4	19·58	1::	{H. vB; h. pF}; cL; E135°± ...	3

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	h.	II.		h m s	s		° ′ ″	″			
3312	3441	......	....	12 49 6·0	+3·326	1	131 2 20·4	+19·58	1	eF; cS; R; gbM; p of 2......	1
3313	3442	......	....	12 49 6·1	3·326	1	131 3 50·4	19·58	1	eF; S; R; gbM; f of 2	3
3314	1481	II. 383	....	12 49 9·7	2·915	1	61 29 48·4	19·58	1	vF; pL......................	2
3315	1483	I. 243	....	12 49 10·9	2·590	2	30 54 17·4	19·58	2	B; pS; vlE; vgbM	2
3316	1482	II. 777	....	12 49 20·0	3·104	1	96 3 28·1	19·57	1	F; S; R; bM	2
3317	3443	......	....	12 49 23·7	3·680	3	154 11 48·1	19·57	3	Cl; pL; pRi; iF; st 10 ...18	3
3318	{1484 = 3445}	II. 549	....	12 49 33·0	3·112	2	97 46 5·1	19·57	2	pB; L; pmE 0°; gbM	3
3319	1485	II. 384	....	12 49 48·9	2·917	1	62 3 50·8	19·56	1	F; cL	2*
3320	......	II. 563	....	12 49 51·1	3·140	1	102 54 35·8	19·56	1	pB; iF; bM..................	1
3321	1486	......	M. 64	12 49 51·8	2·951	3	67 33 15·8	19·56	3	!; vB; vL; vmE 120°±; bMSBN=✱ ?.	10†
3322	3447	......	....	12 50 5·6	3·220	1	116 32 13·8	19·56	1	F; S; R; gbM	1
3323	3446	......	....	12 50 6·4	3·321	1	129 59 36·8	19·56	1	pF; vS; R; sbM ✱17; ✱10, 70°·3	1
3324	1487	II. 346	....	12 50 6·5	2·917	1	62 14 32·8	19·56	1	vF; pL; iF	3
3325	3444	......	Δ. 164	12 50 7·9	3·899	2	160 6 53·5	19·55	2	⊕; B; L; R; g, vsbM; st 12	2
3326	1488	III. 817	....	12 50 11·1	2·680	1	36 56 44·8	19·56	1	vF; S; iR; bM	2
3327	3448	......	....	12 50 15·0	3·375	2	135 30 1·5	19·55	3	F; pL; mE; vgbM	3
3328	1489	......	....	12 50 23·4	2·725	1	40 26 12·5	19·55	1	Neb; ?	1
3329	1490	......	....	12 50 42·3	3·138	1	102 17 56·2	19·54	1	vF; 3Sst sp	1
3330	1491	II. 536	....	12 50 42·5	3·060	1	87 40 23·5	19·55	1	pF; pL; pmE; vgbM; ✱nf 30°	2
3331	1493	II. 387	....	12 50 45·5	2·906	2	60 46 1·2	19·54	2	pF; pL; R; vS✱ att	3
3332	1492	III. 613	....	12 50 47·5	3·087	1	92 51 26·2	19·54	1	cF; E; er; ✱sf 30″	2
3333	1494	II. 386	....	12 50 55·4	2·912	1::	61 49 52·2	19·54	1	F; pL; R....................	2
3334	1495	......	....	12 51 9·4	2·837	1	51 52 3·2	19·54	1	eF	1
3335	3449	......	Δ. 311	12 51 45·5	3·571	2	148 50 34·2	19·54	2	Cl; L; pRi; iR; st 10... ...	2
3336	1496	II. 385	....	12 51 48·6	2·908	2	61 38 24·6	19·52	2	F; S; R; pslbM	3
3337	1497	I. 68	....	12 51 59·8	3·151	1	104 17 25·6	19·52	2	B; R; psmbM; ✱13 np	3*
3338	......	II. 299	....	12 52 1·7	3·152	1	104 31 34·6	19·52	1	pB; pL; mbM.................	1*
3339	......	III. 908	....	12 52 1·7	2·172	1	19 1 32·6	19·52	1	eF; vS; iR; vlbM	1
3340	1499	IV. 30	....	12 52 22·9	2·852	2	54 23 0·3	19·51	2	vF; pL; vmE 30°±; bet 2 st	5†
3341	......	II. 644	....	12 52 24·2	2·832	1	51 56 32·3	19·51	1	pB; S; R; mbM...............	1
3342	1498	I. 162	....	12 52 28·6	2·990	6	75 4 23·3	19·51	6	B; pL; mE 90°; sbMN; S✱inv.	7
3343	1500	......	....	12 52 41·1	2·903	1::	61 16 45·3	19·51	1::	1st of 5; ✱7 n	1
3344	1501	II. 388	....	12 52 49·6	2·903	2	61 17 56·0	19·50	3	cF; S; R; ✱7 n; 2nd of 5 ...	4
3345	......	III. 758	....	12 52 58·9	3·102	1	95 20 33·0	19·50	1	vF; vS; p of 2..............	1
3346	......	III. 759	....	12 52 58·9	3·102	1	95 20 33·0	19·50	1	vF; vS; F of 2, ...	1
3347	1502	II. 389	....	12 53 0·1	2·902	1::	61 16 45·0	19·50	1::	3rd of 5	2
3348	1503	III. 83	....	12 53 9·2	2·999	2	76 46 17·0	19·50	2	cF; pL; R; vglbM; r.........	3
3349	1504	......	....	12 53 17·4	3·106	1	96 5 50·7	19·49	1	vF; S; E....................	1
3350	1505	II. 778	....	12 53 20·0	3·102	1	95 19 0·7	19·49	1	pF; cS; E; psbM; ✱ np......	2
3351	1507	II. 391	....	12 53 24·0	2·901	2	61 15 23·7	19·49	2	pB; pmE; bM; ✱7 n; 4th of 5	3
3352	1506	III. 614	....	12 53 24·1	3·094	1	93 50 3·7	19·49	1	cF; S; iR; bM	2
3353	1508	II. 390	....	12 53 28·3	2·908	1	62 21 3·7	19·49	1	vF	2
3354	1510	III. 363	....	12 53 29·7	2·900	1	61 17 14·7	19·49	2	pF; S; R; ✱7 n; 5th of 5 ...	3
3355	......	II. 300	....	12 53 33·9	3·147	2	103 11 32·7	19·49	2	....	2
3356	1509	I. 143	....	12 53 33·9	3·055	2	86 45 0·7	19·49	3	cB; cE; ✱10 att 135°±	4*†
3357	1512	......	....	12 53 35·2	2·724	1	42 1 50·7	19·49	1	pF; S; R; gbM	1
3358	1511	I. 69	....	12 53 37·9	3·150	1	103 45 56·7	19·49	1	pB; pL; iR; st nr	2*
3359	3450	......	....	12 53 44·3	3·257	1	120 12 2·4	19·48	1	vF; cS; R; ✱ att; p of 2......	1
3360	3451	......	....	12 53 51·3	3·257	1	120 10 2·4	19·48	1	vF; vS; R; slbM; f of 2......	1
3361	......	II. 517	....	12 53 51·5	3·069	2	89 18 31·4	19·48	2	pB; pS; R; bM	2
3362	3452	......	....	12 54 9·0	3·361	1	132 0 55·1	19·47	1	eF; 3 or 4 st 11, 12, f	1
3363	......	V. 3	....	12 54 9·5	3·060	1	87 35 32·4	19·48	1	eF; vL; rr	1*
3364	......	II. 392	....	12 54 10·0	2·899	1::	61 23 44·4	19·48	1::	1st of 3	1
3365	1514	II. 645	....	12 54 10·5	2·823	2	51 54 16·4	19·48	2	pB; cS; R; smbM; ✱17 np..	3
3366	1513	IV. 47	....	12 54 12·7	3·094	1	93 47 23·4	19·48	1	pB; S; R; bM; stellar?......	3
3367	1515	......	....	12 54 26·8	2·719	1	42 1 52·1	19·47	1	eF; S; E; bM................	1
3368	1516	II. 393	....	12 54 39·4	+2·897	1	61 21 47·1	+19·47	2	F; pL; 2nd of 3	3

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	h.	H.		h m s	s		° ′ ″	″			
3369	1518	II. 394	....	12 54 42·9	+2·897	1	61 24 48·1	+19·47	1	vF; 3rd of 3	2
3370	1517	......	....	12 54 50·7	3·154	2	104 13 10·8	19·46	2	cF; L; vlE 45°±	4
3371	1519	II. 779	....	12 54 51·3	3·112	1	96 57 43·8	19·46	1	cF; S	2
3372	......	III. 364	....	12 55 1·9	2·895	1	61 12 29·8	19·46	1	vF	1
3373	3453	II. 190	....	12 55 43·0	3·115	1	97 19 31·2	19·44	1	F; pS; vlE; glbM	4
3374	......	III. 760	....	12 55 51·7	3·115	1	97 22 30·2	19·44	1	cF; vS; R	1
3375	3454	......	....	12 56 14·8	3·358	1	130 39 45·9	19·43	1	vF; R; Δ2st 8, 9, f	1
3376	......	III. 818	....	12 56 20·2	2·663	1	38 47 27·9	19·43	1	cF; S; R; vglbM	1
3377	......	II. 191	....	12 56 27·9	3·136	1	100 44 59·9	19·43	1	pB; pL; iR	2
3378	3456	......	....	12 56 38·5	3·263	2	119 46 26·6	19·42	2	pB; S; R; bM; *f 6ˢ	1
3379	3455	......	....	12 56 42·6	3·425	1	136 28 2·6	19·42	1	eeF; S; R; p of 2	1
3380	1521	......	....	12 56 48·3	2·646	1	37 55 20·6	19·42	1	eF; R; psbM	1
3381	3458	II. 561	....	12 56 56·5	3·129	1	99 35 40·6	19·42	1	pB; L; R; gmbM	2
3382	3457	......	....	12 56 58·2	3·426	1	136 29 22·3	19·41	1	F; S; R; f of 2	1
3383	1520	I. 40	....	12 56 59·1	3·101	2	94 48 23·6	19·42	2	pF; L; E; gbMBN; r	3
3384	......	III. 761	....	12 56 59·5	3·113	1	96 55 28·6	19·42	1	vF; S	1
3385	1522	II. 395	....	12 57 6·0	2·887	1	61 3 42·6	19·42	1	F; S; R; bM; *9 nf 1′	2
3386	3459	......	Δ. 411	12 57 14·1	3·455	1	138 32 6·3	19·41	1	B; vL; vmE 38°·7	1
3387	3460	......	....	12 57 32·1	3·388	2	132 50 51·0	19·40	2	B; pS; R; gpmbM; p of 2	2
3388	3461	......	....	12 57 34·1	3·306	1	124 35 6·0	19·40	1	F; pL; R; vglbM	1
3389	3462	......	....	12 57 40·2	3·387	2	132 45 46·0	19·40	2	eF; S; R; pslbM; f of 2	2
3390	1523	II. 188	....	12 57 43·2	3·107	(2)	95 45 18·0	19·40	2	F; pL; lE; r	3
3391	1524	II. 396	....	12 58 15·4	2·877	4	60 7 28·7	19·39	5	F; S; R; psbM*11	6
3392	3463	......	....	12 58 18·4	3·329	1	126 48 31·4	19·38	2	vF; pS; am 3S st	2
3393	1527	......	....	12 58 23·9	1·656	2	13 50 36·4	19·38	2	vF; S; R; vgbM	2*
3394	3464	......	....	12 58 26·1	3·263	1	119 0 14·4	19·38	1	F; cS; R; gbM	1
3395	1525	II. 413	....	12 58 26·5	2·825	2	54 4 20·7	19·39	2	pB; cS; R; smbM	4
3396	1526	II. 397	....	12 58 27·1	2·888	1	61 40 48·4	19·38	1	F; S; R	2
3397	3465	I. 130	....	12 58 31·6	3·116	1	97 15 49·4	19·38	1	vB; pS; E 0°±; bMBN	3
3399	1528	......	....	12 59 2·4	2·840	1	56 4 6·1	19·37	2	eF; S; R	2
3400	1529	II. 398	....	12 59 4·5	2·885	1	61 30 58·1	19·37	1	F; S; iF	2
3401	......	III. 303	....	12 59 9·8	2·874	1	60 10 26·1	19·37	1	eF; vS	1
3402	1530	II. 663	....	12 59 27·3	2·754	2	47 31 10·8	19·36	2	F; vS; R; stellar; vS*s	3
3403	1532	III. 779	....	12 59 28·4	2·537	1	32 55 57·8	19·36	1	eF; S; lE	3
3404	3466	......	....	12 59 33·6	3·255	2	117 28 37·8	19·36	2	vF; vL; cE; vgbM	2
3405	1531	III. 304	....	12 59 35·3	2·873	2	60 12 10·8	19·36	3	vF; vS; vlE; vglbM; ⁑ sp	4
3406	1533	III. 783	....	12 59 35·4	2·588	1	35 40 40·8	19·36	1	vF; S; E; * att	2
3407	3467	......	....	12 59 37·3	3·221	1	112 55 45·8	19·36	1	F; pL; R; glbM	1
3408	......	III. 765	....	12 59 57·7	3·224	1	113 15 24·5	19·35	1	vF; pL; iF	1
3409	......	III. 937	....	13 0 17·3	1·670	1	13 57 22·5	19·35	1	vF; S; iR; bM	1
3410	......	III. 781	....	13 0 18·4	2·582	1	35 38 40·2	19·34	1	vF; S	1
3411	......	III. 782	....	13 0 32·3	2·580	1	35 36 40·2	19·34	1	vF; S	1
3412	1534	......	....	13 0 36·8	3·099	1	94 16 0·9	19·33	1	vF; vS; R; psbM	1
3413	3468	......	....	13 0 40·4	3·480	1	138 45 23·9	19·33	1	B; pL; R; gmbM	1
3414	......	III. 780	....	13 0 41·9	2·540	1	33 34 22·2	19·34	1	eF; S	1
3415	1535	......	....	13 0 57·8	2·948	2	70 50 8·9	19·33	2	F; vS; R; sbM; stellar	2*
3416	......	III. 346	....	13 1 8·2	2·902	1	64 29 21·6	19·32	1	eF; pL; lE	1
3417	3469	......	....	13 1 29·8	3·264	1	117 53 57·3	19·31	1	eF; cS; R	1
3418	1537	II. 189	....	13 1 32·2	3·111	1	96 1 35·3	19·31	1	B; pL; R; *9 sf	4
3419	......	III. 365	....	13 1 32·5	2·873	1	60 56 21·3	19·31	1	vF	1
3420	1536	II. 301	....	13 1 33·1	3·168	2	104 45 42·3	19·31	2	B; pL; R; psmbM	3*
3421	......	II. 185	....	13 1 40·9	3·102	1	94 36 21·3	19·31	1	F; S; iF; pB*nr	1*
3422	1539	III. 954	....	13 1 50·4	2·743	1	47 34 57·3	19·31	1	vF; vS; R; lbM	2
3423	1538	III. 401	....	13 1 51·4	2·811	2	54 3 8·3	19·31	2	vF; S; R; stellar	3
3424	1542	II. 815	....	13 1 54·5	2·599	1	37 18 52·3	19·31	1	vF; vS; stellar	2
3425	3470	......	....	13 1 54·7	3·404	1	132 21 15·0	19·30	1	vF; S; E; r	1
3426	......	......	Auw. N. 31	13 2 1·0	3·099	...	94 38 51·0	19·30	...	A neb (Markree Obs. Apr. 9, 1852).	0*
3427	1541	......	....	13 2 6·3	2·993	1	77 37 1·0	19·30	1	vF; S; lE; 2Sst s	1
3428	......	III. 766	....	13 2 11·9	3·225	1	112 38 21·7	19·29	1	vF; vS	1
3429	3471	......	....	13 2 15·9	+3·219	1	111 48 8·7	+19·29	1	pF; cS; R; slbM; am st	1

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	h.	H.		h m s	s		° ′ ″	″			
3430	1540 = 3472	I. 42	....	13 2 23·2	+3·118	2	97 5 2·7	+19·29	2	pB; pL; R; vgpmbM; * 8 np	6
3431	......	III. 819	....	13 2 29·4	2·616	1	38 34 19·7	19·29	1	vF	1
3432	1543	II. 537	....	13 2 32·9	3·057	1	87 35 35·7	19·29	1	cF; pL; R; lbM; er	2
3433	1544	III. 366	....	13 3 5·6	2·863	1	60 20 39·4	19·28	1	cF; pS; lE	2
3434	1545	......	....	13 3 37·6	2·558	1	35 44 28·1	19·27	1	pF; S; iR; gbM	1
3435	......	III. 655	....	13 4?15·2	2·704	1	47 27 18·5	19·25	1	vF; pS; lbM	1
3436	1546	III. 305	....	13 4 20·5	2·852	1	59 36 40·5	19·25	2	vF; vS; vlE..................	3
3437	1547	I. 96	....	13 4 28·5	2·781	2	52 11 9·2	19·24	2	vB; vL; vmE 25°; vsbMN...	4
3438	......	III. 848	....	13 4 34·7	2·340	1	27 7 16·5	19·25	1	vF; vS........................	1
3439	......	......	D'Arrest, 93	13 4 37	3·05	[1]	63 51 42	19·24	[1]	pF; pL; R	0
3440	1550	III. 820	....	13 4 45·5	2·610	1	39 9 53·2	19·24	1	vF; R; bet 2 vS st	2
3441	1549	I. 85	....	13 4 48·8	2·907	2	66 20 21·9	19·23	2	pF; cL; E 17°·0; biN; * 9 f	3
3442	1548	......	....	13 4 50·5	3·175	1	105 3 9·9	19·23	1	vF; R; bM; * 10 np 5′	1
3443	3473	......	....	13 4 51·7	3·419	3	132 21 4·9	19·23	3	pB; cS; R; am 4 st............	3
3444	1551	II. 414	....	13 5 2·2	2·787	1	52 58 43·9	19·23	1	pF; S; E; psbM..............	2
3445	1552	II. 637	....	13 5 7·5	3·097	(1)	93 36 2·6	19·22	1	F; cL; iR; lbM	2
3446	......	II. 356	....	13 5 7·9	2·897	1	65 12 15·6	19·22	1	pB; S	1
3447	1553	III. 669	....	13 5 28·3	3·183	1	106 0 52·6	19·22	1	vF; R; bM	2
3448	1554	II. 746	....	13 5 30·7	3·203	1	108 46 4·3	19·21	1	cB; S; R; mbMpBN	2
3449	1555	III. 545	....	13 5 36·5	3·036	1	84 31 7·3	19·21	1	cF; vS; R; er	2
3450	1556	II. 129	....	13 5 42·5	2·982	3	76 39 28·3	19·21	3	cF; cL; vlE; lbM	4
3451	1557	......	....	13 5 46·2	2·663	1	43 4 1·3	19·21	1	pF; cS; R; * 12 nf 90″	1
3452	1559	II. 664	....	13 5 55·8	2·692	2	45 12 45·3	19·21	2	pF; L; mE 20°; vlbM	4
3453	1558	......	M. 53	13 6 2·0	2·941	5	71 5 3·0	19·20	5	!; ⊕; B; vC; ir; vvmbM; st 12...	12
3454	1560	III. 649	....	13 6 10·1	2·826	3	57 26 40·0	19·20	2	vF; S; lE; * 13 n	4
3455	3474	......	....	13 6 13·8	3·425	3	132 12 57·0	19·20	3	pB; pL; R; gbM; * 7 nf ...	3
3456	1561	......	....	13 6 16·8	3·027	2	83 11 34·0	19·20	2	vF; S; R; pgbM...............	2
3457	1562	......	....	13 6 27·1	2·646	1	42 10 39·0	19·20	1	F; vS; R; gbM	1
3458	1563	III. 367	....	13 6 45·9	2·861	2	61 27 30·7	19·19	2	vF; pL; iR	3
3459	1564	I. 97	....	13 6 59·7	2·775	2	52 39 40·4	19·18	2	vB; pl; E 166°·8; smbMvBN; * np.	3
3460	......	III. 909	....	13 7 19·4	1·918	1	18 36 12·4	19·18	1	vF; vS; R	1
3461	1565	II. 510	....	13 7 33·0	3·185	2	105 51 1·8	19·16	2	cF; pS; vlE; bM	3
3462	......	II. 816	....	13 7 49·8	2·568	1	37 58 10·8	19·16	1	F; S; ir; vgmbM	1
3463	3477	......	....	13 7 55·8	3·243	1	113 14 29·5	19·15	1	F; L; R; vgvlbM; * 9 p ...	1
3464	3476	......	....	13 7 56·4	3·744	1	149 19 9·5	19·15	1	Cl; P; E; sc st 11...	1
3465	1566	II. 511	....	13 7 56·9	3·184	1	105 38 49·5	19·15	1	pB; pL; R; bM	3
3466	3475	......	....	13 7 59·4	3·845	1	152 40 25·2	19·14	1	Cl; vL; vRi; st 11...	1
3467	3478	......	....	13 8 26·1	3·283	1	117 40 52·2	19·14	1	pF; R; sp of 2	1
3468	......	III. 670	....	13 8 26·7	3·185	1	105 43 13·2	19·14	1	vF	1
3469	......	II. 512	....	13 8 31·7	3·185	2	105 37 43·2	19·14	2	cF; S	2
3470	3479	......	....	13 8 46·5	3·283	1::	117 35 33·9	19·13	1::	Neb; nf of 2	1
3471	1567	......	....	13 9 0·0	2·837	1	59 33 42·9	19·13	1	vF	1
3472	1569	VI. 7	....	13 9 30·5	2·938	1	71 35 10·3	19·11	2	Cl; vF; pL; iR; vgbM; st 15...	3
3473	1568	II. 513	....	13 9 31·1	3·188	1	105 53 46·3	19·11	1	F; pS; iR	3
3474	1570	......	M. 63	13 9 31·9	2·699	1	47 13 45·3	19·11	1	vB; L; pmE 120°±; vsmbMBN.	3
3475	1571	III. 306	....	13 9 38·0	2·823	1	58 18 9·3	19·11	1	cF; cS; R; p of 2	2
3476	1572	III. 307	....	13 9 52·3	2·821	1	58 13 16·0	19·10	1	cF; cS; R; f of 2	2
3477	3480	I. 138	....	13 10 27·4	3·274	1	116 6 47·7	19·09	1	vB; S; R; vsmbM; * 10 f....	4
3478	3482	......	....	13 10 33·7	3·358	1::	124 41 30·4	19·08	1	eF; vS; E; r	1
3479	3481	......	....	13 10 33·9	3·357	1	124 35 15·4	19·08	1	eF; vS; R; * nr	1
3480	3483	......	....	13 10 46·3	3·519	1	137 10 22·1	19·07	1	B; S; R; pslbM	1
3481	1573	III. 308	....	13 10 57·3	2·817	1	58 10 2·4	19·08	1	vF; cS........................	3
3482	......	III. 312	....	13 11 14·2	3·227	1	110 22 7·8	19·06	1	F; L; iR; bM.................	1
3483	1574	III. 282	....	13 11 54·7	3·178	1	104 6 53·5	19·05	1	vF; pL; pmE 135°±	2*
3484	1575	III. 309	....	13 11 55·9	+2·810	1	57 47 16·5	+19·05	1	eF; vS	3

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	h.	H.		h m s	s		° ′ ″	″			
3485	1576 = 3489	III. 117	....	13 12 6·9	+3·162	2	102 0 4·2	+19·04	2	vF; cS; R; 1st of 3	5
3486	1577 = 3490	II. 193	....	13 12 9·0	3·161	2	101 55 11·2	19·04	2	pB; S; vlE; sbM; 2nd of 3...	5
3487	3484	II. 566	....	13 12 9·6	3·285	1	116 50 8·2	19·04	1	pB; pS; cE; psbM; ✱7·8 f...	3
3488	1578 = 3491	III. 118	....	13 12 14·9	3·162	2	101 57 46·0	19·04	2	cF; pS; vlE; 3rd of 3	5
3489	1578, a	......	R. nova	13 12 ±	3·162	...	101 57 ±	19·04	...	No description, one of a group of *four*.	0
3490	3485	......	....	13 12 26·3	3·285	1	116 40 17·9	19·03	1	vF; S; R; 1st of 4	1
3491	1579	II. 313	....	13 12 42·4	3·236	1	111 4 37·6	19·02	1	cB; cS; vlE 90°±; bf	2
3492	......	II. 780	....	13 12 44·0	3·258	1	113 40 5·6	19·02	1	F; L; R; vglbM	1
3493	3486	......	....	13 12 55·4	3·469	1	132 59 25·6	19·02	1	eF; vS; R; 2nd of 4	1
3494	......	III. 724	....	13 12 55·9	3·226	1	109 52 4·6	19·02	1	cF; vS; iF	1
3495	1580	II. 327	....	13 13 3·1	2·818	2	59 1 33·6	19·02	2	pF; pL; gbM	3
3496	3487	......	....	13 13 4·2	3·470	2	132 58 39·3	19·01	2	pB; pL; R; 3rd of 4	2
3497	3488	......	....	13 13 8·5	3·470	2	132 59 51·3	19·01	2	cF; S; vlE; 4th of 4	2
3498	1583	III. 633	....	13 13 20·1	2·702	1	48 51 23·3	19·01	1	vF; S; R; lbM	2
3499	1581	III. 539	....	13 13 22·1	3·174	1	103 20 52·0	19·00	1	cF; vS; R; gbM	2
3500	1582	......	....	13 13 24·5	3·085	1	91 34 7·0	19·00	1	vF; iR; ✱ 11 sp	1
3501	1584	III. 650	....	13 13 39·1	2·787	1	56 11 1·0	19·00	1	vF; cS; R; bM; sp of 2	3
3502	1585	......	....	13 13 46·8	2·786	1	56 7 15·0	19·00	2	vF; S; bet 2 st; nf of 2	2
3503	3493	II. 567	....	13 14 3·7	3·289	2	116 41 36·7	18·99	2	cB; pS; lE; psbM ✱	4
3504	3492	......	....	13 14 4·5	3·385	1	125 54 9·7	18·99	1	vB; pS; R; svmbM	1
3505	......	II. 665	....	13 14 9·9	2·660	1	46 10 1·7	18·99	1	pB; cS; E	1
3506	......	II. 22	....	13 14 34·0	3·002	1	80 46 29·4	18·98	??	vF; vS	1*
3507	1586	III. 619	....	13 15 6·1	2·717	1	50 42 22·8	18·96	1	vF; S; cE 0°±	2
3508	3494	......	....	13 15 12·3	3·342	1::	121 36 35·5	18·95	1	eeF; p of 2	1
3509	1588	III. 808	....	13 15 14·4	2·367	1	31 37 0·8	18·96	1	cF; S; cE	2
3510	1587	III. 119	....	13 15 32·9	3·168	2	102 14 3·2	18·94	2	cF; cS; iR; glbM	2
3511	1589	II. 646	....	13 15 38·3	2·712	1	50 31 29·5	18·95	1	F; L; iR; vglbM	2†
3512	......	II. 826	....	13 16 2·4	2·359	1	31 34 28·2	18·94	1	F; S; E	1*
3513	3495	......	....	13 16 12·7	3·345	1	121 36 34·6	18·92	1	F; lE; psbM; f of 2	1
3514	1590	III. 368	....	13 16 19·0	2·841	2	62 17 27·6	18·92	2	pF; pS; pmE; glbM; r	3
3515	1592	......	....	13 16 19·0	2·827	1	60 56 38·6	18·92	1	vF; L; Δ 2 st 11 np	1
3516	1591	III. 925	....	13 16 24·5	3·018	1	82 52 33·6	18·92	1	vF; S; R; gbM	3
3517	3497	......	....	13 16 35·7	3·164	1	101 33 30·3	18·91	1	pB; S; lE	1
3518	3496	......	....	13 16 37·7	3·939	1	152 40 50·0	18·90	1	Cl; eR; mC; st 12...16	1
3519	3498	......	....	13 16 46·4	3·409	2	126 57 18·3	18·91	2	cB; P; R; psmbM; r	2
3520	1594	II. 666	....	13 16 59·8	2·646	1	46 10 43·3	18·91	1	pF; S; R; gmbM	2
3521	3499	......	....	13 17 1·9	3·327	2	119 34 56·0	18·90	2	vF; S; vlE	2
3522	1593	......	....	13 17 2·8	2·990	1	79 33 7·0	18·90	1	pF; S; R; gbM	1
3523	3500	......	....	13 17 5·0	3·333	1	119 36 57·0	18·90	1	vF; vS	1
3524	1596	II. 328	....	13 17 15·0	2·790	2	57 42 15·0	18·90	3	pB; pL; R; gmbM; ⁑ p	5
3525	3501	......	Δ. 482	13 17 15·1	3·481	4	132 17 10·7	18·89	4	!!; vB; vL; vmE 122°·5; bifid	4†
3526	1595	II. 653	....	13 17 15·9	2·955	3	75 17 16·7	18·89	3	pB; vS; R; gmbM; ⁑ f	4
3527	1597	II. 314	....	13 17 43·5	3·240	1	110 23 21·4	18·88	1	F; pS; lE; vgbM	2*
3528	3502	......	....	13 17 56·3	3·325	1	119 6 14·4	18·88	1	pB; S; E	1
3529	1598	III. 84	....	13 17 59·6	2·953	1	75 31 20·4	18·88	1	eF; vS; R; psbM	3
3530	3503	......	Δ. 312?	13 18 19·5	3·811	2	148 16 44·8	18·86	2	Cl; Ri; lC; st 11...	2
3531	3504	......	Δ. 440	13 18 24·0	3·554	2	136 34 49·8	18·86	2	!!; ⊕; ω Centauri	2†
3532	3505	......	....	13 18 27·3	3·370	2	123 9 1·8	18·86	2	vF; S; R; glbM	2
3533	1599	III. 402	....	13 18 29·8	2·730	2	52 53 5·8	18·86	2	cF; cS; R; vsmbM ✱; ✱12 sp; sp of 2.	3
3534	1600	III. 403	....	13 18 39·8	2·729	2	52 50 44·8	18·86	2	F; cS; R; vsmbM ✱; n of 2	3
3535	1600, a	......	R. nova	13 18 39·8	2·729	::	52 49 ±	18·86	:::	vF	0
3536	......	IV. 70	....	13 18 48·6	1·743	2	18 45 22·8	18·86	2	○?; cB; S; R; g, slbM	2
3537	1602	*II. 667	....	13 19 4·9	+2·632	1	46 0 37·5	+18·85	1	pB; vS; vlE; glbM	2

No. of Catalogue.	References to: Sir J. H.'s Catalogues of Nebulæ.	Sir W. H.'s Classes and Nos.	Other Authorities.	Right Ascension for 1860, Jan. 0.	Annual Precession in Right Ascension for 1880.	No. of Obs. used.	North Polar Distance for 1860, Jan. 0.	Annual Precession in N.P.D. for 1880.	No. of Obs. used.	Summary Description from a Comparison of all the Observations, Remarks, &c.	Total No. of times of Obs. by h. and H.
	h.	H.		h m s	s		° ′ ″	″			
3538	……	III. 115	…………	13 19 8·3	+3·167	2	101 35 25·2	+18·84	2	vF; vS; stellar …………	2
3539	1601	II. 25	…………	13 19 11·5	3·050	2	87 9 53·2	18·84	2	pB; pL; vlE; vsmbM * 12…	6
3540	1604	III. 404	…………	13 19 47·8	2·730	2	53 19 49·9	18·83	2	cF; pS; E; bM; sp of 2……	3
3541	1603	……	…………	13 19 49·1	2·926	2.	72 23 55·6	18·82	2	vF; S; R; * nf …………	1
3542	3507	……	…………	13 19 51·7	3·328	2	118 50 27·6	18·82	2	cF; S; R; pslbM; * f 2′……	2
3543	3508	……	…………	13 20 2·9	3·329	1::	118 54 24·3	18·81	1::	vF; S; R; p of D neb………	1
3544	3509	……	…………	13 20 4·1	3·329	1	118 53 52·3	18·81	1	pF; S; f of D neb …………	1
3545	1605	III. 405	…………	13 20 7·4	2·728	2	53 16 40·6	18·82	2	vF; pL; R; nf of 2…………	3
3546	3506	……	…………	13 20 8·6	3·977	1	152 41 27·0	18·80	1	Cl; vRi …………………	1
3547	3510	……	…………	13 20 17·7	3·594	2	138 10 46·0	18·80	2	pB; cS; iE; glbM; r ………	2
3548	1606	III. 651	…………	13 20 49·5	2·773	3	57 17 26·7	18·79	3	F; pS; vlE; bM …………	4
3549	1607	……	…………	13 20 59·9	2·916	1	71 29 31·4	18·78	1	vF; R …………………	1
3550	……	……	D'Arrest, 94	13 21 13	3·02	[2]	83 16 42	18·78	[2]	pF. See note………………	0*
3551	3511	……	…………	13 21 19·2	3·373	1	122 26 25·1	18·77	1	pF; L; vmE; pgbM; rr ……	1
3552	……	III. 821	…………	13 21 34·6	2·438	1	36 28 48·1	18·77	1	cF; stellar …………………	1
3553	1609	III. 784	…………	13 21 38·4	2·371	1	33 47 3·1	18·77	1	cF; S; iR…………………	2
3554	1608	……	…………	13 21 47·4	2·770	3	57 14 35·8	18·76	3	pF; pL; lE; lbM; f of 2……	3
3555	3512	……	…………	13 22 3·3	3·898	2	150 12 3·1	18·77	2	Cl; vF; S; vRi; st 15………	2
3556	1611	……	…………	13 22 8·6	2·560	1	42 38 21·5	18·75	1	vF; pS; R …………………	1
3557	1610	V. 22	…………	13 22 17·6	3·221	2	107 14 29·2	18·74	2	cF; L; mE 128°·8; pgbM …	4
3558	1613	……	…………	13 22 28·5	2·920	2	72 13 18·2	18·74	2	F; pL; R; gbM …………	2
3559	1614	III. 672	…………	13 22 28·5	2·558	2	42 41 26·2	18·74	2	F; vS; R; stellar…………	5
3560 3561	1612	III. 45 III. 46	…………	13 22 29·1	2·974	2	78 16 4·2	18·74	2	vF; pL vF; pL } D neb …………	4
3562	1615	III. 71	…………	13 22 37·1	2·922	1	72 26 57·9	18·73	2	vF; S; R; am 3 st; * 7 nf…	3
3563	1616	……	…………	13 22 49·1	2·953	2	75 58 15·9	18·73	2	vF; S; R………………	2
3564	3513	……	…………	13 22 51·6	3·321	2	117 25 21·6	18·72	2	vF; pL; vlE; * 7 nf 10′……	2
3565	1617	II. 679	…………	13 22 54·8	3·081	1	90 59 58·6	18·72	1	F; cS; lE; gbM; p of 2……	3
3566	1618	II. 680	…………	13 22 59·8	3·081	1	90 56 3·6	18·72	1	pF; pL; iR; bM; f of 2……	3
3567	1619	III. 642	…………	13 23 9·3	2·952	1	75 54 40·6	18·72	1	vF; S; iR …………………	2
3568	1620	III. 652	…………	13 23 21·2	2·774	1	58 8 37·3	18·71	2	vF; vS; R; glbM …………	3
3569	3515	……	…………	13 23 31·3	3·400	2	124 3 45·0	18·70	2	F; pL; vlE; vglbM ………	2
3570	3514	……	Δ. 252?	13 23 41·0	4·129	1	155 15 11·7	18·69	1	!; B; pL; cE; bM curved axis; 4 st inv.	1†
3571	1621	……	…………	13 23 50·0	2·908	2	71 8 20·0	18·70	2	eF; S; R; bM; ** f ………	2
3572	1622	……	M. 51	13 23 55·4	2·539	5	42 5 4·0	18·70	4	!!!; nucl & ring (h); spiral (R)	10†
3573	3516	……	…………	13 23 57·1	3·382	1	122 30 27·7	18·69	1	pB; S; R; g, psbM…………	1
3574	1623	I. 186	…………	13 24 4·4	2·536	4	42 0 50·7	18·69	3	B; pS; R; vgbM; f of 2……	6
3575	1625	IV. 63	…………	13 24 12·2	2·264	1	30 51 32·7	18·69	1	pB; cL; iR; gmbM; r ……	2
3576	……	II. 689	…………	13 24 14·0	2·546	3	42 35 13·7	18·69	2	pB; pL; R; mbM …………	3
3577	1624	III. 406	…………	13 24 21·5	2·726	2	54 26 31·4	18·68	2	vF; vS; lE …………………	3
5072	……	……	…………	13 24 32·0	……	…	89 18 29·7	……	…	See No. 5072.	
3578	……	II. 797	…………	13 24 39·9	2·409	2	36 12 12·4	18·68	2	pF; cS; R; vglbM …………	1
3579	3517	III. 507	…………	13 24 52·2	3·142	1	98 3 15·8	18·66	1	vF; cS; R; gbM; r ………	2
3580	3518	……	…………	13 25 18·7	3·610	1	137 24 41·2	18·64	1	F; pL; R; vgbM …………	1
3581	1626	III. 643	…………	13 25 21·9	2·945	1.	75 22 53·5	18·65	1	F; S; cE; * 11 att sp………	2
3582	1627	III. 9	…………	13 25 26·6	3·003	3	81 57 44·2	18·64	4	F; vS; R; psbM; p of 2……	6
3583	1628	III. 10	…………	13 25 42·8	3·003	2	81 57 21·2	18·64	2	F; vS; R; stellar; f of 2……	4
3584	1629	III. 99	…………	13 25 48·2	3·004	1	82 6 15·9	18·63	1	F; S; R; psbMN …………	4
3585	1630	……	…………	13 25 54·1	3·076	1	90 18 50·9	18·63	1	pB; S; R; psmbM …………	1
3586	1631	……	…………	13 26 21·9	3·003	1	81 59 5·6	18·62	1	eF ………………………	1
3587	1632	III. 656	…………	13 26 44·5	2·615	1	47 24 36·3	18·61	1	vF; S; R; lbM …………	2
3588	1633	III. 926	…………	13 27 7·3	3·009	1	82 47 49·7	18·59	1	vF; S; *9 nf inv? (? 28ˢ R.A.)	3*
3589	3519	……	…………	13 27 10·2	3·397	1	122 45 39·4	18·58	1	eF; eS; * S and * p ………	1
3590	1635	II. 841	…………	13 27 14·9	2·075	1	26 33 53·7	18·59	1	pB; S; vlE …………………	3
3591	1634	……	…………	13 27 18·2	2·904	1	71 25 25·4	18·58	1	vF; S; R; bM………………	1
3592	1636	II. 842	…………	13 27 21·3	2·072	1	26 30 43·7	18·59	1	pB; pL; R; gbM …………	3
3593	3520	……	…………	13 27 47·5	3·583	1?	135 11 16·8	18·56	1::	vF; S; R; * n, nr …………	1
3594	3521	……	…………	13 27 59·8	3·399	1	122 44 13·8	18·56	1	vF; S; R; * 10 f…………	1
3595	1637	III. 86	…………	13 28 4·5	2·942	2	75 27 49·8	18·56	2	vF; S; vlE; 1st of 3 ………	4
3596	1638	III. 85	…………	13 28 4·9	+2·943	3	75 32 20·8	+18·56	3	cF; S; R; bM; 2nd of 3……	5

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	h.	H.		h m s	s		° ′ ″	″			
3597	1639	III. 87	....	13 28 5·2	+2·943	1	75 37 21·8	+18·56	2	cF; pL; R; glbM; 3rd of 3..	4
3598	1640	III. 407	....	13 28 6·2	2·713	3	54 34 50·8	18·56	3	F; cS; R; ✱ 10 p; p of 2 ...	5
3599	......	III. 822	....	13 28 9·6	2·422	1	37 46 37·8	18·56	1	cF; pS; iR; lbM..............	1
3600	1641	III. 928	....	13 28 15·0	3·054	(1)	87 53 16·5	18·55	1::	vF; S; R.....................	2
3601	1642	III. 408	....	13 28 16·7	2·711	2	54 29 21·8	18·56	2	vF; vS; R; f of 2	4
3602	1643	......	....	13 28 31·7	2·941	1	75 35 24·2	18·54	1	F; L; E; vgbM	1
3603	1645	III. 425	....	13 28 51·6	2·711	1	54 36 47·9	18·53	1	F; S; R; vS ✱ nr	2
3604	3522	......	....	13 28 52·3	3·666	1	139 6 59·6	18·52	1	eeF; S; lE	1
3605	1644	III. 100	....	13 28 58·3	3·008	3	82 41 27·9	18·53	3	vF; pS; vlE; ✱ 9 sp	4
3606	3523	......	M. 83 = Δ. 628	13 29 9·0	3·360	4	119 9 31·6	18·52	4	!!; (H, h) vB; vL; E 55°·1; esbMN (L) 3-branched spiral.	6†
3607	3524	......	....	13 29 20·8	3·539	4	132 8 2·3	18·51	4	F; pL; cE; vglbM............	4
3608	1646	III. 101	....	13 29 27·9	3·000	1	81 54 44·3	18·51	1	vF; pL; R; er................	4
3609	......	III. 823	....	13 29 26·9	2·410	1	37 39 33·6	18·52	1	cF; pL; R; vlbM	1
3610	......	III. 409	....	13 29 29·9	2·696	1	53 43 3·3	18·51	1	vF; pL; R; lbM...............	1
3611	1647	......	....	13 30 0·9	3·041	1	86 30 48·0	18·50	1	eF; eL.......................	1
3612	1648	III. 620	....	13 30 4·6	2·654	2	50 55 41·0	18·50	2	cF; pL; E 65°; biN?	3
3613	3525	......	....	13 30 17·1	3·591	2	135 9 0·4	18·48	2	vF; S; R; vglbM; ✱ 13 att...	2
3614	1649	II. 297	....	13 30 30·2	3·234	2	107 10 3·1	18·47	2	!!; (H, h) cF; vL; vg, psmbMLN (L) 2-branched spiral.	4†
3615	1650	I. 34	....	13 30 35·1	2·985	1	80 23 32·1	18·47	1	B; L; E 150°; psbMrN	3
3616	1651	III. 72	....	13 30 48·6	2·916	3	73 18 27·1	18·47	3	vF; S; R; bM.................	4
3617	......	II. 817	....	13 30 49·8	2·409	1	38 1 30·1	18·47	1	pB; S; R; vgbM	1
3618	1652	III. 369	....	13 30 53·0	2·795	1	61 51 45·1	18·47	1	vF; S; vlE	2
3619	1653	III. 505	....	13 31 11·6	3·025	(1)	84 46 14·8	18·46	1	vF; S; R; bM.................	4
3620	3526	II. 638	Δ. 623	13 31 59·9	3·390	1	120 55 34·6	18·42	1	B; pL; E 45°±; psmbM ...	2
3621	3527	......	....	13 32 12·4	3·174	1	100 46 55·6	18·42	1	pB; L; pmE; glbM	1
3622	......	III. 803	....	13 32 21·9	2·236	2	32 10 56·6	18·42	2	vF; vS	2
3623	1656	III. 673	....	13 32 32·9	2·465	1	40 59 35·6	18·42	1	eF; vS; R; gbM...............	2
3624	1654	II. 895	....	13 32 45·5	3·059	1	88 26 55·0	18·40	1	vF; S; R; bM; p of D neb...	2
3625	1655	II. 896	....	13 32 49·9	3·058	1	88 27 15·0	18·40	1	F; S; iR; f of D neb	2
3626	1657	......	....	13 33 13·6	3·019	2	84 13 5·7	18·39	2	vF; R; am pB st..............	2
3627	1660	......	....	13 33 14·4	0·995	1	14 14 14·0	18·40	1	eF; S	1
3628	1658	III. 370	....	13 33 26·7	2·775	1	60 53 12·4	18·38	1	cF; S; mE 0°±; ✱ 9 sp......	2
3629	3528	......	....	13 33 43·5	3·374	1	119 12 28·8	18·36	1	vF; pL; R; vlbM	1
3630	1659	III. 410	....	13 33 58·5	2·660	2	52 25 32·8	18·36	2	F; cS; vlE; er...............	3
3631	3529	......	....	13 34 27·7	3·665	2	137 27 41·9	18·33	2	B; pL; vlE; vglbM; 3 st nr..	2
3632	1661	......	....	13 34 32·8	2·628	4	50 30 0·2	18·34	4	F; S; R; gbM; S ✱ np	4
3633	......	......	Auw. N. 32	13 34 43·9	3·196	...	103 9 3·6	18·31	...	A nebula (Markree Obs. 1855)	0
3634	3530	......	....	13 35 3·6	4·112	2	152 11 47·3	18·31	2	Cl; P; L; iF; st 12..........	2
3635	1662	......	....	13 35 5·5	3·026	1	85 1 44·6	18·32	1	eF; S; bet 2 st..............	1
3636	1663	......	M. 3	13 35 40·8	2·769	5	60 55 6·0	18·30	5	!!; ⊕; eB; vL; vsmbM; st 11..	14
3637	1664	I. 98	....	13 35 56·1	2·670	3	53 37 45·7	18·29	3	cB; pL; R; g, psmbM	4
3638	1664, *a*	......	R. nova	13 36 8·6	2·670	::	53 37 45·7	18·29	::	F; S.........................	0
3639	1665	II. 798	....	13 36 24·8	2·248	2	33 37 25·4	18·28	2	F; E 73°·0; D or biN; B ✱ nf	3
3640	3531	......	Δ. 273	13 36 54·7	4·131	2	152 11 45·2	18·24	2	Cl; B; S; pC; iR; st 10...12	2
3641	3532	......	....	13 37 15·3	3·985	1	148 29 24·2	18·24	1	Cl; L; vRi; st 7...16	1
3642	3533	......	Δ. 388	13 37 37·5	3·757	2	140 40 6·6	18·22	2	⊕; vB; pL; R; rrr; st 15...	2
3643	3534	......	....	13 38 48·2	4·237	2	153 59 16·1	18·17	2	Cl; S; C; iR; st 14...........	2
3644	1666	II. 668	....	13 39 9·3	2·561	1	47 47 33·4	18·18	1	vF; vS; lE 90°±; sbM	2
3645	......	I. 170	....	13 39 17·9	2·556	2	47 33 53·1	18·17	2	cB; pL; E 90°±; bMN.....	2
3646	3535	......	....	13 39 25·8	3·397	1	119 41 12·8	18·16	1	vF; R; vlbM; ‡ p	1
3647	3536	......	....	13 39 41·2	3·405	1	120 13 12·2	18·14	1	pF; S; R; 2 st nr	1
3648	......	V. 6	....	13 39 55·7	2·899	1	72 59 14·2	18·14	1	eF; vL; r....................	1
3649	1667	III. 785	....	13 39 57·1	+2·232	1	34 0 24·5	18·15	1	eF; 2 st att or inv	2
3650	......	III. 946	....	13 40 20·5	−0·151	1	9 52 6·7	18·09	1	vF; vS; R	1*
3651	1668, *a*	......	R. nova	13 40 ±	+2·511	::	45 28 ±	18·13	::	R; bM; is sp h. 1668.........	0
3652	1668	I. 180	....	13 40 31·6	2·511	1	45 27 31·9	18·13	1	cB; L; pmE 142°; gbM......	2
3653	3538	......	....	13 40 39·5	+3·401	1	119 44 21·3	+18·11	1	F; S; R; gbM.................	1

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	h.	H.		h m s	s		° ′ ″	″			
3654	3537	......	....	13 40 51·7	+4·064	1	149 14 42·0	+18·10	1	Cl; vL; vRi	1
3655	1669	II. 533	....	13 41 9·9	3·026	2	85 21 35·0	18·10	2	vF; vL; lE; vgbM	4
3656	1670	II. 688	....	13 41 13·7	2·456	1::	43 9 25·1	18·07	1	cF; L; vmE	3
3657	3539	......	....	13 41 22·7	3·405	1	119 47 20·4	18·08	1	F; S; R; gbM	1
3658	1672	III. 681	....	13 41 40·0	2·605	1	51 0 17·4	18·08	1	pF; cS; lE; F * inv	2
3659	1673	III. 621	....	13 41 51·2	2·612	2	51 28 9·4	18·08	1	eF; S; R	3
3660	{1671 = 3540}	II. 306	....	13 41 51·7	3·139	2	96 31 46·8	18·06	2	vF; vS; R; r	3
3661	3541	......	....	13 42 11·8	3·782	1	140 30 34·2	18·04	1	○, or vF; eS; D neb	1†
3662	1674	I. 255	....	13 42 19·9	1·999	1	28 18 58·8	18·06	1	B; pL; mE 57°·4; psbMBEN	2*
5073	......	......	....	13 42 38·3	......	...	89 14 2·4	......	...	See No. 5073.	
3663	1675	II. 710	....	13 42 57·5	2·571	1	49 19 5·9	18·03	1	cF; cS; R; sbM; p of 2	3
3664	1676	III. 422	....	13 43 37·6	2·671	2	55 41 13·3	18·01	2	vF; R; stellar; 1st of 3	3*
3665	1677	II. 711	....	13 43 45·0	2·567	2	49 19 42·0	18·00	2	pB; pS; vlE; glbM; f of 2	3
3666	3542	......	Δ. 282	13 44 7·0	4·155	1	151 9 39·1	17·97	1	Cl; pL; pC; st 11...	1
3667	1678	......	....	13 44 8·2	3·014	1	84 18 29·4	17·98	1	vF; vL; R; vgbM	1
3668	1679	III. 423	....	13 44 22·4	2·667	1	55 36 31·1	17·97	2	F; S; R; psbM; 2nd of 3	3*
3669	1682	II. 669	....	13 44 27·1	2·539	1	47 56 13·1	17·97	1	cF; pL; R; gbM	2
3670	1680	......	....	13 44 28·9	2·668	1	55 39 38·1	17·97	1	eF; pL; R; svmbM*	2
3671	1684	I. 256	....	13 44 33·4	2·012	1	29 6 16·1	17·97	1	vB; pL; iR; psmbM	2
3672	1689	II. 899	....	13 44 38·6	0·426	1	12 28 21·7	17·99	1	vF; pS; lE 0° ±	2
3673	1681	II. 307	....	13 44 43·6	3·129	1	95 21 14·5	17·95	1	cF; L; iR; bM	3
3674	1685	II. 712	....	13 44 51·4	2·570	2	49 44 8·8	17·96	3	cF; S; vlE; sbM	4
3675	1683	II. 685	....	13 44 52·1	3·089	1	91 30 32·5	17·95	1	F; pS; R; 2 st p	3
3676	3543	III. 923	....	13 44 56·4	3·388	2	117 46 54·5	17·95	2	pB; S; R; slbM	3
3677	1686	III. 549	....	13 45 5·2	3·042	1	86 58 26·2	17·94	1	F; vS; R; psbM	2
3678	1687	III. 929	....	13 45 8·1	3·044	1	87 12 15·2	17·94	1	vF; S; E 0°; rr	3
3679	3544	......	....	13 45 39·7	3·737	1	137 48 33·6	17·92	1	vF; vS; R; *8 f	1
3680	......	III. 665	....	13 45 44·8	3·078	2	90 25 29·6	17·92	2	cF; vL; R; lbM; r	2
3681	1688	......	....	13 45 49·7	3·036	1	86 28 58·6	17·92	1	F; iR	1
3682	1690	II. 670	....	13 46 20·2	2·494	1	46 3 51·0	17·90	1	cF; pL; R; psbM	2
3683	1691	III. 698	....	13 46 26·0	2·561	1	49 37 37·0	17·90	1	vF; S; iR; B * p	3
3684	1694	III. 849	....	13 46 43·8	2·006	1	29 25 55·7	17·89	1	eF; vS	2
3685	1692	II. 308	....	13 46 50·5	3·147	1	96 54 11·4	17·88	1	vF; S; R; lbM	3
3686	1693	II. 686	....	13 47 2·3	3·081	1	90 44 38·1	17·87	1	F; S; R; bM	3
3687	1695	II. 424	....	13 47 6·9	2·661	1	55 49 32·1	17·87	1	pF; cL; R; lbM	2
3688	1696	II. 713	....	13 47 25·5	2·544	2	48 56 20·8	17·86	2	cF; pL; bM; B * p; 1st of 4	5
3689	1697	II. 697	....	13 47 26·3	2·588	2	51 23 43·8	17·86	2	cF; L; lE 90°; vgbM	3
3690, 3691	1697, a	......	R. 2 novæ	13 47 ±	......	...	51 23 ±	......	...	{2 neb in a line with h. 1697; bM.}	0
3692	1700	II. 415	....	13 47 32·2	2·617	2	53 10 5·5	17·85	2	cF; S; R; lbM; * nf 90″	3
3693	1698	II. 714	....	13 47 32·9	2·545	3:	49 2 24·5	17·85	1	pB; S; R; 2nd of 4	5
3694	1699	II. 715	....	13 47 33·2	2·545	3:	49 1 24·8	17·86	1	pF; S; R; 3rd of 4	5
3695	1702	III. 699	....	13 47 45·1	2·543	2	48 57 33·2	17·84	3	vF; pS; 4th of 4	3
3696	1701	III. 506	....	13 47 56·1	3·008	5	83 58 28·9	17·83	5	F; pL; vmE 15°; r	7
3697	3546	......	....	13 47 58·1	3·422	1	119 38 52·6	17·82	1	pF; S; R; glbM; bet 2 st 10.	1
3698	1703, a	......	R. nova	13 48 4·7	3·009	::	84 2 32·4	17·81	::	F; S; E (nisi=III. 506)	0
3699	3545	......	....	13 48 30·8	4·752	2	159 43 18·7	17·79	2	Cl; vL; lRi; lC; st 11...	2
3700	......	III. 682	....	13 48 38·9	2·574	1	50 52 52·3	17·81	1	eF; cS; E	1
3701	......	II. 671	....	13 48 58·8	2·520	1	48 3 22·0	17·80	1	pB; pL; E	1
3702	1703	I. 6	....	13 49 5·3	3·009	4	84 3 32·4	17·78	4	B; pL; R; psbM; *8 nf	8
3703	1703, b	......	R. nova	13 49 5·3	3·009	:	84 17 32·4	17·78	:	vF; L	0
3704	1705	II. 534	....	13 49 12·0	3·011	(1)	84 18 20·4	17·78	2	cF; L; R; gbM	4
3705	3547	......	....	13 49 15·5	3·657	2	133 14 39·1	17·77	2	pB; cS; R; pgbM; am st	2
5074	......	......	....	13 49 18·6	......	...	89 31 9·0	......	...	See 5074.	
3706	3548	......	....	13 49 20·9	3·581	3	129 17 44·1	17·77	4	!; vB; vL; vl, vsmbM *	6†
3707	1706	III. 786	....	13 49 24·0	2·192	2	34 58 35·4	17·78	2	F; cS; R; stellar; *16 nf	3
3708	1704	III. 285	....	13 49 33·3	3·125	1	94 48 28·8	17·76	1	vF; vS; R	2
3709	1708	II. 843	....	13 49 38·6	1·943	1	28 37 31·4	17·78	1	F; S	2
3710	1707	II. 716	....	13 49 46·8	+2·531	1	48 49 39·8	+17·76	1	pB; L; R; bMFN	3

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	h.	H.		h m s	s		° ' "	"			
3711	1709	III. 809	....	13 49 48·6	+2·031	1	30 39 35·1	+17·77	1	cF; S; E; ?* inv	2
3712	1710	II. 889	....	13 50 29·0	2·999	4	83 12 55·9	17·73	4	cF; pL; R; vgbM; *11 np...	5
3713	1711	......	....	13 50 36·8	2·715	3	60 8 59·9	17·73	3	pB; pL; R; lbM	3
3714	......	II. 844	....	13 50 38·7	1·987	1	29 47 45·9	17·73	1	pB; cL	1
3715	......	I. 238	....	13 50 40·4	1·986	3	29 48 15·9	17·73	3	cB; pL; vlE; vgmbM.........	3
3716	1712	I. 187	....	13 50 41·1	2·382	4	42 4 55·9	17·73	4	B; L; mE 40°·4; smbMN ...	5
3717	1713	......	....	13 50 47·4	2·576	1	51 31 33·6	17·72	1	pB; lE; vglbM	1†
3718	1714	II. 698	....	13 50 56·6	2·579	3	51 42 17·3	17·71	3	F; cS; R; smbM..............	4
3719	3549	......	....	13 50 57·8	4·119	2	148 54 29·7	17·69	2	Cl; Ri; vC; pL; st 11...12...	2
3720	......	I. 239	....	13 51 6·5	1·973	1	29 35 14·3	17·71	1	pB; pS; E; mbM	3
3721	1715	III. 546	....	13 51 15·6	2·996	1	83 3 41·7	17·69	2	vF; vS; r; stellar	3
3722	1715, a	......	R. nova	13 51 ±	......	...	83 0 ±	......	...	F; S; R	0
3723	1717	I. 181	....	13 51 17·9	2·496	1	47 28 19·0	17·70	1	cB; cL; R; gbM	2
3724	1721	......	....	13 51 19·2	0·414	1	13 7 58·9	17·73	1	Cl; P; S	1
3725	1716	III. 547	....	13 51 22·6	2·996	1	82 58 36·7	17·69	2	vF; vS; biN; r; stellar	3
3726	1719	I. 240	....	13 51 30·0	1·968	1	29 33 27·0	17·70	1	pB; pL; E; mbMN	3
3727	1718	......	....	13 51 31·1	2·524	1	48 52 36·7	17·69	1	F; L; vgbM; *9 nf...........	1
3728	1720	III. 666	....	13 52 10·9	3·101	1	92 31 21·8	17·66	1	vF; cS; R; gbM..............	2*
3729	3550	......	....	13 52 33·0	3·414	1	118 11 17·2	17·64	1	vF; S; R; glbM	1
3730	1722	I. 191	....	13 52 34·9	2·575	2	51 52 3·5	17·65	2	cF; S; np of 2	3
3731	1723	I. 190	....	13 52 38·2	2·575	3	51 53 52·5	17·65	3	cF; cL; E 15°·0; lbM; sf of 2	4
3732	......	III. 125	....	13 52 39·4	2·709	1	60 11 32·5	17·65	1	vF; S; iR; sbM*	1
3733	3551	......	....	13 52 55·7	3·492	1	123 16 22·6	17·62	1	vF; S; R; gbM	1
3734	3552	......	....	13 53 10·9	3·479	1	122 23 20·3	17·61	1	pB; pL; R; vgbM	1
3735	1724	III. 411	....	13 53 24·5	2·618	2	54 32 37·3	17·61	2	eF; vS; pmE 90°	3
3736	......	III. 667	....	13 53 31·9	3·097	2	92 10 41·0	17·60	2	vF; cS	2
3737	1725	III. 412	....	13 53 39·8	2·592	1	53 4 6·0	17·60	1	cF; cS; E	2
3738	1727	III. 810	....	13 53 41·9	1·944	1	29 28 38·3	17·61	1	vF; vS; R	2
3739	1726	III. 683	....	13 53 51·9	2·557	1	51 8 1·0	17·60	2	vF; pL; iF	4
5075	......	......		13 53 58·0	......	...	89 13 53	......	...	See No. 5075.	
3740	1728	II. 699	....	13 54 24·3	2·541	2	50 24 2·1	17·57	2	F; pS; R; lbM	5
3741	1732	III. 684	....	13 54 50·2	2·535	(1)	50 9 17·8	17·56	1	vF; vS; R; bM; in Cl	2
3742	3553	......	....	13 54 50·7	3·632	1	130 44 6·2	17·54	1	eF; E bet 2 vS st..............	1
3743	1729	II. 672	....	13 55 0·0	2·498	1	48 19 36·5	17·55	1	pF; pS; bM....................	2
3744	......	III. 56	....	13 55 0·2	2·958	1	79 54 37·2	17·54	1	eF; vS; E; r	1
3745	1733	......	....	13 55 10·5	1·647	1	24 24 32·5	17·55	1	pF; pS; R; pslbM	1
3746	1730	III. 11	....	13 55 17·4	2·974	4	81 17 11·9	17·53	4	cF; S; R; psbM; *p	5
3747	1731	......	....	13 55 20·0	2·978	1	81 38 22·9	17·53	1	vF; R; bM	1
3748	3554	......	....	13 55 24·3	3·501	3	123 17 44·6	17·52	3	pB; pL; R; gpmbM	3
3749	1736	I. 230	....	13 55 44·4	2·117	2	34 9 20·6	17·52	2	pB; S; pmE 45°±; vsvmbMN.	4
3750	1734	II. 309	....	13 55 59·7	3·134	1	95 18 37·0	17·50	1	pF; cL; R; gmbM; np of 2 ..	3†*
3751	1735	II. 310	....	13 56 3·7	3·134	1	95 21 25·7	17·49	1	pF; cL; R; sf of 2	3*
3752	1738	II. 827	....	13 56 14·1	1·945	1	29 58 35·3	17·51	1	pB; S; iE; mbM..............	2
3753	1737	III. 653	....	13 56 26·8	2·646	2	56 49 5·4	17·48	2	vF; cS; lE 0°; bM	3
3754	1739	II. 416	....	13 56 57·7	2·606	1	54 33 31·8	17·46	1	pF; cS; lE; bM; *11 sp......	3
3755	1740	......	....	13 57 18·2	2·607	1::	54 40 40·5	17·45	1	vF; S	3
3756	1741	II. 417	....	13 57 19·7	2·598	3	54 11 30·5	17·45	3	pB; pL; ivlE; vsmbM	4
3757	1742	III. 413	....	13 57 25·7	2·599	1	54 18 33·2	17·44	1	F; *13 p	2
3758	......	II. 799	H. O. N.	13 57 26·0	2·080	1	33 30 33·5	17·45	1	pF; L; E.........................	2
3759	......	III. 57	....	13 57 30·0	2·953	1	79 42 30·9	17·43	1	eF; eS	1
3760	1744, a	III. 787?	R. nova?	13 57 30·5	2·137	...	35 5 18·3	17·44	...	B; S; R; gmbM; conn with M. 101.	0*
3761	1743	II. 691	....	13 57 30·6	2·293	1	40 9 16·2	17·44	1	pB; cL; vmE 90°±; smbMN	2
3762	1744, b	......	R. nova	13 57 32·8	2·134	...	35 0 28·3	17·44	...	vF; pL; gvlbM } all conn with M. 101.	0*
3763	1744, c	......	R. nova	13 57 33·0	2·137	...	35 5 51·3	17·44	...	F; pS; iR; glbM } all conn with M. 101.	0*
3764	1744, d	......	R. nova	13 57 41·7	+2·131	...	34 57 41·3	17·44	...	vF; pL; iR; vlbM } all conn with M. 101.	0*
3765	1747	III. 947	....	13 57 43·7	−0·245	1::	11 5 51·1	17·47	1	vF; pL; iR; vgvlbM	2
3766	......	III. 767	....	13 57 50·1	+2·133	1	35 3 26·9	17·43	1	vF; vS	1*†
3767	1744, e	......	R. nova	13 57 59·7	2·131	...	35 2 3·3	17·43	...	F; pL; lE; vlbM } conn with M. 101.	0*
3768	1744, f	......	R. nova	13 58 2·8	+2·134	...	35 8 16·3	+17·43	...	pB; pS; R; psbM } conn with M. 101.	0*

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	h.	H.		h m s	s		° ′ ″	″			
3769	......	......	D'Arrest, 95	13 58 8	+2·92	[2]	77 27 4	+17·40	[2]	F; pS	0
3770	1744	......	M. 101	13 58 12·9	2·127	1	34 57 22·3	17·41	1	pB; vL; iR; g, vsmbMBSN..	5*†
3771	1744, g	......	R. nova	13 58 14·1	2·131	...	35 2 42·3	17·41	...	vF; pL; R; vlbM; conn with M. 101.	0*
3772	3555	......	Δ. 431	13 58 39·9	3·807	3	137 38 52·1	17·37	3	Cl; vL; vlC; st 8...	3
3773	1744, h	III. 788?	R. nova?	13 58 49·8	2·125	...	34 59 40·3	17·39	...	B; pS; R; psbM } conn with M. 101.	0*
3774	1744, i	III. 789?	R. nova?	13 58 53·9	2·122	...	34 56 57·3	17·38	...	pB; pL; iR; gbM } conn with M. 101.	0*
3775	3556	......	....	13 59 1·0	3·449	1	119 20 28·8	17·36	1	pF; S; R; pslbM	1
3776	1746	VI. 9	....	13 59 8·7	2·700	2	60 48 11·1	17·37	2	Cl; L; vRi; vmC; st 11...	4
3777	1745	III. 286	....	13 59 14·1	3·129	1	94 46 56·5	17·35	1	F; L; R; vgbM	3
3778	......	III. 788	....	13 59 19·3	2·120	1	34 59 23·1	17·37	1	vF; vS; 2nd of 3 } inv in M. 101.	1†
3779	......	III. 789	....	13 59 26·2	2·118	1	34 57 22·8	17·36	1	vF; vS; 3rd of 3 } inv in M. 101.	1†
3780	......	III. 58	....	13 59 30·1	2·955	1	79 57 25·5	17·35	1	eF; S; lE	1
3781	1745, a	......	R. nova	13 59 30·2	3·129	: :	94 46 56·5	17·36	: :	pB; S; E	0
3782	1748	I. 231	....	13 59 50·7	2·096	1	34 25 50·5	17·35	1	pB; S; R; gbM	3
3783	......	I. 214	....	13 59 59·3	2·149	2	35 40 25·5	17·35	2	pB; L; bM	2
3784	1750	II. 800	....	14 0 20·7	2·061	1	33 35 24·6	17·32	1	pB; S; pmE; bM	2
3785	1749	......	....	14 0 34·6	2·994	1	83 17 26·0	17·30	1	F; mE; vglbM	1
3786	1751	III. 287	....	14 0 49·1	3·138	(2)	95 25 38·7	17·29	1: :	F; pS; iR	3
3787	......	III. 790	....	14 0 54·3	2·104	1	34 52 19·0	17·30	1	vF; pL	1
3788	......	III. 762	....	14 0 55·8	3·085	1	91 1 21·4	17·28	1	vF; vS	1
3789	......	II. 692	....	14 1 24·6	2·225	1	38 37 18·1	17·27	1	F; pS; vgbM; np of 2	1
3790	......	II. 693	....	14 1 42·5	2·223	1	38 37 47·8	17·26	1	F; vS; smbM; stellar; sf of 2	1
3791	......	III. 59	....	14 1 43·2	2·958	1	80 24 17·5	17·25	1	eF; S	1
3792	3557	......	....	14 1 46·3	3·704	1	132 39 20·2	17·24	1	pF; vL; R; vgbM	1
3793	......	III. 791	....	14 2 26·5	2·073	1	34 19 14·9	17·23	1	vF; S; R; p of 2	1
3794	......	I. 232	....	14 2 26·5	2·073	1	34 19 14·9	17·23	1	cB; R; vgbM; f of 2	1
3795	......	II. 801	....	14 2 34·8	2·071	2	34 16 45·6	17·22	2	F; pL	2
3796	3558	......	....	14 2 45·6	3·520	1: :	122 58 35·7	17·19	1: :	F; R; *8 s nr	1
3797	3559	......	....	14 3 10·0	3·775	2	135 25 33·1	17·17	2	vF; S; R; bM	2
3798	1752	III. 32	....	14 3 17·4	2·846	1	71 47 18·4	17·18	2	cF; cS; R; sbMF ⁑	4
3799	1753	II. 890	....	14 3 59·6	2·988	3	82 58 10·5	17·15	3	pB; pS; R; gbM; r	5
3800	1754	II. 876	....	14 4 1·8	2·817	1	69 43 45·5	17·15	1	pB; vS; E	3
3801	1755	IV. 46	....	14 4 12·2	3·126	1	94 22 49·9	17·13	1	pB; vS; R; psmbM*; *18 inv.	2
3802	3560	......	....	14 4 16·2	3·475	1	119 59 46·6	17·12	1	pB; L; R; gbM; rr	1
3803	3561	......	....	14 4 23·5	3·419	1	116 26 51·6	17·12	1	vF; S; R; bM; * sf	1
3804	......	III. 674	....	14 5 5·6	2·265	2	40 46 9·3	17·11	2	cF; cS; iR	1
3805	1756	......	....	14 5 13·3	3·050	1	88 5 4·7	17·09	1	vF; S; rr	1
3806	1757	II. 687	....	14 5 59·1	3·105	1	92 32 43·5	17·05	1	pB; L; E 20° ±; lbM	3
3807	1757, a	......	R. nova	14 6 ±	3·105	...	92 33 ±	17·05	...	3′ dist from h. 1757	0
3808	1758	IV. 49	....	14 6 5·9	3·104	1	92 29 28·5	17·05	1	cF; S; R; stellar	3
3809	1759	II. 877	....	14 6 36·4	2·801	1	68 54 40·9	17·03	1	pB; pL; iR	2
3810	1760	III. 685	....	14 6 51·3	2·482	3	50 1 53·6	17·02	3	vF; S; vlE	5
3811	3562	......	....	14 6 54·8	3·848	2	137 27 28·0	17·00	2	pF; S; R; psbM; S ⁑ nf	2
3812	......	III. 676	....	14 7 49·9	2·196	2	38 59 31·4	16·98	2	F; S; lE; stellar	2
3813	1761	......	....	14 8 20·7	3·010	1	84 56 4·5	16·95	1	F; S; R; bM	1
3814	......	III. 644	....	14 8 23·6	2·871	1	74 13 59·5	16·95	1	vF; vS; E; a D neb	1
3815, 3816, 3817	1762, a	......	R. 3 novæ	14 8 ±	2·725	...	64 0 ±	16·94	...	3 "knots" near h. 1762	0
3818	1762	III. 134	....	14 8 31·0	2·725	3	64 0 55·2	16·94	3	F; pL; pmE 90°; *10 np	5
3819	1764, a	......	R. nova	14 8 47·0	2·53	: :	52 56 57·7	16·93	: :	eeF	0
3820	1763	III. 804 = III. 835	....	14 9 17·5	1·899	1	31 34 24·3	16·91	1	vF; S; E; r	3*
3821	1764, b	......	R. nova	14 9 18·9	2·53	: :	53 6 33·7	16·89	: :	vF	0
3822	1764	III. 414	....	14 9 40·1	2·533	1	53 7 33·7	16·89	1	cF; pL; vmE 110°·3; vgvmbM.	2
3823	3563	......	....	14 9 41·8	3·742	1	132 43 31·1	16·87	1	!; vF; pmE; esvmbM*12	1
3824	......	......	D'Arrest, 96	14 9 53	2·92	[2]	78 28 0	16·87	[2]	F; S; R; III. 47 f 10s·4	0
3825	1765	III. 47	....	14 10 2·1	+2·926	1	78 32 18·1	+16·87	1	vF; vS; R; gbM; r	2

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	h.	H.		h m s	s		° ′ ″	″			
3826	1766	II. 418	....	14 10 10·3	+2·549	3	53 59 51·1	+16·87	3	pB; R; vsmbM; 2 or 3 st inv	4
3827	1768	III. 731	....	14 10 40·3	2·462	1	49 51 34·2	16·84	1	cF; vS; R; p of 2	2
3828	1767	......	....	14 10 43·5	2·960	1	81 10 12·2	16·84	1	F; pL; iF; gbM	1
3829	......	III. 805	....	14 10 44·8	1·789	3	29 20 6·5	16·85	3	cF; vS; R; stellar	3
3830	1770, a	......	R. nova	14 10 45·3	2·968	::	81 39 52·2	16·85	::	vF	0
3831	1769	III. 732	....	14 10 48·4	2·459	3	49 45 41·2	16·84	4	cF; S; R; gbM	5
3832	1770, b	......	R. nova	14 10 53·9	2·968	:	81 46 46·3	16·83	:	vF	0
3833	1771	II. 419	....	14 11 8·5	2·521	2	52 46 41·6	16·82	3	F; pS; E 80°; D or biN	4
3834	1771, a	......	R. nova	14 11 ±	2·521	...	52 46 ±	16·82	...	Makes D or biN neb with h. 1771.	0
3835	1770	......	....	14 11 13·9	2·968	2	81 46 56·3	16·81	2	pB; cS; gbM	2
3836	......	III. 551	....	14 11 29·6	+2·970	2	81 59 ±	16·80	2	....	2*
3837	......	III. 948	....	14 11 36·1	−0·747	1	10 44 46·7	16·79	1	eF; vS; E 0°±	1
3838	1773	II. 194	....	14 11 38·1	+2·720	2	64 12 39·0	16·80	2	cF; pS; R; vsvmbM*	4
3839	1772	III. 552	....	14 11 43·6	2·970	1	81 58 40·7	16·79	1	vF; vS; R	2
3840	1774	......	....	14 11 43·8	2·897	1	76 27 52·7	16·79	1	vF; cS; pmE	1
3841	1775	......	....	14 12 12·9	2·701	1	63 4 49·8	16·76	1	vF; S; lE	1
3842	3564	......	....	14 12 24·5	3·475	1	118 36 20·5	16·75	1	eF; L; S * inv..................	1
3843	1776	I. 99	....	14 12 32·5	2·517	2	52 51 11·8	16·76	2	cB; S; R; vsbM*	4
3844	1777	III. 347	....	14 12 50·8	2·723	1	64 32 41·9	16·73	1*	vF; S; vlE; bM	2*
3845	1778	II. 579	....	14 13 1·0	3·013	2	85 21 16·6	16·72	2	pF; cL; E; gbM	3
3846	1779	I. 144	....	14 13 16·5	3·014	3	85 25 7·3	16·71	3	B; pL; R; psbM; r; *12 nf .	4*
3847	1780	......	....	14 13 20·8	2·541	1	54 13 56·6	16·72	1	pF; R	1
3848	1779, a	......	R. nova	14 13 28·5	3·014	...	85 24 7·3	16·72	...	Place from MS.	0
3849	1781	III. 12	....	14 13 31·7	2·967	(1)	81 50 21·0	16·70	2	F; S; iR	4
3850	1782	I. 145	....	14 13 52·1	3·023	1	86 6 53·4	16·68	1	pF; pS; lE; p of 2	2
3851	1783	I. 146	....	14 14 1·1	3·022	1	86 5 8·4	16·68	1	B; S; R; vsmbM; f of 2......	2
3852	1784	III. 415	....	14 14 31·5	2·536	2	54 9 46·8	16·66	2	vF; cL; p of 2.................	3
3853	1785	......	....	14 14 45·4	2·535	1	54 8 52·2	16·64	1	pB; S; f of 2	1
3854	1786	II. 754	....	14 15 1·5	2·440	3	49 39 28·2	16·64	3	pB; pS; R; bMFN; * sp ...	5
3855	1783, a	......	R. nova	14 15 5·4	*3·02	...	85 53 50	16·64	...	L; F; vmE	0
3856	1790	I. 235	....	14 15 18·1	1·909	1	32 38 5·9	16·63	1	pF; L; iR; vgmbM; r	3
3857	1787	III. 110	....	14 15 30·3	2·879	1	75 26 38·0	16·60	1	F; cS; vlE; *8 sf	3
3858	1789	......	....	14 15 30·7	2·534	1	54 13 57·3	16·61	1	vF; R; gbM; *8 sf	1*
3859	1788	III. 416	....	14 15 30·8	2·531	2	54 5 13·3	16·61	2	vF; S; R; np of 2	3*
3860	1791	III. 417	....	14 15 43·3	2·532	3	54 9 17·0	16·60	3	cF; S; R; bM*; sf of 2	4*
3861	3565	III. 924	....	14 15 47·9	3·474	1	118 1 55·4	16·58	1	F; S; E; gvlbM; r	2
3862	3566	......	Δ. 357	14 16 13·4	4·121	1	144 9 51·1	16·55	1	Cl; vlRi; vlC; st 10	1
3863	......	III. 135	....	14 16 25·3	2·689	1	62 58 37·8	16·56	1	eF; vS; stellar.................	1*
3864	1792	III. 121	....	14 16 32·5	3·291	1	106 5 6·5	16·55	2	F; pL; R; vgbM; p of 2 ...	4
3865	1795	III. 418	....	14 16 41·3	2·489	1	52 14 23·5	16·55	1	eF; S; R; stellar...............	2
3866	1793	III. 122	....	14 16 46·6	3·292	1	106 7 34·2	16·54	2	vF; L; vlE; vglbM; f of 2 ...	4
3867	1796	III. 733	....	14 16 49·7	2·419	1	49 2 30·5	16·55	1	F; vS; R; bM	3
3868	1794	III. 927	....	14 16 53·9	2·978	4	82 46 50·2	16·54	4	F; S; lE	5
3869	1797	II. 177	....	14 17 7·8	2·866	1	75 43 22·6	16·52	1	pB; pS; gbM	3
3870	......	II. 694	....	14 17 17·0	2·134	1	38 49 32·9	16·53	1	pF; pS; lE; mbM	1
3871	1800	III. 734	....	14 17 24·4	2·414	2	48 58 44·6	16·52	2	cF; pS; R; gbM................	3
3872	1799	III. 668	....	14 17 28·1	3·107	1	92 34 12·0	16·50	1	F; pS; R; vgbM*; r	2
3873	1798 = 3569	III. 120	....	14 17 32·1	3·243	2	102 32 27·0	16·50	2	vF; pL; R; vgbM	3
3874	3568	......	Δ. 313	14 17 36·5	4·345	1	148 59 58·4	16·48	1	Cl; S; pC; st L & S	1
3875	......	II. 331	....	14 17 43·9	0·726	2	17 47 31·9	16·53	2	pF; cS; iR; bM; er	2
3876	1801	II. 673	....	14 17 48·1	2·380	1	47 35 13·0	16·50	1	F; pL; lE; vglbM	2
3877	1802	III. 136	....	14 18 3·6	2·714	1	64 45 34·4	16·48	1	vF; S; pmE 0°±; *9 f	4
3878	1803	......	....	14 18 6·3	2·566	1	56 18 59·4	16·48	1	F; S; R; bM	1
3879	3567	......	....	14 18 9·9	6·643	1	167 46 0·3	16·41	1	vF; E; gbM; r	1
3880	1804	II. 420	....	14 18 12·4	2·530	2	54 29 56·4	16·48	2	pB; S; R; smbM	3
3881 3882 3883	1804, a	......	R. 3 novæ	14 18 ±	+2·530	...	54 30 ±	+16·48	...	h. 1804 is D; 2 others near ..	0

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	h.	H.		h m s	s		° ′ ″	″			
3884	1805	III. 419	…………	14 18 31·3	+2·496	1	52 54 5·8	+16·46	1	vF; S; cE; vgbM; er………	2
3885	3570	……	Δ. 302	14 19 23·2	4·417	3	150 5 16·7	16·39	3	Cl; L; pRi; pCM; st 8… …	3
3886	……	III. 768	…………	14 20 1·3	3·095	1	91 37 28·4	16·38	1	eF; S ………………………	1
3887	1806	……	…………	14 20 15·9	3·000	1	84 33 53·8	16·36	1	vF; S; R; vgbM …………	1
3888	……	III. 319	…………	14 20 17·6	0·680	1	17 44 24·0	16·40	1	eF; vS………………………	1*
3889	1807	III. 14	…………	14 20 54·8	2·952	1	81 7 26·2	16·34	1	eeF; L; r……………………	2
3890	1809	III. 677	…………	14 21 3·9	2·177	1	40 48 47·2	16·34	1	vF; pS; vlE; vglbM ………	2
3891	1808	II. 329	…………	14 21 12·5	2·552	1	56 7 10·9	16·33	1	cF; S; R; vsmbM; r ………	4
3892	1810	……	…………	14 21 25·6	2·408	1	49 24 57·6	16·32	1	vF; S; R; gbM ……………	1
3893	3571	……	…………	14 21 35·0	3·508	1	119 7 31·7	16·29	1	eF; S; R ……………………	1
3894	1811	……	…………	14 21 47·6	2·907	1	77 59 10·7	16·29	1	vF; vS; R; *9 sp …………	1
3895	1812	……	…………	14 21 59·6	2·685	2	63 31 30·7	16·29	2	pF; S; R; gbM ……………	2
3896	1814	II. 674	…………	14 22 4·2	2·374	4	48 6 50·7	16·29	4	F; S; E 90°±; gbM ………	5
3897	1820	I. 236	…………	14 22 10·2	1·865	2	32 47 47·7	16·29	2	B; S; R; psbMrN …………	5
3898	……	……	Auw. N. 33	14 22 12·0	3·068	…	89 49 7·8	16·26	…	Neb *11f 150ˢ (Bond, May, 1853).	0
3899	1818	I. 185	…………	14 22 14·2	2·243	2	43 13 13·4	16·28	2	cB; pS; R; pglbM…………	4
3900	1813	I. 70	…………	14 22 14·9	3·146	1	95 20 26·8	16·26	1	⊕; vB; cL; R; gbM; rrr; st 19; *17 sf.	3
3901	1815	III. 132	…………*	14 22 20·8	2·657	1	61 57 59·1	16·27	1	F; S; E; sbM………………	2
3902	1816	II. 580	…………	14 22 34·6	3·020	1	86 5 58·2	16·24	1	eF; cL; R; np of 2…………	2
3903	1819	II. 357	…………	14 22 35·7	2·728	2	66 11 4·5	16·25	2	vF; S; R; vgbM …………	3
3904	1817	II. 581	…………	14 22 36·8	3·020	2	86 8 14·2	16·24	2	cB; pL; R; sf of 2 …………	3
3905	1817, a	……	R. nova	14 23 ±	3·020	…	86 8 ±	16·24	…	Makes a BD neb with h. 1817	0
3906	1821		…………	14 22 41·4	+2·602	1	58 58 19·5	16·25	1	vF; R; *7 p; *11 s…………	1
3907	……	III. 949	…………	14 23 2·9	−1·635	1	9 18 14·0	16·30	1	eF; S; lE ………………	1
3908	1822	III. 126	…………	14 23 11·4	+2·608	2	59 21 35·9	16·23	2	cF; S; * inv; *12 nf………	3
3909	3572	……	Δ. 469	14 23 40·3	3·826	2	133 34 30·1	16·17	2	pB; L; R; vglbM; st inv …	2
3910	1823	II. 150	…………	14 23 43·8	2·964	2	82 5 56·7	16·19	2	cF; pL; iR; gbM …………	5
3911	1824	III. 645	…………	14 23 54·1	2·867	1	75 22 18·4	16·18	1	eF; vS; np of 2 …………	2
3912	……	……	Auw. N. 34	14 24 3·0	3·066	…	89 42 5·1	16·17	…	Neb R (Bond, May, 1853)…	0
3913	1825	II. 891	…………	14 24 4·0	2·982	2	83 23 45·1	16·17	2	pB; pL; vlE; bM …………	3
3914	1826	II. 330	…………	14 24 7·8	2·583	1	58 9 43·1	16·17	1	pF; pS; R; bM ……………	2
3915	1828	III. 420	…………	14 24 14·9	2·478	1	53 0 46·8	16·16	1	F; S; E?; ⁑ inv? …………	2
3916	1827	……	…………	14 24 17·5	2·868	1	75 27 56·8	16·16	1	eeF; sf of 2………………	1
3917	1829	II. 421	…………	14 24 34·5	2·499	3	54 2 59·5	16·15	4	pF; pL; R; mbM; r ………	5
3918	……	……	Auw. N. 35	14 24 48·0	3·067	…	89 45 4·8	16·16	…	Neb; F; E (Bond, May, 1853)	0
3919	1831	……	…………	14 24 52·7	2·686	1	63 59 8·9	16·13	1	eF ……………………	1
3920	1832	II. 695	…………	14 24 53·3	2·122	1	39 45 49·2	16·14	1	pB; L; iR; vgbM …………	2*
3921	1830	II. 892	…………	14 24 59·9	2·977	2	83 7 34·6	16·12	2	vF; pS; iE ………………	3
3922	3573	……	Δ. 342	14 25 7·9	4·256	2	145 56 31·0	16·10	2	Cl; L; pRi; lC; st 9… ……	2*
3923	1833	II. 27	…………	14 25 31·6	2·951	2	81 18 15·0	16·10	2	pB; pL; R; gbM; r ………	5
3924	1834	……	…………	14 26 19·7	2·916	1	78 51 33·5	16·05	1	vF; vS; R; stellar ………	1
3925	……	II. 807	…………	14 26 22·6	1·686	1	29 54 6·1	16·07	1	pB; pS; E 0° ………………	1
3926	1835	II. 574	…………	14 26 22·9	3·002	2	84 55 54·5	16·05	2	F; pS; vlE; *14 inv ………	3
3927	……	II. 79	…………	14 26 28·2	2·924	1	79 28 9·2	16·04	1	F; L; R; lbM; r……………	1
3928	3574	……	…………	14 26 28·4	3·887	2	135 20 29·9	16·03	2	vF; S; cE; bet 2 st………	2
3929	……	III. 882	…………	14 26 35·4	0·855	1	19 44 5·4	16·08	1	vF; pL; R; bM ……………	1
3930	1836	III. 310	…………	14 26 39·7	2·563	1	57 43 21·2	16·04	1	vF; vL; iR; lbM; ⁑ p ……	3
3931	1838	II. 696	…………	14 26 42·1	2·100	1	39 26 14·5	16·05	1	F; S; cE; *15 np …………	2
3932	1837	II. 893	…………	14 26 53·5	2·988	4	83 55 1·9	16·03	4	cF; pS; R; gbM……………	5
3933	1839	II. 422	…………	14 26 53·6	2·470	1	53 4 35·9	16·03	1	F; pS; E; bM………………	2
3934	1843	I. 237	…………	14 27 51·2	1·758	1	31 27 43·7	15·99	1	B; L; lE 0°; vgmbM ………	3
3935	1842	I. 189	…………	14 27 51·3	2·110	1	39 55 14·7	15·99	1	B; L; E 45°±; pgbM; r …	2
3936	1840	III. 283	…………	14 27 55·2	2·678	1	63 54 56·1	15·97	1	vF; vS; R; r; 3 st 9, 10 np…	2
3937	1841	II. 894	…………	14 28 8·6	2·988	2	84 1 3·8	15·96	4	vF; S; R; *12 att …………	6
3938	1844	III. 421	…………	14 30 6·5	2·454	4	52 50 57·8	15·86	4	F; cS; R; bM; p of 2………	5
3939	1845	……	…………	14 30 18·9	2·454	1	52 53 2·5	15·85	1	vF; S; R; f of 2……………	2
3940	1849	II. 808	…………	14 30 26·8	1·902	1	34 53 55·5	15·85	1	pF; S; iF; r; *10 f ………	2
3941	3575	……	…………	14 30 28·4	3·880	1	134 25 46·3	15·81	1	F; S; vgbM; am st…………	1
3942	1848	I. 188	…………	14 30 29·8	+2·120	1	40 38 43·5	+15·85	1	cB; S; E 90°±; psmbM …	3

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	h.	H.		h m s	s		° ′ ″	″			
3943 3944 3945	1848, a	......	R. 3 novæ	14 30 ±	+2·120	...	40 38 ±	+15·85	...	3 novæ, one mottled	0
3946	1846	II. 582	....	14 30 39·7	3·032	1	87 6 6·6	15·82	3	vF; mE or biN 140°±; ✱6·7 p	4
3947	1847	II. 681	....	14 30 42·5	3·070	1	89 46 46·6	15·82	1	pB; pS; lE; gbM	3
3948	1851	II. 423	....	14 31 38·1	2·448	2	52 49 33·4	15·78	3	pB; cS; R; bM; r	4
3949 3950	1851, a	......	R. 2 novæ	14 32 ±	2·488	...	52 50 ±	15·78	...	No description	0
3951	1850	II. 648	....	14 31 38·2	2·321	(2)	47 36 0·4	15·78	1	cF; cS; R; lbM; r	3
3952	1853	II. 675	....	14 31 38·8	2·325	1::	47 45 53·4	15·78	1::	F; vS; R; bM; 4B st p	2*
3953	1852	II. 700	....	14 31 38·9	2·404	2	50 55 57·4	15·78	3	cF; cS; lE; in Δ of st	4
3954	3576	II. 196	....	14 32 4·2	3·477	1	115 55 37·2	15·74	1	cB; cS; R; psbM; r; ✱ nr ...	3
3955	......	III. 127	....	14 32 12·5	2·594	1	59 53 49·2	15·74	1	eF; vS............................	1
3956	1854	II. 575	....	14 32 12·8	2·986	4	84 1 23·2	15·74	4	cB; pS; R; mbM; ✱15 p ...	5
3957	......	III. 894	....	14 32 20·6	2·754	2	68 53 19·9	15·73	2	vF; vS............................	1
3958	......	III. 128	....	14 32 26·5	2·593	1	59 53 48·9	15·73	1	vF; vS; iR	1
3959	1855	II. 649	....	14 32 26·6	2·351	1	48 52 33·2	15·74	1	F; cS; lE 0°±.....................	4
3960	1859	......	....	14 32 46·2	2·351	2	48 56 49·6	15·72	2	F; pL; E 0°±; gbM	2
3961	1856	III. 895	....	14 32 50·2	2·760	1	69 21 2·0	15·70	1	vF; S; vgbM; ⁑ f; p of 2 ...	2
3962	1858	......	....	14 32 55·7	+2·761	1	69 24 52·0	15·70	1	eF; vS; ⁑ att; f of 2	1
3963	......	III. 950	....	14 32 57·9	−1·318	1	10 33 44·4	15·78	1	vF; S; R; S Cl p..............	1
3964	1857*	I. 182	....	14 32 59·9	+3·068	1	89 40 43·0	15·70	1	cB; pL; R; psmbM; r	3
3965	1861	III. 675	....	14 33 2·8	2·175	1	42 44 30·0	15·70	1	vF; pS; iE; ⁑ n; 1st of 3 ...	2
3966	3577	......	Δ. 333	14 33 15·1	4·353	1	146 56 49·8	15·66	1	Cl; L; pRi; CM; st 11...13...	1
3967	......	VI. 8	....	14 33 16·5	3·198	1??	98 30 48·4*	15·68	1??	Cl; pL; eRi; vmC	1*
3968	1860	III. 671	....	14 33 19·0	3·326	2	106 52 1·1	15·67	2	vF; pL; R	3
3969	1864	......	....	14 33 29·2	2·172	2	42 42 29·7	15·69	1	vF; S; R; ⁑ nr; 2nd of 3 ...	2
3970	1862	III. 550	....	14 33 39·6	3·014	1	85 56 1·8	15·66	2	vF; S; R; vglbM; ✱8·9 nf...	3
3971	1863	II. 682	....	14 33 44·9	3·069	1	89 42 24·5	15·65	1	pF; S; lE; bM	3
3972 3973 3974	1865, a	......	R. 3 novæ	14 34 ±	2·170	...	42 41 ±	15·67	...	h. 1865 is quadruple; ? F neb connecting.	0
3975	1865	......	....	14 33 49·6	2·170	1	42 40 59·1	15·67	1	vF; S; R; psbM; 3rd of 3...	1
3976	......	......	D'Arrest, 97	14 33 53	3·03	[1]	87 12 42	15·64	[1]	○?; vF; S; disc; ✱15 n 95″...	0
3977	1866	I. 184	....	14 34 38·1	3·325	2	106 38 34·0	15·60	2	pF; pL; pmE 45°±; mbM; ✱10s.	3*
3978	3578	III. 508	....	14 34 40·0	3·197	2	98 24 7·0	15·60	2	F; pL; E; r........................	3
3979	1867	III. 657	....	14 34 41·8	2·281	1	46 36 11·3	15·61	1	vF; cS; E 90°±	2
3980	1868	III. 658	....	14 34 51·9	2·281	1	46 37 41·0	15·60	1	vF; eS; lE	2
3981	1869	III. 686	....	14 35 2·8	2·387	2	50 45 12·7	15·59	2	vF; S; R; lbM	3
3982	1870	III. 133	....	14 36 29·1	2·598	1	60 40 46·3	15·51	1	vF; L; iR; lbM	2
3983	1871	III. 896	....	14 36 42·8	2·773	1	70 31 5·7	15·49	1	vF; cS; R; vglbM	2
3984	1873	I. 171	....	14 37 12·6	2·297	1	47 34 26·1	15·47	1	pB; S; R; smbM; r; ✱ nr ...	3
3985	1872	II. 538	....	14 37 17·6	3·039	1	87 43 15·8	15·46	1	pB; L; iR; gbM; r............	2
3986	3579	......	....	14 37 23·1	3·276	1	103 20 50·5	15·45	1	vF; S; E; pslbM	1
3987	1874	I. 126	....	14 37 48·7	3·035	1	87 26 59·9	15·43	1	B; L; vmE; bMBN	2
3988	......	III. 48	....	14 38 25·5	2·880	1	77 18 32·0	15·40	1	eF; S	1
3989	3580	......	Δ. 356	14 38 57·4	4·258	1	143 56 9·2	15·34	1	Cl; pL; pRi; lC; st 10...11...	1
3990	1875	I. 183	....	14 39 9·0	3·068	1::	89 38 40·5	15·35	1::	pF; pS; vlE; r	3
3991	1877	II. 809	....	14 39 24·3	1·892	1	35 59 5·8	15·36	1	F; S; vlE; Δ 2 st 10.11 ...	2
3992	1878	III. 687	....	14 39 47·5	2·368	3	50 40 19·9	15·33	3	cF; cS; R; bM	4
3993	3581	......	....	14 39 54·0	3·293	1	104 15 56·3	15·31	1	pB; pL; pmE; gpmbM	1
3994	1876	III. 690	....	14 39 54·2	3·362	1	108 29 45·3	15·31	1	vF; S; iR; lbM	2
3995	1879	III. 885	....	14 41 13·2	2·773	1	70 54 33·2	15·24	1	vF; vS; cE 90°; vglbM	2
3996	3582	......	....	14 43 44·7	4·208	1	142 5 25·1	15·07	1	Cl; vF; vS; vC	1
3997	1880	......	....	14 43 52·3	2·985	1	84 17 59·7	15·09	1	D neb; both eF	1
3998	......	III. 373	....	14 44 58·6	3·104	3	91 57 9·6	15·02	3	F; R; bMFN; S✱ s	3*
3999	1881	II. 576	....	14 46 13·3	3·003	1	85 27 35·5	14·95	1	cF; S; vlE; bM; ?biN	4*
4000	1882	III. 129	....	14 46 16·8	2·551	1	59 35 3·5	14·95	1	vF; S; R; pgbM...............	2
4001	1883	......	....	14 46 17·5	2·295	3	48 49 8·8	14·96	3	pB; pL; lE; pslbM; ✱8 np...	3
4002	1884	III. 130	....	14 46 34·3	+2·551	1	59 37 52·9	+14·93	1	vF; S; R; pgbM...............	2

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	h.	H.		h m s	s		° ′ ″	″			
4003	1885, *a*	.....	R. nova	14 46 50·9	+3·009	::	85 53 7·0	+14·91	:	eF; 2′ p h. 1885	0
4004	1885	III. 554	....	14 46 58·9	3·009	1	85 53 7·0	14·90	1	F; pS; vmE 148°·4; gvlbM...	4
4005	......	......	D'Arrest, 98	14 47 33	3·02	[1]	86 27 8	14·87	[1]	vF; pL; vlbM; *8·9p; 225″ s	0
4006	......	III. 806	....	14 47 33·0	1·555	1	30 26 56·7	14·89	1	vF; vS; lE	1
4007	1886	......	....	14 48 52·2	3·344	1	106 40 4·7	14·79	1	F; S; R; bM; *16 sp.........	1
4008	1887	II. 676	....	14 49 5·8	2·229	2	46 52 39·0	14·80	2	pB; S; R; smbM; stellar ...	3
4009	3583	......	....	14 49 57·8	3·877	1	131 27 21·3	14·71	1::	F; mE; L* sf	1
4010	1888	II. 677	....	14 50 2·4	2·227	1	46 56 4·2	14·74	1	F; cS; R; pslbM	2
4011	1890	III. 976	....	14 50 41·1	2·532	1	59 12 55·0	14·70	1	eF; pS; iF	2
4012	1889	III. 691	....	14 50 51·7	3·382	1	108 42 27·1	14·67	1	pF; S; R; stellar..............	2
4013	......	II. 683	....	14 51 12·5	3·081	1	90 31 47·8	14·66	1	pB; pL; R; mbM; cB*npatt	1
4014	1891	......	....	14 51 21·5	1·979	1	39 44 42·1	14·67	1	pF; S; vsbM*13; 1st of 3 ...	1
4015	1893	III. 678	....	14 51 46·4	1·977	1	39 44 55·2	14·64	1	F; S; vsbM*13; 2nd of 3 ...	2
4016	1892	III. 131	....	14 51 48·3	2·534	1	59 28 16·9	14·63	2	F; S; R; vgbM; *nf (? var)	4*
4017	3584	......	....	14 51 49·4	5·894	1	161 52 27·1	14·57	1	eF; S; R; bM..................	1
4018	3585	......	....	14 52 5·8	4·224	1	141 21 29·4	14·58	1	Cl; pL; pRi; lC	1
4019	1895	III. 679	....	14 52 31·1	1·973	1	39 45 19·0	14·60	1	vF; vS; vsmbM*13; *6 nr; 3rd of 3.	2
4020	1895, *a*	......	R. nova	14 52 ±	1·973	...	39 45 ±	14·60	...	S	0
4021	1894	II. 539	....	14 52 53·9	+3·034	1	87 32 48·8	14·56	1	cB; cL; E 165°±; sbMN ...	2
4022	......	III. 311	....	14 53 0·6	−0·023	1	16 24 35·3	14·61	1	vF; S; iR; bet 2 st	1
4023	3586	......	....	14 53 11·2	+3·296	1	103 36 47·9	14·53	1	vF; S; E; glbM	1
4024	3587	I. 71	....	14 53 28·2	3·185	2	96 53 29·6	14·52	2	cB; S; R; svmbM	4
4025	......	II. 756	....	14 53 51·4	1·781	2	35 37 32·1	14·48	2	pF; pS; iE; r	2*
4026	1896	I. 127	....	14 54 5·4	3·037	1	87 44 16·7	14·49	1	B; pS; R; psmbM	2
4027	1897	......	....	14 54 15·4	3·038	1	87 49 46·1	14·47	1	vF; vS; R	1
4028	......	III. 811	....	14 54 22·9	1·784	1	35 35 32·7	14·49	1	vF; S; E.........................	1
4029	1898	II. 756?	....	14 54 29·9	1·781	1	35 32 4·1	14·47	1	B; R; sbM; splendid ⁑ f ...	1*
4030	1898, *a*	......	R. nova	14 54 50·6	1·781	::	35 32 4·1	14·47	::	vF	0
4031	3588	......	....	14 55 2·0	4·347	1	143 47 24·0	14·40	1	Cl; vL; Ri; lC; st 9...12 ...	1
4032	3589	......	....	14 55 20·2	4·409	1	145 2 17·4	14·38	1	Cl; cL; Ri; lCM; st 13...14	1
4033	1899	II. 540	....	14 57 2·4	3·045	(1)	88 14 41·0	14·30	1	pB; S; mbM	2
4034	......	II. 332	....	14 57 32·3	0·165	2	17 45 19·6	14·32	1	pB; cL; iR; bp; r	2
4035	3590	......	....	14 57 32·5	6·045	3	162 19 4·3	14·21	3	F; cS; lE; glbM; am st	3
4036	1900	......	....	14 57 46·9	+3·675	1	122 34 35·2	14·24	1	eeF(?)	1
4037	......	III. 312	....	14 57 54·8	−0·262	1	15 36 18·3	14·31	1	eF; vS; lE; 2 st inv	1
4038	......	II. 542	....	14 58 21·5	+3·030	1	87 21 21·6	14·22	1	pB	1
4039	......	II. 541	....	14 58 21·9	3·037	1	87 49 21·6	14·22	1	F	?
4040	3592	......	....	14 58 41·7	3·755	2	125 47 8·4	14·18	2	vF; S; lE; vlbM; r	2
4041	3591	......	....	14 58 51·0	5·033	2	154 8 9·5	14·15	2	pB; pL; R; vgvlbM	2
4042	......	III. 511	....	14 59 1·9	3·036	1	87 46 20·4	14·18	1	vF; R; p of 2..................	1
4043 } 4044 }	1901, *a*	......	R. 2 novæ?	14 59 ±	3·038	...	87 51 ±	14·16	...	2 of 6	*
4045	1901	I. 128	....	14 59 23·5	3·038	1	87 50 41·8	14·16	1	vB; pL; R; psbMN; f of 2...	2
4046	......	......	D'Arrest, 99	14 59 29	3·03	[1]	87 26 48	14·15	[1]	eF; S; v close ⁑ f 7′	0
4047	1902	II. 543	....	15 0 2·6	3·039	1	87 54 47·6	14·12	1	cF; S; lE; psbM	2
4048	......	III. 886	....	15 0 16·5	2·848	1	76 36 4·3	14·11	1	eF; vS; np of 2	1*
4049	......	III. 887	....	15 0 16·5	2·848	1	76 36 4·3	14·11	1	eF; vS; sf of 2.................	1*
4050	1903	II. 544	....	15 0 44·0	3·021	3	86 53 20·4	14·08	3	pB; S; vlE; lbM; am st......	5
4051	1905	II. 751	....	15 1 1·8	2·725	(3)	69 54 1·8	14·06	1::	cF; cS; E; p of D neb	4*†
4052	1905	II. 752	....	15 1 9·2	2·725	(3)	69 56 1·8	14·06	1::	pF; pS; E; f of D neb	4†
4053	1904	IV. 71	....	15 1 14·4	2·746	1::	71 1 0·5	14·05	1	*6 in vL neb	2
4054	1906	......	....	15 1 29·3	2·179	2	46 49 50·5	14·05	2	F; S; R; psbM	2
4055	......	II. 192	....	15 1 37·4	3·255	2	100 46 50·6	14·02	2	F; L; E; r	2
4056	1907	II. 585	....	15 2 30·8	3·013	3	86 24 19·8	13·96	3	pF; cS; ilE; gbM; *14f ...	4
4057	......	II. 684	....	15 2 36·1	3·057	1	89 0 5·5	13·95	1	pB; S; iE	1
4058	1909	I. 215	....	15 2 36·3	1·639	1	33 41 39·4	13·98	1	vB; cL; pmE 146°·0; gbM...	2†
4059	1909, *a*	......	R. nova	15 2 ±	1·639	...	33 42 ±	13·98	...	vS	0
4060	......	......	D'Arrest, 100	15 2 41	3·05	[1]	88 55 48	13·95	[1]	eF; H. II. 545, s 3′ 15″	0
4061	1908	II. 545	....	15 2 41·0	3·057	1	88 59 25·5	13·95	1	pF; S; E; psbM...............	5
4062	......	II. 755	....	15 5 10·7	+1·792	1	36 55 21·6	+13·82	1	pB; pL; lE	1

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	h.	H.		h m s	s		° ′ ″	″			
4063	3593	III. 736	....	15 6 3·3	+3·312	1	103 44 18·9	+13·73	1	pB; pL; pmE0°; psmbM; * inv	2
4064	1910	II. 757	....	15 6 9·4	1·548	1	32 27 30·8	13·76	1	cB; S; E; mbMRN; r	4
4065	......	II. 818	....	15 6 27·0	1·128	1	26 30 47·5	13·75	1	pF; cS; R; vgbM	1*
4066	3594	......	....	15 7 19·4	4·059	2	135 7 30·2	13·64	2	○; vS; R; quite sharp	2†
4067	3595	III. 116	....	15 7 30·1	3·238	1	99 33 7·2	13·64	1	F; cL; R; vgbM..............	2
4068	1911	......	....	15 7 38·4	2·199	1	48 14 58·2	13·64	1	F; vS; R; bM.................	1
4069	1912	III. 659	....	15 7 58·6	2·198	1	48 12 57·2	13·64	1	cF; vS; R; bM; r	2
4070	1912, a	......	R. nova	15 8 18·6	2·198	...	48 8 27·2	13·62	...	vF; place from MS.	0
4071	1913	II. 678	....	15 8 29·8	2·175	1	47 31 15·0	13·60	1	F; S; R; r; 3 st nr...........	2
4072, 4073	1913, a	......	R. 2 nova	15 8 ±	2·175	...	47 31 ±	13·60	...	2 nova apparently connected..	0
4074	......	II. 763	....	15 8 37·8	1·353	2	29 40 9·0	13·60	2	pF; pS; E 0°±	2
4075	3596	VI. 19	....	15 9 22·4	3·443	1	110 29 44·6	13·52	1	⊕; pF; L; viR; vgbM; rrr	2
4076	3597	III. 138	....	15 9 53·3	3·504	1	113 31 44·7	13·49	1::	F; S; R; gbM	4
4077	1914	II. 650	....	15 10 0·6	2·167	2	47 26 25·3	13·51	2	cB; pL; pmE; smbMN	5
4078	1915	III. 660	....	15 10 2·9	2·162	2	47 16 49·0	13·50	2	vF; S; vlE; gbM	3
4079	1915, a	......	R. nova	15 10 2·9	2·162	...	47 17 ±	13·50	...	Close to 1915 pos 0°	0
4080	......	III. 737	....	15 10 15·1	1·863	1	39 12 3·0	13·50	1	vF; vS; stellar..................	1
4081	3598	III. 139	....	15 10 24·3	3·505	2	113 31 3·5	13·45	2	cF; S; R; gpmbM	5
4082	......	II. 758	....	15 10 56·2	1·606	1::	34 0 29·5	13·45	1::	pF; pS; iR	1
4083	1916	......	M. 5	15 11 27·2	3·028	1	87 23 8·7	13·39	1	!!; ⊕; vB; L; eCM; st 11...15	11†
4084	......	......	R. nova	15 11 39	1·568	::	33 51 55	13·39	::	One of 2, 15′ apart np & sf...	0
4085	......	II. 760	....	15 11 57·2	1·606	1::	34 6 25·7	13·39	1::	pF; pS; R	1
4086	1917, a	......	R. nova	15 12 ±	15·52	...	32 10 ±	13·38	...	A ray, vmE, par. to h. 1917	0
4087	1917	II. 759	....	15 12 11·7	1·552	2	33 10 5·4	13·38	2	cB; vL; vmE 155°·0; vg, psbMN.	3†
4088	......	......	R. nova	15 12 53	+1·568	::	34 2 31	13·39	::	One of 2, 15′ apart np & sf...	0
4089	......	III. 943	....	15 13 10·9	−0·870	1	14 7 6·1	13·37	1	vF; vS..........................	1
4090	......	II. 400	....	15 13 15·7	+2·681	2	68 36 53·4	13·28	2	vF; S; er.......................	2
4091	......	III. 944	....	15 13 35·7	−0·876	1	14 7 4·9	13·33	1	vF; vS..........................	1
4092	1915 = 3599	III. 374	....	15 13 38·5	+3·109	2	92 4 2·5	13·25	2	vF; pL; vlE; r	3
4093	3600	......	....	15 13 51·6	3·297	1	102 34 48·6	13·22	1	B; S; R; glbM; p of 2	1
4094	3601	......	....	15 13 56·2	3·299	1	102 39 13·6	13·22	1	F; S; lE; glbM; f of 2	1
4095	3602	......	....	15 14 3·2	3·194	1	96 51 8·3	13·21	1	eF; vS; psbM	1
4096	1920	......	....	15 14 36·6	2·018	2	43 36 42·0	13·20	2	cF; L; pmE; glbM; ** s......	2
4097	1919	I. 148	....	15 14 59·9	2·975	4	84 25 34·8	13·16	4	cB; cL; iR; vsbM * 12; am st	5
4098	1922	III. 661	....	15 16 12·3	2·158	(2)	47 50 32·7	13·09	1::	eF; S	2
4099	1921	......	....	15 16 13·2	2·156	4	47 46 29·7	13·09	4	vF; pL; vlE; vgbM	4
4100	3603	......	Δ. 357	15 17 15·2	4·480	3	144 1 39·4	12·98	3	Cl; vL; vRi; lC; st 11...14..	3
4101	3604	......	Δ. 389	15 17 55·5	4·300	2	140 10 47·2	12·94	2	⊕; cB; L; R; vgbM; rrr; st 15	2
4102	1923	II. 874	....	15 19 40·8	2·729	1	71 25 32·5	12·85	1	pB; cS; R; psbM; * 7 n.......	2
4103	1924	......	....	15 21 5·7	2·141	1	47 50 27·1	12·77	1	vF; vS; sp of D neb	1
4104	1925	II. 651	....	15 21 8·7	2·141	4	47 50 15·1	12·77	4	pF; pS; R; nf of D neb......	6
4105	......	II. 130	....	15 23 5·1	2·825	2	76 33 15·6	12·62	2	F; pL; iR; vgbM; r	2
4106	1926 = 3606	II. 401	....	15 23 29·0	3·116	2	92 20 47·7	12·59	2	pB; pS; R; vgbM; 3 st f......	3
4107	3605	......	....	15 23 30·4	5·466	1	156 22 34·9	12·53	1	F; S; am st	1
4108	3607	......	....	15 25 20·2	4·334	2	140 11 14·9	12·43	2	⊕; cB; pL; R; vglbM; rrr; st 16.	2
4109	......	II. 906	....	15 26 3·2	0·808	1	24 45 29·8	12·46	1	F; S; lE 45°±; vglbM	1
4110	......	II. 654	....	15 27 17·7	2·782	1	74 32 25·2	12·34	1	F; pS; E 150°±..............	1
4111	1927	II. 178	....	15 28 2·2	2·777	1	74 20 50·4	12·28	1	pB; cS; p of D neb	4
4112	1927	II. 179	....	15 28 2·2	2·777	1	74 20 50·4	12·28	1	pB; cS; f of D neb............	4
4113	......	II. 399	....	15 28 57·7	2·483	1	60 51 21·6	12·22	1	pF; pL; iR; bM; r............	1
4114	......	II. 761	....	15 29 26·4	1·445	1	33 1 16·3	12·21	1	pF; pS; iF	1
4115	1931	II. 762	....	15 29 57·4	1·431	(1)::	32 50 58·1	12·17	1	cF; cL; lE	2
4116	1928	II. 96	....	15 30 6·8	2·746	1	72 55 36·2	12·14	1	pF; pL; ilE; gbM	2
4117	1931, a	......	R. nova	15 30 31·4	+1·431	::	32 46 22·1	+12·14	::	No description.................	0

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	h.	H.		h m s	s		° ′ ″	″			
4118	1929	......	....	15 30 38·0	+2·953	1	83 33 9·7	+12·09	1	⊕; vF; vL; R; vgbM; rrr...	1†
4119	1930	III. 634	....	15 30 48·4	2·172	2	49 46 12·3	12·11	2	vF; S; R; gbM; 2 st 8 f......	3
4120	3608	......	....	15 31 12·7	7·159	1	165 13 11·1	11·97	1	F; pL; R; vgbM	1
4121	3609	......	....	15 31 20·1	3·694	1	120 5 47·2	12·04	1	vF; L; R; gbM; r	1
4122	......	II. 76	....	15 32 10·2	2·834	1	77 24 9·7	11·99	1	pF; pL; R; rr..................	2
4123	1932	......	....	15 33 25·5	2·396	1	57 46 40·6	11·92	1	vF; vS; R; bM	1
4124	1934, *a*	......	R. nova?	15 34 29·2	1·214	...	30 6 57·2	11·76	...	R; psbM (by diagram)	0
4125	3610	......	....	15 34 53·7	5·014	2	150 46 24·5	11·75	3	!; ○; pF; vS; R; r? am 150 st	3†
4126	1933	II. 655	....	15 35 0·3	2·759	1	73 45 35·7	11·79	1	F; pS; E 0°.....................	2
4127	1934, *b*	......	R. nova?	15 35 33·7	1·214	...	30 8 46·2	11·77	...	F; mE	0
4128	1934	II. 764	....	15 36 18·6	1·214	1	30 11 55·2	11·74	1	cB; S; R; psbM; r............	2
4129	......	II. 656	....	15 36 21·3	2·790	1	75 20 47·7	11·69	1	pB; S; E 135°±; bM	1
4130	......	II. 765	....	15 36 24·1	1·306	1	31 29 46·9	11·73	1	pF; cS	1
4131	......	II. 766	....	15 36 32·5	1·215	1	30 13 46·6	11·72	1	pB; cL; iE; r	1*
4132	3611	......	Δ. 552	15 36 53·4	3·904	2	127 19 24·9	11·63	2	!; ⊕; vB; L; R; vgbM; st 13...15.	2
4133	1934, *c*	......	R. nova	15 37 18·3	1·214	...	30 14 28·0	11·62	...	(? if not =II. 766)...........	0
4134	......	III. 378	....	15 38 47·1	1·169	1	29 47 35·8	11·56	1	vF; vS	1
4135	1935	II. 425	....	15 39 16·4	3·019	2	87 8 54·4	11·48	2	vF; vS; R; gbM	5
4136	1936	III. 635	....	15 39 29·9	2·103	2	48 26 39·7	11·49	2	vF; vS; R; bM; sp of 2......	3
4137	1937	III. 636	....	15 39 33·0	2·102	1	48 26 5·4	11·48	1	cF; vS; R; bM; nf of 2......	2
4138	3613	......	....	15 40 38·9	3·334	1	103 19 13·4	11·38	1	eF; S; R; vS * p	1
4139	1938	II. 97	....	15 40 39·7	2·709	2	71 40 24·0	11·40	2	pF; cS; R; r; bet 2D st......	4
4140	......	VII. 29	....	15 40 56·1	3·664	1	118 10 31·5	11·35	1	Cl; pL; pRi; st vS	1
4141	3612	......	Δ. 343	15 41 7·4	4·716	1	146 2 39·3	11·31	1	Cl; L; pRi; st 12...14	1
4142	3614	......	....	15 41 11·3	3·684	1	118 57 17·9	11·33	1	vF; S; R; sbM	1
4143	......	III. 371	....	15 42 3·8	2·459	1	60 54 51·7	11·29	1	vF; S; R	1
4144	3615	......	Δ. 334	15 44 34·9	4·795	3	147 0 57·8	11·06	3	Cl; pS; pRi; mC; st 16 ...	3
4145	1939	II. 583	....	15 47 11·0	+3·054	1	89 1 54·3	10·91	1	pF; S; E 90° ±; gbM; r ...	3
4146	......	III. 313	....	15 47 35·7	−0·483	2	17 24 55·8	10·96	2	vF; S; E 90°±; vS * f	2
4147	......	II. 657	....	15 47 49·1	+2·772	1	74 59 59·1	10·87	1	F; bet 2 B st	1
4148	1940	......	....	15 49 3·7	2·948	1	83 39 30·4	10·78	1	pB; pL; E	1
4149	......	III. 739	....	15 49 24·0	0·892	1	27 15 50·0	10·80	1	vF; pL; R; vgbM	1
4150	1941	......	....	15 50 19·6	2·946	2	83 35 40·4	10·68	2	!; vF; vS; R; g, smbM	2
4151	1942	III. 646	....	15 51 5·5	2·742	1	73 42 40·9	10·63	1	vF; S; lE; p of 2	2
4152	1943	III. 73	....	15 51 8·4	2·741	1	73 37 40·9	10·63	1	eF; vS; lE; f of 2	3
4153	3516	......	Δ. 304	15 51 50·9	5·056	4	150 6 0·6	10·52	4	Cl; B; vL; pRi; lC; st 7...	4
4154	3617	......	....	15 52 18·5	3·846	1	124 8 38·0	10·50	1	F; S; R; gpmbM; Δ of st np	1
4155	3618	......	Δ. 359	15 56 43·3	4·641	1	143 38 28·8	10·16	1	Cl; S; mC; st 11...14	1
4156	1944	III. 622	....	15 57 35·3	2·182	2	52 15 33·5	10·15	2	vF; S; R; * 10 sf	4
4157	......	III. 33	....	15 58 43·2	+2·658	1	70 16 11·9	10·14	1	eF; (?)	1
4158	......	II. 873	....	15 58 43·5	−0·284	1	18 55 6·9	10·13	1	F; R; bM	1
4159	1945	......	....	15 58 49·9	+2·901	1	81 31 23·2	10·04	1	* 7 in photosphere	1
4160	1946	III. 637	....	15 59 36·9	2·065	2	48 55 52·0	10·00	2	pF; vS; R; stellar	3†
4161	......	III. 140	....	16 0 51·0	2·629	1	69 4 1·0	9·90	1	vF; vS; r; pB * sf	1
4162	3619	......	Δ. 360	16 2 19·2	+4·675	2	143 50 42·2	9·74	3	Cl; vB; vL; vRi; lC; st 10...	3
4163	......	III. 973	....	16 2 40·3	−3·081	1	10 38 44·0	9·90	1	vF; vS; lE 0°; r	1
4164	1947	III. 553	....	16 2 48·9	+3·051	1	88 55 4·2	9·74	1	F; L; pmE; vgbM; r.........	2
4165	......	III. 883	....	16 3 32·5	−0·266	1	19 13 44·8	9·76	1	eF; vS	1
4166	3620	......	....	16 3 44·7	+3·922	1	125 52 42·5	9·65	1	pF; R; vgvlbM; r	1
4167	1948	III. 74	....	16 4 5·4	+2·713	1::	72 55 51·5	9·65	1	vF; S; r	2*
4168	......	III. 884	....	16 5 50·0	−0·147	1	19 59 34·4	9·58	1	vF; vS............................	1
4169	3621	......	....	16 6 33·5	+3·866	1	123 53 2·9	9·43	1	eF; S; E; lbM	1
4170	3622	......	Δ. 326	16 7 16·0	4·936	2	147 32 42·5	9·35	2	Cl; B; L; lC; st 7...10 ...	2
4171	......	III. 812	....	16 7 18·1	1·199	1	32 8 30·2	9·44	1	vF; vS; lE	1
4172	1949	III. 889	....	16 7 18·4	2·293	1	56 35 35·9	9·33	1	vF; S; R; bM..................	2
4173	3624	......	M. 80	16 8 41·9	+3·567	2	112 37 34·1	9·27	2	!!!; ⊕; vB; L; vmbM(var *); rrr; st 14.	2*
4174	......	III. 314	....	16 8 49·3	−0·740	2	17 10 21·4	9·38	2	vF; vS; lE	2
4175	3623	......	Δ. 68	16 9 54·3	+6·720	3	161 51 51·0	+ 9·10	3	⊕; pF; L; iR; vgbM; rrr; st 14...	3

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	h.	H.		h m s	s		° ′ ″	″			
4176	1950	III. 888	....	16 10 18·4	+2·322	2	57 41 3·1	+9·17	2	vF; S; R; vglbM	3
4177	1951	III. 688	....	16 11 21·9	2·206	2	53 56 19·0	9·10	2	vF; S; iR	3
4178	1952	II. 151	....	16 11 58·8	2·911	1	82 15 5·9	9·03	1	F; pL; lE; vgbM; r	2
4179	3625	......	....	16 13 42·5	4·589	1	141 36 46·8	8·86	1	Cl; eL; eRi.....................	1
4180	1953	II. 402	....	16 14 31·8	3·114	1	91 56 56·9	8·83	1	vF; cL; cE 45°±; r	2
4181	1954	......	....	16 14 43·9	2·132	1	51 52 57·9	8·83	1	vF; eS; R	1
4182	1955	III. 623	....	16 14 50·5	2·133	2	51 54 7·9	8·83	2	vF; vS; R; ⁑ nf...............	4
4183	......	......	M. 4= B.A.C.5455	16 15 3·6	3·665	...	116 11 11·1	8·77	...	Cl; 8 or 10 L st in line, with 5 st; rrr.	4
4184	3626	......	Δ. 514	16 16 1·3	4·097	1	130 20 6·7	8·69	1	Cl; B; L; pRi; lCM; st 9...11	2
4185	......	II. 810	....	16 16 4·4	1·154	1	32 2 48·5	8·75	1	pF; pS; lE	1
4186	......	III. 891	....	16 16 46·1	2·121	1	51 40 47·1	8·67	1	eF; vS; R; lbM................	1
4187	3627	......	Δ. 412	16 17 20·5	4·456	2	138 49 29·1	8·57	2	Cl; cL; pRi; lCM; st 13...15	2
4188	1956	III. 624	....	16 18 3·7	2·121	2	51 44 50·1	8·57	2	F; S; iR; bM..................	4
4189	3628	......	Δ. 536	16 18 14·8	4·037	3	128 30 59·3	8·51	4	B; pL; R; psbM; rr	4
4190	......	III. 740	....	16 18 19·3	0·401	1	24 17 36·7	8·59	1	cF; pL; iR	1
4191	......	III. 892	....	16 18 23·5	2·144	1	52 26 49·2	8·54	1	eF; S; bM	1
4192	......	II. 811	....	16 18 42·4	1·320	1	34 34 35·2	8·54	1	pB; iR; vgvlbM..............	1
4193	3629	VI. 10	....	16 18 43·0	3·657	1	115 43 5·4	8·48	1	Cl; cL; mC; gbM; rrr	2
4194	1957	......	....	16 20 20·8	2·009	1	48 44 41·0	8·40	1	F; R; bM	1
4195	1958	III. 638	....	16 20 27·2	2·010	2	48 47 10·4	8·38	2	eF; vS; R; bM	3
4196, 4197	1958, a	......	R. 2 novæ	16 20 ±	2·010	...	48 47 ±	8·38	...	2 novæ, one eF; one S	0
4198	1959	III. 639	....	16 21 10·4	2·025	1	49 13 2·9	8·33	1	vF; vS; R	2
4199	3630	......	....	16 21 41·1	7·067	1	162 57 2·8	8·16	1	vF; vS; ✱9 nr.................	1
4200	3631	......	....	16 21 49·9	4·654	1	142 18 59·3	8·21	1	Cl; L; lC; st L	1
4201	......	III. 680	....	16 22 6·9	1·623	2	39 48 49·8	8·26	2	vF; S; R; lbM; er...........	2
4202	......	II. 690	....	16 22 34·5	1·687	2	41 17 48·6	8·22	2	F; pS; iF; gbM	2
4203	3632	......	....	16 22 36·1	5·219	2	150 18 1·9	8·13	2	pF; pL; vlE; gbM...........	2
4204	1960	II. 652	....	16 23 0·8	2·004	2	48 45 15·4	8·18	2	cF; pL; R; gbM; r	3
4205	......	II. 647	....	16 23 2·7	2·057	2	50 13 17·4	8·18	2	F; S; iF	2
4206	3633	......	....	16 23 22·7	4·423	1::	137 48 2·7	8·09	1::	eF; (?); ⁑ f nr	1
4207	3634	......	....	16 23 39·8	4·426	1	137 51 20·8	8·06	1	F; cS; lE; vglbM; ⁑ p	1
4208	1961	II. 875	....	16 23 50·4	2·053	1	50 8 9·6	8·12	1	pF; S; vlE; vgmbM	2
4209	3635	......	Δ. 400	16 23 50·9	4·506	1	139 27 48·5	8·05	1	Cl; L; lC; iF	1
4210	3636	......	....	16 24 7·3	4·246	1	133 44 18·2	8·04	1	Cl; μ *Normæ* inv	1
4211	3637	VI. 40	Mechain	16 24 43·0	3·350	1	102 44 17·3	8·01	1	⊕; L; vRi; vmC; R; rrr ...	2
4212	1962	III. 640	....	16 24 58·0	2·004	1	48 52 16·6	8·02	2	cF; vS; R; bM	3
4213	1962, a	......	R. nova	16 25 ±	2·004	...	48 52 ±	8·02	...	No description; near h. 1962	0
4214	1963	III. 641	....	16 25 18·7	2·010	1	49 3 57·0	8·00	1	vF; vS; R	3
4215	1964	III. 890	....	16 25 34·0	2·203	1	54 38 1·1	7·97	2	vF; pL; iE; rr; ✱ nr	3
4216	3638	......	....	16 25 36·3	4·316	2	135 19 19·6	7·92	2	Cl; B; S; st pL	2
4217	1964, a	......	R. nova	16 25 47·9	2·203	...	54 35 11·1	7·91	...	No description; 4′ nf 1964...	0
4218	......	II. 753	....	16 26 37·0	2·624	2	69 55 19·4	7·88	3	pB; pL; vlE; pgmbM	2
4219	......	III. 813	....	16 26 50·0	1·264	1	34 9 56·7	7·89	1	vF; vS; iR	1
4220	3639	......	....	16 26 50·4	6·305	1	159 4 52·8	7·76	1	vF; eS; R; gbM..............	1
4221	1965	......	....	16 28 15·0	2·190	1	54 21 40·8	7·76	1	F; S; R; gbM; ✱ 11 np......	1
4222	......	III. 730	....	16 28 18·6	2·579	1	68 9 52·2	7·74	1	eF; vS; E	1
4223	3640	......	....	16 29 14·5	4·483	1::	138 43 48·6	7·62	1::	!; F; vL; viE; B ✱ inv	1
4224	3641	......	Δ. 483	16 30 29·3	4·233	2	133 5 18·9	7·53	2	Cl; cL; pRi; iR; st 11...14..	2†
4225	3642	......	Δ. 413	16 30 52·1	4·475	1	138 29 3·7	7·49	1	Cl; vL; lRi; lC; rrr; F neb inv.	1
4226	1966	III. 893	....	16 31 43·6	2·058	2	50 41 28·4	7·48	2	vF; S; R; gbM; bet 2 st ...	4
4227	1967	......	....	16 32 19·0	2·155	1	53 31 20·9	7·43	1	vF; vS; sbM ✱12	1
4228	3643	......	....	16 33 50·3	4·420	1	137 11 50·2	7·24	1	Cl; (in M. Way).............	1
4229	3644	......	Δ. 442	16 36 6·6	4·404	2	136 45 36·8	7·06	2	Cl; pRi; eiCM; st 11...12...	2†
4230	1968	......	M. 13, Halley	16 36 41·2	2·140	3	53 16 19·4	7·08	3	!!; ⊕; eB; vRi; vgeCM; st 11...20.	14†
4231	1969	II. 701	....	16 38 12·4	2·125	(1)	52 54 7·5	6·95	1	pB; pL; E 45°±; vgmbM...	2
4232	3646	......	Δ. 364	16 38 18·4	4·776	3	143 33 29·1	6·87	3	Cl; L; Ri; lCM; st 9...12 ...	2
4233	3645	......	....	16 38 23·8	+7·032	1	162 20 32·0	+6·80	1	vF; pL; vgvlbM..............	2

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	h.	H.		h m s	s		° ′ ″	″			
4234	1970	......	Σ.5 = Lal. 30510	16 38 36·0	+2·513	1	65 56 10·0	+6·90	1	○; vB; vS; R; disc and border.	1†*
4235	3647	......	......	16 39 3·5	+5·147	1	148 44 47·7	6·79	1	pF; R; vglbM; *5 p 79ˢ ...	1
4236	......	I. 280	......	16 39 16·1	−3·036	3	11 33 21·3	7·01	3	B; cL; lE; slbM	3
4237	3648	......	Δ. 454	16 39 19·8	+4·308	3	134 28 15·7	6·79	3	Cl; pS; pRi; pC; st 12...15..	3
4238	1971	......	M. 12	16 39 58·1	3·110	1	91 41 47·4	6·78	2	!!; ⊕; vB; vL; iR; gmbM; rrr; st 10...	7
4239	3649	......	......	16 40 39·6	5·171	2	148 58 2·1	6·67	2	⊕; pB; cL; R; glbM; rr ...	3
4240	3650	......	Δ. 456?	16 40 41·4	4·311	1	134 28 33·4	6·68	1	Cl; vL; vRi; lbM; st 12...13	1
4241	......	......	D'Arrest, 101	16 41 24	0·72	[2]	28 9 18	6·72	[2]	F; S; R; mbM	0
4242	......	......	D'Arrest, 102	16 41 47	0·68	...	27 46 0	6·69	...	F; S; makes Δ with 2 S st 12 and 14 m.	0
4243	3651	......	......	16 41 51·9	4·170	1	130 58 41·7	6·59	1	Cl; cL; eRi (in M. Way) ...	1
4244	......	IV. 50	......	16 43 6·4	1·678	1	42 8 38·8	6·56	1	vB; L; R; Disc + F, r, border	1
4245	3652	......	Δ. 499	16 44 14·5	4·197	1	131 33 37·7	6·39	1	Cl; B; cL; pRi; st 10...13 ..	1
4246	3653	II. 584	......	16 45 1·0	3·584	1	111 55 47·5	6·35	1	pB; cL; iR; rrr; st 14...16 ..	2
4247	......	III. 727	......	16 45 54·8	1·932	1	48 1 25·6	6·32	1	cF; S; E 90°	1*
4248	......	III. 735	......	16 46 0·4	1·775	1	44 20 25·3	6·31	1	eF; pS	1
4249	3654	......	Δ. 520	16 46 3·5	4·412	2	129 16 12·2	6·24	2	Cl; B; L; Ri; st 8...11	3
4250	......	......	D'Arrest, 103	16 47 1	0·56	...	26 47 5	6·26	...	F; pS; irr	0
4251	3655	......	......	16 47 33·9	4·327	2	134 33 17·6	6·12	2	Cl; pRi; vlC; iF; st L & S...	2
4252	3656	......	......	16 47 40·8	+4·378	2	135 42 21·0	6·10	2	Cl; L; lRi; lC; st 8...12	2
4253	......	III. 974	......	16 47 55·2	−6·988	1	7 9 0·3	6·41	1	cF; S; bM; p of 2	1
4254	......	III. 975	......	16 47 58·3	−7·058	1	7 5 59·7	6·39	1	vF; vS; f of 2	1
4255	3657	......	Δ. 374?	16 48 3·0	+4·734	1	142 28 51·8	6·06	1	Cl; S; Δ ar; st 13	1
4256	1972 = 3659	......	M. 10	16 49 47·6	3·159	3	93 52 6·8	5·96	3	!; ⊕; B; vL; R; gvmbM; rrr; st 10...15.	7
4257	1973	III. 689	......	16 49 47·6	2·121	1	53 16 46·7	5·99	1	eF; cL; E 90°	2
4258	3658	......	......	16 50 8·6	4·032	2	126 53 32·3	5·91	2	⊕; vF; vL; iR; vgbM; rrr; st 20.	2
4259	1974	......	......	16 50 11·5	2·008	1	50 9 37·8	5·96	1	vF (?)	1*
4260	3660	......	Δ. 456	16 50 37·1	4·326	3	134 26 53·8	5·86	3	!; Cl; B; vL; vRi; st 11...	2
4261	3661	......	M. 62 = Δ. 627	16 52 18·7	3·810	5	119 53 42·9	5·73	5	!; ⊕; vB; L; gmbM; rrr; st 14...16.	5†
4262	......	III. 123	......	16 52 22·2	2·523	1	66 46 53·8	5·76	1	vF; pL; R; lbM	1
4263	3662	......	Δ. 521	16 52 26·0	4·130	2	129 30 46·3	5·71	2	Cl; B; pL; cRi; st 10	2
4264	1975 = 3663	......	M. 19	16 53 59·2	3·701	3	116 3 13·0	5·60	2	⊕; vB; L; R; vCM; rrr; st 16... red.	7
4265	3664	......	Δ. 556	16 55 16·3	4·067	1	127 40 56·4	5·48	1	Cl; L; pRi; lC; st 9...11 ...	1
4266	......	III. 124	......	16 55 22·1	2·521	1	66 46 39·3	5·51	1	vF; stellar	1
4267	......	III. 728	......	16 55 52·8	1·534	1	39 51 36·0	5·50	1	vF; cS; iR	1
4268	1976 = 3665	VI. 11	......	16 55 55·3	3·661	2	114 33 37·9	5·43	2	⊕; B; L; R; gCM; rrr; st 16.	3
4269	3666	II. 195	......	16 56 43·5	3·606	1	112 30 9·8	5·36	1	⊕; cB; L; R; gpmCM; rrr; st 16.	3
4270	1977 = 3667	VI. 12	......	17 1 28·2	3·716	2	116 23 13·1	4·97	2	⊕; vB; L; R; psbM; rrr; st 16; F neb f.	4
4271	1978	......	......	17 1 34·2	3·715	1	116 22 33·5	4·95	1	F; S; vgbM; ⊕ p	1
4272	......	......	D'Arrest, 104	17 3 40	0·57	[1]	27 21 54	4·86	[1]	vF; vS; R	0
4273	3668	......	......	17 3 51·8	5·588	2	152 39 3·3	4·71	2	F; vL; vlE; am st; 2 st inv...	2
4274	......	IV. 57	......	17 4 10·8	1·883	2	47 29 24·4	4·78	2	F; stellar	2
4275	3670	I. 147	......	17 5 40·2	3·803	2	119 17 18·0	4·60	2	⊕; B; cL; R; s, vglbM; rrr; st 16...17.	3
4276	3669	......	......	17 5 46·4	5·242	1	149 0 21·5	4·55	1	vF; vS; R; glbM	1
4277	......	......	D'Arrest, 105	17 6 3	0·73	[1]	29 3 54	4·59	[1]	pF; vS; R; *13 nr	0
4278	......	......	D'Arrest, 106	17 6 15	+0·71	[1]	28 49 54	+4·58	[1]	F; pL; lE	0

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	h.	H.		h m s	s		° ′ ″	″			
4279	3671	I. 45	....	17 7 49·0	+3·765	2	117 58 27·9	+4·43	2	⊕; cB; pS; R; gvmbM; rrr; st 16...17.	4
4280	3672	......	Δ. 522	17 8 4·2	4·140	1::	129 17 11·4	4·38	1::	Cl; pL; Ri; R; gbM; st 12...14.	1
4281	3673	......	....	17 8 45·0	+4·279	2	132 43 29·9	4·33	2	Cl; vL; pRi; lC (✱ nf taken)	2
4282	......	III. 945	....	17 9 14·5	−2·000	1	14 23 21·1	4·47	1	vF; S; E; S ✱ s	1
4283	3676	......	....	17 9 30·9	+3·643	1	113 35 51·4	4·28	1	pF; L; R; rr	1
4284	3675	......	....	17 9 41·0	4·722	2	141 35 30·9	4·23	2	!!!; ○; pB; vS; R	3†
4285	3674	......	....	17 9 57·7	+5·861	2	154 51 25·4	4·18	2	vF; vS; vlE; glbM	2
4286	......	III. 951	....	17 10 45·4	−3·497	...	11 12 41·4	4·38	1	eF; S	1
4287	1979 = 3677	......	M. 9	17 10 57·6	+3·507	2	108 22 59·8	4·16	2	⊕; B; L; R; eCM; rrr; st 14	6
4288	3678	......	....	17 11 31·9	4·021	2	125 54 52·0	4·10	2	eF; vL; icE; vglbf; ✱ 8 inv .	2
4289	3679	......	....	17 11 35·8	3·828	1::	120 0 3·0	4·10	1::	Diffused neb in patches	1
4290	3680	......	....	17 12 40·1	4·109	3	128 20 0·0	4·00	3	!!!; ◉; eF; S; am St	3†
4291	......	II. 812	....	17 12 40·7	+1·011	1	32 24 7·4	4·08	1	F; S; R; vglbM	1
4292	1980	II. 767	....	17 12 44·1	−1·062	1	17 32 6·2	4·14	1	cF; pL; R; vgmbM	2
4293	......	I. 149	....	17 12 47·8	+3·535	1	109 25 12·0	4·00	1	cB; pS; lE; er	1
4294	......	......	M. 92	17 12 56·9	1·840	...	46 43 31·2	4·04	...	⊕; vB; vL; eCM; rrr; st S..	8*
4295	3681	I. 46	....	17 15 16·2	3·719	1	116 12 43·4	3·78	1	cF; L; R; gbM; rrr	2
4296	3683	I. 48	....	17 15 23·2	3·491	1	107 40 23·4	3·78	1	⊕; vB; cL; vgvmbM; rrr; st 20.	3
4297	3682	......	....	17 15 28·0	3·960	1	124 3 6·8	3·76	1	F; L; E; vglbM; ⁑ inv	1
4298	......	......	D'Arrest, 107	17 16 23	0·61	[1]	28 4 54	3·77	[1]	pB; S; R; bMN = ✱ 12	0
4299	3685	......	....	17 16 28·0	3·827	1	119 51 37·1	3·67	1	Neb in patches (M. Way) ...	1
4300	3684	......	Δ. 225	17 17 23·3	6·165	2	156 55 47·9	3·53	2	⊕; cB; L; vgmbM; rrr; st 14...17.	2
4301	......	......	Auw. N. 36	17 20 19·2	3·184	...	94 57 2·0	3·37	...	F; L; vlbM (Winnecke, April 12, 1860).	0
4302	1981 = 3686	IV. 11	....	17 20 50·4	3·650	2	113 37 47·3	3·21	3	!!; ; pB; S; R	5*†
4303	......	III. 137	....	17 22 2·6	2·412	1	63 25 22·2	3·24	1	vF; pL; iF	1
4304	3687	......	....	17 22 58·1	3·913	1	122 28 48·3	3·11	1	Cl; S; P; B ✱ inv	1
4305	3688	......	....	17 24 47·4	4·142	1	128 58 1·5	2·95	1	eF; pS; lE; ✱ 9 att	2†
4306	3689	......	....	17 25 34·1	3·915	3	122 28 38·7	2·89	3	Cl; st 6·7, 13	2
4307	3690	......	Δ. 457	17 26 3·2	4·380	2	134 38 32·2	2·84	2	⊕; vB; L; R; pg, psvmbM; rrr; st 17...	2
4308	......	II. 901	....	17 26 3·6	2·680	1	73 29 2·4	2·88	1	F; S; iF; er	1
4309	3691	......	....	17 27 57·9	6·657	2	159 41 19·0	2·60	2	cF; S; R; glbM; ✱ 13 sp ...	2
4310	3693	......	....	17 28 51·0	3·998	1	124 54 49·0	2·60	1	Cl; pL; lRi; lC	1
4311	3692	......	Δ. 366	17 29 17·3	4·871	2	143 35 5·2	2·54	2	⊕; B; vL; Ri; st 13	3
4312	3694	......	....	17 29 48·6	5·527	1	151 36 5·1	2·47	1	eF; S; R; p of 2	1
4313	3696	......	Δ. 568	17 30 1·9	4·067	1	126 51 8·0	2·50	1	Cl; pL; pRi; iR; st 9...10...	1
4314	1982 = 3697	I. 44	....	17 30 6·9	3·658	4	113 48 55·0	2·50	4	pB; pL; R; ✱ 12 f inv	6
4315	1983 = 3698	......	M. 14	17 30 16·0	3·146	2	93 9 25·0	2·50	2	!; ⊕; B; vL; R; eRi; vgmbM; rrr; st 15...16.	7
4316	3695	......	....	17 30 18·8	5·528	1::	151 36 3·9	2·43	1	eeF; f of 2	1
4317	4020	h. o. n.	....	17 30 26·3	8·939	1	123 9 16·8	2·46	1	Cl; F; L; pRi; lC; st 13...15	1
4318	3699	......	M. 6	17 30 55·8	3·905	1	122 7 4·9	2·43	1	Cl; L; iR; lC; st 7, 10... ...	4
4319	3700	......	....	17 32 15·0	5·436	1	150 39 39·1	2·27	1	eF; S; R; 3 st nr	2
4320	......	......	D'Arrest, 108	17 33 56	+0·69	[2]	29 6 12	2·26	[2]	vS; gbM	0
4321	......	VI. 41	....	17 34 7·6	−2·177	1	14 11 11·9	2·33	1	⊕; cL; R; vgbM; rr.........	1
4322	3701′	......	....	17 35 0·1	+4·002	1::	124 56 32·1	2·07	1	Nebulous portion of M. Way	1
4323	3702	......	Δ. 612	17 35 9·3	3·912	2	122 17 0·5	2·05	2	Cl; vL; Ri; lC	2
4324	3702′	......	....	17 36 30·1	3·957	1	123 37 33·2	1·94	1	Cl; vL; pRi; st 8...12	1
4325	......	II. 587	....	17 37 53·6	+2·997	1	86 44 25·5	+1·85	1	F; cL; iF	1

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	h.	H.		h m s	s		° ′ ″	″			
4326	3703	......	....	17 37 56·4	+3·887	1	121 27 58·6	+1·82	1	Cl; pS; lRi; lC; st 10...12 ..	1
4327	1984	......	....	17 38 46·0	+3·689	1	114 49 51·5	1·75	1	Cl; st vS	1
4328	1987	III. 741	....	17 39 11·8	−1·069	1	17 49 22·7	1·69	1	vF; vS; R; stellar; ✱8 s......	2
4329	3704	......	....	17 39 39·5	+4·019	1::	125 22 49·8	1·66	1::	Cl; F; eL; vS st+neb	1
4330	3701	......	....	17 39 43·4	19·744	1	175 24 24·0	1·20	1	pB; R; vgbM....................	1
4331	1985	I. 150	....	17 40 32·5	3·565	1	110 18 21·0	1·60	1	pB; pL; R; bM	3
4332	3705	......	Δ. 557	17 40 41·6	4·077	2	127 0 0·1	1·57	2	⊕; vB; pL; R; vgmbM; rrr; st 18...20.	2
4333	1986	II. 586	....	17 40 54·7	3·557	2	109 58 3·8	1·56	2	pB; pS; R; gbM; r; ✱15 np	3
4334	3706	......	Δ. 597 ?	17 40 55·0	3·999	1::	124 48 46·5	1·55	1::	Cl; vL; vRi; st 12...13	1
4335	3707	VI. 13	....	17 41 40·5	3·847	2	120 10 12·7	1·49	2	Cl; pL; pRi; bifid; st 12...	3†
4336	3708	......	....	17 41 59·7	3·991	1::	124 34 43·8	1·46	1::	cL; iR; pmbM; r	1
4337	3709	......	....	17 42 37·5	4·018	1::	125 20 42·0	1·40	1::	Cl; rr; st eS+neb	1
4338	1988	......	....	17 44 16·8	3·705	1	115 21 32·1	1·27	1	eF; S; (?)	1
4339	3711	......	....	17 44 29·7	3·620	1	112 18 23·5	1·25	1	Cl; pRi (in M. Way)	1
4340	3710	......	M.7 (Lacaille)	17 44 39·6	3·999	1	124 46 37·9	1·23	1	Cl; vB; pRi; lC; st 7...12 ...	3
4341	3712	......	....	17 44 54·5	3·815	1::	119 5 36·3	1·21	1::	Neb or nebulous patch of M. Way.	1
4342	3713′	......	....	17 45 36·6	3·855	1	120 23 56·5	1·15	1	Neb or nebulous patch of M. Way.	1†
4343	1989	......	....	17 45 57·6	2·502	1	66 53 15·5	1·15	1	!; vF; S; R; vsvmbMvSRN.	1†
4344	3713	......	....	17 46 1·3	5·767	1	153 38 2·5	1·05	1	F; S; E; bM; bet 2 st 10 ...	1
4345	3714	......	....	17 48 31·2	6·132	1	156 24 42·9	0·83	1	pF; S; pmE 90°; ✱12 f, att...	1
4346	1990	......	M. 23	17 48 40·9	3·532	3	108 59 43·7	0·89	3	Cl; B; vL; pRi; lC; st 9·10, 11...13.	6
4347	3715	......	Δ. 460 ?	17 48 51·4	4·372	2	134 13 52·2	0·84	2	Neb+Cl; pL; mE; gwlbM...	2
4348	......	III. 957	....	17 49 25·1	2·631	1	71 42 59·5	0·85	1	vF; vS; p of 2	1
4349	......	III. 958	....	17 49 31·1	2·629	1	71 38 58·5	0·85	1	vF; vS; f of 2	1
4350	3716	......	....	17 50 20·0	+5·991	2	155 23 58·1	0·67	2	vF; vS; f ✱ of ⁑ inv	2
4351	......	......	Auw. N. 37	17 50 54·1	−0·638	...	19 49 12·3	0·81	...	pF; L; mE (Auwers, July 22, 1854).	0
4352	3717	......	....	17 51 10·6	+3·685	1	114 38 24·1	0·67	1	Cl; Ri; eL; vlC	1
4353	......	VIII. 53	....	17 51 30·0	3·491	1	107 23 18·2	0·64	1	Cl; pS; lRi; lC	1
4354	......	......	D'Arrest, 109	17 53 43	0·49	[2]	27 20 24	0·53	[2]	vF; R; 1st of 3	0
4355	{1991 = 3718}	{IV. 41 V. 10, 11, 12}	M. 20	17 53 51·8	3·640	3	113 1 39·9	0·43	3	!!!; vB; vL; trifid; ⁑ inv ...	8 Mon.†
4356	......	......	D'Arrest, 110	17 54 9	0·48	[2]	27 18 30	0·50	[2]	vF; vS; 2nd of 3	0
4357	3719	II. 199	....	17 54 13·9	3·282	2	98 56 37·3	0·41	2	pB; pL; R; rr..................	3
4358	3721	VII. 7	....	17 54 36·4	3·779	1	117 53 32·8	0·36	1	Cl; pS; Ri; lC; st 9...13 ...	3
4359	3720	I. 49	....	17 54 37·2	3·845	2	120 1 29·8	0·36	2	⊕; B; pL; R; gvmbM; rrr; st 16...17.	3
4360	......	......	D'Arrest, 111	17 54 38	0·49	[2]	27 22 0	0·46	2	F; pL; 3rd of 3	0
4361	3722	......	M. 8	17 55 17·9	3·677	3	114 21 15·3	0·31	3	!!!; vB; eL; eiF; with LCl...	8 Mon.†
4362	1992	......	....	17 55 27·9	2·811	1	78 57 6·6	0·32	1	....	1
4363	......	V. 9	....	17 55 39·8	3·652	1	113 27 28·4	0·28	1	F; L; cE.......................	1
4364	3723	II. 200	....	17 55 51·7	3·846	2	120 3 24·5	0·25	2	⊕; pF; cS; R; gbM; rrr; st 16...17.	3*
4365	3724	......	Δ. 569	17 56 1·6	4·054	1::	126 18 6·9	0·23	1::	Cl (in M. Way)	1
4366	3725	......	....	17 56 6·0	3·677	2	114 19 58·9	0·23	2	Cl; B; L; pRi; vL neb p ...	2
4367	1993	......	M. 21	17 56 13·8	3·626	1:	112 30 8·6	0·22	1	Cl; pRi; lC; st 9...12........	2
4368	......	V. 13	....	17 56 32·3	3·692	1	114 53 23·7	0·19	1	eL; eiF; st f	1*
4369	......	......	Auw. N. 38	17 56 38·5	3·076	...	90 17 44·9	0·20	...	pF; vS; vS neb ✱ p (Hind, Ap. 1852).	0
4370	......	......	Auw. N. 39	17 57 15·0	3·247	...	97 34 58·4	0·15	...	(Brorsen, 1856.) No description.	0
4371	......	II. 198	....	17 57 27·3	3·778	1	117 49 20·6	0·12	1	pF; S; iE; er or Cl	1
4372	3726	......	Δ. 473	17 57 50·9	+ 4·350	2	133 43 25·8	0·06	2	⊕; B; R; eC; gbM; rrr; st 15...16.	2*
4373	......	IV. 37	....	17 58 20·0	− 0·023	1	23 22 9·5	+0·15	1	○; vB; pS; sbMvSN.........	1
4374	1994	II. 197	....	17 58 42·4	+ 3·696	1	115 0 52·0	0·00	1	cF; pL; iR; r	2

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	h.	H.		h m s	s		° ′ ″	″			
4375	3727	......	......	17 58 45·8	+5·788	1	153 47 28·2	−0·06	1	eeF; eeS; R	1†
4376	3729	......	......	17 58 47·5	3·646	1::	113 14 0·0	0·00	1::	Cl; vL; vRi	1
4377	......	III. 555	......	17 59 39·1	2·624	1	71 27 7·8	−0·04	1	cF; S; lE; r	1
4378	3730	IV. 12	......	18 0 44·2	3·722	1	115 56 17·6	0·18	1	⊕; F; L; lE; vglbM; rr; st 20.	3
4379	1995	......	......	18 0 50·8	3·518	1	108 26 46·6	0·18	1	Cl; pRi; vlC; st L & S	1
4380	3732	......	......	18 1 9·8	3·769	1	117 32 31·7	0·21	1	F; vL; cE; lbM; rr	1
4381	3728	......	......	18 1 11·4	8·690	1	166 36 39·2	0·36	1	vF; vS; R; glbM	1
4382	3731	......	......	18 1 12·2	3·900	3	121 46 50·4	0·22	3	⊕; pB; pL; R; glbM; rrr; st 16	3
4383	......	II. 902	......	18 1 18·7	2·649	1	72 25 54·3	0·19	1	F; L; R; vglbM	1
4384	1996 = 3733	......	......	18 1 23·9	3·671	2	114 7 30·1	0·23	2	vF; vL; lE; ⁑ inv	2
4385	1997	VIII. 54	......	18 2 24·7	3·476	1	106 48 57·7	0·31	1	Cl; L; lC; st cL	3
4386	3734	......	......	18 2 47·5	3·970	1	123 53 29·2	0·36	2	○; F; L; cE; hazy border...	2
4387	......	......	D'Arrest, 112	18 3 51	1·34	[1]	37 44 0	0·38	[1]	eF; vS; R; ✱16 nr	0
4388	1998 = 3735	VII. 30	......	18 4 20·6	3·602	2	111 37 33·6	0·48	2	Cl; vL; lC	3
4389	3736	II. 201	Δ. 619	18 4 33·3	3·902	3	121 51 0·7	0·51	3	⊕; cB; L; R; rrr; st 15	5
4390	2000	......	Σ. 6	18 5 17·8	2·912	1	83 10 53·5	0·55	2	○; vB; vS; R; l hazy	3*
4391	1999	......	......	18 5 23·5	3·617	1:	112 10 47·6	0·58	1	Cl; st vS	1
4392	2001 = 3739	VII. 31	......	18 7 24·1	3·616	2	112 10 21·5	0·75	2	Cl; pRi; pC; cE; st 13	3
4393	3737	......	Δ. 376	18 7 24·6	4·797	2	142 15 13·3	0·79	2	⊕; cB; cL; R; gmbM; rrr; st 15	2
4394	3738	......	......	18 7 56·9	5·793	1	153 51 18·2	0·86	1	eF; S; ✱6, sp	1
4395	2002	......	......	18 8 47·2	3·529	1	109 55 8·6	0·88	3	F; pL; cE; ⁑ inv	3†
4396	2003	VIII. 55	......	18 9 28·2	3·473	1	106 41 22·1	0·93	1	Cl; lC	2
4397	2004	......	M. 24	18 10 13·7	3·518	2	108 28 7·3	0·99	2	!; Cl; vRi; vmC; R; st 15 (M. Way).	4*
4398	3740	VIII. 15	......	18 10 14·2	3·363	1	102 17 10·3	0·99	1	Cl; lRi; lC	2
4399	2005	......	......	18 10 20·9	3·429	1	104 59 37·7	1·01	1	Cl; lRi; lC; st 10...12	1
4400	2006	......	M. 16	18 10 57·0	3·401	1::	103 50 2·2	1·06	1	Cl; at least 100 st L & S ...	3
4401	2007	......	M. 18	18 11 44·6	3·485	1::	107 11 8·1	1·13	1	Cl; P; vlC	4
4402	3741	......	......	18 11 45·3	5·724	2	153 17 47·0	1·20	2	vF; S; R; gvlbM; ✱9 p	2
4403	2008	......	M. 17	18 12 33·1	3·460	2	106 13 36·0	1·20	2	!!!; B; eL; eiF; 2-hooked...	9†
4404	3742	I. 50	......	18 14 41·9	3·855	3	120 25 26·0	1·40	3	⊕; vB; pL; R; rrr; st 16 ...	4
4405	2009	......	......	18 15 20·7	3·358	1::	102 5 58·8	1·44	1	lC; lRi; lC; st 11...12	1
4406	2010 = 3743	......	M. 28	18 15 55·4	3·692	2	114 56 30·0	1·50	2	!; ⊕; vB; L; R; geCM; rrr; st 14...16.	8
4407	3744	II. 204	......	18 17 13·4	3·645	1	113 16 30·4	1·62	1	○ or ⊕; pB; eeS; R	2
4408	3745	......	......	18 19 12·0	5·726	1	153 22 22·5	1·85	1	pF; S; R; gbM	1
4409	3746	......	......	18 19 23·4	3·358	1	102 6 42·3	1·79	1	Cl; pL; pRi; st 12...15	1
4410	......	VIII. 72	C. H.	18 20 43·1	2·921	1	83 31 15·3	1·89	1	Cl; lC; st L	3
5076	......	......	......	18 20 44·6	......	...	123 30 27·3	......	...	See No. 5076	0
4411	......	......	M. 69=Δ. 613	18 22 13·2	3·917	3	122 26 33·5	2·05	3	⊕; B; L; R; rrr; st 14...16	4*
4412	3748	I. 51	......	18 22 16·6	3·708	1	115 34 55·5	2·05	1	⊕; B; S; R; rr	2
4413	2011	......	......	18 23 4·6	3·385	1	103 15 12·7	2·11	1	Cl (in M. Way)	1
4414	2012 = 3749	II. 205	......	18 23 24·2	+3·651	3	113 33 57·5	2·15	3	⊕; pB; pL; iR; gpmbM; rrr; st 16.	4
4415	......	......	Auw. N. 40	18 23 35·4	−1·719	...	15 29 47·7	2·01	...	!!; pB; pL; E 50°; 2 st p *Var.* (Tuttle.)	0*
4416	2013	VI. 23	......	18 24 31·5	+3·477	1	106 59 3·8	2·24	1	Cl; pL; vRi; pC; st 11...15	2
4417	......	II. 907	......	18 24 55·8	1·967	1	50 14 52·8	2·24	1	F; S; iF	1
4418	2014	VIII. 14	......	18 25 4·6	3·488	1::	107 26 2·3	2·29	1	Cl; L; Ri; lC; st vS	2
4419	......	......	Auw. N. 41	18 25 4·9	0·230	...	25 5 44·2	2·20	...	S; pmE; ⁑ inv (Σ. neb No. 7)	0
4420	3751	......	......	18 25 44·1	3·317	1	100 29 32·8	2·34	1	Cl; P; lC; pS; st 9·10, 12...13	1
4421	3752	......	Δ. 607	18 26 32·0	+3·936	4	123 5 5·1	−2·43	4	B; S; lE; rrr; st 15	5

No. of Cata-logue.	References to: Sir J. H.'s Catalogues of Nebulæ.	References to: Sir W. H.'s Classes and Nos.	References to: Other Authorities.	Right Ascension for 1860, Jan. 0.	Annual Precession in Right Ascension for 1880.	No. of Obs. used.	North Polar Distance for 1860, Jan. 0.	Annual Precession in N.P.D. for 1880.	No. of Obs. used.	Summary Description from a Comparison of all the Observations, Remarks, &c.	Total No. of times of Obs. by h. and H.
	h.	H.		h m s	s		° ′ ″	″			
4422	3750	......	....	18 27 8·1	+7·511	1	163 22 49·3	−2·59	1	vF; S; lE; glbM..............	1
4423	......	......	Auw. N. 42	18 27 19·3	3·210	...	96 4 46·7	2·45	...	pF; vS; E (Winnecke, June 1855).	0
4424	2015 = 3753	......	M. 22	18 27 52·1	3·662	4	114 0 25·8	2·54	4	!!; ⊕; vB; vL; R; vRi; vmC; st 11...15.	10
4425	2016	......	....	18 28 5·5	2·496	1	66 32 22·1	2·53	1	Cl; P; lC........................	1
4426	3754	VIII. 12	....	18 29 5·1	3·266	1	98 20 0·1	2·63	1	Cl; L; pRi; vlC	2
4427	3755	......	....	18 32 2·0	5·603	2	152 25 36·5	2·95	2	pF; S; R; psbM; r..............	2
4428	3756	......	M. 70=Δ. 614	18 34 4·9	3·910	3	122 25 12·6	3·08	3	⊕; B; pL; R; gbM; st 14...17	4*
4429	2017	......	....	18 34 9·9	3·185	1	94 53 29·9	3·07	1	Cl; L; Ri; st 10...18	1
4430	2018	......	....	18 34 41·1	3·219	1	96 21 12·4	3·12	1	Cl; vRi; vlC (in M. Way)...	1
4431	3757	......	....	18 35 9·5	5·941	2	155 19 18·8	3·24	2	vB; pL; R; vg, psvmbM; *7 p	2
4432	3758	......	M. 26	18 37 32·6	3·293	1	99 31 57·9	3·37	1	Cl; cL; pRi; pC; st 12...15..	6
4433	... ..	VI. 15	....	18 39 37·8	3·715	1	116 3 40·2	3·56	1	Cl (?); vF; cL	1
4434	3759	......	....	18 40 6·8	5·131	1	147 27 46·8	3·64	1	pF; pS; lE 90°; pslbM	1
4435	......	......	Auw. N. 43	18 43 18·8	3·192	...	95 22 6·4	3·86	...	Cl; B; 60 st 13 (Winnecke, 1854).	0
4436	3760	......	....	18 43 36·6	5·679	1	153 19 39·2	3·96	1	Neb. No description.........	1
4437	2019	......	M. 11	18 43 37·2	3·219	2	96 26 7·6	3·88	2	!; Cl; vB; L; iR; Ri; st 1 L, 11..	8†
4438	3761	......	....	18 44 2·1	4·873	2	143 58 46·9	3·97	2	F; S; vlE; gbM	3
4439	4021	h. o. n.	....	18 44 18·1	4·866	2	143 53 19·3	3·99	2	pF; S; R; glMbM; last of gr	2
4440	2020	......	....	18 44 52·5	2·836	1	79 49 0·6	3·98	1	Cl; pRi; lC; iF	1
4441	3762	I. 47	....	18 45 29·2	3·276	1	98 52 8·5	4·05	1	⊕; pB; vL; ir; vglbM; rrr	2
4442	3763	......	M. 54=Δ. 624	18 46 6·2	3·846	3	120 38 29·7	4·11	3	⊕; vB; L; R; g, smbM; rrr; st 15.	6
4443	2021	......	....	18 46 17·5	3·549	1	110 4 8·1	4·13	1	Cl; pRi; st 9...13	1
4444	2022 = 3766	III. 143	....	18 46 39·8	3·623	2	112 52 17·2	4·16	2	F; S; rr Cl + neb	3
4445	3764	......	....	18 47 25·9	6·042	2	156 17 14·0	4·30	2	vF; S; R; glbM; *9 sp......	2
4446	3765	......	....	18 48 2·6	6·430	1	158 47 1·2	4·36	2	vF; pL; R; vgvlbM	2
4447	2023	......	M. 57 D'Arquier	18 48 20·1	2·228	2	57 8 57·2	4·26	3	!!!; ◎; B; pL; cE (in β Lyra).	14†
4448	3767	......	....	18 48 51·4	5·153	1	147 56 49·3	4·39	1	pF; cS; R; vmbM	1
4449	3768	......	....	18 49 59·0	5·874	1	155 5 21·7	4·51	1	pF; S; E; glbM; 2 st 8 p ...	1
4450	3770	......	Δ. 573	18 50 5·5	4·047	1	126 48 45·9	4·47	1	⊕; vL; vlE; vgbM; rrr; st 14...16.	1
4451	2024	......	....	18 50 25·1	2·838	1:	79 49 16·2	4·46	1	Cluster........................	1
4452	3769	......	....	18 50 26·1	4·871	1:	144 7 32·7	4·51	1	eF; pL; R	1
4453	......	VIII. 13	....	18 52 20·9	3·281	1	99 7 33·8	4·64	1	Cl; vL; P	1
4454	3771	......	....	18 52 32·0	6·472	1	159 6 45·8	4·74	1	vF; S; R; pmbM; * 7, 8 nf	1
4455	3772	......	....	18 53 15·0	5·552	1	152 23 37·6	4·78	1	eeF; vglbM; v difficult	1
4456	3773	......	....	18 53 29·2	5·938	1	155 39 40·7	4·81	1	vF; S; R; glbM; p of 2......	1
4457	2025	......	....	18 53 33·1	3·086	1	90 39 5·1	4·73	1	Cl; vL; P; st 12...	1
4458	3774	......	....	18 53 44·6	5·934	1	155 38 10·4	4·82	1	eF; S; R; glbM; f of 2	1
4459	2026	......	....	18 53 59·2	3·512	1	108 44 8·6	4·78	1	Cl; pL; pRi; R; st 12...15...	1
4460	2027	......	....	18 54 50·0	2·810	1	78 35 18·1	4·83	1	Cl; P; lC.......................	1
4461	3775	......	....	18 55 7·2	5·463	2	151 34 23·8	4·94	2	cF; vS; cE; psbM; 3 st p ...	2
4462	......	III. 742	D'Arrest, 113	18 55 32·0	1·618	1	41 45 14·2	4·86	1	vF; stellar	1*
4463	2028	......	....	18 55 58·2	2·351	1	60 55 35·4	4·92	1	Cl; pL; P; st 11...12..........	1
4464	3776	......	Δ. 262	18 56 26·8	5·730	2	154 3 58·5	5·05	2	cB; cL; R; vg, svmbM; r ...	2
4465	3777	......	....	18 57 31·7	5·518	1	152 10 36·1	5·13	1	eF; cS; R; glbM	1
4466	2029	......	....	18 57 56·6	3·035	1	88 25 11·0	5·10	1	Cl; L; lC; st L & S	1
4467	3778	......	Δ. 295	18 58 28·0	5·324	5	150 11 49·7	5·21	5	⊕; B; vL; iR; rrr; st 11...16	5
4468	3779	......	....	18 59 37·5	5·085	1	147 15 30·0	5·30	1	pB; pL; R; gbM	1
4469	3780	......	....	19 0 33·6	4·653	2	140 51 32·9	5·37	2	pF; pL; mE 63°; vglbM......	2
4470	2030	VII. 19	....	19 0 50·5	2·981	3	85 59 17·5	5·35	3	Cl; vL; vRi; pC; st 12...14..	5
4471	2031	VII. 62	....	19 1 46·5	2·971	2	85 32 43·1	5·43	1	Cl; S; Ri; lC; st 11...12 ...	5
4472	3781	......	....	19 2 15·9	5·014	1	146 32 24·1	5·53	1	pB; S; R........................	1
4473	......	......	Auw. N. 44	19 4 4·8	+3·051	Au	89 11 51	−5·62	Au	!!; pB; pL; gbM; *Var.?* (Hind)	0*

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	h.	H.		h m s	s		° ′ ″	″			
4474	3782	......	....	19 4 14·3	+4·647	2	140 53 11·6	−5·68	2	vF; pS; iR	2
4475	3786	......	....	19 6 50·4	4·160	1	130 25 44·6	5·88	1	vF; S; R; pslbM	1
4476	3783	......	....	19 6 53·8	5·349	1	150 44 16·4	5·92	1	vF; S; R; lbM; 1st of 3......	1
4477	3784	......	....	19 7 6·3	5·349	1	150 44 46·1	5·93	1	eF; vS; 2nd of 3..............	1
4478	3785	......	....	19 7 7·3	5·351	1	150 46 25·8	5·94	1	eF; S; 3rd of 3	1
4479	2032	IV. 14	....	19 7 14·8	3·137	2	92 57 5·3	5·89	2	vF; L; R; vvlbM; r	3
4480	2033	......	....	19 8 3·7	2·968	1	85 22 9·5	5·95	1	Cl; P; lC........................	1
4481	2034	......	....	19 8 38·7	3·449	1	106 30 34·4	6·02	1	Cl; vL; lC	2
4482	2035	......	....	19 9 37·9	3·097	1	91 9 48·3	6·09	1	Cl; P; lC; st 10...11	1
4483	3787	......	....	19 10 22·9	5·690	1	154 8 23·1	6·23	1	pB; S; R; pgbM	1
4484	......	......	Lac. I. 13	19 10 45·5	6·917	...	161 45 35·7	6·21	...	Neb without st (Lac. Auw. 40)	0
4485	2036	......	M. 56	19 11 7·2	2·339	4	60 3 41·6	6·18	6	⊕; B; L; iR; gvmCM; rrr; st 11...14.	14
4486	3788	......	....	19 11 17·6	4·954	1	146 1 45·6	6·28	1	vF; L; R; vglbM	1
4487	2037	III. 743	....	19 11 37·3	2·931	1	83 42 46·5	6·25	1	○; F; L; R; vsbM disc; S* nf	3*†
4488	3789	......	....	19 11 39·2	5·283	3	150 10 43·4	6·32	3	cF; cS; R; lbM; * 9 s	3
4489	3790	......	....	19 12 48·3	5·896	2	155 52 59·1	6·43	2	eeeF; pS; am S st	3
4490	2038	......	....	19 13 23·1	3·102	1:	91 21 22·0	6·40	1	eS; stellar	1
4491	3791	......	....	19 14 30·8	4·890	2	145 13 36·8	6·54	2	pB; S; mE; pslbM	2
4492	......	......	Auw. N. 45	19 15 55·9	2·098	...	52 28 32·1	6·53	...	vF (Winnecke, Dec. 1853)	0
4493	2039	VIII. 81	....	19 17 13·9	2·565	3	68 6 5·0	6·70	3	Cl; P; lC........................	4
4494	3792	......	....	19 18 23·8	4·094	1	129 9 44·8	6·84	1	eF; pS; R; vgvlbM	1
4495	2040	......	....	19 19 6·7	3·000	1	86 44 46·9	6·87	1	Cl; Ri; bet 2 st 9	1
4496	3793	......	....	19 20 47·4	4·941	2	146 11 39·2	7·06	2	eF; vS; R; lbM; 3 vS st nr	2
4497	2041	VIII. 21	....	19 21 22·5	2·491	1	65 8 23·8	7·04	1	Cl; vL; pRi; vlC; st 10...	3
4498	2042	VI. 14	....	19 24 29·7	2·628	3	70 1 4·0	7·30	3	Cl; L; vC; E 0°; st 14...18	5
4499	2043	VI. 38	....	19 24 53·0	2·875	4	81 3 37·8	7·34	4	cB; S; iR; rrr..................	5
4500	3796	......	....	19 27 18·0	4·068	1	128 51 32·9	7·57	1	eF; R; vgbM	1
4501	3795	......	....	19 27 18·9	4·209	1	132 36 18·9	7·57	1	eF; vS; * 14 att	1
4502	3794	......	....	19 28 30·3	6·638	2	160 57 27·8	7·74	2	pB; E; biN; * 8 f	2
4503	3798	......	M. 55=Δ. 620	19 31 7·9	3·817	2	121 15 44·2	7·86	2	{⊕; pB; L; R; vRi; vgbM; st 12...15.}	5
4504	3797	......	....	19 31 43·0	5·109	2	148 58 44·5	7·95	2	pS; R; vgbM	2
4505	2044	......	....	19 33 58·6	1·790	2	43 44 48·8	8·04	2	Cl; L; pRi; lC; st 11...14...	2
4506	3799	......	....	19 34 7·2	4·858	1	145 40 17·8	8·14	1	pB; pS; pmE; glbM	1
4507	2045	III. 744	....	19 35 0·0	3·300	...	100 38 29·9	8·17	...	pF; pL; R; bM; r	2
4508	2046	......	....	19 35 6·9	2·462	1:	63 30 55·5	8·15	1::	Cl; vL; pRi; lC; st 10...15	1
4509	3800	......	....	19 35 16·0	3·744	1	118 52 41·0	8·20	1	eF; pS; R; vlbM; * np......	1
4510	2047	IV. 51	....	19 36 3·0	3·386	3	104 28 52·5	8·25	3	○; B; vS; R	8†
4511	2048	......	Harding	19 36 24·3	2·053	1	50 7 59·5	8·25	1	Cl; vL; vRi; st 11...15 (Harding, 1827).	1
4512	2049	VII. 18	....	19 37 13·6	2·555	1	67 1 48·4	8·32	1	Cl; cRi; E; st 11...12	2
4513	......	II. 878	....	19 40 47·3	1·298	1	34 18 29·2	8·56	1	pB; iF; bM......................	1
4514	2050	IV. 73	....	19 41 7·5	1·622	1	39 49 41·7	8·61	1	⊙; B; pL; R; * 11 M	2†
4515	2051	VIII. 73	....	19 43 30·0	2·912	1	82 26 25·1	8·83	2	Cl; P; lC........................	3
4516	2052	VII. 9	....	19 45 4·3	2·569	1	67 15 45·5	8·95	1	Cl; L; pRi; pC; st 11...12 ..	2
4517	2053	......	....	19 45 43·7	1·072	1	30 55 55·2	8·96	1	Cl; vL; lC; st 7, ...	1
4518	2054	VIII. 16	....	19 46 33·8	2·408	1	60 57 6·5	9·05	1	Cl; P; lC; st 11...12	2
4519	2055	VIII. 18	....	19 46 53·0	2·832	1:	78 40 27·3	9·09	1::	Cl; S; P	2
4520	2056	......	M. 71	19 47 28·8	2·674	3	71 34 55·1	9·13	3	Cl; vL; vRi; pmC; st 11...16	13
4521	2057	VI. 16?	....	19 48 13·9	2·696	1:	72 28 24·0	9·20	1::	Cl; vS; vC	3
4522	2058	VIII. 19	....	19 48 41·7	2·824	1	78 15 35·1	9·23	1	Cl; P; lC........................	3
4523	3802	......	....	19 48 57·9	3·818	1	122 11 30·6	9·28	1	vF; S; R; psbM	1
4524	2059	......	....	19 49 31·7	2·823	1	78 12 33·0	9·30	1	Cl; S; P	1
4525	3801	......	....	19 49 42·9	5·672	1	155 36 45·6	9·38	1	eF; vS; R; psbM; * 11 np...	1
4526	3803	......	....	19 50 49·6	4·355	1	137 27 28·8	9·44	1	vF; S; vlE; glbM	2
4527	......	II. 202	....	19 51 22·9	2·417	1	61 1 41·1	9·43	1	Neb; r	1
4528	3804	......	....	19 51 33·2	4·845	2	146 28 18·7	9·51	2	cF; cL; R; vglbM; 2 st f ...	2
4529	3805	......	....	19 51 47·6	4·079	1	130 35 26·7	9·51	1	pB; S; R; vS * np	1
4530	3806	......	....	19 52 27·6	4·760	1	145 13 47·9	9·57	1	vF; S; R; bM..................	1
4531	3807	......	....	19 53 19·4	+4·402	1	138 39 10·1	−9·63	1	pF; S; vlE; psbM	1

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	h.	H.		h m s	s		° ′ ″	″			
4532	2060	......	M. 27	19 53 29·3	+2·588	4	67 39 43·0	−9·60	3	!!!; vB; vL; BiN; iE (Dumb-bell N).	13†
4533	3808	......	....	19 54 40·9	4·723	2	144 45 44·5	9·75	2	F; S; vlE; glbM	2
4534	3809	......	....	19 55 31·7	4·849	1	146 47 1·7	9·81	1	pF; S; R	1
4535	2063	......	....	19 56 8·8	1·354	1	34 15 9·9	9·77	1	Cl; pS; pmC; iR; st 12...16	1
4536	2062	III. 144	....	19 56 23·1	2·309	3	56 50 48·7	9·81	3	F; am M. Way st	4*
4537	2061	......	....	19 56 23·6	2·847	1	79 7 24·4	9·82	1	Cl; cL; E; pRi; st 13... ...	1
4538	......	......	Auw. N. 46	19 56 39·0	3·068	...	89 56 49·0	9·85	...	F; *10 p 1s, s 1′ 29″ (Bond, Nov. 1852).	0
4539	3810	......	....	19 56 41·0	5·207	1	151 29 30·7	9·91	1	F; pS; gbM	1
4540	3811	......	Δ. 425	19 57 5·0	4·395	2	138 46 5·7	9·91	2	B; S; cE; gpmbM	2
4541	3812	......	....	19 57 39·9	4·849	1	146 47 18·2	9·96	1	F; S; lE; glbM	1
4542	2065	......	....	19 57 46·5	3·151	1	93 57 0·8	9·94	1	Cl; S; vmC; st 19	1
4543	2064	......	M. 75	19 57 49·1	3·547	3	112 18 47·5	9·95	3	⊕; B; pL; R; vmbMBN; rr	10
4544	2066	VII. 59	....	19 59 11·2	1·967	1	46 23 49·7	10·01	1	Cl; L; vRi; cC	2
4545	3813	......	....	19 59 31·7	4·733	1	145 11 4·4	10·12	1	eeF; L; pmE	1
4546	3814	......	....	19 59 40·6	4·387	2	138 46 26·4	10·12	2	vB; S; R; pgvmbM	2
4547	3815	......	....	19 59 57·7	4·382	2	138 41 20·8	10·14	2	cF; cS; E 90°; gbM	2
4548	2067	......	Σ. 2630	20 0 38·3	2·258	1	54 37 13·1	10·13	1	Cl; st L & S; * inv	1
4549	3816	......	....	20 2 2·9	6·426	2	161 11 51·8	10·34	2	F; pS; lE; glbM; *9 p 10s·5; 1st of 4.	2
4550	2068	......	Σ. 2631	20 2 4·8	2·636	1	69 17 54·5	10·25	1	Cl; lC; st 10...13; * inv	1
4551	2069	VIII. 86	....	20 2 44·4	2·179	1	52 9 34·3	10·29	1	Cl; P; lC	2
4552	3819	......	....	20 3 15·0	4·278	1	136 34 21·6	10·38	1	F; vS; R; vgmbM; *7 nf ...	1
4553	3817	......	....	20 3 27·6	6·429	2	161 17 6·5	10·45	2	pB; S; R; eS*sf; 2nd of 4...	2
4554	3818	......	....	20 3 44·5	6·428	2	161 17 10·6	10·48	2	vF; vS; R; 3rd of 4	2
4555	3821	......	....	20 4 2·8	4·209	1	134 56 31·8	10·44	1	vF; pL; R; glbM	1
4556	3820	......	....	20 4 40·6	6·419	2	161 17 3·8	10·54	2	F; S; R; r; vS*att; 4th of 4	2
4557	......	VIII. 22	....	20 5 58·2	2·509	1	63 42 34·8	10·54	1	Cl; P; lC	1
4558	2070	......	....	20 5 59·3	2·261	1	54 34 19·8	10·54	1	Cl; pRi; * inv	1
4559	2071	VIII. 20	....	20 6 7·1	2·514	1	63 55 43·5	10·55	1	Cl; vB; vL; Ri; lC; st 6...11	2
4560	3822	......	....	20 6 37·5	4·584	1	143 12 32·8	10·64	1	pF; cL; pmE; glbM	1
4561	......	IV. 72	....	20 7 22·0	2·185	1	52 1 28·8	10·64	1	F; vL; vmE; * att	1
4562	3823	......	....	20 8 5·5	4·644	1	144 23 0·5	10·75	1	vF; L; lE	1
4563	3824	......	....	20 8 29·7	4·206	2	135 13 51·9	10·77	2	pF; S; R; vglbM	2
4564	3825	......	....	20 10 43·4	4·344	2	138 40 53·1	10·93	2	pF; S; R; svbM*12	2
4565	2072	IV. 13	....	20 10 45·8	2·419	2	59 51 33·3	10·89	2	!!; ◉; F; S; vvlE	7†
4566	......	VIII. 83	....	20 12 23·7	1·746	1	40 12 6·0	11·00	1	Cl; pRi; lC	1
4567	......	......	D'Arrest, 114	20 12 26	2·42	[2]	59 47 42	11·01	[2]	Cl + neb; S; st vS	0
4568	3826	......	....	20 14 4·0	4·433	1	140 52 0·6	11·18	1	F; S; R; glbM; am st	1
4569	3827	......	....	20 14 52·7	4·143	1	134 5 51·8	11·24	1	F; cS; R; bM	1
4570	2073	......	....	20 15 40·2	3·468	3	109 45 13·6	11·28	3	cL; E; bM*17; *10 att	3*
4571	2074	......	....	20 15 46·1	2·547	1	64 41 32·2	11·26	1	Cl; S; vlC; st 10...11	1
4572	2075	IV. 16	....	20 16 7·9	2·676	3	70 20 19·3	11·29	3	!!; ○; B; pS; R; 4Sst nr	6†
4573	2076	III. 141	....	20 16 44·2	3·590	1	115 14 51·2	11·36	1	cF; cL; vlE; vglbM; r; 3st p	3
4574	3828	......	....	20 17 43·6	4·267	1	137 28 57·8	11·44	1	pB; pL; gbM; 2st 10 nr	1
4575	2077	VIII. 56	....	20 18 5·5	2·137	2	49 40 14·4	11·42	2	Cl; pB; pS; P; pC; st 10...12	5
4576	2078	......	M. 29	20 18 51·9	2·212	1	51 56 3·6	11·48	1	Cl; P; lC; st L & S	6
4577	3830	......	....	20 20 50·3	4·274	1	137 56 31·9	11·67	1	vF; *12 att sp	1
4578	3831	......	....	20 22 0·0	4·141	1	134 40 55·8	11·74	1	eF; pS; R; vgvlbM	1
4579	3829	......	....	20 22 36·0	9·441	1	170 28 50·4	11·92	1	pB; cS; R; psmbM	1
4580	3832	......	....	20 23 2·0	3·727	4	121 17 38·7	11·81	4	pF; cS; R; gbM; bet 2 st ...	4
4581	3834	......	....	20 25 35·6	3·752	3	122 26 56·3	11·99	3	cB; L; mE 6°·0; pslbM	3
4582	2079	III. 142	....	20 25 52·1	3·118	2	92 29 44·3	11·99	2	vF; pL; E 0° or biN; p of 2	5
4583	2080	......	....	20 26 8·3	3·118	2	92 30 28·7	12·01	2	vF; vS; sf of 2	2
4584	3833	......	....	20 26 29·1	6·798	1	164 6 50·1	12·13	1	F; S; R; gbM; 5 st p	1
4585	......	I. 103	....	20 26 43·0	2·935	1	82 44 5·5	12·05	1	vB; L; gmbM; er	1
4586	2081	......	....	20 27 19·8	2·941	3	83 3 41·3	12·09	3	⊕; B; L; R; rrr; st 16...; *9 p	3*
4587	3835	......	....	20 27 59·4	4·457	2	142 35 10·9	12·17	2	pB; cL; R; glbM; r	2
4588	3836	......	....	20 28 25·0	4·458	2	142 38 11·7	12·21	2	vF; cS; R; slbM; f of 2	2
4589	2082	VIII. 17	....	20 28 35·0	+2·646	1	68 13 54·9	−12·17	1::	Cl; vL; P; vlC	4

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	h.	H.		h m s	s		° ′ ″	″			
4590	2083	VI. 42	....	20 28 36·4	+1·211	3	29 49 56·8	−12·14	3	Cl; pL; eRi; pCM; st 11...16	5
4591	......	VII. 8	....	20 28 42·5	2·510	5	62 9 51·9	12·19	5	Cl; vB; vL; vRi; cC; st pL..	5
4592	3837	......	....	20 30 3·7	4·563	1	144 47 44·4	12·32	1	pB; pL; R; pslbM	1
4593	3838	......	....	20 31 3·8	5·831	1	159 14 17·4	12·42	1	pF; L; mE; vglbMvS*	1
4594	2084	IV. 76	....	20 31 57·3	1·269	3	30 20 17·9	12·37	3	vF; vL; vg, vsbM; rr.........	4†
4595	3839	......	....	20 32 30·6	3·750	1	122 58 55·9	12·47	1	vF; L; R; gbM	1
4596	3840	......	....	20 33 4·6	4·497	2	143 51 23·1	12·53	2	vF; pS; cE; lbM	2
4597	2085	VIII. 23	....	20 34 40·7	2·769	1	73 50 36·0	12·60	1	Cl; P; vlC	2
4598	2086	III. 219	....	20 37 14·2	2·852	1	77 59 34·9	12·77	1	vF; S; stellar; ⁑ att	2
4599	3841	......	....	20 39 39·2	3·882	1	128 30 26·2	12·96	1	B; cS; R; pgmbM; 4 st p ...	1
4600	2088	V. 15	....	20 39 53·0	2·478	3	59 47 14·8	12·94	2	!!; pB; cL; eiF; *k* Cygni inv	6†
4601	2087	II. 426	....	20 40 7·8	3·074	2	90 11 12·9	12·97	2	cF; S; R; bM................	4
4602, 4603, 4604	2087, *a*	......	R. 3 novæ	20 40 ±	3·074	...	90 11 ±	12·97	...	Group of 5 with many st ...	0
4605	2089	II. 427	....	20 40 13·6	3·074	2	90 12 34·6	12·98	2	F; vS; R; bM................	4
4606	3842	......	....	20 42 15·1	4·243	1	139 17 41·8	13·14	1	pB; S; lE; gbM	1
4607	......	II. 206	....	20 45 16·1	2·454	1	58 23 48·0	13·30	1	F; S; iE; r	1
4608	2090	......	M. 72	20 45 44·9	3·302	3	103 3 35·8	13·34	3	⊕; pB; pL; R; gmCM; rrr..	10
4609	3843	......	....	20 47 8·1	4·356	2	142 24 21·2	13·46	2	vF; S; E; p of 2	2
4610	3844	......	....	20 47 18·9	4·043	1	134 30 30·2	13·46	1	eF; cS; R	1
4611	3845	......	....	20 47 48·0	4·353	3	142 24 12·7	13·51	3	F; pL; vlE; vgbM; f of 2 ...	3
4612	3846	......	....	20 48 20·9	4·211	2	139 10 13·8	13·54	2	pF; S; vlE; gpmbM; vB * p	2
4613	......	VIII. 82	....	20 49 13·4	2·092	1	45 15 32·5	13·55	1	Cl; cL; st pS	1
4614	3847	......	....	20 49 27·4	4·529	2	146 6 24·7	13·61	2	eeF; vS; vmE0°; *13 att, n	2
4615	2091	VIII. 76	....	20 49 51·7	2·024	1	43 15 22·6	13·58	1	Cl; L; P; vlC	3
4616	2092	V. 14	....	20 50 35·2	2·478	2	58 50 11·8	13·64	1	!!; eF; eL; eE; eiF; bifurcate	4†
4617	......	......	M. 73	20 51 16·0	3·299	...	103 10 31·0	13·70	...	Cl??; eP; vlC; no neb	3
4618	2093	......	....	20 51 19·8	2·490	1	59 19 15·3	13·69	1	F; eL; neb & st	1†*
4619	2094	......	....	20 51 30·9	2·095	1	45 3 59·3	13 69	1	Cl; P; lC.....................	2
4620	......	VIII. 58	....	20 51 33·5	2·123	3	45 53 24·0	13·70	3	Cl; P; lC; st L	3
4621	2096	V. 37?	....	20 53 48·2	2·142	(1)	46 13 4·8	13·84	1::	F; eeL; diff nebulosity	3
4622	2095	......	....	20 53 54·5	3·083	1	90 44 34·9	13·87	1	eF; S; E 0°..................	1
4623	3848	......	....	20 53 57·1	4·201	2	139 35 0·3	13·89	2	cF; cS; R; bM	2
4624	3849	......	....	20 54 13·9	4·203	1	139 40 27·7	13·91	1	eF; R; lbM; *11 f...........	1
4625	2097	I. 52	....	20 54 43·8	2·801	(3)	74 21 51·7	13·91	1	B; pL; R; gbM	4
4626	3850	......	....	20 55 23·1	4·345	3	143 6 3·6	13·98	3	pB; S; R; psbM; am st.......	3
4627	2099	I. 192	....	20 56 17·5	1·749	1	35 59 39·6	13·98	2	cB; L; E 45°±; r; ⁑ att ...	5†
4628	2098	IV. 1	Lal. 40765	20 56 31·2	3·273	4	101 55 4·8	14·04	4	!!!; ○; vB; S; elliptic	23*†
4629	2100	......	....	20 57 5·8	3·291	1	103 3 8·9	14·07	1	eF; pL; R; r	1
4630	2101	......	....	20 57 8·1	2·054	1	43 13 48·5	14·05	1	Cluster; no description	1
4631	3851	......	....	20 57 21·1	4·033	2	135 22 12·0	14·10	2	F; pL; E; vgvlbM; * p.......	2
4632	2102	II. 203	....	20 57 38·9	2·535	2	60 39 30·3	14·09	2	pB; cS; R; psbM; pB*np...	4
4633	3852	......	....	20 58 18·4	4·111	1	137 44 44·2	14·16	1	pF; S; R; bM; 2 st 12 n ...	1
4634	......	IV. 74	....	20 59 22·9	0·771	1	22 26 51·2	14·16	1	eF; *7 m in neb (?)	1
4635	3853	......	....	20 59 38·5	5·136	1	154 35 45·2	14·26	1	pB; cS; lE; pgbM	1
4636	3854	......	....	20 59 39·2	5·000	1	154 5 34·2	14·26	1	pF; cS; R; psbM; *7·8 p ...	1
4637	3855	......	....	20 59 45·4	4·185	1	139 52 10·5	14·25	1	eeF; S; R; B ⁑* sf	1
4638	2103	VIII. 57	....	21 0 46·2	2·255	2	49 3 58·9	14·27	2	Cl; P; lC; st 10...	3
4639	3856	......	....	21 2 7·3	4·173	1	139 51 26·0	14·40	1	B; cS; R; pgmbM	1
4640	2105	VIII. 74	....	21 2 45·7	1·948	(1)	39 43 13·3	14·39	1	Cl of triple st; lC	2
4641	3857	......	....	21 2 47·1	5·457	1	158 51 57·9	14·47	1	vF; cS; R; glbM	1
4642	2104	......	....	21 3 36·5	2·825	1	75 7 5·2	14·46	1	Cl; lC	1
4643	2106	......	....	21 5 3·4	2·471	1	56 50 48·0	14·60	1	Cl; pRi; iF; st 11...15	1
4644	3858	......	....	21 5 39·3	4·079	2	137 47 45·7	14·61	2	pB; pL; lE; gbM	2
4645	2107	......	....	21 6 14·3	2·150	1	44 53 32·7	14·61	1	Cl; vL; pRi; E; st 10... ...	1
4646	......	III. 209	....	21 6 52·3	2·860	1	76 58 16·2	14·66	1	vF; S; R......................	1
4647	3859	......	....	21 6 55·9	4·114	2	138 56 41·3	14·69	2	B; cS; cE; psmbM; *10 f...	2
4648	2110	VI. 24	....	21 7 45·4	2·253	2	48 4 44·0	14·70	2	Cl; vF; pL; vRi; vC; st 15...18	4
4649	2108	......	....	21 7 48·6	3·008	1	86 3 33·4	14·72	1	eF	1
4650	2109	III. 858	....	21 7 52·9	+3·035	2	87 44 17·4	−14·72	2	eF; pL; R; lbM..............	3

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	h.	H.		h m s	s		° ′ ″	″			
4651	3860	......	Δ. 406	21 9 26·5	+4·109	2	139 8 52·8	−14·84	2	vB; pS; E; mbM	2
4652	2111	......	....	21 9 33·0	2·424	1	54 23 16·7	14·81	1	Cl; no description	1
4653	2112	III. 145	....	21 12 36·6	2·640	1	64 9 17·3	14·99	1	F; S; vlE; r	3
4654	2113	......	....	21 14 20·1	3·216	1	99 22 29·7	15·11	1	vF; R; gbM; ⁑ nr	1*
4655	2114	......	....	21 15 26·3	1·717	1	32 59 50·8	15·14	1	Cl; F; pS; P	1
4656	3861	......	....	21 15 58·1	3·884	1	133 3 41·7	15·21	1	eF; vS; R; p of 2	1
4657	2115	......	....	21 16 16·6	2·021	1	39 47 4·3	15·19	1	Cl; P; lC............	1
4658	3862	......	....	21 16 41·6	4·607	1	150 37 16·9	15·27	1	B; pL; lC; gpmbM	1
4659	3863	......	....	21 16 53·5	3·879	1	133 0 26·2	15·26	1	vF; pS; R; f of 2	1
4660	3864	......	....	21 17 58·9	4·085	1	139 40 10·1	15·33	1	eeF; vS; R	1
4661	2116	VII. 51	....	21 18 12·1	2·181	3	44 13 3·7	15·31	3	Cl; pS; pRi; pC; st 13... ...	4
4662	2117	......	....	21 18 44·9	2·447	1	54 5 44·1	15·33	1	Cl; P; st 10...	1
4663	3865	......	....	21 19 9·1	4·222	1	143 23 17·3	15·39	1	eF; pL; vmE 90°·8; ✱s	1
4664	2118	VII. 50	....	21 19 37·1	2·135	1	42 34 45·6	15·38	1	Cl; P; ? neb	2
4665	3866	......	....	21 21 26·5	3·879	2	133 41 50·4	15·52	2	F; cL; lE; gvlbM; p of 2 ...	2
4666	2119	......	....	21 21 31·0	2·148	1	42 40 21·6	15·48	1	Cl; S; C; cE	1
4667	3867	......	....	21 21 37·5	3·880	2	133 45 57·1	15·53	2	F; S; R; vglbM; f of 2	2
4668	3868	......	....	21 22 49·4	3·757	1	129 14 1·3	15·59	2	cF; cS; R; pgbM	2
4669	......	III. 936	....	21 22 51·7	1·459	1	27 43 29·5	15·55	1	vB; cr	1
4670	2120	......	M. 15= Lal. 40815	21 23 9·9	2·895	1	78 27 22·3	15·59	2	!; ⊕; vB; vL; iR; vsmbM; rrr; st vS.	16
4671	3869	......	....	21 23 31·9	3·898	1	134 40 55·1	15·63	1	B; R; cS; psbM............	1
4672	2121	III. 859	....	21 24 18·3	3·043	1	88 7 23·2	15·66	1	F; S; R; mbM; ✱14s	2
4673	2122	VII. 52	....	21 24 22·5	2·188	1	43 31 7·5	15·65	1	Cl; L; cRi; lC; st 10...13 ...	2
4674	3870	......	Δ. 263 ?	21 24 38·7	4·809	2	154 31 11·7	15·71	2	pF; cL; vlE; vgpmbM; r ...	2
4675	2123	......	....	21 25 11·3	2·819	1	73 11 28·0	15·70	1	Cl; lC	1
4676	2124	VI. 32	....	21 25 42·8	2·044	2	39 1 55·7	15·71	2	Cl; cL; vRi; pC; st 11...16	3
4677	3871	......	....	21 25 44·2	3·800	2	131 26 32·5	15·75	2	cF; S; R; gbM	2
4678	2125	......	M. 2= Lal. 41928	21 26 12·5	3·091	2	91 26 37·2	15·76	3	!!; ⊕; B; vL; g, pmbM; rrr; st eS.	19†
4679	3872	......	....	21 26 36·3	4·251	1	145 10 46·0	15·80	1	pB; pL; vmE 127°·1; g, pslbM	1
4680	3873	......	....	21 26 51·0	3·699	1::	127 24 5·7	15·81	1::	eF; pL; vgbM; ✱6 f 40ˢ......	1
4681	2126	......	M. 39	21 27 12·6	2·160	1	42 10 58·0	15·80	1	Cl; vL; vP; vlC; st 7...10...	3
4682	2127	......	....	21 29 5·1	2·244	1	44 37 6·0	15·90	1	Cl; P; lC............	1
4683	3875	......	....	21 30 3·9	3·827	2	133 10 21·6	15·98	1	F; pL; R; vglbM; ✱13 inv...	2
4684	3874	......	....	21 30 20·1	4·755	1	154 32 2·4	16·02	1	vF; S; R; vS⁑ nf	1
4685	3877	......	....	21 31 24·1	3·821	2	133 10 19·5	16·05	2	B; S; vlE; mbM............	2
4686	3876	......	....	21 31 59·5	6·179	1	165 44 25·1	16·13	1	pF; R; g, psmbM; am st ...	1
4687	2128 = 3878	......	M. 30	21 32 26·0	3·422	2	113 47 59·0	16·10	2	!; ⊕; B; L; lE; gpmbM; st 12...16.	3†
4688	3879	......	....	21 32 59·7	4·134	1	143 20 29·8	16·14	1	eF; cS; lE; vglbM............	1
4689	3880	......	....	21 33 22·2	3·873	1	135 25 37·2	16·16	1	vF; cL; R; vglbM............	1
4690	3881	......	....	21 33 31·6	3·626	1	125 4 56·2	16·16	1	eF; vS; am st	1
4691	3882	......	....	21 33 48·4	3·620	1	124 48 14·9	16·17	2	F; S; R; bM	2
4692	3883	......	....	21 36 32·5	3·965	2	139 3 59·4	16·32	2	F; S; R; glbM; p of 2	2
4693	3884	......	....	21 36 57·1	3·961	2	138 59 48·8	16·34	2	F; S; R; glbM; f of 2	2
4694	3885	......	....	21 37 10·8	3·904	1	137 9 38·5	16·35	1	F; S; R; gbM	1
4695	......	......	Auw. N. 47	21 38 22·8	3·296	...	99 28 19·2	16·36	...	Nebulous ✱10·11 or vSCl (Cooper).	0
4696	3886	......	....	21 38 39·8	5·278	1	160 58 43·5	16·45	1	pB; S; R; vgbM; ✱9 f	1
4697	3888	......	....	21 38 45·1	4·022	3	141 12 49·8	16·44	3	pB; L; pmE; vgbM	3
4698	3887	......	....	21 38 56·2	4·466	1	151 21 18·5	16·45	1	eF; pL; R; p of 2	1
4699	3889	......	....	21 38 58·8	4·460	1	151 15 13·5	16·45	1	pB; pS; lE; gbM; f of 2 ...	1
4700	2129	......	....	21 39 8·8	2·013	1	36 1 22·4	16·42	1	Cl; S; P; lC	1
4701	2130	VII. 40	....	21 39 21·8	2·049	1	36 56 49·1	16·43	1	Cl; S; pRi; has a ruby ✱ ...	2
4702	2131	IV. 75	....	21 39 45·0	1·387	2	24 32 15·8	16·44	2	!; cF; pL; gbM ⁑✱	4
4703	3890	......	....	21 39 51·6	3·605	1	125 5 43·6	16·48	1	pB; S; R; glbM............	1
4704	3891	......	....	21 41 22·0	3·609	2	125 31 43·2	16·56	2	pB; pL; R; vgbM; ✱14 att p	2
4705	2132	II. 261	....	21 41 44·2	2·773	2	68 29 7·2	16·56	2	F; pS; R; vglbM; r	3
4706	......	III. 696	....	21 42 15·3	+1·576	4	26 49 29·9	−16·57	4	vF; cS; R; r	4

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	h.	H.		h m s	s		° ′ ″	″			
4707	3892	......	....	21 42 28·6	+4·229	1	147 12 6·1	−16·63	1	pF; cS; R; bM	1
4708	3893	......	....	21 42 30·4	4·187	1	146 13 34·1	16·63	1	F; L; R; g, psmbM	1
4709	2134	VII. 66	....	21 42 34·2	1·441	1	24 50 39·6	16·58	1	Cl; cL; cRi; pC; st 11...14	3
4710	2133	......	....	21 42 41·2	2·650	1	60 41 28·0	16·60	1	vF; ? a ⁑ inv in neb	1*
4711	3894	......	....	21 43 34·1	3·921	2	138 54 28·9	16·67	2	vB; pS; R; mbMN	2
4712	3895	......	....	21 44 13·2	3·907	1	138 32 26·0	16·70	1	B; S; R; in Δ of st 13.........	1
5077	......	......	....	21 45 12·5	......	...	40 53 46·4	......	...	See No. 5077.	
4713	3896	......	....	21 45 46·0	3·984	3	141 19 11·6	16·78	3	vF; pL; lE; vgbM; r.........	3
4714	3897	......	....	21 45 50·0	3·493	1	119 56 36·9	16·77	1	eeF; vS	1*
4715	3898	......	....	21 46 30·0	3·486	1	119 41 30·0	16·80	1	eF; S; E towards eF *	1
4716	3900	......	....	21 46 58·7	3·588	2	125 28 21·1	16·83	2	B; pL; iR; glbM; r	2
4717	3899	......	....	21 46 59·2	3·940	2	140 10 49·8	16·84	2	pB; S; lE; mbM...............	2
4718	2135	III. 452	....	21 47 29·0	3·042	3	87 42 49·8	16·84	3	F; pL; R; bM; r	4
4719	2136	VIII. 67	....	21 49 48·4	1·722	1	28 3 3·1	16·93	1	Cl; P; viC	2
4720	......	......	D'Arrest, 115	21 49 53	3·04	[2]	87 42 42	16·95	[2]	Cl; vS; st 19 m; bet 2 st 16...	0
4721	3901	......	....	21 50 58·1	3·752	1	133 58 28·4	17·02	1	cF; cL; cE; glbM	2
4722	3902	......	....	21 51 10·5	3·521	3	122 33 8·4	17·02	3	F; pL; vlE; vglbM	3
4723	2137	III. 930	....	21 51 49·3	3·289	1	107 10 39·5	17·05	1	eF	2*
4724	3903	......	....	21 51 53·6	3·750	1	134 3 32·2	17·06	1	cB; S; vlE; svbMN	2
4725	3905	......	....	21 52 37·2	3·401	1	115 18 18·3	17·09	1	F; pS; R; vglbM; *10 f ...	1
4726	3904	......	....	21 52 54·3	3·975	3	142 25 3·4	17·12	3	pB; S; R; psbM...............	3
4727	3906	......	....	21 53 6·8	3·852	1	138 21 46·4	17·12	1	eF; S; R; *8 np...............	1
4728	2138	III. 692	....	21 53 29·4	3·243	1	103 57 3·1	17·13	3	vF; cL; E 135°±; vgbM ...	4
4729	3908	......	....	21 53 52·3	3·513	4	122 32 38·5	17·15	4	pB; pL; lE; gbM; 1st of 4...	4†
4730	3909	......	....	21 53 53·1	3·514	4	122 38 8·5	17·15	4	cB; cS; R; sbM*; 2nd of 4	4†
4731	3910	......	....	21 53 55·3	3·515	2	122 39 58·5	17·15	2	cF; S; R; p of D neb; 3rd of 4	2†
4732	2141	......	....	21 53 55·9	2·106	1	35 50 51·1	17·13	1	Cl; vL; pRi; lC	1
4733	3911	......	....	21 53 59·0	3·514	6	122 39 31·5	17·15	6	B; pL; R; f of D neb; 4th of 4	6†
4734	2139	II. 247	....	21 54 0·1	2·858	1	72 56 7·5	17·15	3	pB; pS; R; bMN; r; *sp...	4†
4735	3912	......	....	21 54 6·3	3·582	1	126 28 45·2	17·16	1	eF; S; R; *8 s 2′	1
4736	3907	......	....	21 54 21·4	4·525	2	154 43 13·6	17·18	2	cF; pS; vgbM..................	2
4737	2140	III. 693	....	21 54 26·8	3·339	1	111 13 26·9	17·17	1	vF; S; R; lbM; p of 2	2
4738	2142	II. 595	....	21 54 38·9	3·316	1	109 35 22·6	17·18	2	vF; pS; vlE 90°; lbM.........	4
4739	2143	II. 1	....	21 54 51·2	3·341	1	111 28 50·3	17·19	1	pB; pL; mE 64°·3 bet 3 st; er	3
4740	2144	......	....	21 55 8·7	3·337	1	111 8 59·7	17·21	1	vF; pL; iR; vglbM; f of 2...	1
4741	......	III. 165	....	21 55 13·6	2·599	1	55 33 55·0	17·20	1	vF; am 5 or 6 st	1
4742	3913	.....?	....	21 56 20·4	4·543	1	155 19 8·9	17·27	1	vF; S; lE; vgbM	1
4743	3914	......	....	21 56 24·7	4·521	2	154 59 23·6	17·28	2	pB; S; R; pmbM	2
4744	2145	......	....	21 56 42·8	2·949	1	79 51 21·9	17·27	1	Cl; lRi; lC; st 9...10	1
4745	3915	......	....	21 56 50·6	3·900	3	140 47 57·3	17·29	4	cB; S; R; am st	4
4746	2146	II. 599	....	21 57 7·6	2·493	1	49 37 29·6	17·28	1	F; cS; cE; vglbM; er	2
4747	3916	......	....	21 58 4·3	4·529	2	155 22 59·5	17·35	2	vF; S; R; pslbM; *11 p 3′...	2
4748	3917	......	....	21 58 7·9	3·889	1	140 40 49·5	17·35	1	pF; S; R; smbM	1
4749	3918	......	....	21 58 27·0	3·488	3	121 55 30·5	17·35	3	F; R; gbM; 1st of 4	3
4750	3920	......	....	21 58 39·2	3·486	1	121 51 49·2	17·36	1	eF; S; stellar; 2nd of 4	1
4751	3921	......	....	21 58 40·2	3·488	3	121 49 42·2	17·36	3	cF; R; stellar; 3rd of 4	3
4752	3922	......	....	21 58 47·8	3·484	1	121 43 31·9	17·37	1	pB; L; lE; gbM; 4th of 4...	1
4753	3919	......	....	21 58 53·7	4·140	1	148 6 29·6	17·38	1	pB; L; cE; gpslbM	2
4754	3923	......	....	21 59 24·6	3·451	1	119 43 57·3	17·39	1	vF; vS; R; almost a ○	1
4755	2147	VII. 53	....	21 59 42·2	2·384	2	44 11 36·3	17·39	2	Cl; L; cRi; pC; st 9...12 ...	4
4756	2148	......	....	21 59 57·7	2·741	1	63 34 17·7	17·41	1	eF; R; bM; vF ⁑ np	1*
4757	3924	......	....	22 0 29·0	3·798	1	137 50 36·5	17·45	1	vB; pS; R; gbM...............	1
4758	3926	......	....	22 1 8·3	3·427	1	118 29 42·9	17·47	1	⊕; pL; iR; rr..................	1
4759	3925	......	....	22 1 30·8	4·794	1	159 20 46·7	17·51	1	pF; S; R; gbM	1
4760	2149	II. 207	....	22 1 37·4	2·682	1	59 20 35·6	17·48	1	B; pL; gbM; er	2
4761	2150	II. 897	....	22 2 34·9	3·273	1	107 19 55·1	17·53	1	pB; lE; r........................	2
4762	3927	......	....	22 2 47·3	4·487	2	155 32 6·2	17·56	2	pB; S; R; 2 st nr............	2
4763	3928	......	....	22 3 13·8	3·462	4	121 14 22·2	17·56	4	F; S; R; gbM; r; 2vS st nr...	4
4764	2151	III. 862	....	22 4 16·2	2·525	1	49 40 48·3	17·59	1	eF; pS; lE; r; am 3 st.........	2
4765	3929	......	....	22 5 15·4	3·392	1	116 50 37·8	17·64	1	pF; S; lE; bM	1
4766	3930	......	....	22 6 4·9	+3·436	4	120 4 3·9	−17·67	4	F; pL; R; vglbM	4

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	h.	H.		h m s	s		° ′ ″	″			
4767	2152	III. 931	....	22 6 37·7	+3·272	1	107 45 40·3	−17·69	1	vF; S; R; bM	2
4768	2153	II. 606	....	22 6 43·4	2·447	1	45 20 52·3	17·69	1	eF; S; er	4
4769	3931	......	....	22 6 57·4	3·731	1	136 32 31·7	17·71	1	pB; S; pmE; psbM; p of 2...	1
4770	3932	......	....	22 7 7·9	3·730	1	136 32 16·4	17·72	1	F; vS; R; *8 f; f of 2	1
4771	......	VIII. 63	....	22 7 8·2	2·127	1	33 42 56·0	17·70	1	Cl; S; P; lC	1
4772	2154	......	....	22 7 34·6	2·119	2	33 25 12·4	17·72	2	Cl; pC; has a ruby *10	2
4773	2155	VIII. 75	....	22 9 41·1	2·358	1	40 48 55·7	17·81	1	Cl; L; P; lC; st vL	3
4774	2157	VI. 29	....	22 10 2·5	2·236	1	36 22 2·4	17·82	1	Cl; C; st eS	2
4775	2156	III. 932	....	22 10 8·7	3·249	1	106 16 47·8	17·84	1	vF; S; vlE; vgbM; *13n ...	2*
4776	......	III. 863	....	22 11 0·4	2·563	1	50 10 13·2	17·86	1	vF; vS; mbM	1
4777	3933	......	....	22 11 26·0	3·957	1	145 48 53·0	17·90	1	eeF; R; (?)	1
4778	......	III. 864	....	22 12 28·8	2·569	1	50 8 9·4	17·92	1	vF; S; mE 165°±	1
4779	2158	III. 933	....	22 12 53·8	3·247	2	106 28 21·5	17·95	2	F; pS; R; gpmbM	4
4780	3934	III. 458	....	22 12 57·6	3·353	1	115 22 44·5	17·95	1	F; S; R; er	2
4781	3935	......	....	22 15 6·5	3·395	1	119 3 25·8	18·04	1	vF; S; E; glbM; ?biN	1
4782	3936	......	....	22 15 10·0	3·403	1	119 39 16·8	18·04	1	eF; pL; R; vlbM	1
4783	2159	......	....	22 15 23·6	2·153	1	32 36 51·1	18·03	1	Cl; L; pRi; lC	1
4784	3937	......	....	22 15 26·0	3·450	1	123 3 31·5	18·05	1	eF; S; R; lbM	1
4785	3938	......	....	22 16 21·1	3·466	2	124 24 1·6	18·08	1	cB; pS; vlE; glbM; B ⁂ sp	2
4786	3939	......	....	22 16 44·2	3·429	3	121 53 45·0	18·10	3	F; cS; vlE; p of 2	4
4787	3940	......	....	22 18 15·5	3·424	4	121 51 23·5	18·15	3	F; cS; vlE; f of 2	4
4788	3941	......	....	22 18 57·4	4·079	1	150 52 54·3	18·19	1	eeF; lE; vgvlbM; 3 st sf ...	1
4789	3942	......	....	22 19 7·4	3·478	2	125 51 19·3	18·19	2	vF; pS; R; vgvlbM	2
4790	2160	II. 248	....	22 19 38·1	2·916	2	74 33 55·0	18·20	2	F; cS; R; gbMS*; 3 st nr...	4
4791	2161	......	....	22 19 39·4	2·198	1	32 52 26·3	18·19	1	Cl; L; pRi; lC; st 10...16...	1
4792	3943	II. 469	....	22 20 50·7	3·336	2	115 34 20·5	18·25	2	cF; cS; lE; r; ⁂ inv	3
5078	......	......	....	22 20 50±	......	...	115 34 ±	......	...	(See No. 5078)	0
4793	2162	......	....	22 20 52·2	2·770	1	61 36 48·8	18·24	1	vF; S; R; am st	1
4794	3944	......	....	22 21 12·0	3·474	1	126 10 21·2	18·26	1	vF; S; R; gbM	1
4795	......	......	Auw. N. 48	22 22 5·6	3·285	...	111 32 59·8	18·29	...	!; pF; vL; E or biN (Harding)	0
4796	2163	......	....	22 22 21·2	2·365	1	37 53 26·3	18·29	1	Cl; P; lC; st 12...13	1
4797	......	VII. 41	....	22 22 37·4	2·380	2	38 24 45·0	18·30	2	Cl; iR; lC; st vS	2
4798	3945	......	....	22 22 59·4	3·503	1	128 33 0·1	18·33	1	eF; S; R; p of 2	2
4799	2165, a	......	R. nova	22 23 13·6	3·213	::	104 40 30·3	18·34	::	vF; E np to sf (nisi=h.2164)	0
4800	3946	......	....	22 23 23·8	3·501	1	128 32 9·8	18·34	1	eF; S; R; f of 2	2
4801	2164	......	....	22 23 28·3	3·213	2	104 44 11·8	18·34	2	vF; cS; vglbM	2
4802	2165	IV. 31	....	22 24 53·0	3·213	4	104 50 30·3	18·39	4	F; pS; R; vsbMFSRN	5
4803	......	......	D'Arrest, 116	22 24 57	2·76	[1]	59 45 18	18·38	[1]	vF; pL; vlbM; h.2166 dist 2′	0
4804	2166	......	....	22 24 58·6	2·760	2	59 45 25·3	18·39	2	vF; S; R; gvlbM	2
4805	3948	......	....	22 25 28·5	3·352	1	117 58 4·4	18·42	1	vF; S; lE; *11 p	1
4806	3947	......	....	22 25 32·6	3·541	1	131 39 53·4	18·42	1	F; pL; pmE	1
4807	2167	II. 476	....	22 26 55·7	3·173	5	101 4 53·2	18·46	5	vF; pL; R; glbM; r	6
4808	2168	II. 428	....	22 27 3·9	3·027	2	85 9 17·9	18·47	2	pF; S; R; psbM; r	4
4809	2169	III. 180	....	22 27 40·9	2·886	1::	70 23 45·6	18·48	1::	eF; vS; R; *9 s	2
4810	3949	......	....	22 28 2·3	3·331	2	116 45 55·0	18·50	3	cB; L; mE 0°; vlbM	3
4811	2170	III. 237	....	22 29 46·2	2·877	1	69 5 50·5	18·55	1	F; S; iR; vgvlbM	2
4812	3950	......	....	22 29 46·2	3·466	2	127 57 13·5	18·55	2	vF; S; vlE; gbM	2
4813	2171	......	....	22 30 32·5	2·984	3	80 11 4·6	18·58	4	eF; pS; lE 90°; vglbM	4
4814	3951	......	....	22 30 37·2	4·257	1	157 12 16·0	18·60	1	pB; pS; mE 90°	1
4815	2172	I. 53	....	22 30 39·5	2·736	1	56 20 5·6	18·58	3	B; pL; pmE 160°; smbM ...	4†
4816 4817 4818 4819 4820	2172, a	......	R. 5 novæ	22 30 ±	2·736	...	56 20 ±	18·58	...	5 near; positions measured; no distances.	0*
4821	2173	II. 233	....	22 30 44·0	2·857	2	66 55 34·6	18·58	2	cB; S; mE 163°·0; vsmbM *11; p of 2.	5
4822	3950	(No. 2?)	....	22 30 50·5	3·466	1	127 55 48·4	18·62	1	eF; ??	1 ?
4823	2174	III. 166	....	22 30 54·0	2·736	1	56 17 51·3	18·59	1	eF; vS; E	2
4824	2175	II. 234	....	22 31 5·7	+2·858	2	66 55 58·3	−18·59	2	F; pS; mE 90°±; vglbM; f of 2.	5

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	h.	H.		h m s	s		° ′ ″	″			
4825	2176	......	....	22 32 57·9	+2·982	1	79 42 40·2	−18·66	1	eF; pL; E	2
4826	2177	......	....	22 34 15·4	2·333	1	33 20 2·3	18·69	1	Cl; vL; pRi; vlC	1
4827	2178	II. 705	....	22 35 6·6	2·218	1	29 27 5·4	18·72	1	B; S; R; pgvlbM; er.........	2
4828	3952	......	....	22 35 26·3	3·451	1	128 36 27·8	18·74	1	eeF; S; R; ⁑ f 90°, 40ˢ	1
4829	3953	......	Δ. 255?	22 36 9·6	4·117	1	155 51 19·9	18·77	1	F; S; R; bM	1
4830	3954	......	....	22 36 30·8	3·351	1	120 47 3·9	18·77	1	F; pL; vmE0°; vgvlbM......	1
4831	2179	II. 442	....	22 37 14·0	3·079	2	90 53 42·0	18·80	2	F; S; R; psbM	5
4832	3955	......	....	22 37 30·5	3·475	1	130 4 35·7	18·81	1	F; cS; lE; glbM	1
4833	2180	II. 477	....	22 38 40·4	3·166	4	101 44 29·8	18·84	5	vF; pL; R; lbM	6
4834	2181	II. 598	....	22 40 12·8	3·262	3	113 2 10·3	18·89	3	pB; S; vlE; vgmbM; S ✱ nr	4
4835											
4836											
4837											
4838	2181, *a*	......	R. 7 novæ	22 40 ±	3·262	...	113 2 ±	18·89	...	No descriptions	0
4839											
4840											
4841											
4842	2182	VIII. 77	C. H.	22 41 23·2	2·374	2	32 39 8·7	18·91	2	Cl; pL; pRi; lC; st 9...13 ..	3
4843	3956	......	....	22 42 23·0	3·408	1	127 34 56·5	18·95	1	eF; vS; R; ✱ 12 att np	1
4844	......	......	D'Arrest, 117= R. nova	22 42 30	2·99	[2]::	79 10 30	18·94	[2]	vF; vS; R; III. 216 nf	0
4845	2183	III. 216	....	22 42 56·0	2·988	1	79 7 46·2	18·96	2	cF; S; R; glbM; ✱11 np ...	5
4846	2184	III. 217	....	22 43 2·5	2·987	1	79 2 36·2	18·96	2	cF; S; R; pgbM; f of 2......	5
4847	......	......	D'Arrest, 118	22 43 20	2·99	[2]	79 6 0	18·97	[2]	eF; vS; R; 2 st 11, s	0
4848	2184, *a*	......	R. nova	22 43 22·5	2·99	...	79 8 2·2	18·96	...	One of 5. See h. 2183, 2184, D'Arrest, 117, 118.	0*
4849	2185	II. 443	....	22 43 25·0	3·088	2	92 16 54·6	18·98	2	cF; cS; R; sbM ✱ 13; ✱ np ..	4
4850	2186	II. 702	....	22 44 15·4	3·238	3	111 21 1·0	19·00	3	pB; pS; lE 120° ±; mbM ...	4
4851	2187	II. 453	....	22 44 21·1	3·118	2	96 18 6·0	19·00	2	vF; pL; lE; vgbM; r.........	3
4852	2188	......	....	22 44 36·2	2·534	1	38 33 54·0	19·00	1	Cl; vP	1
4853	2189	......	....	22 45 12·8	3·069	2	89 38 56·4	19·02	2	pF; pS; R; gbM	2
4854											
4855	2189, *a*	......	R. 3 novæ	22 45 ±	3·069	...	89 39 ±	19·02	...	"A group of 4," incl h. 2189	0
4856											
4857	3957	......	....	22 45 42·2	3·506	1	136 5 36·8	19·04	1	pF; lE; glbM; vS ✱ inv	1
4858	3958	......	....	22 46 23·9	3·420	1	130 3 30·2	19·06	1	vF; S; R	1
4859	3959	......	....	22 46 49·0	3·930	1	154 26 29·6	19·08	2	pB; pS; R; vglbM	1
4860	3960	......	Δ. 518	22 47 2·8	3·421	2	130 24 35·6	19·08	2	cB; L; vmE 43°·3; mbM ...	2
4861	3961	......	....	22 47 45·7	3·456	1	133 23 49·0	19·10	1	eF; vL; ✱ 7 nf..................	1
4862	3962	......	....	22 48 42·2	3·940	1	155 46 26·1	19·13	1	pB; cS; R; gpmbM	2
4863	3963	......	....	22 48 42·4	3·383	3	127 46 23·4	19·12	3	cB; vL; vlE; vglbM	3
4864	2190	VII. 43	....	22 48 46·6	2·356	2	29 55 6·3	19·09	2	Cl; pRi; cC.....................	3
4865	3964	......	....	22 49 1·9	3·385	3	128 5 31·1	19·13	4	cB; L; vlE; gpmbM; rr......	4
4866	2191	III. 745	....	22 49 18·9	2·466	1	33 38 14·1	19·13	1	vF; pL; iF; er	2
4867	3965	......	....	22 49 21·4	3·428	2	131 49 4·8	19·14	2	F; cL; vlE; vgmbM	2
4868	2192	III. 576	....	22 49 28·5	2·784	1	54 22 38·8	19·14	1	vF; cS; R; stellar; ⁑ p	3
4869	2193	......	....	22 50 18·4	2·404	1	30 45 23·2	19·16	1	Cl; P; pC; st 9...11	1
4870	2194	III. 465	....	22 51 3·9	2·986	1	77 37 14·6	19·18	1	eF; S; R	4
4871	2195	III. 243	....	22 51 12·3	2·886	1	64 37 4·6	19·18	2	F; pS; E 90°; gbM; er	3
4872	2195, *a*	......	R. 2 novæ	22 51 ±	2·886	...	64 37 ±	19·18	...	2 of a group of 3; inv in F neb.	0
4873											
4874	2196	......	....	22 51 36·5	2·543	1	36 23 55·3	19·19	1	Cl; vL; E	1
4875	......	......	D'Arrest, 119	22 52 32	2·97	[2]	75 12 12	19·21	[2]	pF; R; bet 2 st 16; ✱ 13 nr...	0
4876	2197	II. 450	....	22 52 46·6	3·163	2	103 33 10·4	19·22	2	F; vS; vlE; smbM; er; p of 2	4†
4877	2198	II. 451	....	22 52 46·9	3·163	2	103 34 46·4	19·22	3	F; vS; vlE; smbM; er; f of 2	5†
4878	......	......	Auw. N. 49	22 53 6·3	3·144	...	101 16 41·1	19·23	...	✱ 11·12 in neb (Markree Obs. Oct. 8, 1855).	0
4879	2199	II. 251	....	22 53 8·0	2·968	2	74 46 10·1	19·23	2	pB; L; E 175°; vgbM	3
4880	......	II. 249	....	22 54 3·1	2·967	2	74 21 17·2	19·26	2	F; cS; lE; lbM; pB ✱ p......	2
4881	3966	......	....	22 54 13·6	+3·387	1	130 19 19·2	−19·26	1	F; L; mE 33°·8; vglbM......	1

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	h.	H.		h m s	s		° ′ ″	″			
4882	3967	......	....	22 54 16·6	+3·387	1	130 20 14·9	−19·27	1	eF; vL; vlbM	1
4883	2201	II. 212	....	22 54 17·6	2·861	1	60 36 32·2	19·26	1	cB; cL; lE; gmbM; r; 2Sst n	2
4884	2200	II. 590	....	22 54 20·5	3·065	2	88 59 35·2	19·26	2	cF; cS; psbM ...*...........	3
4885	3968	......	....	22 54 55·7	3·397	2	131 34 57·6	19·28	2	cF; pS; vmE 5°±; * 11 np...	2
4886	2202	III. 210	....	22 54 56·3	2·971	1	74 46 22·6	19·28	1	vF; S; lE; p of 2............	2
4887	2203	III. 211	....	22 55 5·3	2·971	1	74 46 59·6	19·28	1	vF; vS; f of 2	2
4888	2204	III. 230	....	22 56 12·2	3·020	1	81 52 19·7	19·31	1	vF; vS; vsmbM * 12	2
4889	3969	......	....	22 56 13·3	3·514	1	140 52 10·7	19·31	1	eF; pL; R; glbM; *11 np ...	1
4890	......	III. 202	....	22 56 22·3	2·969	1	74 8 43·7	19·31	1	eF; vS........................	1
4891	3970	......	....	22 57 19·6	3·411	2	133 51 40·8	19·34	2	F; S; R; Δ with 2 st 7	2
4892	2205	I. 55	....	22 57 56·4	2·999	4	78 25 53·5	19·35	4	pB; cL; mE 11°·9; bet 2 st...	7*†
4893	2206	......	....	22 58 40·4	3·055	1	87 12 42·2	19·36	1	vF; S; E; psbM	1
4894	3971 = 3972	......	....	22 59 21·2	3·330	4	127 1 37·6	19·38	4	pB; S; R; lbM; *8·9 att s...	4*
4895	2207	......	....	22 59 23·7	2·845	1	56 39 4·6	19·38	1	vF; S; R; bM; *10 p	1
4896	2208	III. 558	....	23 1 2·4	3·169	1	106 22 33·4	19·42	1	eF; L; bet 2 D st	2
4897	3973	......	....	23 1 56·3	3·379	2	134 10 34·8	19·44	2	pB; cL; lE; vgbM * 13	2
4898	2209	III. 203	....	23 2 5·5	2·971	2	72 35 4·8	19·44	2	vF; L; pmE 45°; lbM.........	3
4899	2210	III. 184	....	23 4 29·6	3·087	2	92 54 44·3	19·48	2	cF; vS; R; sbM * 15	4
4900	2211 = 3974	II. 2	....	23 4 33·6	3·247	2	119 17 53·3	19·49	2	pB; cS; R; psvmbM; *10 np	5
4901	2212	......	....	23 4 45·0	3·004	2	77 49 34·0	19·50	2	eF; bM*; (?)	2
4902	2213	VII. 44	....	23 5 26·7	2·536	1	30 11 23·7	19·51	2	Cl; pRi; pC; fan-sh; st pB...	4
4903	2214	III. 220	....	23 5 46·8	3·006	2	78 4 51·4	19·52	3	F; cS; R; vglbM; r	7
4904	......	III. 470	....	23 6 36·7	3·125	1	99 57 29·1	19·53	1	eF; vS........................	1
4905	3975	......	....	23 7 0·8	3·364	1	134 21 49·8	19·54	1	pB; S; lE; pgbM	1
4906	2215	II. 429	....	23 7 28·8	3·052	2	86 15 44·5	19·55	2	vF; cS; R; bM; sp of 2	3
4907	......	II. 706	....	23 7 34·3	2·537	2	29 14 59·5	19·55	2	vF; L; 2 pB st inv	2
4908	2217	......	....	23 7 36·2	2·945	1	67 4 19·5	19·55	1	F; S; R; psbM	1
4909	2216	II. 430	....	23 7 37·8	3·052	2	86 13 35·5	19·55	2	B; L; mE 97°·5 (2 est); mbM; nf of 2.	3
4910	3976	......	....	23 7 49·1	3·313	1	129 17 54·2	19·56	2	F; S; vlE; vgvlbM; *10 att .	2
4911	2218	......	....	23 8 5·0	2·975	1	71 46 56·2	19·56	1	np of 2 neb	1
4912, 4913	2218, a	......	R. 2 novæ	23 8 ±	2·975	...	71 47 ±	19·56	...	2 of 4 incl h. 2218, 2219 ...	0
4914	......	......	D'Arrest, 120	23 8 17	2·94	[2]	65 29 18	19·57	[2]	vF; vS; (d'Arrest) *16 p 11ˢ	0
4915	2219	III. 181	....	23 8 17·5	2·975	1	71 47 55·9	19·57	1	cF; S; R; sf of 2............	2
4916	3977	......	Δ. 475 ?	23 8 25·2	3·346	1	133 21 29·9	19·57	1	B; S; mE 90°±; vsbM*13...	1
4917	2224, a	......	R. nova	23 8 27·0	3·041	::	84 3 21·6	19·57	::	No description	0
4918	2221	......	....	23 8 30·3	3·010	1::	78 11 12·9	19·57	1::	F; R; bM	1
4919	2220	II. 235	....	23 8 32·3	3·087	1	93 9 4·9	19·57	1	cF; pL; R; B ** f	3
4920	2222	III. 221	....	23 8 44·3	3·007	3	77 27 35·6	19·58	3	F; cS; R; bM*6; p of 2......	6
4921	2224	II. 467	....	23 8 51·2	3·041	1	84 4 21·6	19·58	1	cB; pS; iR; psbM	2
4922	2223	III. 222	....	23 8 53·6	3·008	2	77 34 16·6	19·58	4	cF; cS; R; sbM*16	6*
4923	......	III. 185	....	23 9 8·7	3·087	2	93 6 55·6	19·58	2	vF; pS; E; er; 3S st inv.......	2
4924	......	III. 238	....	23 9 43·0	3·007	1	77 17 25·3	19·59	1	eF; eS	1
4925	......	II. 454	....	23 10 14·0	3·099	1	95 30 25·0	19·60	1	F; S; smbM	1
4926	2225	III. 182	....	23 10 14·2	2·980	1	72 4 20·0	19·60	1	vF; am vS st	3
4927	3978	......	Δ. 476 ?	23 10 39·7	3·330	1	132 52 58·7	19·61	1	pB; L; pmE; gbM.............	1
4928	2226 = 3979	II. 236	....	23 10 47·0	3·098	2	95 24 42·7	19·61	2	pB; pS; iR; gbM	6
4929	3980	......	Δ. 477, 1	23 11 9·4	3·329	1	132 59 53·4	19·62	1	pB; pL; pmE; gbM; p of 2..	1
4930	......	III. 186	....	23 11 21·9	3·097	1	95 12 23·4	19·62	1	eF; vS	1
4931	3981	......	Δ. 477, 2	23 11 36·8	3·326	1	133 1 8·1	19·63	1	F; pL; pmE; gbM; f of 2 ...	1
4932	2227	II. 431	....	23 11 38·9	3·112	1	98 19 42·1	19·63	2	cF; S; R; pspmbM............	4
4933	2228 = 3982	I. 104	....	23 11 49·7	+3·116	2	99 15 31·1	−19·63	2	pF; cL; pmE 0°±	3*

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	h.	H.		h m s	s		° ′ ″	″			
4934	......	......	D'Arrest, 121	23 12 30	+3·04	[2]	82 42 24	−19·64	[2]	F; S; R; Δ with 2 st 19, n ...	0
4935	2229	......	....	23 12 51·0	3·036	1	82 21 19·5	19·65	1	eF; eS	1
4936	2230	II. 439	....	23 13 9·5	3·037	2	82 33 59·5	19·65	2	cB; pS; R; psbM	3
4937	3983	......	....	23 13 21·1	3·584	1	152 53 32·2	19·66	1	eF; eS; am 5 st; (?)	1
4938	2231	III. 435	....	23 13 25·2	3·036	1	82 21 49·2	19·66	1	F; vS; R; psbM	2
4939	2232	II. 250	....	23 13 27·2	2·994	2	73 32 34·2	19·66	2	pB; cS; R; smbM	4
4940	2233	II. 440	....	23 13 38·0	3·037	2	82 32 58·2	19·66	1	cB; pS; R; psbM	3
4941	......	......	D'Arrest	23 14 5·8	2·998	[4]	74 12 23·9	19·67	[4]	cB; pS; bM✱ (D'Arrest, Resultate).	0*
4942	2231, *a*	......	R. nova	23 14 13·7	3·036	::	82 21 49	19·67	::	E p and f	0
4943	......	......	D'Arrest, 122	23 14 22	3·04	[2]	82 33 6	19·67	[2]	vF; vS	0
4944	3985	......	....	23 14 21·5	3·314	2	133 14 52·6	19·68	2	F; S; R; lbM	2
4945	3986	......	....	23 14 30·5	3·717	2	158 25 49·6	19·68	2	F; vS; E 90°; psbM	2
4946	2234	II. 441	....	23 14 36·4	3·035	1	81 52 38·6	19·68	1	F; S; F ⁑ att	2
4947	2235	IV. 52	....	23 14 38·8	2·616	1	29 35 10·6	19·68	1	vF; ✱9 inv a l excentric	3
4948	3987	......	....	23 15 3·6	3·219	1	120 2 41·3	19·69	1	eF; S; R; sbM	1
4949	3984	......	....	23 15 6·0	4·986	1	172 40 19·0	19·70	1	vF; pL; R; vlbM; ✱ nr	1
4950	2236	II. 600	....	23 15 23·6	2·858	2	49 54 46·3	19·69	2	cF; L; mE 0° ±; vlbM; r ...	4†
4951	3988	......	....	23 16 17·5	3·215	1	120 8 51·7	19·71	1	vF; S; R; glbM	1
4952	......	III. 473	....	23 16 47·9	3·002	1	74 0 18·4	19·72	1	eF; cL; sc st f; (?)	1
4953	......	III. 218	....	23 16 48·7	3·032	1	80 50 18·4	19·72	1	eF; pS; lE	1
4954	3989	......	....	23 17 17·8	3·464	2	148 33 48·1	19·73	2	pF; pS; R; glbM; p of 2 ...	2
4955	3990	......	....	23 17 37·9	3·462	1	148 39 59·1	19·73	1	eF; S; R; f of 2	1
4956	2237	......	....	23 17 51·8	3·010	1	75 29 12·1	19·73	1	vF; pS; R; gbM	2
4957	2238	......	M. 52	23 18 3·2	2·643	1	29 10 20·1	19·73	1	Cl; L; Ri; mCM; R; st 9...13	8
4958	3991	......	....	23 18 14·6	3·674	1	158 47 39·8	19·74	1	eF; vS; R; pslbM; ✱10 p 22ˢ	2
4959	3992	......	....	23 18 40·6	3·452	1	148 34 57·5	19·75	1	eF; R	1
4960	3994	......	....	23 18 48·0	3·266	1	129 59 55·5	19·75	2	{eF; S; R / eF; S; R} D neb; 4 st p..	2
4961	2239	III. 212	....	23 18 52·9	3·016	1	76 33 33·5	19·75	1	vF; vS; R; psbM	2
4962	2240	......	....	23 18 52·9	2·956	1	63 44 7·5	19·75	1	F; vS; psmbM; ✱10 p	1
4963	3993	......	....	23 18 56·4	3·590	1	156 2 49·5	19·75	1	eF; cL; R; vgvlbM	1
4964	2241	IV. 18	....	23 19 9·9	2·864	5	48 13 57·5	19·75	5	!!!; ○; vB; pS; R; blue......	16*†
4965	......	III. 438	....	23 20 0·9	3·112	1	100 11 16·2	19·76	1	eF; S; stellar	1
4966	2242	III. 226	....	23 20 15·0	3·025	4	78 18 33·9	19·77	4	{(H.) vF / (h.) pB} S; R; vsmbM; ✱9 p (? var).	6*
4967	2242, *a*	......	R. nova	23 20 15·0	3·025	::	78 24 33·9	19·77	::	vF; S; 6′ s of h. 2242	0
4968	2243	......	....	23 20 49·8	3·040	1	81 59 10·6	19·78	1	F; cS; gbM; p of 2............	1
4969	2244	......	....	23 20 59·8	3·041	1	81 59 55·6	19·78	1	vF; S; R; gbM; f of 2	1
4970	3995	......	....	23 21 2·6	3·457	1	150 29 2·6	19·78	1	B; S; lE; vsvmbM✱11	1
4971	2245	II. 226	....	23 21 28·0	2·985	2	68 20 38·3	17·79	2	vF; pL; vlE; lbM; am 4 st...	3†
4972	2246	III. 860	....	23 21 36·7	2·938	(1)	58 25 6·3	19·79	1::	vF; S; R; lbM; r	2
4973	2247	II. 242	....	23 21 52·1	3·008	2	73 28 4·3	19·79	2	vF; S; iR; r; ⁑ f	4
4974	......	......	D'Arrest, 123	23 21 55	3·06	[1]	87 19 4	19·79	[2]	eF; ✱14 p 13ˢ·7, l n............	0
4975	2248	III. 426	....	23 23 21·9	3·061	1	86 52 7·7	19·81	1	eF; cL; R ✱ gbM; ⁑ nr	3
4976	2249	VIII. 69	....	23 23 29·0	2·839	2	41 38 50·7	19·81	2	Cl; P; lC; st 7...11	4
4977	......	......	D'Arrest, 124	23 23 47	3·06	[2]	87 12 42	19·81	[2]	vF; vS; ✱11 f 1ˢ, n 85″	0
4978	3996	......	Δ. 347 ?	23 24 48·4	3·352	1	144 52 22·1	19·83	1	pF; L; R; vgbM	1
4979	3997	......	....	23 25 20·0	3·325	2	142 28 11·8	19·84	2	cB; S; lE; psbM; ✱8 f	2
4980	2250	III. 213	....	23 25 20·6	3·019	1	74 55 35·8	19·84	1	eF; pL; Δ with 2 st 10	2*
5079	......	......	....	23 25 33·1	......	...	96 22 12·0	......	...	See No. 5079.	
4981	......	III. 187	....	23 25 56·9	3·084	2	93 31 10·5	19·85	2	eF; pL; stellar	1
4982	......	......	D'Arrest, 125	23 26 4	3·09	[1]	93 39 12	19·85	[1]	eF; pL; 3 st 11 & 12 f.,........	0
4983	3998	......	....	23 26 48·5	3·494	1	156 19 38·2	19·86	1	eeF; pL (certain)	1
4984	......	III. 188	....	23 27 3·0	3·084	1	93 41 10·2	19·86	1	eF; stellar	1
4985	3999	......	....	23 27 43·9	3·347	2	146 47 2·9	19·87	2	B; cS; E; g, sbM; ✱8·9 p ...	2
4986	2251	......	....	23 27 44·2	3·022	1	74 42 26·9	19·87	1	vF; vS; gbM; ✱ nf 1′.........	1
4987	2252	......	....	23 27 53·0	3·059	2	85 52 26·9	19·87	2	eF; ✱12 p; sp of 2	2
4988	2253	......	....	23 28 2·0	3·058	1	85 49 1·9	19·87	1	vF; nf of 2	1
4989	2254	III. 579	....	23 28 2·6	2·900	2	46 27 42·9	19·87	2	eF; S; R; ✱9·10 p, v nr......	3
4990	2255	VIII. 62	....	23 28 15·5	+2·510	2	17 51 23·9	−19·87	2	Cl; L; P; lC; st 8, 10...15...	5

No. of Catalogue.	References to Sir J. H.'s Catalogues of Nebulæ.	References to Sir W. H.'s Classes and Nos.	References to Other Authorities.	Right Ascension for 1860, Jan. 0.	Annual Precession in Right Ascension for 1880.	No. of Obs. used.	North Polar Distance for 1860, Jan. 0.	Annual Precession in N.P.D. for 1880.	No. of Obs. used.	Summary Description from a Comparison of all the Observations, Remarks, &c.	Total No. of times of Obs. by h. and H.
	h.	H.		h m s	s		° ′ ″	″			
4991	2256	II. 244	....	23 28 35·2	+3·026	1	75 27 48·6	−19·88	2	F; S; R; psbM; stellar	4
4992	4000	......	....	23 28 56·7	3·209	1	128 13 2·6	19·88	1	pB; L; E; vgbM...............	1
4993	2257	......	....	23 29 2·6	3·067	1	88 36 51·3	19·89	1	pB; S; R; psbM; *12 sp ...	1
4994	2257, a	......	R. nova	23 29 10·6	3·067	::	88 36 51·3	19·89	::	No description	0
4995	2258	......	....	23 29 20·0	3·073	1	90 28 43·3	19·89	1	F; pL; lE; gbM; *10 s	1
4996	2259	III. 146	....	23 31 28·2	2·992	1	63 45 34·7	19·91	1	F; S; lE; bM; am st	2
4997	2260	II. 432	....	23 31 35·5	3·092	3	97 17 32·7	19·91	3	pF; cL; E 12°±; vgbM......	7
4998	2261	I. 110	....	23 31 41·7	3·111	2	103 43 56·7	19·91	2	{(H.) cB; (h.) eF}; cL; E; gmbM; r (? var.)	4*
4999	......	III. 189	....	23 32 21·2	3·087	1	95 24 5·4	19·92	1	eeF	1
5000	2262	I. 111	....	23 32 38·8	3·107	2	103 4 12·4	19·92	2	pB; pL; iR; mbM	4
5001	4001	......	....	23 34 41·4	3·397	1	156 45 1·5	19·95	1	eF; S; R; p of 2	1
5002	4002	......	....	23 34 49·4	3·396	1	156 44 21·5	19·95	1	eF; cS; R; f of 2	1
5003	2263	II. 208?	....	23 35 17·3	3·006	1	64 32 13·5	19·95	1	vF; *14 att 255°............	1*
5004	......	II. 208	....	23 36 45·2	3·010	1	64 38 3·2	19·96	1	cL; R; *10·11 np	1*
5005	2264	II. 255	....	23 37 8·5	3·049	3	80 0 26·2	19·96	5	cB; cS; gmbM; r; B * f......	7
5006	2265	II. 256	....	23 37 13·4	3·051	1	80 50 20·2	19·96	3	pF; S; R; *15 sf	4
5007	4003	......	....	23 37 35·5	3·190	3	133 41 27·9	19·97	3	cB; S; vlE; svmbM*14......	3
5008	2266	......	....	23 38 26·1	2·757	1	21 1 30·9	19·97	1	vL; surrounds *8	1
5009	4004	......	....	23 38 29·5	3·141	2	120 17 4·6	19·98	2	vF; S; R; gmbM; *12 f ...	2
5010	2267	III. 427	....	23 39 27·3	3·066	2	86 58 39·6	19·98	2	cF; pL; vlE 0°; lbM; 2 B st nr	4
5011	2268	II. 213	....	23 40 2·3	3·011	1	61 17 42·3	19·99	1	cF; cL; vlE; vglbM; r	2
5012	4005	......	....	23 40 35·8	3·137	2	121 17 45·3	19·99	2	B; cL; R; psmbM	2
5013	2269	III. 437	....	23 40 50·1	3·064	1	83 54 46·3	19·99	2	F; S; R; gbM; er	3
5014	2270	......	....	23 41 35·4	3·066	1	86 36 8·0	20·00	2	vF; cL; R; vglbM; *13 n ...	2
5015	2271	III. 854	....	23 42 7·7	3·015	5	59 47 45·0	20·00	5	cB; vS; R; psbM; rr	7*
5016	2272	VII. 55	....	23 43 4·8	2·849	1	22 45 49·7	20·01	1	Cl; pRi; pC; st 11...15	4
5017	4006	......	....	23 43 34·1	3·151	1	131 31 2·7	20·01	1	B; pL; R; gbM	1
5018	2273, a	......	R. nova	23 43 ±	3·028	...	63 37 ±	20·01	...	vvF; a little np h. 2273	0
5019	2273	......	....	23 43 52·2	3·028	1	63 37 55·7	20·01	1	vF; S; iF; vF ** inv	1
5020	2274	II. 230	....	23 43 55·7	3·041	2	70 37 26·7	20·01	2	pF; pS; R; mbM; sp of 2 ...	4*
5021	2274, a	......	R. nova	23 44 13·9	3·04	...	70 40 43·7	20·01	...	Seen and meas. with h. 2274, 2275.	0
5022	2275	II. 231	....	23 44 17·1	3·042	2	70 39 16·7	20·01	2	pB; pL; E 90°; bM; nf of 2	4
5023	2276	......	....	23 44 38·0	3·049	1	74 31 22·4	20·02	1	Cl of sc st 10 m	1
5024	2277	II. 851	....	23 45 4·8	3·024	2	59 31 6·4	20·02	3	vF; cS; R; * nf	5
5025	2278	III. 231	....	23 46 11·5	3·063	2	82 54 5·4	20·02	2	cF; S; R; psbM; stellar; 1st of 4.	3
5026	2279	III. 232	....	23 46 19·0	3·063	2	82 53 50·4	20·02	2	pF; S; R; psbM; stellar; 2nd of 4.	3
5027	2280	......	....	23 46 38·2	3·063	1	82 54 18·1	20·03	1	F; S; R; 3rd of 4	1
5028	2281	III. 233	....	23 46 45·2	3·063	2	82 48 10·1	20·03	2	pF; pL; lE; glbM; 4th of 4	4
5029	2282	II. 468	....	23 48 10·1	3·066	4	84 51 58·1	20·03	4	pB; pS; iR; psbM; r; *7 p 30s	5
5030	2283	......	....	23 49 42·8	2·975	2	29 23 16·8	20·04	2	Cl; S; pRi; vC; st 10, 13...	2
5031	2284	VI. 30	C. H.	23 49 58·5	2·994	1	34 3 46·8	20·04	1	Cl; vL; vRi; vmC; st 11...18	3
5032	2285	VII. 56	....	23 50 0·7	2·979	1	29 33 50·8	20·04	1	Cl; pRi; pC....................	2
5033	2286	......	....	23 50 47·4	3·063	1	80 0 18·8	20·04	1	vF; vS; (?)	1
5034	2287	......	....	23 51 34·3	2·999	1	30 45 41·8	20·04	1	Cl; vL; P; lC; st 7, 10...	1
5035	4009	......	....	23 51 45·2	3·136	1	146 14 16·8	20·04	1	pB; cS; R; gmbM	1
5036	2288	III. 466	....	23 51 47·0	3·065	1:	80 2 53·5	20·05	1	vF; pS; iR	2
5037	2289	III. 867	....	23 51 50·3	3·070	1	87 8 41·5	20·05	1	eF; pS; iR; lbM............	2
5038	2290	II. 232	....	23 52 17·4	3·058	1	70 0 13·5	20·05	1	pF; S; R; sbM; * 10 sp......	3
5039	2291	II. 10	....	33 52 28·1	3·063	1	75 58 24·5	20·05	1	F; pS; iE 15°±	3
5040	2292	......	....	23 53 17·8	3·033	1	40 3 38·5	20·05	1	Cl; pRi; pC; st 9...	1
5041	2293	......	....	23 53 49·6	3·069	1	84 32 7·5	20·05	1	vF; S; R; psbM............	1
5042	2294	III. 855	....	23 54 14·4	3·056	2	59 20 47·5	20·05	3	eF; S; R; sbM; stellar sp of 2	4
5043	2295	III. 856	....	23 54 17·0	3·056	3	59 20 0·5	20·05	3	eF; S; R; stellar; nf of 2 ...	4
5044	2296	III. 984	....	23 55 10·4	3·067	2	77 48 0·5	20·05	3	eF; stellar; Δ with 2 st	4*
5045	4010	......	....	23 55 43·1	3·085	1	125 1 38·5	20·05	1	vF; S; R; am st	1
5046	2297	II. 240	....	23 56 4·5	3·067	1	74 37 31·5	20·05	1	cB; cL; ir; vgbM	2†
5047	2298	III. 436	....	23 56 32·1	+3·070	1::	83 25 16·5	−20·05	1	vF; pL; R; lbM............	2

No. of Cata-logue.	References to Sir J. H.'s Catalogues of Nebulæ.	Sir W. H.'s Classes and Nos.	Other Authorities.	Right Ascension for 1860, Jan. 0.	Annual Precession in Right Ascension for 1880.	No. of Obs. used.	North Polar Distance for 1860, Jan. 0.	Annual Precession in N.P.D. for 1880.	No. of Obs. used.	Summary Description from a Comparison of all the Observations, Remarks, &c.	Total No. of times of Obs. by h. and H.
	h.	H.		h m s	s		° ′ ″	″			
5048	2299	......	....	23 56 39·9	+3·070	1	83 17 43·5	−20·05	1	vF; pL; R; gbM...............	•1
5049	2300	II. 227	....	23 56 48·7	3·067	1	70 0 3·5	20·05	1	pF; cL; mE 45°±; lbM......	4
5050	2301	......	....	23 57 19·7	3·072	1	85 34 38·5	20·05	1	pB; vS; mE; vsmbM	2
5051	2302	......	....	23 57 32·6	3·051	1	22 6 18·2	20·06	1	!; eeF; eeL.......................	1*
5052	4011	......	....	23 57 35·2	3·087	2	152 50 42·2	20·06	2	F; S; R; gbM.................	2
5053	2303	......	....	23 57 55·7	3·072	1	83 51 42·2	20·06	1	pF; S; R; ✱ 9 np	1
5054	2304	......	....	23 57 56·2	3·072	1	85 34 12·2	20·06	1	vF; S; gbM	1
5055	2305	VIII. 29	....	23 58 1·9	3·070	1	111 29 37·2	20·06	1	Cl; vP; vlC........................	2
5056	2306	......	....	23 58 17·7	3·072	1	85 33 47·2	20·06	1	vF; S; R; ✱ nf	1
5057	4013	III. 190	....	23 59 17·4	+3·072	1	94 29 48·2	−20·06	1	vF; vS; R; vg, psmbM; 2 st 9 sf.	2

SUPPLEMENTARY LIST OF NEBULÆ AND CLUSTERS.

5058	......	......	G. P. Bond	0 35 1·0	+3·073	...	89 50 6·6	−19·80	...	F; S; R; ✱ 11 sp 1′; disc Sept. 16, 1862.	0
5059	79, *b*	......	R. nova	0 50 7·9	3·241	...	60 23 27	19·56	...	No description; γ in Lord R.'s diagram.	0
5060	......	......	S. Coolidge	3 6 55·2	3·089	...	89 4 27·5	13·67	...	F; disc Jan 25, 1860	0
5061	......	......	S. Coolidge	3 16 29·0	+3·086	...	89 19 3·0	13·07	...	F; disc Dec. 16, 1859	0
5062	(123)	......	....	4 57 18·4	−0·385	1	159 36 32·1	− 5·43	1	No description.................	1
5063	(374)	......	....	5 20 52·9	−0·469	1::	159 36 35·4	− 3·42	1::	No description.................	1
5064	......	......	J. H. Safford	6 5 7:	+3·093	...	88 50 39:	+ 0·43	...	2 Cls; near 2 st 9.10 & 10.11; disc Mar. 19, 1863.	0
5065	......	......	J. H. Safford	6 6 40·9	3·093	...	88 58 11·2	0·44	...	Cl; bet 2 st 9.10 & 10.11; disc Mar. 19, 1863.	0
5066	......	......	G. P. Bond	7 55 12·5	3·509	...	65 25 19·4	9·73	...	vF; cometic; disc Sept. 1, 1852.	0
5067	......	......	S. Coolidge	9 59 51·1	3·079	...	89 15 7·3	17·38	...	Neb; no description; disc Mar. 31, 1859.	0
5068	......	......	S. Coolidge	10 16 14·1	3·079	...	89 13 46·0	18·05	...	F; disc Mar. 31, 1859	0
5069	939, *c*	......	R. nova	11 33 ±	3·122	...	71 29 ±	19·93	...	No description	0
5070	2849, *a*	......	D'Arrest	12 12 41	3·06	...	83 46 42	20·02	...	A nebula; no description ...	0*
5071	......	......	S. Coolidge	12 31 0·5	3·069	...	89 2 46·5	19·87	...	✱ 12, in F neb; disc May 3, 1859	0
5072	......	......	S. Coolidge	13 24 32·0	3·067	...	89 18 29·7	18·67	...	✱ 12, in F neb; disc Apr. 30, 1859.	0
5073	......	......	S. Coolidge	13 42 38·3	3·065	...	89 14 2·4	18·03	...	✱ 12, in F neb; disc Apr. 30 1859.	0
5074	......	......	G. P. Bond	13 49 18·6	3·068	...	89 31 9·0	17·77	...	S; R; ✱ 92′; disc June 8, 1855.	0
5075	......	......	S. Coolidge	13 53 58·0	3·065	...	89 13 53·3	+17·60	...	✱ 12, in neb; disc Apr. 29, 1859.	0
5076	......	......	Lac. I. 11	18 20 44·6	3·952	...	123 30 27·3	− 1·89	...	Neb. without stars	0
5077	......	......	G. P. Bond	21 45 12·5	2·217	...	40 53 46·4	16·67	...	Neb; no description; disc Feb. 10, 1848.	0
5078	3943, *a*	......	Lassell	22 20 50:	3·336	...	115 34 ±	18·25	...	Neb' ✱ 1′ dist from h. 3943 ..	0
5079	......	......	G. P. Bond	23 25 33·1	+3·093	...	96 22 12·0	−19·83	...	Neb; ✱ 9·10 sf; disc Oct. 23 1848.	0

Of this supplementary list, the objects Nos. 5058, 5060, 5061, 5064, 5065, 5066, 5067, 5068, 5071, 5072, 5073, 5074, 5075, 5077, and 5079 were communicated to me by Professor Bond, Director of the Observatory of Harvard College, U.S., too late for insertion in the body of the Catalogue.

ERRATA.

In page 7, lines 13, 14, *for* 5063 *read* 5079, and *for* six *read* 22.

II. *On the Spectra of some of the Chemical Elements.*

By WILLIAM HUGGINS, *Esq.*, *F.R.A.S.* *Communicated by Dr.* W. A. MILLER, *Treas. R.S.*

Received November 5,—Read December 10, 1863.

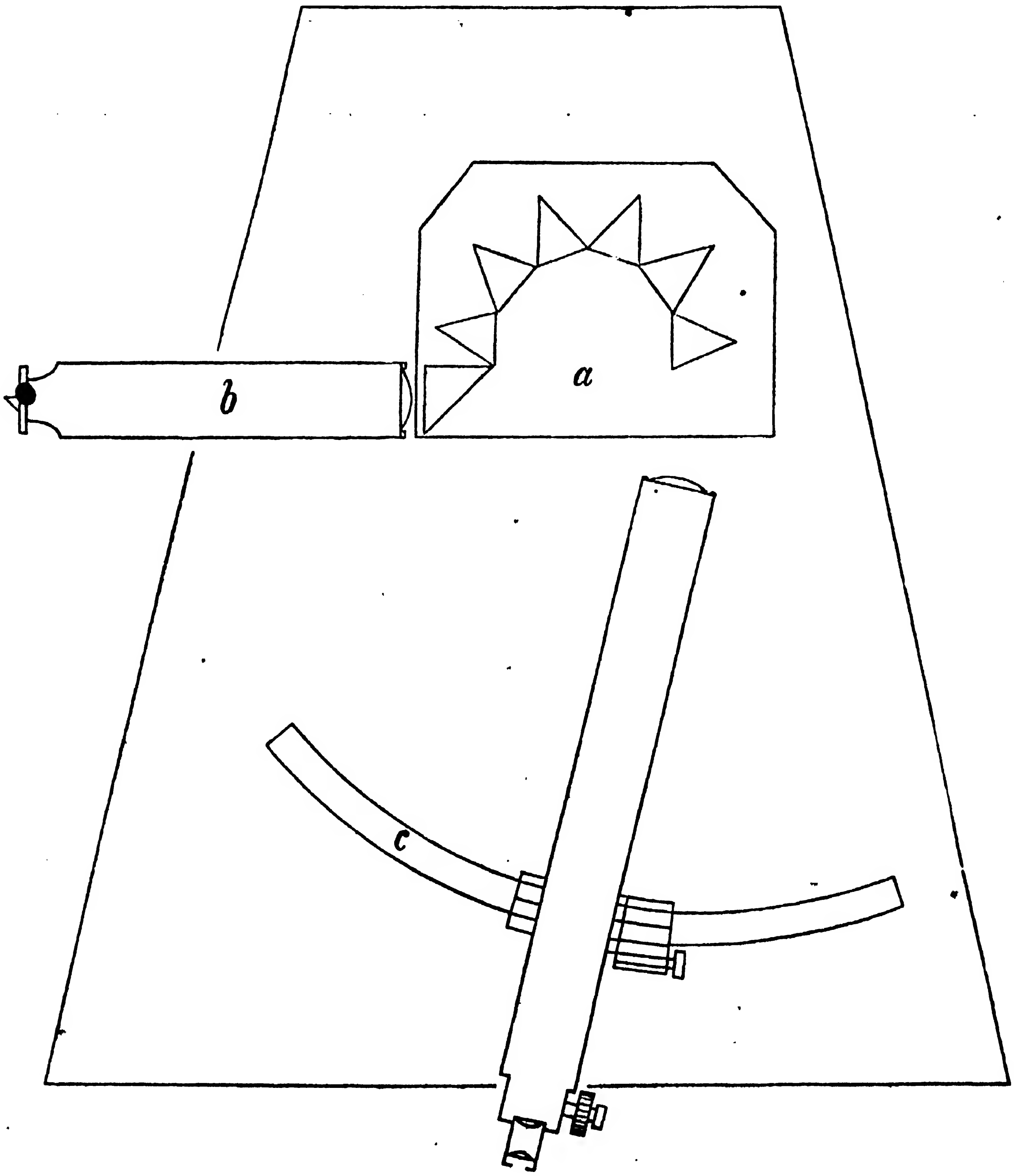
1. I HAVE been engaged for some time, in association with Professor W. A. MILLER, in observing the spectra of the fixed stars. For the purpose of accurately determining the position of the stellar lines, and their possible coincidence with some of the bright lines of the terrestrial elements, I constructed an apparatus in which the spectrum of a star can be observed directly with any desired spectrum. To carry out this comparison, we found no maps of the spectra of the chemical elements that were conveniently available. The minutely detailed and most accurate maps and tables of KIRCHHOFF were confined to a portion of the spectrum, and to some only of the elementary bodies; and in the maps of both the first and the second part of his investigations, the elements which are described are not all given with equal completeness in different parts of the spectrum. But these maps were the less available for our purpose because, since the bright lines of the metals are laid down relatively to the dark lines of the solar spectrum, there is some uncertainty in determining their position at night, and also in circumstances when the solar spectrum cannot be conveniently compared simultaneously with them. Moreover, in consequence of the difference in the dispersive power of prisms, and the uncertainty of their being placed exactly at the same angle relatively to the incident rays, tables of numbers obtained with one instrument are not alone sufficient to determine lines from their position with any other instrument.

It appeared to me that a standard scale of comparison such as was required, and which, unlike the solar spectrum, would be always at hand, is to be found in the lines of the spectrum of common air. Since in this spectrum about a hundred lines are visible in the interval between *a* and H, they are sufficiently numerous to become the fiducial points of a standard scale to which the bright lines of the elements can be referred. The air-spectrum has also the great advantage of being visible, together with the spectra of the bodies under observation, without any increased complication of apparatus.

2. The optical part of the apparatus employed in these observations consists of a spectroscope of six prisms of heavy glass. The prisms were purchased of Mr. BROWNING, optician, of the Minories, and are similar in size and in quality of glass to those furnished by him with the Gassiot spectroscope. They all have a refracting angle of 45°. They increase in size from the collimator; their faces vary from 1·7 inch by 1·7 inch to 1·7 inch by 2 inches.

The six dispersing prisms and one reflecting prism were carefully levelled, and the

former adjusted at the position of minimum deviation for the sodium line D. The train of prisms was then enclosed in a case of mahogany, marked *a* in the diagram, having two openings, one for the rays from the collimator *b*, and the other for their emergence after having been refracted by the prisms. These openings are closed with shutters when the apparatus is not in use. By this arrangement the prisms have not required cleansing from dust, and their adjustments are less liable to derangement. The colli-



mator *b* has an achromatic object-glass by Ross of 1·75 inch diameter, and of 10·5 inches focal length. The object-glass of the telescope, which is of the same diameter, has a focal length of 16·5 inches. The telescope moves along a divided arc of brass, marked in the diagram *c*. The centre of motion of the telescope is nearly under the centre of the last face of the last prism. The eyepiece was removed from the telescope, and the centre of motion was so adjusted that the image of the illuminated lens of the colli-

mator, seen through the train of prisms, remained approximatively concentric with the object-glass of the telescope whilst the latter was moved through an extent of arc equal to the visible spectrum. All the pencils emerging from the last prism, therefore, with the exception of those of the extreme refrangible portion of the spectrum, are received nearly centrically on the object-glass of the telescope. The total deviation of the light in passing through the train of prisms is, for the ray D, about 198°. The interval from A to H corresponds to about 21° 14′ of arc upon the brass-scale.

3. The measuring-part of the apparatus consists of an arc of brass, marked *c* in the figure, divided to intervals of 15″. The distance traversed by the telescope in passing from one to the other of the components of the double sodium line D, is measured by five divisions of 15″ each. These are read by a vernier.

Attached to the telescope is a wire micrometer by DOLLOND. This records 60 parts of one revolution of the screw for the interval of the double sodium line. Twelve of these divisions of the micrometer, therefore, are equal to one division of the scale upon the arc of brass. The micrometer has a cross of strong wires placed at an angle of 45° nearly with the lines of the spectrum. The point of intersection of these wires may be brought upon the line to be measured by the micrometer screw, or by a screw attached to the arm carrying the telescope. For the most part the observations were read off from the scale, and the micrometer has been only occasionally employed in the verification of the measures of small intervals. The sexagesimal readings of the scale, giving five divisions to the interval of the double line D, have been reduced to a decimal form, the units of which are intervals of 15″, and these are the numbers given in the Tables. An attempt was made to reduce the measures to the scale of KIRCHHOFF's Tables, but the spectra are not found to be superposable on his. This is due, in great part, probably to the prisms in his observations having been varied in their adjustment for different parts of the spectrum. The eyepieces are of the positive form of construction. One, giving the power of 15, is by DOLLOND; the other, of about 35, is by COOK.

4. The excellent performance of the apparatus is shown by the great distinctness and separation of the finer lines of the solar spectrum. All those mapped by KIRCHHOFF are easily seen, and many others in addition to these. The *whole* spectrum is very distinct. The numerous fine lines between *a* and A are well defined. So also are the groups of lines about and beyond G. H is seen, but with less distinctness.

As, with the exception of the double potassium line near A, no lines have been observed less refrangible than *a*, the Maps and Tables commence with the line *a* of the solar spectrum and extend to H.

The observations are probably a little less accurate and complete near the most refrangible limit. Owing to the feebleness of the illumination of this part of the spectrum, the slit has to be widened, and moreover, the cross wires being seen with difficulty, the bisection of a line exactly is less certain.

5. For all the observations the spark of an induction coil has been employed. This coil has about fifteen miles of secondary wire, and was excited by a battery of GROVE's

construction, sometimes two, at others four cells having been employed. Each of these cells has 33 square inches of acting surface of platinum. With two such cells the induction spark is 3 inches in length. A condenser is connected with the primary circuit, and in the secondary a battery of Leyden jars is introduced. Nine Leyden jars, each surface of each of which exposes 140 square inches of metallic coating, were employed. These are arranged in three batteries of three jars each, and the batteries are connected in polar series.

The metals were held in the usual way with forceps. The nearness of the electrodes to each other, their distance from the slit, and the breadth of the latter were varied to obtain in each case the greatest distinctness. The amount of separation of the electrodes was always such that the metallic lines under observation extended across the spectrum. The two sets of discharging-points were arranged in the circuit in series.

6. Some delay was occasioned by the want of accordance of the earlier measures, though the apparatus had remained in one place and could have suffered no derangement. These differences are supposed to arise from the effect of changes of temperature upon the prisms and other parts of the apparatus. This source of error could not be met by a correction applied to the zero-point of measurement, as the discordances observed corresponded, for the most part, to an irregular shortening and elongation of the whole spectrum.

The principal air lines were measured at one time of observing, during which there was satisfactory evidence that the values of the measures had not sensibly altered; and these numbers have been preserved as the fiducial points of the scale of measures. The lines of the spectra of the metals have been referred to the nearest standard air line, so that only this comparatively small interval has been liable to be affected by differences of temperature. Upon these intervals the effect of such changes of temperature as the apparatus is liable to be subjected to is not, I believe, of sensible amount with the scale of measurement adopted. Ordinarily, for the brighter portion of the spectrum, the width of the slit seldom exceeded $\frac{1}{400}$ inch; when this width had to be increased in consequence of the feebler illumination towards the ends of the spectrum, the measure of the nearest air line as seen in the compound spectrum was again taken, and the places of the lines of the metal under observation were reckoned relatively to this known line.

By this method of frequent reference to the principal air lines the measures are not sensibly affected by the errors which might have been introduced from the shifting of the lines in absolute position in consequence of alterations either in the width of the slit, in the place and direction of the discharge before the slit, or in the apparatus from variations of temperature, flexure or other causes.

The usual place of the electrodes was about ·7 inch from the slit, though occasionally they were brought nearer to the slit. When they are placed in such close proximity, the sparks charge the spectroscope by induction, but the inconvenience of sparks striking from the eyepiece to the observer may be prevented by placing the hand upon the apparatus, or putting the latter into metallic communication with the earth.

The spectrum of comparison was received by reflexion from a prism placed in the usual manner over one-half of the slit. As the spectrum of the discharge between points of platinum, when these are not too close, is, with the exception of two or three easily recognized lines, a pure air-spectrum, this was usually employed as a convenient spectrum of comparison for distinguishing those lines in the compound spectrum which were due to the particular metal employed as electrodes. The measures, however, of all the lines, including those of the air-spectrum itself, were invariably taken from the light received into the instrument directly, and in no case has the position of a line been obtained by measures of it taken in the spectrum of the light reflected into the slit by the prism.

The measures of all the lines were taken more than once; and when any discordance was observed between the different sets, the lines were again observed. The spectra of most of the metals were re-measured at different times of observing. In the measurement of the solar lines for their coordination with the standard air-spectrum, the observations were repeated on several different occasions during the progress of the experiments. The line G of the solar Table is the one so marked by KIRCHHOFF*. When no change in the instrument could be detected, the measures came out very closely accordant, for the most part identical. The discordances due to small alterations in the instrument itself were never greater than 5 or 6 of the units of measurement in the whole arc of 4955 units. As the apparatus remained in one place free from all apparent derangement, these alterations are probably due to changes of temperature. The method employed to eliminate these discordances has been described.

Throughout the whole of the bright portion of the spectrum the probable error of the measures of the narrow and well-defined lines does not, I believe, exceed one unit of the scale.

In the case of lines of sensible breadth and of nebulous bands, the point of intersection of the wires of the micrometer was brought as nearly as possible upon the centre of the lines.

7. It is well known that the lines of different metals as a whole, as well as the lines of the same metal amongst themselves, differ greatly in their characters. For example, the narrow sharply-defined lines of cobalt and iron contrast strikingly with the broader and nebulously edged lines of antimony and arsenic. The spectrum of zinc affords a good example of the differences in this respect between lines of the same metal. In general, it may be that the less volatile a metal is, the narrower and more sharp are the lines, though indeed in the case of the metals barium, calcium, and strontium many of the lines are of hair-like narrowness and sharply defined.

In the spectra of many of the metals bands of light also exist, generally rather broad and faint, which are not resolvable with my instrument into lines. Many of these have the appearance of being true nebulous bands, whilst others under careful scrutiny present indications of being probably composed of lines.

* Untersuchungen ü. d. Sonnenspectrum, 2 Theil, Taf. iii. Berlin, 1863.

These characteristic differences of the lines deserve more careful scrutiny than it was needful, in accordance with my present purpose, to bestow upon them. As approximative indications of their character, the following abbreviations are placed against the numbers in the Tables:—

A line sharply defined at the edges, and narrow when the slit is narrow . . .	s
A band of light, defined *as a line*, but remaining even with a narrow slit, nebulous at the edges	n
A haze of light irresolvable into lines	h
Double, too close for measurement	d

The comparative intensity of the lines is indicated by the smaller figures, which are placed in the position of exponents against the numbers in the Tables. I purposed to limit these estimations to the first ten figures, but so many faint lines were seen that the scale has been extended by adding fractional parts of unity. These figures may be accepted as approximative estimations of the relative intensity of the lines of *each* spectrum. But as the spectra were not, for this purpose, compared one with another, and so many circumstances affect eye-estimations of brightness, these figures must not be taken otherwise than as roughly indicating the values in intensity of the lines of *different* spectra.

In many cases some of the lines of one metal will be seen to be very closely approximated in position to those of another metal, though they do not actually coincide. In the Tables there are lines of different metals having the same numbers, these may with a greater dispersive power be found to be only very near each other. In the case of some, there may be small errors of observation; for to have compared each spectrum with all the others would have involved very great labour.

8. I am indebted to the kindness of Professor W. A. Miller for the loan of specimens of gold, silver, thallium, cadmium, lead, tin, bismuth, antimony, arsenic, and palladium. Dr. Matthiessen has furnished me with lithium, calcium, and strontium* and purified tin, cadmium, lead, bismuth, antimony, and iron. I have procured from Messrs. Johnson and Matthey tellurium, palladium, osmium, rhodium, iridium, and pure platinum. I have electro-deposited upon platinum, from the solutions of their salts, silver, manganese, chromium, lead, tin, cadmium, cobalt, bismuth, nickel, antimony, and iron. I have also prepared by the voltaic method, amalgams of sodium, potassium, barium, and strontium.

9. *The air-spectrum.*—The lines given in this spectrum are present with all electrodes when the spark is taken in air at the common pressure. To distinguish the lines which belong to air, the spectrum between electrodes of platinum was observed simultaneously with that between points of gold. The lines common to both these spectra were measured as those due to the components of air. The spectrum thus obtained remains invariably

* Dr. Matthiessen informs me that "the calcium, strontium, and lithium were prepared from the pure chlorides as described in the Quart. Journ. Chem. Soc., vol. viii. pp. 107, 143."

constant, with reference to the position and relative characteristics of its lines, with all the metals which have been employed. The air-spectrum *as a whole*, however, varies considerably in intensity and distinctness with electrodes of different metals. As the lines are due to the stratum of air separating the points of the electrodes, it is to be expected that these lines will appear strongest and most distinct when those metals are employed which, being less volatile, will therefore in a less degree displace the air between the electrodes with their own special vapour. This consideration appears to be confirmed by observation. The air-spectrum is especially intense and distinct when the spark is taken between points of platinum, gold, iridium, and rhodium; whilst, of all the metals which I have employed, mercury and sodium, perhaps, are those with which the intensity of the air-spectrum is most diminished. With these comparatively very volatile metals, the air between and about the electrodes must be, to a very considerable extent, replaced by the metals themselves in a state of vapour. It accords with this suggested explanation of the differences in brightness of the air lines with different metals, that, if the electrodes be mercury or sodium and a platinum wire, the air-spectrum is observed to be weaker when the current is so directed that the greater heating effect of the discharge shall be at the mercury or sodium electrode, and to become perceptibly stronger when the current is reversed. It is known that, within certain limits, the air-spectrum is rendered more intense by the separation of the electrodes.

The following experiments have been made to refer the lines of this compound spectrum to the components of common air to which they severally belong:—

a. *Hydrogen.*—The strong line of the air-spectrum at 589·5 is coincident with FRAUNHOFER'S C, and with the red line of hydrogen.

When the spark is taken in air that has passed over sulphuric acid, this line becomes very faint. A larger surface of acid being employed, the line faded out so completely that no trace of it could be perceived. Steam was then mixed with air, when this line became much brighter and the other lines of hydrogen appeared.

The presence and comparative brightness of this line form a delicate test for aqueous vapour.

b. *Carbonic acid.*—Air that had passed through a solution of caustic potash was examined, but its spectrum was not observed to differ from that of ordinary air. When carbonic acid is added to air, several prominent lines make their appearance. These are due to carbon, since they coincide with lines in the spectrum of graphite. One of the strongest and most characteristic of these lines, and a test for carbonic acid, is a red line a little less refrangible than the hydrogen line. Its number is 580·5.

[Though a good indication of the oxygen and nitrogen compounds of carbon, the absence of this line must not always be accepted as a proof that no carbon is present. I have recently found that, when carbon is subjected to the induction spark in the presence of hydrogen, this line in the red is not seen. Further details of these experiments will be given when the spectrum of carbon is described.—February 7, 1864.]

c. *Nitrogen.*—In the spectrum of the electric spark when taken in a current of pure

nitrogen, a few of the lines of common air are wanting, but no new lines appear. The lines of the air-spectrum which remain in nitrogen preserve unaltered their relative brightness and their distinctive characters. In the Tables these lines are distinguished by the letter N.

The nitrogen was prepared by causing air freed from carbonic acid by potash, to pass over red-hot finely divided copper which had been previously reduced from the oxide by hydrogen. The nitrogen was then dried by sulphuric acid. The freeness of the nitrogen from oxygen and from moisture was shown by the total extinction of all the lines which did not retain their usual brightness, and the absence of any trace of the strong hydrogen line. Subsequently a fresh portion of nitrogen was prepared by the same method, and a portion of it sealed up at the common pressure in a glass tube of suitable form, pierced with platinum electrodes. This tube continues to give results identical with those obtained in the current of nitrogen.

d. *Oxygen*.—When a current of oxygen from fused chlorate of potash was substituted for nitrogen, the numerous lines of the nitrogen spectrum faded out, and those which were extinguished by nitrogen reappeared with an intensity greater than they possess when the spark passes in air. These are distinguished in the Tables with the letter O.

No new lines were added to the spectrum, but an unexpected result was observed. Two (it may be, three) of the lines visible in nitrogen remained also in oxygen. The most noticeable of these is the double line 2642. This in the air-spectrum is not quite so strong as the line next in greater refrangibility. This brighter line became extinct in oxygen at the same time that the double line remained fully as brilliant as in air, if not a little exalted in intensity. This result, therefore, could not be due to any oxygen remaining in the nitrogen, or of nitrogen in the oxygen. The other line, which behaves similarly in oxygen and nitrogen, is the hazy one in the red, 807. The line in the Tables marked with the symbols of nitrogen and oxygen, at 3456, is in the air-spectrum a double line. The narrow defined line of nitrogen is superposed upon the broader nebulous line of oxygen. Oxygen and nitrogen from other sources were then examined. Nitrogen was evolved from a mixture of nitrite of potash and chloride of ammonium. Oxygen was obtained from peroxide of manganese and sulphuric acid, also from bichromate of potash and sulphuric acid, and also from oxide of mercury. The gases thus prepared were identical in their action upon the spectrum with those previously examined. I have not at present carried this inquiry further.

[I have carefully re-examined the lines which are apparently common to nitrogen and to oxygen. I now regard them as due to the superposition in the air-spectrum of lines of oxygen and of nitrogen. When the most remarkable of these, the double line 2642, is closely observed with the eyepiece of a power of 35 times, the double line, as a whole, appears to become in a slight degree more refrangible when the air is replaced by oxygen. As the oxygen lines of the air-spectrum become more brilliant in oxygen, the phenomenon observed may be explained by supposing a pair of unequally bright oxygen lines to

be closely approximated in position to, but a little more refrangible than, a similar pair of nitrogen lines.

In air these four lines would form an ill-defined double line, while in oxygen the exaltation in brilliancy of the lines due to oxygen would make up for the extinction of those of nitrogen, thus leaving a pair similar to that seen in air, but now a little more refrangible, from the loss of the less refrangible line of nitrogen, and the greater brightness of the faint and more refrangible of the oxygen lines. This explanation exactly corresponds with the changes in appearance and position of the double line. The observations have been repeated several times with oxygen from chlorate of potash, and also with oxygen from bichromate of potash and sulphuric acid. The change in position as observed relatively to the corresponding air line in the spectrum of comparison was not relied upon. The fixed cross of the micrometer was made to coincide with the oxygen line next in less refrangibility, 2626, the moveable cross was then brought upon the centre of the brighter of the pair 2642. When a current of pure oxygen was made to pass through the glass tube in which the platinum electrodes were sealed, the double line was seen to have moved from the point of intersection of the wires towards the more refrangible end of the spectrum. To restore the cross to a position similar to that which it before occupied, namely, upon the centre nearly of the brighter of the pair of lines, required that the screw should be turned through a part of a revolution corresponding to a little more than two units of the scale. This measure is greater than the *apparent* change in position would have suggested, for in oxygen the lines are rather broader and more nebulous. The distance between the components of the double line is greater in oxygen. The alterations of position and of character are much better seen when the spectra of oxygen and nitrogen are viewed simultaneously.

A similar explanation is to be given of the nebulous band in the red at 807. In oxygen the position of greatest brightness is more refrangible than it is in air and in nitrogen, though the band itself does not advance beyond the more refrangible limit of the corresponding band in air. The line at 629·5 is a pure nitrogen one and fades out completely in oxygen, but then a nebulous line appears at a little distance, about 638. Of this, in the air-spectrum, a faint trace only can be perceived.—Feb. 1864.]

10. *Sodium.*—When the spark was passed between electrodes of sodium, in addition to the well-known double line, three other pairs of lines and a nebulous band made their appearance in the spectrum. The two more prominent of these are not far from air lines, and with an instrument of insufficient dispersive power might easily be confounded with them. As these lines might be occasioned by impurities in the commercial sodium employed, I prepared an amalgam of sodium, by making mercury the negative electrode in a solution of pure chloride of sodium. The mercury had been examined, and its spectrum was known. When the spark passed between this amalgam and a platinum wire, the same lines were seen, with their peculiar characteristics of relative position and intensity. Cotton moistened with solutions of chloride of sodium and of nitrate of soda was then used as one electrode, the other being a platinum wire.

With both these salts the pairs at 820 and 1170 were satisfactorily observed, though it was with some difficulty, and only by occasional glimpses.

I then compared the sodium-spectrum directly with that of the sun. So numerous are the fine lines of the solar spectrum, and so difficult is it to be certain of absolute coincidence, that I hesitate to say more than that the pair of lines 818 and 821 appeared to agree in position with KIRCHHOFF's lines 864·4 and 867·1; and of the pair 1169 and 1174, one appears to coincide with a line sharply seen in the solar spectrum, but not marked in KIRCHHOFF's Map, which would be about 1150·2 of his scale, and the other with KIRCHHOFF's line 1154·2. The other pair and the nebulous band are too faint to admit of satisfactory comparison with solar lines.

11. *Potassium.*—When commercial potassium is employed as an electrode, about 16 lines are seen in addition to the pair near A of the solar spectrum. Four quite distinct specimens of potassium gave identical results, the same lines being visible in all, and no other lines. I then prepared by electricity an amalgam of potassium, but, with the exception of the line 840 occasionally visible, the lines were not seen. As the potassium lines are fainter than those of sodium, this negative result does not appear to be conclusive, since the great intensity of the mercury-spectrum might overpower the feebler lines of potassium, especially when this was present only in small quantity and not in the concentrated metallic form. One electrode was then surrounded with cotton containing concentrated solution of chloride of potassium, and afterwards with cotton containing that of caustic potash. With both these, rather more easily with the latter, the lines 840, 1049, 1065, and 1073 were occasionally and faintly perceived.

[This great diminution in the brilliancy and number of the lines when, in the place of metallic potassium, solutions of its salts are substituted, may be due to the unfavourable condition of the latter for the production of potassium vapour. The large volume of the gases formed by the decomposition of the water must disperse and attenuate the comparatively small volume of vapour of the element forming the base of the salt, and also the great expansion in the gaseous state of the constituents of the water would lower the temperature of the vapour of potassium mingled with them. The salts should be subjected to the discharge free from water, and in a condition in which they conduct the current. If dry, or fused upon the wires, they are disrupted and scattered.

A platinum wire was coiled at one extremity into a little cup-like cage. Chloride of potassium was placed in this and fused. This wire, with the fused bead of chloride, was placed *above* the platinum wire forming the other electrode. A spirit-lamp is placed beneath the wires; as soon as the bead is in a state of fusion, the lamp is withdrawn and contact immediately made. During the few seconds that the chloride remains fused, most of the lines of metallic potassium are seen. Of the lines 1328 and 840 the observation is less certain, and is very doubtful of 763 and of 727.

Protochloride of tin similarly employed gives a brilliant spectrum of tin.—Feb. 1864.]

12. *Calcium.*—The spectrum was obtained from electrodes of metallic calcium, supplied to me by Dr. MATTHIESSEN. The colour of the spark, as seen by the eye, is bril-

liant red purple. The contrast is exceedingly beautiful between this and the intense green light of thallium. Two or three nebulous bands in the red present indications of resolvability. There is also a diffused green light from 1297 to 1375. The line 1506 is in a small degree more refrangible than the strong thallium line. The strong line 1260 is very near a tin line, but the contrast between the sharp calcium line and the nebulous tin line is very marked. A pair of strong lines is seen near the extreme refrangible end of the spectrum, which may coincide with those of FRAUNHOFER'S H. This specimen of calcium produced also the lines of magnesium; these were of course omitted, as on the chemical analysis of this specimen of calcium it was found to contain magnesium.

13. *Barium.*—As I could not obtain barium in the metallic state, I prepared an amalgam of barium by the electrolysis of chloride of barium. The mercury was a portion of the same used in the other experiments, and which had been examined in the spectroscope. The spectrum is one of great beauty, and the lines are for the most part sharp, narrow, and intense. There is a very strong line in the indigo, near a line of platinum; the latter is furnished also by my specimens of iridium and rhodium.

The line next in greater refrangibility appears to agree very nearly in position with one of tin.

14. *Strontium.*—Metallic strontium prepared by Dr. MATTHIESSEN was employed. The spectrum is exceedingly brilliant, the lines being numerous, narrow, and intense. It is remarkable for several bright nebulous columns in the red and orange; these present indications of containing numerous fine lines.

This metallic strontium contains calcium, the lines of which have been eliminated. An amalgam of strontium was prepared, and with this all the principal lines of the spectrum from the metal were confirmed. As might be expected, many of the fainter lines were not recognized in the spectrum of the amalgam.

15. *Manganese.*—The lines were obtained from an electro-deposit of manganese from a solution of the chloride of manganese. Upon comparing this with a specimen of manganese which I was informed had been reduced by charcoal, all the lines of the electro-deposited manganese were seen in the other; but this contained, in addition, the numerous lines of the iron-spectrum. The most characteristic groups are a triple line from 909 to 915·5, the five lines from 2267 to 2401, and the close group extending from 3097 to 3133.

There are two remarkable broad nebulous bands, one at 840 and the other at 1565; the former, I suspect, is double. As the deposited manganese is brittle, the lines were fitful in consequence of the disruption of portions of the deposit by the spark. This may be the reason that some of the finer lines were not observed.

16. *Thallium.*—The specimen of thallium was lent me by Professor W. A. MILLER, who received it as pure thallium from Mr. CROOKES. With the exception of a few faint lines, one in the red rather strong, and a distinct line near the most refrangible end, the spectrum agreed with the description in Professor MILLER'S "Note" on Thallium *.

* Proceedings of the Royal Society, January 1863, vol. xii. p. 407.

17. *Silver.*—The spectrum is that of electrotype silver, obtained from pure nitrate of silver in cyanide of potassium.

18. *Tellurium.*—This metal was supplied to me as pure by Messrs. JOHNSON and MATTHEY. It contains many strong and characteristic lines. The strong line in the red is very near the strong line in cadmium, but the latter is in a small degree less refrangible.

19. *Tin.*—The spectrum was obtained from purified tin, and confirmed by comparison with electrotype tin; one line, not observed in the spectrum of the latter, has been omitted.

20. *Iron.*—Electrotype iron was employed. This spectrum agreed exactly with a specimen received from Dr. MATTHIESSEN as very nearly, if not quite, pure iron.

21. *Cadmium.*—The spectrum of purified cadmium was confirmed by comparison with cadmium electro-deposited.

22. *Antimony.*—The numerous and strong lines of this spectrum are, for the most part, nebulous at their boundaries. The spectrum is that of electro-deposited antimony.

23. *Gold.*—The specimen of which the spectrum is given was received from Professor MILLER. It was reduced by him from the pure chloride, and fused under bisulphate of potash.

24. *Bismuth.*—Electro-deposited from the nitrate of bismuth.

25. *Mercury.*—Commercially pure mercury was washed with nitric acid, and then distilled. A portion of this was placed in a small cup made from glass tube, into which a platinum wire was sealed. The other electrode was a platinum wire.

26. *Cobalt.*—Electrotype cobalt from the chloride was employed. The lines are numerous, sharp, and narrow, and in their groupings there is considerable resemblance to the spectrum of iron.

27. *Arsenic.*—From a specimen of carefully re-sublimed arsenic received from Professor MILLER. The strong line 1814 is very near, but not quite so refrangible as, one of the strong lines of copper. The strong line in the red, 812, is near the hazy band of the air-spectrum.

28. *Lead.*—The lead was obtained by electrolysis from the nitrate of lead.

29. *Zinc.*—Electrotype zinc was used. This spectrum is remarkable for the strong contrast between the nebulous lines, and others near them sharply defined.

30. *Chromium.*—The chromium was electro-deposited. The triple nebulous band from 1081 to 1090 is remarkable. The groups of lines in the blue and indigo, which for the most part fall between air lines, are very beautiful, and in a marked manner characteristic of this metal.

31. *Osmium.*—Received as pure from Messrs. JOHNSON and MATTHEY. Iridium and rhodium have also been measured, but, as these have lines in common, their spectra are deferred.

32. *Palladium.*—A specimen prepared by Dr. WOLLASTON was observed simultaneously with palladium received as pure from Messrs. JOHNSON and MATTHEY. The

latter contained several lines which were not in the Wollaston specimen. The lines only which were common to both spectra were measured, and are given in the Tables.

Nebulous bands, probably resolvable, are seen at 1000, and from 1219 to 1233.

33. *Platinum.*—The lines of platinum are not easily observed, as several of them are fainter than the air lines near which they occur. The points of platinum must be brought near each other. The spectrum was mapped from electrodes of platinum wire specially prepared by Messrs. JOHNSON and MATTHEY as "pure" platinum.

There are two bands of fine lines at 913 and 939.

34. The spectrum of *Lithium* was observed from electrodes of metallic lithium. Only one line of moderate intensity was seen in addition to the three strong lines which are known. The numbers are $521{\cdot}5^{6}$ s, 856^{8} s, 2013^{2} s, 2732^{5} n.

35. Several other spectra have been measured, or are in progress; these are reserved until the remaining metals and elements, as far as may be practicable, have been investigated.

NOTE TO THE TABLES.

Upon a re-examination of the Tables I found that it frequently occurred that lines of two or more metals were denoted by the same number. It appeared probable that these lines having a common number were not coincident, but only approximated in position within the limits of one unit of the scale employed; and besides, there might be small errors of observation. I therefore selected about fifty of these groups of lines denoted by common numbers, and compared the lines of each group the one with the other, by a simultaneous observation of the different metals to which they belong. Some of the lines were found to be too faint and ill-defined to admit of being more accurately determined in position relatively to each other.

The following lines appear with my instrument to be coincident.

Zn, As 909	Na, Ba 1005	O, As 1737
Na, Pb 1000	Te, N 1366	Cr, N 2336

Of a much larger number of groups, the lines were, by careful scrutiny, observed to differ in position by very small quantities, corresponding for the most part to fractional parts of the unit of measurement adopted in the Tables. These are—

Sn 459	Ba 621·5	Te 657	Bi 837·3	Co 937	Sb 1081	Te 1485	Zn 1797
Sb 458·8	Bi 621	Cd 656	Sb 837	Sb 937·5	Au 1081·5	Fe 1485·3	Pd 1798
Ca 515	Ca 622	Fe 696·2	Cd 889	Au 981	Ca 1256	Tl 1505	Tl 1851
Au 516	Fe 623	Zn 696	Sb 889·5	Sb 981·5	Co 1257	Mn 1505·5	Bi 1851·3
Te 545·5	Au 643	Sb 765	As 908·8	Ca 1031	Fe 1276	Pd 1548	Sb 1900
Sb 545	Ca 642	Te 765·3	Mn 909	Pd 1031·5	Ag 1276·3	Fe 1548·2	Pb 1900·3
	Fe 641·5			Ag 1031·2			N 1900
				Pb 1031·1			
Sn 581	Ca 649	Na 818·3	Ca 921·2	Te 1030·3	Fe 1438	Pb 1593·3	
Bi 581·5	Su 648	Ca 818	Tl 921		Te 1438·3	Fe 1593	
			Co 921				
			Sb 921·1				

TABLE I.

Solar.	Air.	Na.	K.	Ca.	Ba.	Sr.	Mn.	Tl.	Ag.	Te.	Sn.	Fe.
a 322												
						365^{2} s						
			418^{2} s									
			426^{2} s									
					$441^{\cdot7}$ s	444^{2} s						
B 449	....	......	......	......	......	......	......	......	......	......	459^{3} s	......
					$486^{\cdot7}$ s	480^{2} s						
											491^{1} s	......
				$515^{\cdot7}$ s	$523^{\cdot7}$ s							
					$532^{\cdot7}$ s	......	......	......	......	......	......	......
						$549^{\cdot5}$ s	......	......	......	$545{\cdot}3^{4}$n	......	......
	N $565{\cdot}5^{2}$	......	......	......	......	563^{1} s	......	......	......	......	......	......
					573^{1} s	......	......	......	......	......	581^{1} s	......
C 589·5	H $589{\cdot}5^{4}$ h	......	......	......	......	595^{2} s	......	596^{4} s	......	......	......	......
					608^{1} s							
						619^{2} s	......	......	......	......	......	......
	N $629{\cdot}5^{2}$	......	......	$622^{\cdot5}$ s	$621{\cdot}5^{7}$ s	......	......	......	......	......	......	623^{1} s
				$625^{1.5}$ s	......	......	......	......	......	......	......	......
				$637^{\cdot7}$ s	......	655 h	......	......	......	......	......	$641{\cdot}5^{\cdot5}$ s
				642^{2} s	645^{1} s	......	......	......	......	......	648^{7} s	$667^{\cdot5}$ s
				$649^{\cdot7}$ s	......	......	......	......	......	$657{\cdot}3^{7}$ s	......	......
						669^{4} s	......	......	......	......	......	673^{1} s
			$727^{\cdot5}$ s	655^{3} s	......	681^{1} s	......	......	......	......	......	$674^{\cdot5}$ n
				$699^{\cdot2}$ s	......	684^{1} s	......	......	......	......	......	$683^{\cdot5}$ s
				$709^{\cdot8}$ s	704^{1} s	$692^{\cdot5}$ s	......	......	$690^{\cdot2}$ s	693^{1} s	......	$696{\cdot}2^{\cdot5}$ s
			$763^{\cdot5}$ s	$723^{\cdot2}$ s	......	$703^{\cdot2}$ s	$704^{\cdot2}$ s	......	......	......	......	......
						$705{\cdot}5^{\cdot5}$ s	......	......	......	$703^{\cdot5}$ n	......	$709^{\cdot7}$ s
				813^{1} s	......	723 h	......	......	......	......	......	$719^{\cdot5}$ s
				818^{4} s	......	$745^{\cdot2}$ s	......	......	......	735^{2} s	......	$726^{\cdot5}$ s
	NO 807^{2} h, d	......	......	......	......	760 h	......	......	$762^{\cdot2}$ s	$765{\cdot}3^{3}$n	......	$758^{\cdot5}$ s
				843^{3} s	847^{2} s	777 h	......	768^{1} h	......	774^{3} s	......	$763^{\cdot5}$ s
		$818^{1.5}$ s	......	......	879^{2} s	$807^{\cdot5}$ s	837 } h	......	......	......	......	772^{1} s
		821^{1} s	......	$859^{\cdot2}$ s	......	856^{1} s	843 }	......	......	......	......	795^{1} s
				$863^{\cdot2}$ s	......	......	......	......	......	......	......	829^{2} s
			$840^{1.5}$ s	......	......	......	......	......	......	......	......	$852^{\cdot5}$ s
				882 h	......	......	......	......	......	894^{2} s	......	$869{\cdot}5^{\cdot5}$ s
				$921{\cdot}2^{\cdot2}$ s	......	......	......	......	......	......	......	$909^{\cdot5}$ s
					$908^{1.5}$ s	......	$909^{3.5}$ s	......	$899^{\cdot5}$ s	917^{1} s	......	$937^{\cdot5}$ s
	N 959^{1}	......	......	934 h	$925^{1.5}$ s	924 h	$913^{2.5}$ s	......	......	$927^{\cdot5}$ n	......	$953^{\cdot5}$ s
	N 967^{6}	......	......	......	$943^{1.5}$ s	941^{1} h	$915^{2}{\cdot}5$ s	921^{2} s	$943^{\cdot5}$ s	$945{\cdot}3^{6}$ s	......	$995^{\cdot2}$ s
	N 975^{4}	......	......	......	$993^{\cdot5}$ s	945^{3} s	......	......	......	$971^{1}{\cdot}5$ s	......	......
	N 978^{1}	......	......	......	......	......	......	960^{6} s	......	......	......	......
D^{1} 1000	....	1000^{5} s	......	......	......	......	......	......	......	......	......	......
D^{2} 1005	....	1005^{8} s	......	......	1005^{1} n	......	......	......	......	......	......	......
Solar.	Air.	Na.	K.	Ca.	Ba.	Sr.	Mn.	Tl.	Ag.	Te.	Sn.	Fe.

From *a* to D.

Cd.	Sb.	Au.	Bi.	Hg.	Co.	As.	Pb.	Zn.	Cr.	Os.	Pd.	Pt.
	396^{3} n											
......	$458{\cdot}5^{1}$ n											
......	475^{3} s	......	473^{4} s									
502^{4} s	487^{3} s	......	......	......	......	......	480^{3} s					
......	$501^{\cdot 5}$ n											
......	......	$516^{\cdot 5}$ s										
.....	517^{1} s	......	......	......	......	......	$542{\cdot}5^{7}$ s					
......	......	$535^{\cdot 5}$ s										
......	545^{1} n	$541^{\cdot 5}$ s	572^{3} s	......	......	......	......	......	541^{1} s			
......	614^{1} n	......	$581{\cdot}5^{1}$ h	......	......	......	......	$577^{\cdot 5}$ n				
......	620^{1} n											
639^{2} n	......	......	621^{4} s	......	645^{1} n, d	......	......	......	$621^{\cdot 5}$ n			
......	640^{1} n											
656^{8} s	......	$643^{\cdot 5}$ s	......	......	......	......	......	696^{7} s	$640^{\cdot 5}$ n	641^{1} s		
......	......	......	......	......	......	672^{1} n						
......	......	$659^{\cdot 5}$ s	......	......	......	......	......	......	654^{1} n			
......	679^{2} h	......	......	685^{1} n	$701^{\cdot 5}$ s							
......	......	......	......	......	......	707^{1} n	......	......	......	......	$685^{\cdot 5}$ s	
......	......	......	......	697^{1} n	$731^{\cdot 2}$ s	......	......	......	......	......	......	$689^{\cdot 5}$ s
......	......	......	......	......	......	759^{1} n						
......	719^{1} n	......	......	......	$745^{\cdot 5}$ s	......	......	782^{1} s	......	$741^{\cdot 7}$ s		
......	729^{2} n	$727^{\cdot 5}$ s	......	......	......	......	......	......	......	......	$762^{\cdot 2}$ s	
......	739^{2} n	$734^{\cdot 5}$ s	......	......	$763^{\cdot 2}$ s							
......	765^{3} n	......	......	......	......	812^{6} n	898^{1} s	......	817^{1} n			
......	......	747^{3} s	......	826^{7} n	844 h	833^{1} n	......	855^{5} n	......	......	$837^{\cdot 2}$ s	
......	787^{1} h	......	$837{\cdot}3^{5}$ n	......	......	850^{3} n	......	......	$843^{1\cdot 5}$ n			
......	$796^{\cdot 5}$ h	......	884^{1} n	$863^{\cdot 2}$ n	$865^{\cdot 7}$ s							
......	819^{2} n	......	......	......	$891^{\cdot 7}$ s	870^{2} n	......	......	$856^{\cdot 7}$ n			
$889{\cdot}5^{\cdot 5}$ n	837^{7} n	......	$887^{\cdot 5}$ s	......	......	......	......	895^{1} s				
918^{1} n	871^{7} n											
953^{1} s	889^{3} n	......	899^{1} s	......	......	$908{\cdot}8^{3}$ n	......	909^{5} n	......	......	......	913 h
986^{1} s	$921{\cdot}1^{8}$ n	$951^{\cdot 5}$ s	939^{1} h	......	921^{1} s	......	924^{2} n					
......	$937{\cdot}5^{2}$ n	$956^{\cdot 5}$ s	943^{1} h	......	$923^{\cdot 2}$ s	......	......	......	......	$929^{\cdot 7}$ s		
......	$981{\cdot}5^{1}$ n	$981^{\cdot 7}$ s	......	......	$931^{\cdot 5}$ s	......	......	......	......	......	......	939 h
......	$988{\cdot}5^{3}$ n	......	......	......	$937^{\cdot 5}$ s	......	......	......	......	......	......	$950^{\cdot 5}$ s
......	......	......	......	......	985^{2} s	......	......	......	......	......	995 } h	$958^{\cdot 5}$ s
......	$1000{\cdot}5^{3}$ n	......	......	......	......	......	1000^{1} n	1001^{2} s				
											1005 }	
Cd.	Sb.	Au.	Bi.	Hg.	Co.	As	Pb.	Zn.	Cr.	Os.	Pd.	Pt.

TABLE II.

Solar.	Air.	Na.	K.	Ca.	Ba.	Sr.	Mn.	Tl.	Ag.	Te.	Sn.	Fe.
D^2 1005												
												$1011^{\cdot 2}$ s
				$1031^{3.5}$ s	......	......	......	......	......	......	......	......
			1049^{1} n	......	1034^{6} s	......	......	1055^{2} s	$1031{\cdot}2^{\cdot 2}$ s	$1030{\cdot}3^{\cdot 5}$ s	......	$1030^{\cdot 2}$ s
			$1065^{\cdot 5}$ n	......	1057^{1} s	$1061^{\cdot 5}$ s	......	......	......	1035^{2} s	......	......
			$1073^{1.5}$ n	......	1096^{1} s	......	......	......	......	......	......	$1090^{\cdot 5}$ s
	N $1100^{\cdot 5}$	......	......	......	......	......	......	$1099^{\cdot 5}$ n	......	......	1076^{10} n	......
	N 1118^{1}	......	......	......	......	$1102^{\cdot 5}$ n	......	......	......	1111^{7} n	......	......
	N $1135^{\cdot 2}$	1169^{2} s	......	......	1119^{1} s	......	......	......	......	1122^{1} s	......	......
	N 1150^{2}	1174^{1} s	......	......	......	......	....	......	......	......	......	......
	N 1171^{1}	......	......	......	......	......	......	......	......	1151^{7} n	......	......
	N 1177^{8}	......	......	......	......	......	......	......	......	......	......	......
	N 1180^{1}	......	......	......	......	......	......	......	......	......	......	......
	N 1187^{7}	......	......	......	......	......	......	......	......	......	......	$1225^{\cdot 5}$ s
										......	......	......
										1204^{6} n	......	$1236^{1.5}$ s
	N $1294^{\cdot 5}$	......	......	......	......	1203^{1} n	......	......	$1207^{\cdot 5}$ n	......	......	$1247^{\cdot 5}$ s
	N $1302^{\cdot 5}$	......	......	$1247^{\cdot 5}$ s	......	......	.....	......	......	......	1219^{2} s	$1251^{\cdot 2}$ s
	N $1310^{1.5}$	......	......	$1249^{\cdot 5}$ s	.. ...	$1227^{1.5}$ s,t	......	......	$1223^{\cdot 2}$ n	......	......	1261^{2} s
	N $1314{\cdot}5^{\cdot 5}$	......	......	$1252^{1.5}$ s	......	......	......	.....	$1227^{\cdot 7}$ n	1230^{1} s	......	$1274^{\cdot 7}$ s
	N $1319^{\cdot 2}$	......	......	$1256^{2.5}$ s	......	......	......	......	$1240^{\cdot 3}$ s	......	1260^{8} n	$1276^{\cdot 2}$ s
	N $1349^{1.2}$	......	......	$1258{\cdot}5^{\cdot 5}$ s	......	$1268^{\cdot 2}$ s	1289^{1} s	......	$1257^{\cdot 7}$ s	1270^{4} s	......	$1338^{\cdot 7}$ s, d
				$1260^{\cdot 4}$ s	......	......	......	......	$1276{\cdot}3^{\cdot 5}$ s	......	......	$1383^{\cdot 5}$ s
	N 1366^{1}	......	......	$1265^{\cdot 7}$ s	......	......	......	......	......	......	......	$1391^{\cdot 7}$ s
			1328^{1} n	......	1271^{4} s	......	......	......	$1286{\cdot}5^{3}$ s	......	1284^{8} n	$1400^{\cdot 7}$ s
	N $1383^{\cdot 5}$	......	......	......	......	......	......	......	......	......	......	$1413^{\cdot 5}$ s
	N $1394^{\cdot 7}$	......	......	......	1307^{1} s	1301^{2} s	......	......	......	......	......	$1419^{\cdot 7}$ s
				$1335^{\cdot 7}$ s	......	$1305^{2.5}$ s	$1329^{1.5}$ n	......	.....	......	......	$1421{\cdot}5^{\cdot 7}$ s
						1311^{3} s	......	......	......	......	......	$1434^{\cdot 5}$ s
						1324^{3} s	......	......	......	......	......	$1438^{\cdot 5}$ s
					$1351^{\cdot 5}$ s	......	......	......	......	......	......	$1445^{\cdot 7}$ s
						1341^{3} s	......	.. ...	......	......	......	$1446^{\cdot 7}$ s
						$1349^{\cdot 7}$ h	......	......	......	......	......	$1456^{\cdot 7}$ s
						1359^{3} s	......	1356^{1} s	......	1357^{4} s	......	$1459{\cdot}5^{\cdot 2}$ s
						1365^{4} s	1376^{1} s	......	1372^{2} s	1366^{4} s	......	$1467^{\cdot 7}$ s
						$1397^{2.5}$ s	$1413^{\cdot 7}$ s	......	1380^{7} s	1396^{4} s	......	1481^{1} s, d
							1428^{1} s	......	$1421^{\cdot 3}$ s	......	......	$1485{\cdot}3^{\cdot 5}$ s
						$1425^{\cdot 2}$ s	$1438^{\cdot 7}$ s	......	$1435^{\cdot 3}$ s	$1438{\cdot}3^{4}$ s	......	$1486^{\cdot 2}$ s
						$1467^{1.5}$ h	$1443^{\cdot 7}$ s	......	1446 h	......	1484^{1} s	$1486{\cdot}5^{\cdot 2}$ s
	N $1502^{\cdot 2}$	......	......	1506^{1} s	......	......	1452^{2} s	......	......	......	......	$1488^{\cdot 2}$ s
	N $1516^{\cdot 2}$	......	......	......	......	......	$1456^{\cdot 2}$ s	......	......	......	......	1532^{2} s
							$1473{\cdot}5^{3}$ s	......	......	1485^{4} s	......	$1537^{\cdot 7}$ s
	N $1537^{\cdot 2}$	......	......	......	......	......	$1505{\cdot}5^{\cdot 5}$ s	1505^{9} s	......	......	1506^{2} n	$1541^{\cdot 5}$ s
							1515^{4} s	......	......	$1548{\cdot}2^{3}$ s	1520^{1} h	$1545^{\cdot 5}$ s
											1524^{4} n	$1560^{\cdot 2}$ s
							1559 } h	......	......	......	1576^{1} n	$1574^{\cdot 2}$ s
							1571 }	......	......	1562^{2} s	......	$1582^{\cdot 7}$ s,d
E { 1599	......	......	......	$1599{\cdot}5^{4}$ s	......	......	......	......	......	......	......	$1593^{\cdot 5}$ s
E { 1600	......	......	......	......	......	......	......	......	......	......	......	$1599^{\cdot 7}$ s
												$1600^{\cdot 7}$ s
Solar.	Air.	Na.	K.	Ca.	Ba.	Sr.	Mn.	Tl.	Ag.	Te.	Sn.	Fe.

From D to E.

Cd.	Sb.	Au.	Bi.	Hg.	Co.	As.	Pb.	Zn.	Cr.	Os.	Pd.	Pt.
......	......	1011^{1} n	......	$1008^{1.5}$ n	......	......	1015^{3} n					
......	......	1025^{2} s	1026^{2} n	$1019^{\cdot 5}$ n	......	......	......	......	......	1029^{1} s	$1023^{\cdot 3}$ s	
......	......	......	......	......	......	......	$1031{\cdot}1^{4}$ s	......	......	......	$1031{\cdot}5$ h	
......	$1041^{\cdot 5}$ s	1045^{4} s	1059^{1} n	......	......	1042^{1} n	......	......	......	......	......	$1041^{\cdot 5}$ s
......	$1057^{\cdot 5}$ s	......	......	$1060^{\cdot 3}$ n	......	......	......	......	......	......	......	$1045^{\cdot 5}$ s
......	......	......	......	......	......	......	......	......	......	......	$1056^{\cdot 7}$ s	
......	1081^{1} n	$1081{\cdot}5^{\cdot 5}$ s	......	1074 h	......	......	$1055^{\cdot 5}$ n					
......	......	......	......	......	......	......	......	$1062^{\cdot 5}$ s	1081^{1} h	......	1068 h	
......	$1145^{\cdot 5}$ n	1109^{1} s	......	$1083{\cdot}5^{8}$ n	......	$1090^{\cdot 3}$ n	1094^{1} n	......	1087^{1} h	......	......	$1073^{\cdot 7}$ d
......	......	......	......	$1100{\cdot}5^{8}$ n	......	......	......	$1110^{\cdot 5}$ s	1090^{1} h	$1093^{\cdot 5}$ s	1084 h	
......	......	......	1143^{6} n									
......	1158^{1} n	......	......	1177^{2} n	......	......	......	$1122^{\cdot 5}$ n				
......	$1189^{\cdot 5}$ n	......	1197^{3} n	......	......	......	......	......	......	1141^{1} s	$1127^{\cdot 3}$ s	
......	......	$1199^{\cdot 5}$ s	......	......	......	......	......	......	.. ...	......	1129^{1} s	
......	......	......	......	......	......	1203^{7} n	......	......	1212^{1} s	......	$1185^{\cdot 5}$ s	
......	1207^{1} h											
......	1214^{4} n	......	......	......	......	......	......	......	$1242^{1.5}$ s	......	$1199^{\cdot 5}$ s	
......	1220^{1} n		......	1252^{1} n								
......	......	$1266^{\cdot 5}$ n										
......	1279^{4} n											
......	......	......	1293^{1} s	......	$1039^{\cdot 2}$ s							
......	1383^{3} n	......	......	......	$1043^{\cdot 2}$ s	......	......	......	......	......	$1212^{\cdot 5}$ s	
......	......	......	$1305^{\cdot 5}$ s	......	$1207^{\cdot 5}$ s	$1231^{\cdot 5}$ n	1240^{5}					
......	......	.. ...	......	......	$1217^{\cdot 5}$ s	$1257^{\cdot 5}$ n	......	......	......	1264^{1} s	1219 } h	
......	......	......	......	......	$1257^{\cdot 7}$ s	......	1279^{5} n	$1269^{\cdot 5}$ n	......	......	1233 }	
......	......	......	......	......	......	......	......	......	......	......	$1240{\cdot}5^{\cdot 5}$ s	
......	......	......	......	......	......	......	......	1283^{1} n	......	......	$1248^{\cdot 5}$ s	
......	......	......	......	......	1361^{1} s	1291^{6} n	......	......	......	......	$1259{\cdot}5^{\cdot 7}$ s, d	
......	......	......	......	......	$1401^{1.5}$ s							
......	......	......	......	......	$1470^{\cdot 5}$ s	......	......	......	......	1322^{2} s	$1281^{1.5}$ s	
......	......	......	......	......	1483^{1} s	......	......	......	......	......	1299^{1} s	
......	......	......	......	1385^{10} n	1491^{1} s	1348^{6} n	......	......	......	......	$1303^{1.5}$ n	
......	......	......	......	......	$1496^{\cdot 2}$ s	......	......	......	......	......	$1331^{1.5}$ n	$1367^{\cdot 7}$ s
......	......	......	1395^{1} h	......	......	......	......	......	......	$1405^{\cdot 5}$ s	$1380^{1.5}$ s	
......	1457 h	......	......	......	$1500{\cdot}5^{2}$ s							
......	......	......	......	$1421^{1.5}$ n	$1501{\cdot}5^{2}$ s	......	......	......	$1439^{1.5}$ s	$1432^{1.5}$ s	$1412^{\cdot 5}$ s	
1473^{10} n	1471^{3} n	......	1453^{1} h	......	$1508^{\cdot 2}$ s	1443^{1} n	......	......	......	......	1456^{2} s	$1459^{\cdot 5}$ s
......	......	......	......	$1487^{\cdot 5}$ n	1514^{4} s	......	......	......	$1507{\cdot}5^{\cdot 7}$ s	......	$1492^{\cdot 5}$ s	$1484^{\cdot 5}$ s
......	......	......	......	..... .	$1525^{\cdot 5}$ n, d	1465^{1} n	......	......	1510^{1} s	.		
......	......	......	1495^{1} h	......	$1534^{\cdot 5}$ s	......	1479^{7} n	......	......	......	$1511^{\cdot 5}$ s	
......	1501	......	......	......	$1539^{\cdot 3}$ s	......	......	......	......	......	$1548^{\cdot 5}$ s	
1517^{10} n	......	......	......	......	$1543^{\cdot 2}$ s	......	......	......	1532^{1} s	......	......	1561^{2} s
......	......	......	......	......	$1549^{\cdot 5}$ s	......	......	$1519^{\cdot 5}$ n	......	......	1569^{3} s	
1556^{1} n	......	......	......	......	$1573^{\cdot 2}$ s	1529^{6} n	......	......	1567^{1} s, d			
......	......	......	......	......	$1579^{\cdot 2}$ s							
......	......	......	......	$1583^{\cdot 5}$ n	$1584^{\cdot 2}$ s	1577^{1} n	......	......	$1594^{\cdot 7}$ s			
......	......	......	1598^{7} n	......	$1586^{1.5}$ s							
......	......	......	......	......	$1591^{\cdot 5}$ s	......	$1593{\cdot}3^{\cdot 5}$ n					
Cd.	Sb.	Au.	Bi.	Hg.	Co.	As.	Pb.	Zn.	Cr.	Os.	Pd.	Pt.

N.B. In the column Ba the line 1271^{4} should be 1308^{4} and the line 1307^{1}, 1327^{1}.

TABLE III.

Solar.	Air.	Na.	K.	Ca.	Ba.	Sr.	Mn.	Tl.	Ag.	Te.	Sn.	Fe.
E { 1599												
1600												
				1605^{3} s	......	......	......	......	......	......	......	1603·^{6} s
				1609·^{5} s	......	1617^{3} s	1617·^{5} s	......	......	......	......	1608·^{5} s
				1612^{1,5} s	......	1638^{2} s	......	......	......	1658·^{5} n	......	1613·^{3} s
	O 1678·^{5}	......	......	......	......	1651^{1,5} s	......	......	.... .	......	1657^{2} n	1621·^{3} s
						1656^{1,5} s	.... .	......	1675^{5} n	......	......	1632·^{3} s
	O 1699·^{5}	......	......	......	......	1659^{1,5} s	......	......	......	......	......	1645^{1} s
	N 1713 h	......	......	1702^{1} s	......	1665^{1} s	......	......	......	......	......	1653^{1} s
b { 1708	N 1718 h *	......	......	......	......	1745^{1} s	......	......	......	......	......	1662·^{5} s, d
1723	N 1721 h	......	......	......	......	......	......	1747^{4} n	......	......	......	1691·^{3} s
1731·5	... ..	......	......	......	......	......	......	......	......	......	......	1696·^{3} s
	O 1737·^{3}	......	......	......	......	......	......	......	......	1773^{1} n	......	1698·^{5} s
		1746^{1} n	......	......	......	......	......	......	......	......	......	1713·^{7} s
		1753·^{5} n	......	.. ...	......	......	......	......	......	......	......	1728^{1} s
												1731·^{5} s
												1753·^{3} s
						1817·^{5}	......	1851^{1} n	......	......	......	1767·^{7} s
	N 1860·^{5}	......	.... .	......	......	......	......	......	......	......	1821^{3} n	1775·^{5} s
												1821 h
	N 1900^{1}	......	......	1907·^{7} s	......	......	......	1885^{1} s	......	1909·^{5} n	......	......
	N 1929·^{7}	......	......	......	......	......	......	......	......	......	......	......
	N 1941·^{5}	......	......	1935·^{3} s	......	......	......	......	......	......	......	......
	N 1951·^{5}	......	......	......	......	......	......	......	.. ...	......	......	......
	N 1956·^{5}	......	......	......	......	......	......	......	......	......	......	1940^{1}·5 s
	N 1960·5^{10}											
	N 1967^{10}											
	N 1978·^{5}											
	N 1990·^{3}	......	......	......	......	......	......	......	......	......	......	......
		1991 h										
								1999^{2} h	......	......	......	......
	O 2043·^{5}	......	......	......	......	2021^{1} s	......	......	......	......	......	2036^{1,5} s
	O 2060·^{7}	......	......	......	......	2029^{2} s						
	N 2079·^{5}	......	......	......	......	2060 h	......	2146^{2} s				
	O 2089·^{5}	......	......	......	2075^{3} n	2145^{1} s	......	......	......	......	......	......
	O 2119·^{5}											
	N 2140·^{5}	......	......	......	. ....	2176^{1,5} s	......	......	......	......	......	2092^{1} s
	O 2145·^{5}	......	......	......	2133^{4} n	......	......	......	......	......	......	2098·^{7} s, d
	N 2168·^{5}	......	......	......	......	......	......	......	......	......	......	......
												2147^{1} s
				2172·^{7} s								
	O 2181·^{5}	......	......	......	......	2180^{1,5} s						
						2185^{1} s	......	......	......	......	......	......
	N 2192·^{3}	......	......	......	......	......	......	......	......	......	......	......
F 2200										2191^{1} h	......	......
Solar.	Air.	Na.	K.	Ca.	Ba.	Sr.	Mn.	Tl.	Ag.	Te.	Sn.	Fe.

* When the induction spark is taken in oxygen, a faint line is seen nearly in the position of the nitrogen line 1718. Since the lines of oxygen have a diminished intensity when the spark passes in air, this line would be too faint to be distinctly observed in the air-spectrum, in which it occurs in a position of close proximity to brighter lines of nitrogen.

From E to F.

Cd.	Sb.	Au.	Bi.	Hg.	Co.	As.	Pb.	Zn.	Cr.	Os.	Pd.	Pt.
......	......	......	......	......	$1602^{\cdot3}$ s	......	......	......	$1605{\cdot}5^{1}$ s			
......	......	......	......	......	1604^{3} s	......	......	......	1607^{1} s	......	$1617^{\cdot5}$ s	
......	1636^{3} h	......	......	......	$1617^{\cdot5}$ s	1648^{4} n	......	......	$1619^{\cdot6}$ s	......	$1622^{\cdot5}$ s	
......	......	1647^{5} s	......	......	$1619^{\cdot3}$ s	......	......	1626^{1} n	$1626^{\cdot6}$ s	......	$1642^{1\cdot5}$ s	1653^{1} s
......	1661^{1} s	......	......	$1662^{\cdot3}$ h	$1622^{\cdot5}$ s	......	......	......	$1640^{\cdot6}$ s	......	$1674^{\cdot7}$ s	
......	......	......	1675^{10} n	......	$1626^{\cdot3}$ s	......	......	$1645^{\cdot5}$ n	......	$1683^{\cdot5}$ s		
......	......	......	1685^{1} n	......	$1642^{\cdot5}$ s	......	1685^{1} s	......	$1657^{\cdot7}$ s			
......	1715^{3} h	......	......	......	$1650^{\cdot5}$ s	......	......	......	1677^{4} s			
1747^{1} s	......	......	1759^{7} n	......	$1670^{1\cdot5}$ s	1737^{1} n	1698^{1} n	......	1680^{3} s	......	......	$1689^{\cdot3}$ s
......	1765^{3} h	......	......	$1777^{\cdot5}$ h	$1685^{\cdot5}$ s	......	......	......	$1681{\cdot}5^{3}$ s			
......	......	......	1787^{6} n	......	$1699^{\cdot5}$ s	......	......	$1743^{\cdot5}$ n	......	......	1735^{2} s	
......	......	......	......	......	$1707^{\cdot5}$ s	......	$1735^{\cdot5}$ n	......	1749^{2} n			
......	1803^{3} h	......	......	......	$1743^{\cdot5}$ s	......	......	$1790^{\cdot5}$ n				
......	......	......	1834^{1} n	......	$1756^{\cdot5}$ s	......	......	$1797^{\cdot5}$ n	......	......	$1798^{1\cdot3}$ s	
1843^{10} s	......	......	......	......	1781 n							
......	1849^{1} s	......	$1851{\cdot}3^{5}$ n	......	1813 h	1814^{5} n	......	......	$1815^{\cdot7}$ n	......	1807^{1} s	
......	......	$1869^{\cdot5}$ s	. ....	......	......	......	......	$1845^{\cdot5}$ n				
......	......	......	......	......	$1857^{\cdot5}$ h	......	......	1859^{1} n	......	$1859^{\cdot5}$ s		
......	1900^{2} h	......	......	......	$1876^{\cdot3}$ s	......	$1900{\cdot}3^{2}$ n	$1893^{\cdot5}$ n	......	......	$1873^{\cdot5}$ s	1879^{1} s
......	......	......	......	......	$1887^{\cdot3}$ s							
......	1919^{2} h											
......	......	......	......	......	$1925^{\cdot5}$ s							
......	......	......	1979^{1} h									
......	......	......	......	......	......	1993^{2} n						
			2015^{1} h	......	$2021^{\cdot5}$ s							
......	......	......	......	2033^{1} h	......	......	......	$2016^{\cdot5}$ n				
......	2051^{3} n	......	......	......	......	......	......	2091^{8} n	2097^{1} s			
......	......	......	......	$2101^{\cdot5}$ h	......	......	......	2110^{8} n				
......	......	......	$2105^{\cdot5}$ n									
......	......	......	2119^{1} n									
......	......	......	......	......	......	2153^{1} n	......	... ..	2156^{1} s	......	$2175^{1\cdot5}$ s	
......	2171^{3} n	......	......	......	......	......	......	......	$2175^{\cdot7}$ s			
......	......	......	......	......	2186^{3} s	......	......	......	$2181^{\cdot3}$ s			
......	......	......	......	......	......	......	......	$2191^{\cdot5}$ n	$2198^{\cdot5}$ s			
Cd.	Sb.	Au.	Bi.	Hg.	Co.	As.	Pb.	Zn.	Cr.	Os.	Pd.	Pt.

TABLE IV.

Solar.	Air.	Na.	K.	Ca.	Ba.	Sr.	Mn.	Tl.	Ag.	Te.	Sn.	Fe.
F 2200	N $2205^{.5}$	……	……	……	……	2213^{1} s						
	O $2218^{.3}$											
	N $2221^{.3}$											
	N 2305^{2}	……	……	……	……	$2254^{1.5}$ s	2267^{6} s	……	……	2245^{1} h	2205^{3} n	……
	N 2336^{1}	……	……	……	……	$2291^{1.7}$ s	2343^{5} s	……	……	$2341^{.5}$ h	……	……
	N $2350^{.7}$	……	2260^{2} n	……	……	$2343^{1.5}$ s	2379^{6} s	……	……	2497 h	……	……
	O $2502^{.7}$ n	……	……	……	……	$2410^{.5}$ s	2385^{5} s	2379^{2} n	……	……	……	……
	O $2512^{.7}$ n	……	……	……	……	2427^{1} s	2401^{5} s	……	……	2595^{1} n	……	……
	O $2563^{.5}$ n	……	……	……	$2459^{.5}$ s	2469^{1} s	2433^{1} s	2437^{4} n	……	2613^{1} n	……	……
	O $2597^{.5}$ n	……	……	……	……	2726^{5} n	2456^{1} s	……	……	……	……	……
	O 2626^{2}	……	……	……	……	……	2492 h	……	……	……	2777^{2} n	$2781^{.5}$ s
	N, O 2642^{2} d	……	……	……	$2535^{.5}$ s	……	……	……	……	2730^{1} n	……	……
	N 2669^{1}	……	……	……	……	……	……	……	……	2739^{1} n	……	……
	N 2689^{1}	……	……	……	……	……	……	……	……	……	……	……
	N 2707^{1}											
	N $2722^{1.5}$											
	N $2738^{1.5}$	……	……	……	……	……	……	……	……	……	……	……
	O 2748^{1}	……	……	$2777^{.5}$ s	……	……	……	……	……	……	……	……
	O 2766^{1}	……	……	$2784^{.3}$ n	……	……	……	……	……	……	……	……
	N 2856 h	……	……	$2792^{.2}$ s	2856^{9} s	……	……	……	……	……	……	……
	N 2904 } h	……	……	……	……	……	……	……	……	2875 h	……	……
	N 2978 }	……	……	……	……	……	$2987^{.7}$ s	……	……	……	……	……
	N 3009^{1}	……	……	……	2931^{3} n	……	$2999^{.7}$ s	……	……	……	……	……
	N 3011^{1}	……	……	……	……	……	3021^{1} s	……	……	……	……	$3272^{1.5}$ s
	N 3056 h											
	O 3086 h	……	……	……	……	……	3054^{1} s	……	……	……	2931^{4} n	……
	N 3144 d	……	……	3124^{3} s	……	……	3097^{1} s	……	……	3051^{1} h	……	3341^{2} s
	N 3174 } h	……	……	3181^{3} s	……	3169^{1} s	3102^{2} s	……	……	……	……	$3532^{1.5}$ s
	N 3219 } h	……	……	……	……	……	3114^{1} s	……	……	$3435^{.5}$ n	……	……
	O 3238^{1} n	……	……	3212^{2} s	……	……	3120^{1} s	……	……	……	……	……
	O 3241^{1} n	……	……	……	……	$3389^{.5}$ n	$3131{\cdot}5^{.3}$ s	……	……	……	……	……
	N 3292 h	……	3328^{2} n	3561^{3} s	……	$3409^{.5}$ n	3133^{2} s	……	……	……	……	……
	O 3395^{1} n	……	……	$3602{\cdot}5^{2.3}$ s	……	……	3141^{1} s	……	……	……	……	……
	O, N 3456^{2} n	……	……	3617^{1} s	……	3489^{1} n	3180^{1} s	……	……	……	……	……
G 3597	O 3560^{1} n,d	……	$3591^{1.5}$ n	$3628^{2.5}$ s	……	3553^{1} h	3242^{1} s	……	……	……	……	$3597^{1.5}$ s
	O 3710 h	……	……	3665^{3} s	……	3604^{5} n	3691^{1} s	……	……	……	……	$3610^{1.5}$ s
	N 3863 h	……	3762^{2} n	$3692^{2.5}$ s	……	……	3749^{1} s	……	……	3619^{2} n	……	$3623^{.5}$ s
	N 3991 h	……	……	……	……	3952^{5} n	3782^{1} s	……	……	……	……	$3645^{.5}$ s
	O 4059^{1}	……	……	3909^{6} s	……	……	……	……	……	……	……	3728^{2} s
	O 4087^{1}	……	4082^{3} n	……	……	4181^{3} n	3870^{1} s	……	……	……	……	$3773^{1.5}$ s
	N 4145 h	……	……	……	4167^{2} n	……	……	……	……	3779 h	……	$3812^{1.5}$ s
	O 4232	……	……	……	……	……	……	……	……	……	……	……
	N 4263^{1} n	……	……	……	……	……	……	……	……	……	……	4009^{1} s
	N 4330 h	……	……	……	4332^{5} n	……	……	……	……	……	……	4019^{1} s
	O 4395^{2}	……	……	……	……	……	……	……	……	……	……	4221^{1} s
	N 4473^{1} n	……	……	……	……	……	……	4443^{3} n	……	……	……	4267^{1} s
	N 4505^{1} n	……	……	……	……	……	……	……	……	4703^{3} h	……	4328^{1} s
	O 4615^{1} n	……	……	……	……	4599^{3} n	……	……	……	……	……	4633^{1} s
	O 4639^{1} n	……	4791^{3} n	……	……	……	……	……	……	……	……	4671^{1} s
	N 4821 h	……	……	……	……	……	……	……	……	……	……	4781^{1} s
	N $5077^{.7}$	……	……	……	……	……	……	……	……	……	……	……
H 5277	………	……	……	5277^{6} n								
Solar.	Air.	Na.	K.	Ca.	Ba.	Sr.	Mn.	Tl.	Ag.	Te.	Sn.	Fe.

From F. to H.

Cd.	Sb.	Au.	Bi.	Hg.	Co.	As.	Pb.	Zn.	Cr.	Os.	Pd.	Pt.
......	2251^{2} n	......	......	......	......	......	......	......	$2257^{\cdot 5}$ s			
2315^{8} s	2339^{2} h	......	......	......	......	......	......	......	$2266^{1\cdot 5}$ s			
......	2377^{2} h	$2291^{\cdot 5}$ s	......	......	2236^{3} s	......	......	......	......	......	$2279{\cdot}5^{2}$ s	
......	2397^{2} n	......	......	$2263^{\cdot 5}$ n	......	......	......	2294^{7} s	$2336^{\cdot 7}$ s			
2562^{6} s	2440^{2} n	2326^{4} s	2317^{1} n	......	2286^{2} s							
......	2488^{3} n	......	......	......	2325^{1} s	......	2384^{3} n					
......	......	......	2408^{1} n	......	$2409^{\cdot 5}$ s	......	......	......	2400^{1} s			
......	2529^{3} n	.. ...	2467^{5} n	......	$2438^{\cdot 5}$ s	2450^{1} n	......	......	$2406^{\cdot 5}$ s			
......	2687^{1} h	......	2453^{1} n	......	$2471^{\cdot 5}$ s	......	......	2469^{6} s	$2435^{\cdot 7}$ s			
......	2740^{1} h	......	2502^{1} n	......	$2550^{\cdot 5}$ s	......	......	......	$2452^{\cdot 7}$ s			
......	2763^{2} n	......	......	......	......	......	......	......	2474^{1} s			
......	......	......	......	......	......	......	......	2559^{4} s				
......	......	......	......	......	2785^{2} n							
......	......	......	......	......	......	......	......	......	2619^{1} s			
......	......	......	2837^{5} n	......	......	......	......	......	$2627^{\cdot 7}$ s			
......	2977^{4} n	......	......	......	2823^{1} n	2859^{1} h	......	......	$2632^{1\cdot 5}$ s, d	......	......	2857^{1} s
......	......	......	3060^{1} n	......	......	2897^{1} h	......	......	$2663^{\cdot 5}$ s			
......	......	......	......	......	2862^{1} s	......	......	......	$2701^{\cdot 7}$ s			
......	......	......	......	......	......	......	......	......	......	......	......	$2936^{\cdot 5}$ s
......	......	$3026^{2\cdot 5}$ s	......	......	2910^{3} s	......	......	......	$2740^{\cdot 7}$ s			
......	3115^{1} n	......	......	......	......	......	......	......	$2768^{\cdot 7}$ s	......	......	$2999^{\cdot 7}$ s
......	......	......	......	......	......	......	......	......	......	......	$3065^{\cdot 7}$ s	
3239^{4} s	......	......	......	......	......	$3006^{1\cdot 5}$ h	......	......	$2840^{\cdot 7}$ s	$2861^{\cdot 7}$ s		
......	3359^{1} h	......	......	......	......	......	......	......	$2871^{\cdot 7}$ s			
......	......	......	......	......	......	......	......	......	$2887^{\cdot 7}$ s	3225^{2} s	......	3156^{1} s
......	......	......	3315^{2} n	......	......	$3097^{2\cdot 5}$ h	......	......	$2899^{\cdot 7}$ s	3421^{1} s		
......	......	......	......	......	......	......	......	......	2914^{1} s			
......	3446^{4} n	......	3481^{3} n	3421^{8} n	......	......	......	......	2927^{1} s	3583^{1} s		
......	......	......	......	......	......	......	3329^{7} n	......	3007^{1} s			
......	......	......	3519^{2} n	......	......	......	......	......	3444^{2} s	3645^{1} s	......	3525^{1} s
......	3756^{3} n	......	......	......	......	3381^{1} n	......	......	3465^{1} s			
......	3819^{1} n	......	3619^{5} n	......	......	......	......	......	3473^{1} s			
......	$4043^{1\cdot 5}$ n	......	......	......	......	3497^{1} n	......	......	3489^{1} s			
......	......	......	3778^{5} n	......	......	......	3730^{2} n	......	3663^{3} s	3773^{3} s		
......	......	......	......	......	......	......	......	......	3719^{3} s			
......	......	......	......	......	......	......	......	......	3797^{3} s	......	3963^{4} s	
......	......	......	......	......	......	......	3831^{6} n	......	3905^{1} s			
......	......	......	......	......	......	......	......	......	3951^{1} n			
......	......	......	......	......	4388^{3} n							
......	......	......	4378^{4} n	4775^{4} n	4394^{3} n							
......	......	......	......	......	4437^{1} n							
......	......	......	4603^{3} n									
......	......	......	......	......	4523^{1} n	......	4713^{7} n					
......	......	......	......	5158^{3} n								
Cd.	Sb.	Au.	Bi.	Hg.	Co.	As.	Pb.	Zn.	Cr.	Os.	Pd.	Pt.

NOTE TO PLATES I. AND II.

The scale upon which the spectra have been laid down limits the intensity that can be given, in the engraving, to the stronger lines. From this cause the spectra, as engraved, appear too faint. If greater force had been given to the lines, by making them broader, they would, in several spectra, have occupied singly the space in which two or more lines have to be laid down. This deficiency in strength of some of the lines is more appreciated by the eye, in consequence of the shortness of the lines of the spectra, with the exception of those of the air-spectrum. The narrowness of the spectra of the metals is unavoidable, if the great advantage of having all the spectra upon one Plate is retained.

In some of the spectra bands of unresolved light occur; these, in the Plates, are crossed with lines that they may be distinguished from groups of fine lines.

III. *Account of Magnetic Observations made in the years* 1858–61 *inclusive, in British Columbia, Washington Territory, and Vancouver Island. By Captain* R. W. Haig, *R.A. Communicated by General* Sabine, *P.R.S.*

Received November 4,—Read November 26, 1863.

In 1858 a Commission was appointed for the purpose of determining and marking the forty-ninth parallel of north latitude from the Pacific to the Rocky Mountains. At the suggestion of General Sabine, this Commission was provided with a set of portable magnetic instruments adapted for the determination of the three magnetic elements, Dip, Declination, and Total Force. These instruments were similar in kind to those which had been used on the Magnetic Survey of the United Kingdom. Before delivery to the Boundary Commission they were examined at the Kew Observatory, and several constants and tables for facilitating the computations were determined and prepared there.

The method of transporting the instruments from place to place, and indeed everything appertaining to the Boundary Commission, was by means of packet mules. Two boxes (a very light load for one mule) contained all the magnetic instruments, which throughout four years of such rough usage retained their original efficiency. Some of the needles became somewhat rusted; but I can suggest no alteration in the construction of such instruments, such as would increase their portability. The declinometer was, I think, unsatisfactory as regarded its capability of determining azimuths of the sun: when at an astronomical station, I necessarily had a meridian mark for the transit instrument, and I referred the direction of the magnet to such meridian.

In assembling the results and deducing from them the directions and positions of the lines of equal dip, force, and declination, no notice has been taken of secular change. The only station at which we have data for judging of the extent of secular change is Fort Vancouver on the Columbia River. As regards dip, we find there

1830.	Dip	69° 39′·7	Douglas.
1839.	Dip	69 22·2	Sir E. Belcher.
1860.	Dip	69 17·4	Present observations.

These figures show an annual diminution of dip of less than 1′ per annum. The mean results of the present observations may be assumed to belong to the year 1860. The method of assembling the results and determining from them the position and direction of the lines of equal dip, force, and declination, is the same as that adopted in the Survey of the United Kingdom.

It consists in referring all the results to the point of mean longitude and latitude among all the stations, and assuming the differences of longitude and latitude expressed in geographical miles to be plane rectangular coordinates of distance from the origin. It is also assumed that the three magnetic elements vary uniformly over the whole district surveyed.

Dip.

Table I. gives the individual results at each station, as well as the partial results from the needle when magnetized in opposite directions. Table II. shows the mean results, along with the most probable dip at each station computed by the method of minimum squares. By the method of minimum squares the variation of dip for one mile of longitude $x=+0'\cdot272$, and the variation for one mile of latitude $y=+0'\cdot776$; from these values we get $u=70^\circ\ 42'$ for the angle which an isoclinal line makes with the meridian measured from the north round by west, and $\frac{1}{r}$ or $\frac{1}{\sqrt{x^2+y^2}}=1\cdot216$ mile for the distance between isoclinals whose difference of dip is 1′. Column 5 contains the most probable dip at each station (θ'), obtained from equations $\theta'=\theta_1+ax+by$. The probable error of the computed dip at each station is nearly equal to $\pm5'$.

Intensity of the Magnetic Force.

The stations where these observations were made are the same as the dip stations. At ten of them observations of deflection and vibration for the horizontal component were made with the unifilar, and observations by LLOYD's statical method for variation of the total force; at the remaining eleven stations the statical method only was employed.

Observations at the first ten stations furnish values of the constant log A, of which a mean value might be adopted for use at the other eleven; but from an examination of these values of log A, it appears that those belonging to 1861 are generally larger than preceding values, owing, I believe, to the weighted needle having become rusted. I have therefore adopted the mean of all values of log A previous to 1861 for statical observations up to that period, and the mean of those in 1861 for observations in that year. The first value is log A$=0\cdot91931$, and that for 1861 is log A$=0\cdot92032$.

Table III. shows the unifilar observations, and the values of X, the horizontal component of the magnetic intensity, derived from them. Column 12 of this Table contains the dips at the several stations, and column 13 the total force $\varphi=X\sec\theta$.

Panama, the first station in this Table, is not included in the general assemblage of results in Table IV., its distance from the other stations being too great. Table IV. shows the combination of all observations for force to determine the direction and distance apart of the isodynamic lines. Column 3 in this Table corresponds to column 13 in Table III. We find (x) the variation in total force for one mile of longitude $=+0\cdot000925$, and (y) the variation for one mile of latitude $=+0\cdot000896$; $\frac{1}{r}$ the distance between the isodynamic lines a unit of force apart$=776\cdot6$ miles, or for a tenth

part of a unit$=77\cdot66$ miles; u, the angle which isodynamic lines make with the meridian measured from the north round by west, is 44° 6′. Column 11 of Table IV. contains the most probable value of the total force at each station, and the probable error of one such value is $\pm0\cdot044$.

Declination.

Table V. is similar in character to Tables II. and IV.; by it we find (x) the increase of declination for one mile of longitude $=+0'\cdot230$, and (y) the increase of declination for one mile of latitude $=+0'\cdot423$; $\frac{1}{r}$ the distance between lines of equal variation 1′ apart $=2\cdot0756$ miles; therefore, for 1° of difference in the declination, the distance is 124·54 miles; u, the angle which lines of equal declination make with the meridian measured from the north round by west, is 61° 27′. The most probable declinations are shown in column 5 of Table V., and the probable error of one such result is $\pm27'$.

The results contained in Tables II., IV. and V. are represented graphicaly in Plate III., which exhibits a map of the country surveyed, with the lines of equal inclination, declination, and intensity drawn upon it.

Table I.—Dip.

1. Date.	2. Needle.	3. Station.	4. Poles direct.	5. Poles reversed.	6. Dip.	7. Mean dip.
1858. August 19 ...	1	Esquimalt	71° 18′·94	71° 42′·06	71° 30′·5	° ′
1859. March 18 ...	4	„	71 31·55	71 42·78	71 37·2	71 34·2
1860. March 22 ...	1	„	71 23·00	71 46·20	71 34·9	
1858. Oct. 4 ...	1	Sumass Prairie	72 12·08	72 29·15	72 20·6	72 22·0
5 ...	4	„	72 16·66	72 29·88	72 23·3	
1859. Jan. 31 ...	4	Nisqually	70 29·62	70 49·56	70 40·0	
July 4 ...	1	Schweltza Lake	71 57·31	72 13·68	72 05·8	72 03·9
5 ...	1	„	71 50·44	72 13·25	72 01·9	
Sept. 7 ...	1	Chilukweyuk Lake	72 19·60	72 42·40	72 31·0	
1860. May 3 ...	1	Fort Vancouver, W. T.	69 02·83	69 31·88	69 17·4	
21 ...	1	Dalles, W. T., 3-mile camp......	69 29·90	69 53·80	69 41·8	
June 1 ...	1	„ 8-mile camp......	69 55·12	70 13·66	70 04·5	
July 9 ...	4	On Ashtnolou River	72 29·09	72 44·79	72 36·9	
August 18 ...	1	Ashtnolou Station	72 16·80	72 37·20	72 27·0	
Nov. 13 ...	4	Inshwointum	72 34·75	73 02·90	72 48·8	
1861. March 26 ...	4	Fort Colville	72 51·81	73 33·93	72 42·9	
April 2 ...	4	„	72 44·60	72 56·50	72 50·6	72 41·9
12 ...	4	„	72 32·40	72 44·70	72 38·5	
23 ...	4	„	72 34·20	72 36·70	72 35·5	
May 19 ...	1	Chomikane River	71 45·40	72 24·60	72 04·2	
31 ...	4	Sinyakwateen	72 29·60	72 42·00	72 35·8	72 34·9
31 ...	1	„	72 17·31	72 50·61	72 34·0	
June 19 ...	1	Pack River	72 31·62	72 59·44	72 45·5	
23 ...	1	Chelemta	72 54·56	73 21·50	73 08·0	
July 6 ...	1	South Crossing (Kootenay)......	72 37·50	72 59·00	72 48·1	
12 ...	1	On Kootenay River	72 57·31	73 17·06	73 07·2	
August 19 ...	1	Tobacco Plains (Kootenay)......	73 16·06	73 29·44	73 22·9	
14 ...	1	Wigwam River Station	73 27·06	73 34·62	73 31·0	
2 ...	1	Akamina Station	73 34·12	73 51·31	73 42·7	

Table II.—Dip.

1.	2.	3.	4.	5.
Station.	W. Longitude. μ.	N. Latitude. λ.	Observed dip. θ.	Computed dip. θ'.
Esquimalt	123 27	48 26	71 34	71 30
Sumass Prairie	122 12	49 01	72 22	72 11
Nisqually	122 25	47 07	70 40	70 39
Schweltza Lake Station	122 00	49 02	72 04	72 14
Chilukweyuk Lake	121 23	49 02	72 31	72 21
Fort Vancouver	122 28	45 38	69 17	69 28
Dalles, 3-mile camp	120 49	45 35	69 42	69 45
Dalles, 8-mile camp	120 49	45 40	70 05	69 49
On Ashtnolou River	120 00	49 10	72 37	72 42
Ashtnolou Station	120 00	49 00	72 27	72 34
Inshwointum Station	118 28	49 00	72 49	72 50
Colville B. B. C. Barracks Station	118 05	48 40	72 42	72 39
Chemikane River	117 45	48 00	72 04	72 12
Sinyakwateen	116 44	48 09	72 35	72 30
Pack River	116 28	48 22	72 46	72 43
Chelemta	116 19	48 41	73 08	72 59
South Crossing (Kootenay)	115 21	48 22	72 48	72 55
On Kootenay River	115 17	48 40	73 07	73 09
Tobacco Plains (Kootenay)	115 08	48 57	73 23	73 24
Wigwam River Station	114 45	49 00	73 31	73 30
Akamina Station	114 04	49 01	73 43	73 38
	393 57	172 33	45 55	
μ_1, λ_1, and θ_1 respectively	118 45	48 13	72 11	
Probable error of a single observation $=\pm 4{\cdot}93$*.				

* This of course includes the effects of local irregularities in terrestrial magnetism as well as actual errors of observation. A similar remark applies to Tables IV. and V.

TABLE III.—Intensity of the Magnetic Force.

Observations with the Unifilar Magnetometer. Vibrations and Deflections.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
Date.	Station.	N. Latitude.	W. Longitude.	r_0.	u_0.	$\log \frac{m}{x}$.	log T².	log π² K.	log *m*X.	X.	*i*.	φ.
				feet.								
1858. April 29	Panama	8° 57′	79° 31′	1	9° 10′ 41″·0	8·90771	......	......	......	......	32° 30′	
30	....	......	......	1	9 8 29·0	8·90594	1·60697	1·66866	0·66353	7·6283	......	9·0447
30	....	......	......	...	......	......	1·59077	1·66861	0·67986			
May 1	....	......	......	1	9 8 49·0	8·90636	1·59065	1·66859	0·68000	7·7058	......	9·1367
2	....	......	......	1	9 6 49·0	8·90479	1·59117	1·66858	0·67949	7·6270	......	9·0433
3	Taboga Island	8 48	79 32	1	9 10 53·0	8·90730	1·59272	1·66858	0·67794	7·6793	32 12	9·0753
Oct. 13	Sumass Prairie	49 01	122 12	1	11 46 53·5	9·01242	......	......	......	......	72 22	
21	....	......	......	1	11 49 02·1	9·01324						
21	....	......	......	1·3	5 21 33·6	9·01348						
Nov. 1	....	......	......	...	......	......	1·44048	1·66836	0·22788			
10	....	......	......	...	......	......	1·43994	1·66837	0·22843	4·0509	......	13·3727
1859. Jan. 24	Esquimalt, V. I............	48 26	123 27	1	11 42 19·6	9·00831	1·43593	1·66835	0·23242	4·0935	71 34·1	12·9474
24	....	......	......	1·3	5 18 11·5	9·00812						
March 21	....	......	......	1	11 33 05·4	9·00374						
21	....	......	......	1·3	5 14 17·7	9·00375						
22	....	......	......	...	......	......	1·42962	1·66835	0·23876	4·1449	......	13·1097
1860. May 3	Fort Vancouver, W. T....	45 38	122 28	1	10 20 29·2	8·95700	......	......	......	4·6180	69 17·4	13·0585
8	....	......	......	...	......	......	1·38249	1·66839	0·28590			
Aug. 17	Ashtnolou Station	49 00	120 00	1	11 52 20·4	9·01711	1·44514	1·66865	0·22351	4·0105	72 37·0	13·3003
1861. April 18	Fort Colville	48 40	118 05	1	11 49 30·3	9·01583	......	......	......	......	72 41·9	13·4239
23	....	......	......	1	11 54 39·6	9·01679	1·44954	1·66834	0·21880			
23	....	......	......	...	......	......	1·44964	1·66835	0·21871	3·9923		
May 31	Sinyakwateen	48 09	116 44	1	11 48 59·7	9·01459	1·44705	1·66847	0·22142	4·0125	72 34·9	13·4045
June 19	Pack River	48 22	116 28	1	11 57 17·0	9·01957	1·45167	1·66845	0·21678	3·9683	72 45·5	13·3881
23	Chelemta River	48 41	116 19	1	12 09 22·0	9·02602	1·45766	1·66839	0·21073	3·9116	73 08·0	13·4816
July 6	South Crossing (Kootenay)	48 22	115 21	1	11 55 50·0	9·01860	1·45159	1·66845	0·21686	3·9731	72 48·2	13·4384
15	Wigwam River	49 00	114 45	1	12 21 41·0	9·03485	1·46796	1·66849	0·20053	3·8268	73 30·8	13·4850

Table IV.—Total Force.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
Year.	Station.	ϕ by unifilar.	log ϕ.	$\log\sqrt{\dfrac{\cos v}{\sin u \sin u'}}$	log A.	Mean log A.	log ϕ.	Statical ϕ.	Adopted ϕ.	ϕ'.
1859–60	Esquimalt	13·1097	1·11759	0·19781	0·91978	0·91931	1·11712	13·0955	13·103	13·148
1858	Sumass Prairie	13·3727	1·12622	0·20647	0·91975	0·91931	1·12578	13·3591	13·366	13·226
1859	Nisqually	......	......	0·19848	......	0·91931	1·11779	13·1158	13·116	13·111
do.	Schweltza Lake Station	......	......	0·20406	......	0·91931	1·12337	13·2853	13·285	13·234
do.	Chilukweyuk Lake	......	......	0·20062	......	0·91931	1·11993	13·1803	13·180	13·257
1860	Fort Vancouver	13·0585	1·11589	0·19614	0·91975	0·91931	1·11545	13·0451	13·052	13·026
do.	Dalles, 3-mile camp	......	......	0·19964	......	0·91931	1·11895	13·1506	13·151	13·087
do.	Dalles, 8-mile camp	......	......	0·19472	......	0·91931	1·11403	13·0026	13·003	13·091
do.	On Ashtnolou River	......	......	0·20210	......	0·91931	1·12141	13·2255	13·226	13·315
do.	Ashtnolou Station	13·3003	1·12386	0·20589	0·91797	0·91931	1·12520	13·3412	13·321	13·306
do.	Inshwointum Station	......	......	0·20364	......	0·91931	1·12295	13·2724	13·272	13·361
1861	Colville B. B. C. Barracks	13·4239	1·12788	0·20584	0·92204	0·92032	1·12616	13·3709	13·397	13·357
do.	Chemikane River	......	......	0·20493	......	0·92032	1·12525	13·3428	13·343	13·334
do.	Sinyakwateen	13·4045	1·12725	0·20471	0·92254	0·92032	1·12503	13·3361	13·370	13·238
do.	Pack River	13·3881	1·12672	0·20660	0·92012	0·92032	1·12692	13·3944	13·391	13·401
do.	Chelemta	13·4816	1·12974	0·20863	0·92111	0·92032	1·12895	13·4570	13·469	13·423
do.	South Crossing (Kootenay)	13·4384	1·12835	0·20779	0·92056	0·92032	1·12811	13·4309	13·435	13·443
do.	On Kootenay River	......	......	0·20816	......	0·92032	1·12848	13·4425	13·443	13·460
do.	Tobacco Plains (Kootenay)	......	......	0·20927	......	0·92032	1·12959	13·4769	13·477	13·481
do.	Wigwam River Station	13·4850	1·12985	0·21029	0·91956	0·92032	1·13061	13·5085	13·497	13·496
do.	Akamina Station	......	......	0·21281	......	0·92032	1·13313	13·5872	13·587	13·522

6484

$\phi_1 = 13{\cdot}309$

Probable error of a single observation $= \pm 0{\cdot}044$

Table V.—Declination.

1.	2.	3.	4.	5.
Station.	W. Longitude. μ.	N. Latitude. λ.	Declination. v.	v'.
Esquimalt	123° 27′	48° 26′	21° 58′	21° 20′
Sumass Prairie	122 12	49 01	21 30	21 42
Nisqually	122 25	47 07	21 23	20 51
Schweltza Lake	122 00	49 02	21 37	21 44
Fort Vancouver	122 28	45 38	20 05	20 13
Dalles, 3-mile camp	120 49	45 35	20 37	20 27
On Ashtnolou River	120 00	49 10	22 10	22 06
On Ashtnolou River	120 00	49 07	21 50	22 04
Ashtnolou Station	120 00	49 00	22 44	22 12
Osoyoos Station	119 24	49 00	22 14	22 07
Inshwointum	118 28	49 00	20 17	22 15
Colville B. B. C. Barracks	118 05	48 40	21 40	22 11
Chemikane River	117 45	48 00	21 28	21 57
Sinyakwateen	116 44	48 09	21 16	22 10
Pack River	116 28	48 22	22 51	22 19
Chelemta	116 19	48 41	22 11	22 27
South Crossing (Kootenay)	115 21	48 22	22 16	22 28
On Kootenay River	115 17	48 40	23 24	22 36
Wigwam Station	114 45	49 00	23 52	22 50
Akamina Station	114 04	49 01	23 12	22 56
	376 01	167 01	38 35	
	118 48	48 21	21 56	
Dalles, 8-mile camp	120 49	45 40	18 44	Rejected.

Probable error of a single result $= \pm 27{\cdot}06$.

IV. *On the Influence of Temperature on the Electric Conducting-Power of Alloys. By* AUGUSTUS MATTHIESSEN, *F.R.S., Lecturer on Chemistry in St. Mary's Hospital, and* CARL VOGT, *Ph.D.*

Received June 11,—Read June 18, 1863.

THE influence of temperature on the electric conducting-power of the pure metals in a solid state has been proved to be very great*, and as very little is as yet known with regard to the influence of temperature on the electric conducting-power of alloys, we undertook this research in order, if possible, to discover the law which regulates this property.

For the sake of clearness, we have thought it advisable to divide this subject into four parts, and they will be treated of in the following order:—

1. Experiments on the influence of temperature on the electric conducting-power of alloys composed of two metals.
2. Experiments on the influence of temperature on the electric conducting-power of some alloys composed of three metals.
3. On a method by which the conducting-power of a pure metal may be deduced from that of the impure one.
4. Miscellaneous and general remarks.

I. *Experiments on the Influence of Temperature on the Electric Conducting-power of Alloys composed of two Metals.*

It will be as well to mention that, from the few experiments already published on the influence of temperature on the conducting-power of alloys, we had at the commencement of the research some idea of the law which regulates this property, and having found after a few experiments our supposition confirmed, we were able to shape the course we intended to pursue, in such a manner as to curtail the number of alloys to be experimented with. Thus, with the alloys made of the metals lead, tin, cadmium, and zinc with one another, instead of using the alloys,

$$Pb_6Sn,\ Pb_4Sn,\ Pb_2Sn,\ PbSn,\ PbSn_2,\ PbSn_4,\ PbSn_6,$$

and testing in the same manner the tin-cadmium, tin-zinc, cadmium-zinc alloys, we only used the following,

$$Sn_6Pb,\ Sn_4Cd,\ Sn_2Zn,\ PbSn,\ ZnCd_2,\ SnCd_4,\ CdPb_6,$$

thus forming a mixed but complete series. Other groups of alloys have been treated in

* Philosophical Transactions, 1862, p. 1.

a similar manner. The reason for grouping alloys made of different metals under different heads has already been elsewhere discussed*. It has also been only considered necessary to experiment on one wire of each alloy, as the results obtained agree, in most cases, very closely with those calculated, and as it has been proved by a few determinations, which are given in Table I., that the same values were obtained for the percentage decrement in the conducting-power of the alloy between 0° and 100°, when series of determinations were made with two wires of the same alloy.

TABLE I.

Alloy.	Volumes per cent.	Percentage decrement observed between 0° and 100°.	Remarks.
Gold-copper (hard drawn) ...	98·63 of Au	21·87	Series made with wires of different specimens of the alloy.
Gold-copper (hard drawn) ...	98·38 „	21·75	
Gold-silver † (hard drawn)...	52·08 „	6·50	
Gold-silver (hard drawn) ...	52·08 „	6·48	
Gold-silver (annealed).........	52·08 „	6·72	
Gold-silver (annealed).........	52·08 „	6·70	Two series of determinations made with the same wire.
Gold-silver (annealed).........	52·08 „	6·71	
Gold-silver (annealed).........	79·86 „	10·15	
Gold-silver (annealed).........	79·86 „	10·21	
Tin-cadmium	23·50 of Sn	28·89	Series made with different wires of the same specimen of the alloy.
Tin-cadmium	23·50 „	29·08	

The method and apparatus employed for the determination of the conducting-power at different temperatures was the same as that described and used for the experiments on the pure metals†. We have, however, in many cases only taken observations at three intervals, as we found that almost the same formula was deduced from observations made at three different temperatures as from seven, especially when the temperature of the second observation was the mean of the other two; now as three or more observations were made at each interval, it was easy to obtain the wished-for temperature as the mean of several determinations. Thus the formulæ deduced for correction of conducting-power for temperature of the alloy $Cd\,Pb_6$ were—

From seven observations . . . $\lambda=9{\cdot}287-0{\cdot}032501t+0{\cdot}00006743t^2$.
From three observations . . . $\lambda=9{\cdot}286-0{\cdot}032450t+0{\cdot}00006683t^2$.

Again, those deduced for the alloy $Sn_2\,Zn$ were—

From seven observations . . . $\lambda=16{\cdot}876-0{\cdot}065544t+0{\cdot}0001471t^2$,
From three observations . . . $\lambda=16{\cdot}899-0{\cdot}065790t+0{\cdot}0001454t^2$,

where λ represents the conducting-power at $t°$ C.

We have here taken, as in former papers, the conducting-power of a hard-drawn silver wire at 0°=100 as defining our unit. The normal wires were made of german silver, the resistances of which were determined by comparing them with the gold-silver alloy‡, the conducting-power of a hard-drawn wire of which is equal to 15·03 at 0°.

* Philosophical Transactions, 1860, p. 162. † Ibid. 1862, p. 1.

‡ Philosophical Magazine for February 1861.

Table II. contains the conducting-powers, specific gravities, and equivalents of the metals used for making the alloys. These values are those which have been used in calculating the results given in this paper.

TABLE II.

Metal.	Conducting-power at 0°.	Specific gravity.	Equivalent.
Silver (hard drawn).........	100·00	10·468	108·0
Silver (annealed)............	108·57	....	......
Copper (hard drawn)	99·95	8·950	31·7
Gold (hard drawn)	77·96	19·265	197·0
Gold (annealed)	79·33	....	......
Zinc	29·02	7·148	32·6
Cadmium	23·72	8·655	56·0
Palladium (hard drawn)...	18·45	11·500	......
Platinum (hard drawn) ...	17·99	21·400	......
Iron (hard drawn)	16·81	7·790	......
Nickel	13·11	8·50	......
Tin	12·36	7·294	58·0
Thallium	9·16	11·900	......
Lead	8·32	11·376	103·7
Bismuth	1·245	9·822	208·0

Tables III., IV., V., and VI. contain the results obtained with the alloys belonging to the different groups. The alloys marked thus (†) are those which were made and used for former experiments; in all cases, however, fresh wires were made. All the rest have been re-made and analyzed. In Table III. the results are given which were obtained with some alloys made of those metals which, when alloyed with one another, conduct electricity in the ratio of their relative volumes; in Table V. those with some alloys of those metals which, when alloyed with one another, do not conduct electricity in the ratio of their relative volumes, but always in a lower degree than the mean of their volumes; in Table IV. those with some alloys made with the metals belonging to the alloys given in Table III. with those in Table V.; and in Table VI. those with some alloys whose places in the foregoing Tables we have not yet been able to assign.

TABLE III.

1.

†Sn_6Pb, containing 16·04 volumes per cent. of lead.

Length 435·5 millims.; diameter 0·793 millim.

		Reduced to 0°*.
Conducting-power found before heating the wire	11·782 at 13°·7	12·423
Ditto, after being kept at 100° for 1 day	12·052 at 9·3	12·494
Ditto, for 2 days..................	12·088 at 9·1	12·522

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
10°·03	12·043	12·033	+0·010
24·56	11·371	11·381	−0·010
39·27	10·760	10·768	−0·008
55·00	10·168	10·165	+0·003
67·73	9·720	9·716	+0·004
84·93	9·175	9·165	+0·010
98·87	8·757	8·766	−0·009

$\lambda = 12{\cdot}510 - 0{\cdot}048619t + 0{\cdot}0001087t^2.$

2.

†Sn_4Cd, containing 83·10 volumes per cent. of tin.

Length 285 millims.; diameter 0·417 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	14·259 at 8°·8	14·658
Ditto, after being kept at 100° for 1 day	14·207 at 6·2	14·569
Ditto, for 2 days..................	14·072 at 7·7	14·517

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
8°·72	13·986	13·985	+0·001
25·52	13·089	13·092	−0·003
39·50	12·419	12·423	−0·004
54·96	11·770	11·761	+0·009
69·40	11·218	11·217	+0·001
84·02	10·733	10·740	−0·007
98·85	10·333	10·330	+0·003

$\lambda = 14{\cdot}487 - 0{\cdot}059047t + 0{\cdot}0001720t^2.$

* These and all similar values were reduced to 0° as described in the paper "On the Influence of Temperature on the Electric Conducting-power of the Pure Metals," Philosophical Transactions, 1862, p. 10.

TABLE III. (continued).

3.

†Sn_2Zn, containing 77·71 volumes per cent. of tin.

Length 276·5 millims.; diameter 0·555 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	16·289 at 10°·9	16·991
Ditto, after being kept at 100° for 1 day	15·862 at 15·1	16·815
Ditto, for 2 days	16·201 at 10·9	16·899

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
11°·08	16·188	16·168	+0·020
24·42	15·339	15·363	−0·024
39·27	14·516	14·529	−0·013
54·23	13·759	13·754	+0·005
69·40	13·055	13·037	+0·018
84·11	12·414	12·404	+0·010
96·65	11·899	11·915	−0·016

$$\lambda = 16{\cdot}876 - 0{\cdot}065544t + 0{\cdot}0001471t^2.$$

4.

†Pb Sn, containing 53·41 volumes per cent. of lead.

Length 359 millims.; diameter 0·844 millim.*

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
9°·12	10·073	10·071	+0·002
24·45	9·510	9·511	−0·001
39·73	8·992	8·995	−0·003
55·26	8·509	8·512	−0·003
69·61	8·108	8·103	+0·005
84·36	7·724	7·721	+0·003
98·73	7·382	7·385	−0·003

$$\lambda = 10{\cdot}423 - 0{\cdot}039433t + 0{\cdot}00008775t^2.$$

5.

†$Zn\,Cd_2$, containing 26·06 volumes per cent. of zinc.

Length 577 millims.; diameter 0·629 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	24·774 at 11°·1	25·834
Ditto, after being kept at 100° for 1 day	25·101 at 10·1	26·077
Ditto, for 2 days	24·916 at 10·5	25·924

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
11°·60	24·817	24·796	+0·021
24·28	23·600	23·647	−0·047
39·86	22·322	22·324	−0·002
54·00	21·232	21·215	+0·017
68·90	20·164	20·133	+0·031
83·41	19·167	19·168	−0·001
98·23	18·255	18·272	−0·017

$$\lambda = 25{\cdot}906 - 0{\cdot}098065t + 0{\cdot}0002072t^2.$$

TABLE III. (continued).

6.

†$Sn\,Cd_4$, containing 23·50 volumes per cent. of tin.

Length 512·5 millims.; diameter 0·670 millim.

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
12°·57	21·096	21·086	+0·010
25·55	20·068	20·084	−0·016
40 20	19·033	19·037	−0·004
54·30	18·127	18·113	+0·014
69·33	17·219	17·220	−0·001
80·96	16·589	16·594	−0·005
91·30	16·086	16·084	+0·002

$$\lambda = 22{\cdot}123 - 0{\cdot}085159t + 0{\cdot}0002082t^2.$$

7.

†$Cd\,Pb_6$, containing 10·57 vols. per cent. of cadmium.

Length 224 millims.; diameter 0·644 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	9·068 at 6°·1	9·264
Ditto, after being kept at 100° for 1 day	9·490 at 2·5	9·574
Ditto, for 2 days	9·039 at 7·7	9·285
Ditto, for 3 days	8·964 at 10·1	9·285

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
11°·50	8·922	8·922	0·000
25·03	8·516	8·516	0·000
40·35	8·083	8·085	−0·002
54·75	7·710	7·710	0·000
70·00	7·342	7·342	0·000
85·55	7·001	7·000	+0·001
98·57	6·737	6·738	−0·001

$$\lambda = 9{\cdot}287 - 0{\cdot}032501t + 0{\cdot}00006743t^2.$$

TABLE IV.

1.

†$Pb_{20}\,Ag$, containing 94·64 volumes per cent. of lead.

Length 372 millims.; diameter 0·704 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	8·508 at 13°·7	8·938
Ditto, after being kept at 100° for 1 day	8·578 at 15·2	9·060
Ditto, for 2 days	8·640 at 14·3	9·096
Ditto, for 3 days	8·731 at 15·7	9·238
Ditto, for 4 days	8·760 at 14·7	9·236

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
10°·47	8·900	8·901	−0·001
24·93	8·459	8·455	+0·004
39·70	8·026	8·031	−0·005
55·03	7·625	7·625	0·000
70·26	7·256	7·256	0 000
85·16	6·933	6·927	+0·006
98·47	6·658	6·662	−0·004

$$\lambda = 9{\cdot}244 - 0{\cdot}033467t + 0{\cdot}00007360t^2.$$

* The reason why here and in some cases in the following Tables no determinations of the effect of heating the wire on its conducting-power are given, is that the wire unfortunately, from some cause or another, became unsoldered after it had been heated to 100° for one or more days.

Table IV. (continued).

2.

†Pb Ag, containing 46·90 volumes per cent. of lead.

Length 267 millims.; diameter 0·584 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	13·009 at 14°·9	13·391
Ditto, after being kept at 100° for 1 day	13·072 at 15·9	13·482
Ditto, for 2 days..................	13·087 at 15·1	13·477

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
14°·10	18·099	13·100	−0·001
24·70	12·841	12·837	+0·004
39·88	12·478	12·477	+0·001
54·61	12·141	12·146	−0·005
70·05	11·818	11·818	0·000
83·88	11·546	11·542	+0·004
99·37	11·250	11·251	−0·001

$\lambda = 13·464 - 0·026424t + 0·00004174t^2$.

3.

Pb Ag_2, containing 30·64 volumes per cent. of lead.

Length 373 millims.; diameter 0·634 millim.

		Reduced to 0°.
Conducting-power found after heating the wire for 2 days...	21·186 at 16°·1	21·874
Ditto, for 3 days..................	21·160 at 16·5	21·863

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
15°·82	21·191	21·190	+0·001
24·96	20·811	20·813	−0·002
39·48	20·236	20·232	+0·004
54·17	19·669	19·669	0·000
69·78	19·089	19·098	−0·009
84·27	18·602	18·593	+0·009
100·00	18·069	18·071	−0·002

$\lambda = 21·866 - 0·043636t + 0·00005686t^2$.

4.

†Sn_{12} Au, containing 90·32 volumes per cent. of tin.

		Reduced to 0°.
Conducting-power found before heating the wire	7·9495 at 11°·8	8·2418
Ditto, after being kept at 100° for 1 day	7·9479 at 13·0	8·2702

T.	Conducting-power.
14°·0	7·9224
57·0	6·9935
100·0	6·2676

$\lambda = 8·2687 - 0·025501t + 0·00005490t^2$.

Table IV. (continued).

5.

†Sn_5 Au, containing 79·54 volumes per cent. of tin.

Length 222 millims.; diameter 0·599 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	4·8386 at 14°·3	5·0427
Ditto, after being kept at 100° for 1 day	4·8432 at 14·6	5·0518
Ditto, for 2 days..................	4·8741 at 13·0	5·0608

T.	Conducting-power.
14°·0	4·8593
57·0	4·3212
100·0	3·9009

$\lambda = 5·0599 - 0·014776t + 0·00003186t^2$.

6.

Tin-copper alloy, containing 93·57 volumes per cent. of tin.

Length 274·5 millims; diameter 0·667 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	11·264 at 18°·1	12·034
Ditto, after being kept at 100° for 1 day	11·498 at 16·9	12·231
Ditto, for 2 days..................	11·445 at 18·3	12·237
Ditto, for 3 days..................	11·549 at 16·3	12·259
Ditto, for 4 days..................	11·571 at 16·3	12·282
Ditto, for 5 days..................	11·558 at 17·1	12·304

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
15°·58	11·622	11·618	+0·004
24·70	11·242	11·242	0·000
38·91	10·679	10·688	−0·009
54·96	10·109	10·111	−0·002
70·29	9·615	9·609	+0·006
85·68	9·160	9·152	+0·008
99·40	8·777	8·784	−0·007

$\lambda = 12·299 - 0·045304t + 0·00009997t^2$.

7.

Tin-copper alloy, containing 83·60 volumes per cent. of tin.

Length 201 millims.; diameter 0·581 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	12·119 at 15°·7	12·764
Ditto, after being kept at 100° for 1 day	12·264 at 15·3	12·900
Ditto, for 2 days..................	12·389 at 15·3	13·031
Ditto, for 3 days..................	12·420 at 14·7	13·038
Ditto, for 4 days..................	12·384 at 15·7	13·043

T.	Conducting-power.		Difference.
	Observed	Calculated.	
8°·27	12·688	12·689	−0·001
25·28	12·009	12·002	+0·007
39·43	11·460	11·470	−0·010
54·31	10·943	10·949	−0·006
70·13	10·444	10·437	+0·007
84·18	10·032	10·022	+0·010
99·28	9·607	9·614	−0·007

[illegible]

Table IV. (continued).

8.

Tin-copper alloy, containing 14·91 vols. per cent. of tin.

Length 141 millims.; diameter 0·501 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	8·7481 at 15°·5	8·8223
Ditto, after being kept at 100° for 1 day	8·8372 at 16·5	8·9170
Ditto, for 2 days	8·8451 at 17·3	8·9288
Ditto, for 3 days	8·8441 at 17·0	8·9264

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
16°·58	8·8565	8·8560	+0·0005
34·85	8·7687	8·7692	−0·0005
56·33	8·6684	8·6693	−0·0009
77·40	8·5753	8·5737	+0·0016
99·48	8·4754	8·4760	−0·0006

$\lambda = 8·9364 - 0·0048890t + 0·000002626t^2$.

9.

Tin-copper alloy, containing 12·35 vols. per cent. of tin.

Length 429 millims.; diameter 0·627 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	10·037 at 17°·9	10·154
Ditto, after being kept at 100° for 1 day	10·076 at 18·2	10·196
Ditto, for 2 days	10·084 at 17·2	10·197
Ditto, for 3 days	10·084 at 16·6	10·193

T.	Conducting-power.
11°·0	10·1386
55·5	9·8710
100·0	9·6526

$\lambda = 10·212 - 0·0068043t + 0·00001210t^2$.

10.

Tin-copper alloy, containing 11·61 vols. per cent. of tin.

Length 322·5 millims.; diameter 0·524 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	12·003 at 12°·1	12·102
Ditto, after being kept at 100° for 1 day	12·069 at 11·5	12·165
Ditto, for 2 days	12·083 at 12·5	12·188
Ditto, for 3 days	12·070 at 14·3	12·190

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
13°·43	12·058	12·057	+0·001
23·40	11·990	11·991	−0·001
40·35	11·852	11·853	−0·001
54·75	11·737	11·736	+0·001
69·78	11·619	11·617	+0·002
84·66	11·499	11·500	−0·001
98·70	11·391	11·391	0·000

$\lambda = 12·186 - 0·008468t + 0·000003700t^2$.

11.

Tin-copper alloy, containing 6·02 vols. per cent. of tin.

Length 210 millims.; diameter 0·456 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	19·382 at 15°·5	19·682
Ditto, after being kept at 100° for 1 day	19·517 at 15·5	19·819
Ditto, for 2 days	19·496 at 16·4	19·816

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
17°·23	19·484	19·484	0·000
24·03	19·355	19·354	+0·001
40·03	19·050	19·052	−0·002
55·47	18·771	18·769	+0·002
69·70	18·511	18·513	−0·002
83·16	18·279	18·276	+0·003
98·87	18·004	18·006	−0·002

$\lambda = 19·820 - 0·019729t + 0·00001397t^2$.

12.

Tin-copper alloy, containing 1·41 vol. per cent. of tin.

Length 599 millims.; diameter 0·449 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	60·105 at 14°·5	62·463
Ditto, after being kept at 100° for 1 day	60·827 at 12·5	62·881
Ditto, for 2 days	60·687 at 14·1	63·001
Ditto, for 3 days	60·579 at 15·1	63·055
Ditto, for 4 days	60·690 at 14·3	63·038

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
15°·53	60·470	60·455	+0·015
24·68	59·011	59·029	−0·018
39·03	56·897	56·900	−0·003
54·98	54·681	54·686	−0·005
68·73	52·924	52·906	+0·018
84·25	51·036	51·041	−0·005
99·70	49·334	49·336	−0·002

$\lambda = 62·997 - 0·16856t + 0·0003163t^2$.

13.

Tin-silver alloy, containing 96·52 vols. per cent. of tin.

Length 304 millims.; diameter 0·478 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	11·646 at 16°·3	12·390
Ditto, after being kept at 100° for 1 day	11·686 at 16·3	12·433
Ditto, for 2 days	11·685 at 17·0	12·464
Ditto, for 3 days	11·668 at 17·6	12·475

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
11°·12	11·983	11·971	+0·012
24·90	11·353	11·364	−0·011
39·40	10·751	10·768	−0·017
54·60	10·193	10·189	+0·004
69·81	9·676	9·657	+0·019
84·88	9·178	9·177	+0·001
99·68	8·743	8·751	−0·008

$\lambda = 12·488 - 0·047691t + 0·0001023t^2$.

TABLE IV. (continued).

14.

Tin-silver alloy, containing 75·51 vols. per cent. of tin.

Length 273 millims.; diameter 0·467 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	12·982 at 17°·6	13·866
Ditto, after being kept at 100° for 1 day	13·054 at 17·1	13·917
Ditto, for 2 days	13·334 at 16·5	14·184
Ditto, for 3 days	13·415 at 15·5	14·217
Ditto, for 4 days	13·402 at 16·5	14·256

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
11°·53	13·651	13·646	+0·005
25·51	12·955	12·958	−0·003
40·26	12·283	12·283	0·000
53·86	11·700	11·708	−0·008
69·58	11·099	11·099	0·000
84·98	10·572	10·561	+0·011
99·48	10·103	10·108	−0·005

$$\lambda = 14{\cdot}250 - 0{\cdot}053772t + 0{\cdot}0001219t^2.$$

15.

Zinc-copper alloy, containing 42·06 vols. per cent. of zinc.

Length 296·6 millims.; diameter 0·516 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	21·356 at 14°·8	21·793
Ditto, after being kept at 100° for 1 day	21·701 at 12·9	22·088
Ditto, for 2 days	21·873 at 13·1	22·269
Ditto, for 3 days	21·824 at 14·9	22·273

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
13°·72	21·807	21·801	+0·006
23·75	21·562	21·564	−0·002
39·28	21·116	21·118	−0·002
54·38	20·693	20·698	−0·005
69·31	20·300	20·297	+0·003
84·63	19·897	19·898	−0·001
99·43	19·527	19·526	+0·001

$$\lambda = 22{\cdot}274 - 0{\cdot}030601t + 0{\cdot}00002980t^2.$$

16.

Zinc-copper alloy, containing 29·45 vols. per cent. of zinc.

Length 190 millims.; diameter 0·381 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	21·235 at 17°·4	21·708
Ditto, after being kept at 100° for 1 day	21·424 at 15·9	21·859
Ditto, for 2 days	21·597 at 15·9	22·036
Ditto, for 3 days	21·625 at 15·9	22·065
Ditto, for 4 days	21·720 at 12·8	22·075

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
13°·47	21·704	21·702	+0·002
24·07	21·413	21·416	−0·003
39·21	21·020	21·017	+0·003
53·65	20·647	20·647	0·000
69·03	20·268	20·269	−0·001
83·71	19·915	19·916	−0·001
98·97	19·565	19·564	+0·001

$$\lambda = 22{\cdot}076 - 0{\cdot}028100t + 0{\cdot}00002745t^2.$$

17.

Zinc-copper alloy, containing 23·61 vols. per cent. of zinc.

Length 365 millims.; diameter 0·379 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	27·784 at 13°·0	28·298
Ditto, after being kept at 100° for 1 day	27·754 at 14·9	28·343
Ditto, for 2 days	27·738 at 15·3	28·342

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
15°·97	27·719	27·714	+0·005
23·80	27·408	27·412	−0·004
39·28	26·828	26·829	−0·001
54·82	26·259	26·262	−0·003
68·66	25·777	25·772	+0·005
83·75	25·258	25·256	+0·002
98·22	24·774	24·776	−0·002

$$\lambda = 28{\cdot}345 - 0{\cdot}040104t + 0{\cdot}00003839t^2.$$

18.

Zinc-copper alloy, containing 10·88 vols. per cent. of zinc.

Length 449 millims.; diameter 0·448 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	45·545 at 14°·8	46·934
Ditto, after being kept at 100° for 1 day	45·807 at 14·0	47·128
Ditto, for 2 days	45·896 at 14·6	47·276
Ditto, for 3 days	45·971 at 13·7	47·268

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
14°·33	45·912	45·912	0·000
23·71	45·059	45·056	+0·003
39·80	43·638	43·648	−0·010
54·33	42·442	42·440	+0·002
69·48	41·246	41·245	+0·001
84·38	40·145	40·134	+0·011
98·95	39·100	39·109	−0·009

$$\lambda = 47{\cdot}267 - 0{\cdot}096627t + 0{\cdot}0001433t^2.$$

19.

Zinc-copper alloy, containing 5·03 vols. per cent. of zinc.

Length 642 millims.; diameter 0·479 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	58·152 at 15°·3	60·376
Ditto, after being kept at 100° for 1 day	58·546 at 14·3	60·637
Ditto, for 2 days	58·665 at 14·0	60·716
Ditto, for 3 days	58·598 at 14·3	60·691

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
15°·17	58·522	58·494	+0·028
23·57	57·277	57·301	−0·024
40·03	55·071	55·093	−0·022
54·91	53·211	53·213	−0·002
67·88	51·679	51·664	+0·015
84·15	49·856	49·839	+0·017
99·45	48·228	48·243	−0·015

$$\lambda = 60{\cdot}697 - 0{\cdot}14995t + 0{\cdot}0002486t^2.$$

Table V.

1.

Gold-copper alloy, containing 98·63 volumes per cent. of gold (hard drawn).

Length 1121·5 millims.; diameter 0·582 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	53·694 at 18°·8	56·122
Ditto, after being kept at 100° for 1 day	53·796 at 16·5	56·184
Ditto, for 2 days	53·835 at 16·7	56·268

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
15°·52	53·972	53·980	−0·008
25·10	52·676	52·653	+0·023
39·74	50·684	50·715	−0·031
55·66	48·739	48·740	−0·001
69·83	47·106	47·092	+0·014
85·00	45·451	45·443	+0·008
95·35	43·986	43·994	−0·008

$\lambda = 56·232 - 0·14916t + 0·0002616t^2$.

2.

Gold-copper alloy, containing 81·66 volumes per cent. of gold (hard drawn).

Length 450 millims.; diameter 0·501 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	15·919 at 13°·6	16·083
Ditto, after being kept at 100° for 1 day	15·935 at 11·1	16·068
Ditto, for 2 days	15·895 at 12·2	16·041
Ditto, for 3 days	15·894 at 11·0	16·026
Ditto, for 4 days	15·887 at 11·4	16·024

T.	Conducting-power.
12°·0	15·880
56·0	15·356
100·0	14·837

$\lambda = 16·024 - 0·011997t + 0·000001291t^2$.

3.

Gold-silver alloy, containing 79·86 volumes per cent. of gold (hard drawn).

Length 605·7 millims.; diameter 0·704 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	21·010 at 11°·7	21·279
Ditto, after being kept at 100° for 1 day	21·038 at 10·8	21·286
Ditto, for 2 days	21·072 at 10·4	21·311
Ditto, for 3 days	21·066 at 10·2	21·301

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
11°·45	21·013	21·030	+0·001
26·04	20·698	20·701	−0·003
40·04	20·391	20·392	−0·001
55·26	20·065	20·064	+0·001
67·73	19·806	19·802	+0·004
84·13	19·463	19·464	−0·001
98·45	19·175	19·176	−0·001

$\lambda = 21·293 - 0·023166t + 0·00001691t^2$.

Table V. (continued).

4.

Gold-silver alloy, containing 79·86 volumes per cent. of gold (annealed*).

Length 596 millims.; diameter 0·704 millim.

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
7°·64	21·342	21·341	+0·001
25·27	20·920	20·924	−0·001
40·71	20·570	20·572	−0·002
54·61	20·265	20·264	+0·001
70·35	19·930	19·928	+0·002
85·25	19·622	19·622	0·000
99·50	19·338	19·339	−0·001

$\lambda = 21·527 - 0·024475t + 0·00002500t^2$.

5.

Gold-silver alloy, containing 19·86 volumes per cent. of gold (hard drawn).

Length 161·5 millims.; diameter 0·351 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	21·835 at 11°·7	22·062
Ditto, after being kept at 100° for 1 day	21·872 at 11·8	22·101
Ditto, for 2 days	21·841 at 12·5	22·083

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
13°·02	21·838	21·833	+0·005
23·90	21·620	21·625	−0·005
38·03	21·355	21·359	−0·004
54·42	21·158	21·056	+0·002
68·95	20·795	20·794	+0·001
82·37	20·557	20·555	+0·002
98·15	20·279	20·280	−0·001

$\lambda = 22·085 - 0·019538t + 0·00001173t^2$.

6.

Gold-silver alloy, containing 19·86 volumes per cent of gold (annealed*).

Length 161·5 millims.; diameter 0·351 millim.

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
14°·95	21·829	21·827	+0·002
24·56	21·637	21·640	−0·003
40·33	21·335	21·337	−0·002
55·38	21·059	21·055	+0·004
69·06	20·806	20·805	+0·001
84·48	20·527	20·528	−0·001
97·53	20·299	20·300	−0·001

$\lambda = 22·125 - 0·020097t + 0·00001419t^2$.

* The conducting-power of these wires did not alter after being kept at 100° for one day.

Table V. (continued).

7.

Gold-copper alloy, containing 19·17 volumes per cent. of gold (hard drawn).

Length 534 millims.; diameter 0·550 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	20·300 at 12°·2	20·504
Ditto, after being kept at 100° for 1 day	20·296 at 12·0	20·517
Ditto, for 2 days	20·287 at 12·4	20·505

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
13°·40	20·272	20·278	−0·006
24·38	20·088	20·088	0·000
40·01	19·838	19·824	+0·014
55·03	19·569	19·573	−0·004
70·11	19·325	19·328	−0·003
84·98	19·088	19·092	−0·004
99·87	18·865	18·861	+0·004

$\lambda = 20{\cdot}513 - 0{\cdot}017718t + 0{\cdot}00091170t^2$.

8.

Gold-copper alloy, containing 0·71 volume per cent. of gold (hard drawn).

Length 1049 millims.; diameter 0·366 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	79·884 at 13°·3	84·008
Ditto, after being kept at 100° for 1 day	80·389 at 14·3	84·261
Ditto, for 2 days	80·014 at 15·5	84·200
Ditto, for 3 days	79·844 at 16·6	84·322

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
17°·27	79·709	79·670	+0·039
23·98	77·952	77·962	−0·010
39·55	74·154	74·212	−0·058
54·26	70·294	70·913	−0·019
69·29	67·920	67·879	+0·041
83·86	65·213	65·175	+0·038
98·78	62·645	62·677	−0·032

$\lambda = 84{\cdot}322 - 0{\cdot}27999t + 0{\cdot}0006162t^2$.

9.

Platinum-silver alloy, containing 19·65 volumes per cent. of platinum (hard drawn).

Length 169 millims.; diameter 0·518 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	6·6565 at 18°·0	6·6960
Ditto, after being kept at 100° for 1 day	6·6616 at 17·9	6·7008
Ditto, for 2 days	6·6654 at 17·2	6·7031

T.	Conducting-power.
8°·27	6·6850
54·00	6·5876
99·90	6·4957

$\lambda = 6{\cdot}7032 - 0{\cdot}0022167t + 0{\cdot}000001394t^2$.

10.

Platinum-silver alloy, containing 5·05 volumes per cent. of platinum (hard drawn).

Length 708 millims.; diameter 0·626 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	17·812 at 15°·9	18·031
Ditto, after being kept at 100° for 1 day	17·801 at 17·1	18·036

T.	Conducting-power.
8°·0	17·920
54·5	17·319
100·0	16·767

$\lambda = 18{\cdot}045 - 0{\cdot}013960t + 0{\cdot}00001183t^2$.

11.

Platinum-silver alloy, containing 2·51 volumes per cent. of platinum (hard drawn).

Length 381·5 millims.; diameter 0·451 millim.

T.	Conducting-power.
12°·0	31·173
56·0	29·550
100·0	28·068

$\lambda = 31{\cdot}640 - 0{\cdot}039363t + 0{\cdot}00003642t^2$.

12.

Palladium-silver alloy, containing 23·28 volumes per cent. of palladium (hard drawn).

Length 520 millims.; diameter 0·802 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	8·4936 at 10°·0	8·5214
Ditto, after being kept at 100° for 1 day	8·5147 at 10·0	8·5426
Ditto, for 2 days	8·5052 at 9·1	8·5305
Ditto, for 3 days	8·4918 at 8·6	8·5157
Ditto, for 4 days	8·4868 at 10·0	8·5146

T.	Conducting-power.
11°·0	8·4846
55·5	8·3577
100·0	8·2256

$\lambda = 8{\cdot}5152 - 0{\cdot}0027644t - 0{\cdot}000001313t^2$.

13.

Copper-silver alloy, containing 98·35 volumes per cent. of copper (hard drawn).

Length 1198 millims.; diameter 0·572 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	86·674 at 9°·5	89·544
Ditto, after being kept at 100° for 1 day	88·210 at 6·5	90·202
Ditto, for 2 days	87·336 at 9·3	90·165

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
10°·48	86·919	86·846	+0·073
25·27	82·583	82·634	−0·051
39·57	78·763	78·861	−0·098
53·96	75·317	75·361	−0·044
69·73	72·007	71·868	+0·139
85·12	68·875	68·802	+0·073
98·35	66·348	66·442	−0·094

$\lambda = 90{\cdot}021 - 0{\cdot}31050t + 0{\cdot}0007193t^2$.

Table V. (continued).

14.

Copper-silver alloy, containing 95·17 volumes per cent. of copper (hard drawn).

Length 929 millims.; diameter 0·489 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	78·165 at 16°·0	82·300
Ditto, after being kept at 100° for 1 day	78·286 at 14·3	81·981
Ditto, for 2 days....................	78·102 at 15·9	82·207
Ditto, for 3 days....................	77·666 at 17·8	82·245

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
15°·43	78·226	78·219	+0·007
24·26	76·066	76·059	+0·001
39·16	72·601	72·616	−0·015
54·62	69·301	69·312	−0·011
69·48	66·406	66·393	+0·013
83·53	63·885	63·866	+0·019
99·00	61·319	61·343	−0·014

$\lambda = 82{\cdot}207 - 0{\cdot}26728t + 0{\cdot}0005711t^2$.

15.

Copper-silver alloy, containing 77·64 volumes per cent. of copper (hard drawn).

Length 623 millims.; diameter 0·374 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	66·807 at 14°·6	69·811
Ditto, after being kept at 100° for 1 day	66·601 at 17·3	70·158
Ditto, for 2 days....................	66·550 at 17·2	70·084
Ditto, for 3 days....................	66·707 at 17·0	70·208
Ditto, for 4 days....................	66·694 at 17·6	70·319

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
15°·15	67·155	67·192	−0·037
24·21	65·433	65·410	+0·023
39·48	62·583	62·565	+0·018
54·90	59·873	59·894	−0·021
69·48	57·557	57·556	+0·001
84·28	55·375	55·365	+0·001
99·90	53·259	53·262	−0·003

$\lambda = 70{\cdot}328 - 0{\cdot}21351t + 0{\cdot}0004271t^2$.

16.

Copper-silver alloy, containing 46·67 volumes per cent. of copper (hard drawn).

Length 1256 millims.; diameter 0·437 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	72·036 at 14°·2	74·940
Ditto, after being kept at 100° for 1 day	73·170 at 14·6	76·204
Ditto, for 2 days....................	73·653 at 12·6	76·284

T.	Conducting-power.
13°·0	73·529
56·5	65·449
100·0	58·894

$\lambda = 76{\cdot}240 - 0{\cdot}21375t + 0{\cdot}0004030t^2$.

Table V. (continued).

17.

Copper-silver alloy, containing 8·25 volumes per cent. of copper (hard drawn).

Length 2328 millims.; diameter 0·525 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	78·323 at 9°·0	80·284
Ditto, after being kept at 100° for 1 day	78·855 at 8·5	80·718
Ditto, for 2 days....................	78·398 at 10·2	[illegible]

T.	Conducting-power.
12°·0	87·015
56·0	69·301
100·0	61·949

$\lambda = 80{\cdot}628 - 0{\cdot}22196t + 0{\cdot}00035 18t^2$.

18.

Copper-silver alloy, containing 1·53 volume per cent. of copper (hard drawn).

Length 2139 millims.; diameter 0·542 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	94·554 at 9°·8	97·708
Ditto, after being kept at 100° for 1 day	95·314 at 9·0	98·231
Ditto, for 2 days....................	94·968 at 9·9	98·168

T.	Conducting-power.
10°·0	94·940
55·0	82·126
100·0	72·146

$\lambda = 98{\cdot}172 - 0{\cdot}033024t + 0{\cdot}0006998t^2$.

19.

Iron-gold alloy, containing 27·93 volumes per cent. of iron (hard drawn).

Length 145 millims.; diameter 0·758 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	2·5815 at 14°·6	2·7160
Ditto, after being kept at 100° for 1 day	2·6193 at 14·4	2·7539
Ditto, for 2 days....................	2·6309 at 14·2	2·7641
Ditto, for 3 days....................	2·6286 at 14·4	2·7636

T.	Conducting-power.
13°·0	2·6239
57·5	2·2732
100·0	1·9926

$\lambda = 2{\cdot}7645 - 0{\cdot}0096586t + 0{\cdot}00001940t^2$.

		Reduced to 0°.
The conducting-power of a second wire was found	2·6177 at 12°·6	2·7451

Table V. (continued).

20.

Iron-gold alloy, containing 21·18 volumes per cent. of iron (hard drawn).

Length 184 millims.; diameter 0·943 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	1·9299 at 14°·6	2·0121
Ditto, after being kept at 100° for 1 day	1·9981 at 10·8	2·0614
Ditto, for 2 days	1·9866 at 13·2	2·0621

T.	Conducting-power.
14°·0	1·9822
57·0	1·7951
100·7	1·7010

$$\lambda=2{\cdot}0632-0{\cdot}0061367t+0{\cdot}00002515t^2.$$

		Reduced to 0°.
The conducting-power of a second wire was found	1·8745 at 17°·2	1·9681

21.

Iron-gold alloy, containing 10·96 volumes per cent. of iron (hard drawn).

Length 226 millims.; diameter 0·470 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	2·3450 at 15°·6	2·3624
Ditto, after being kept at 100° for 1 day	2·3549 at 13·8	2·3704
Ditto, for 2 days	2·3585 at 10·4	2·3703

T.	Conducting-power.
12°·0	2·3573
56·0	2·3138
100·0	2·2798

$$\lambda=2{\cdot}3708-0{\cdot}0011555t+0{\cdot}0000002454t^2.$$

		Reduced to 0°.
The conducting-power of a second wire was found	2·2397 at 17°·2	2·2580

22.

Iron-copper alloy, containing 0·46 volume per cent. of iron (hard drawn).

Length 573·5 millims.; diameter 0·358 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	38·315 at 9°·0	38·852
Ditto, after being kept at 100° for 1 day	39·055 at 9·4	39·626
Ditto, for 2 days	39·124 at 10·4	39·758
Ditto, for 3 days	39·241 at 10·0	39·852
Ditto, for 4 days	39·313 at 11·0	39·986
Ditto, for 5 days	39·384 at 8·8	39·887

T.	Conducting-power.
10°·0	39·283
55·0	36·739
100·0	34·583

$$\lambda=39{\cdot}894-0{\cdot}061958t+0{\cdot}00008346t^2.$$

Table VI.

1.

†Phosphorus-copper, containing 2·5 per cent. by weight of phosphorus (hard drawn).

Length 124 millims.; diameter 0·355 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	7·2993 at 12°·6	7·3432
Ditto, after being kept at 100° for 1 day	7·3287 at 12·3	7·3717
Ditto, for 2 days	7·3424 at 13·6	7·3901

T.	Conducting-power. Observed.	Conducting-power. Calculated.	Difference.
14°·52	7·3395	7·3391	+0·0004
34·22	7·2696	7·2708	−0·0012
56·25	7·1963	7·1954	+0·0009
77·35	7·1243	7·1241	+0·0002
99·08	7·0515	7·0517	−0·0002

$$\lambda=7{\cdot}3900-0{\cdot}0035194t+0{\cdot}000001062t^2.$$

2.

†Phosphorus-copper, containing 0·95 per cent. by weight of phosphorus (hard drawn).

Length 265·5 millims.; diameter 0·396 millim.

T.	Conducting-power. Observed.	Conducting-power. Calculated.	Difference.
11°·05	23·028	23·032	−0·004
24·50	22·637	22·635	+0·002
39·72	22·209	22·203	+0·006
54·96	21·785	21·787	−0·002
69·48	21·407	21·408	−0·001
84·92	21·017	21·023	−0·006
99·83	20·673	20·668	+0·005

$$\lambda=23{\cdot}368-0{\cdot}030873t+0{\cdot}00003836t^2.$$

3.

†Arsenic-copper, containing 5·4 per cent. by weight of arsenic (hard drawn).

Length 225 millims.; diameter 0·289 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	6·3518 at 9°·3	6·3739
Ditto, after being kept at 100° for 1 day	6·3780 at 8·4	6·3980
Ditto, for 2 days	6·3800 at 7·9	6·3988

T.	Conducting-power. Observed.	Conducting-power. Calculated.	Difference.
10°·52	6·3742	6·3738	+0·0004
31·80	6·3230	6·3235	−0·0005
54·20	6·2707	6·2713	−0·0006
75·58	6·2232	6·2220	+0·0012
98·05	6·1703	6·1708	−0·0005

$$\lambda=6{\cdot}3989-0{\cdot}0023880t+0{\cdot}0000006331t^2.$$

TABLE VI. (continued).

4.

†Arsenic-copper, containing 2·8 per cent. by weight of arsenic (hard drawn).

Length 547 millims.; diameter 0·431 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	12·2980 at 8°·9	12·3787
Ditto, after being kept at 100° for 1 day	12·2648 at 9·5	12·3507
Ditto, for 2 days	12·2369 at 11·9	12·3443

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
13°·60	12·1933	12·1930	+0·0003
24·75	12·0937	12·0945	−0·0008
3[illegible]·78	11·9648	11·9635	+0·0013
54·72	11·8364	11·8358	+0·0006
69·11	11·7152	11·7151	+0·0001
84·72	11·5837	11·5867	−0·0030
100·28	11·4631	11·4614	+0·0017

$\lambda=12\cdot3156-0\cdot0090694t+0\cdot000005496t^2$.

5.

†Arsenic-copper, containing traces of arsenic (hard drawn).

Length 381 millims.; diameter 0·364 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	58·680 at 16°·4	61·255
Ditto, after being kept at 100° for 1 day	58·924 at 14·5	61·207
Ditto, for 2 days	59·286 at 12·7	61·295
Ditto, for 3 days	59·013 at 14·1	61·236

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
14°·65	58·948	58·931	+0·017
23·85	57·533	57·546	−0·013
39·95	55·226	55·244	−0·018
54·48	53·298	53·299	−0·001
69·26	51·464	51·448	+0·016
83·47	49·801	49·790	+0·011
98·62	48·141	48·154	−0·013

$\lambda=61\cdot238-0\cdot16183t+0\cdot00002957t^2$.

Tables VII., VIII., IX., and X. contain (1) the values found in a former research*, reduced to 0° with the help of the formula given in the above Tables for some of the alloys (column A.), (2) the values taken from the above Tables, namely, the first observed conducting-power reduced to 0° (column B.), and (3) the formulæ for the correction of the conducting-power for temperature, taking the mean of the values in the columns A. and B. as the conducting-power at 0°.

TABLE VII.

Alloy.	Volumes per cent.	Conducting-power.		Formulæ for the correction of the conducting-power for temperature.
		A.	B.	
Sn_6 Pb	83·96 of Sn	11·582	12·423	$\lambda=12\cdot002-0\cdot046645t+0\cdot0001042t^2$
Sn_4 Cd	83·10 of Sn	14·459	14·658	$\lambda=14\cdot558-0\cdot059337t+0\cdot0001728t^2$
Sn_2 Zn	77·71 of Sn	16·504	16·991	$\lambda=16\cdot747-0\cdot065044t+0\cdot0001460t^2$
Pb Sn	53·41 of Pb	9·855	10·423	$\lambda=10\cdot139-0\cdot038358t+0\cdot00008536t^2$
Zn Cd_2	26·06 of Zn	25·405	25·834	$\lambda=25\cdot619-0\cdot096978t+0\cdot0002049t^2$
Sn Cd_4	23·50 of Sn	21·194	22·123	$\lambda=21\cdot658-0\cdot083368t+0\cdot0002038t^2$
Cd Pb_6	10·57 of Cd	9·047	9·264	$\lambda=9\cdot155-0\cdot032041t+0\cdot00006647t^2$

TABLE VIII.

Alloy.	Volumes per cent.	Conducting-power.		Formulæ for the correction of the conducting-power for temperature.
		A.	B.	
Lead-silver	94·64 of Pb	8·823	8·938	$\lambda=8\cdot880-0\cdot032149t+0\cdot00007070t^2$
Lead-silver	46·90 of Pb	12·071	13·391	$\lambda=12\cdot731-0\cdot024986t+0\cdot00003947t^2$
Lead-silver	30·64 of Pb	....	21·874	$\lambda=21\cdot874-0\cdot043652t+0\cdot00005687t^2$
Tin-gold	90·32 of Sn	8·2418	....	$\lambda=8\cdot2418-0\cdot025418t+0\cdot00005472t^2$
Tin-gold	79·54 of Sn	4·5500	5·0427	$\lambda=4\cdot7963-0\cdot014006t+0\cdot00003020t^2$
Tin-copper (hard drawn)	93·57 of Sn	....	12·034	$\lambda=12\cdot034-0\cdot044328t+0\cdot00009781t^2$
Tin-copper (hard drawn)	83·60 of Sn	....	12·764	$\lambda=12\cdot764-0\cdot042457t+0\cdot00008734t^2$
Tin-copper (hard drawn)	14·91 of Sn	....	8·823	$\lambda=8\cdot8223-0\cdot0048266t+0\cdot000002593t^2$
Tin-copper (hard drawn)	12·35 of Sn	....	10·154	$\lambda=10\cdot154-0\cdot0067656t+0\cdot00001208t^2$
Tin-copper (hard drawn)	11·61 of Sn	....	12·102	$\lambda=12\cdot102-0\cdot0083587t+0\cdot000003674t^2$
Tin-copper (hard drawn)	6·02 of Sn	19·750	19·682	$\lambda=19\cdot716-0\cdot019626t+0\cdot00001390t^2$
Tin-copper (hard drawn)	1·41 of Sn	....	62·463	$\lambda=62\cdot463-0\cdot16713t+0\cdot0003136t^2$
Tin-silver	96·52 of Sn	12·378	12·390	$\lambda=12\cdot384-0\cdot047293t+0\cdot0001014t^2$
Tin-silver	75·51 of Sn	13·547	13·866	$\lambda=13\cdot706-0\cdot051720t+0\cdot0001172t^2$
Zinc-copper (hard drawn)	42·06 of Zn	....	21·793	$\lambda=21\cdot793-0\cdot029939t+0\cdot00002916t^2$
Zinc-copper (hard drawn)	29·45 of Zn	....	21·708	$\lambda=21\cdot708-0\cdot027632t+0\cdot00002698t^2$
Zinc-copper (hard drawn)	23·61 of Zn	....	28·298	$\lambda=28\cdot298-0\cdot040039t+0\cdot00003832t^2$
Zinc-copper (hard drawn)	10·88 of Zn	....	46·934	$\lambda=46\cdot934-0\cdot095947t+0\cdot0001423t^2$
Zinc-copper (hard drawn)	5·03 of Zn	....	60·376	$\lambda=60\cdot376-0\cdot14916t+0\cdot0002473t^2$

* Philosophical Transactions, 1860, p. 166.

TABLE IX.

Alloy.	Volumes per cent.	Conducting-power.		Formulæ for the correction of the conducting-power for temperature.
		A.	B.	
Gold-copper (hard drawn)	98·63 of Au	....	56·122	$\lambda=56{\cdot}122-0{\cdot}14887t+0{\cdot}0002611t^2$
Gold-copper (hard drawn)	81·66 of Au	....	16·083	$\lambda=16{\cdot}083-0{\cdot}012041t+0{\cdot}000001296t^2$
Gold-silver (hard drawn)	79·86 of Au	21·393	21·279	$\lambda=21{\cdot}335-0{\cdot}023212t+0{\cdot}00001694t^2$
Gold-silver (annealed)...............	79·86 of Au	....	21·527	$\lambda=21{\cdot}584^{*}-0{\cdot}024539t+0{\cdot}00002506t^2$
Gold-silver (hard drawn)	52·08 of Au	15·030	....	$\lambda=15{\cdot}030-0{\cdot}010120t+0{\cdot}000003697t^2$
Gold-silver (annealed)...............	52·08 of Au	15·080	....	$\lambda=15{\cdot}080-0{\cdot}010864t+0{\cdot}000007457t^2$
Gold-silver (hard drawn)	19·86 of Au	21·305	22·062	$\lambda=21{\cdot}684-0{\cdot}019185t+0{\cdot}00001152t^2$
Gold-silver (annealed)...............	19·86 of Au	....	22·125	$\lambda=21{\cdot}746^{*}-0{\cdot}019753t+0{\cdot}00001395t^2$
Gold-copper (hard drawn)	19·17 of Au	....	20·514	$\lambda=20{\cdot}514-0{\cdot}017718t+0{\cdot}00001170t^2$
Gold-copper (hard drawn)	0·71 of Au	....	84·008	$\lambda=84{\cdot}008-0{\cdot}27895t+0{\cdot}0006139t^2$
Platinum-silver (hard drawn) ...	19·65 of Pt	....	6·6960	$\lambda=6{\cdot}6960-0{\cdot}0022143t+0{\cdot}000001393t^2$
Platinum-silver (hard drawn) ...	5·05 of Pt	....	18·031	$\lambda=18{\cdot}031-0{\cdot}013949t+0{\cdot}00001182t^2$
Platinum-silver (hard drawn) ...	2·51 of Pt	....	31·640	$\lambda=31{\cdot}640-0{\cdot}039363t+0{\cdot}00003642t^2$
Palladium-silver (hard drawn) ...	23·28 of Pd	....	8·5214	$\lambda=8{\cdot}5214-0{\cdot}002764t-0{\cdot}000001314t^2$
Copper-silver (hard drawn)†	98·35 of Cu	....	89·544	$\lambda=89{\cdot}544-0{\cdot}30886t+0{\cdot}0007155t^2$
Copper-silver (hard drawn)†	95·17 of Cu	....	82·300	$\lambda=82{\cdot}300-0{\cdot}26758t+0{\cdot}0005717t^2$
Copper-silver (hard drawn)†	77·64 of Cu	....	69·311	$\lambda=69{\cdot}811-0{\cdot}21194t+0{\cdot}0004240t^2$
Copper-silver (hard drawn)†	46·67 of Cu	....	74·940	$\lambda=74{\cdot}940-0{\cdot}21011t+0{\cdot}0003961t^2$
Copper-silver (hard drawn)†	8·25 of Cu	....	80·284	$\lambda=80{\cdot}284-0{\cdot}22101t+0{\cdot}0003503t^2$
Copper-silver (hard drawn)†	1·53 of Cu	....	97·708	$\lambda=97{\cdot}708-0{\cdot}32868t+0{\cdot}0006965t^2$
Iron-gold (hard drawn)	27·93 of Fe	....	2·7350	$\lambda=2{\cdot}7350-0{\cdot}0095555t+0{\cdot}00001919t^2$
Iron-gold (hard drawn)	21·18 of Fe	....	1·9901	$\lambda=1{\cdot}9901-0{\cdot}0059194t+0{\cdot}00002426t^2$
Iron-gold (hard drawn)	10·96 of Fe	....	2·3102	$\lambda=2{\cdot}3102-0{\cdot}0011260t+0{\cdot}0000002391t^2$
Iron-copper (hard drawn)	0·46 of Fe	....	38·852	$\lambda=38{\cdot}852-0{\cdot}060341t+0{\cdot}00008128t^2$

TABLE X.

Alloy.	Weight per cent.	Conducting-power.		Formulæ for the correction of the conducting-power for temperature.
		A.	B.	
Phosphorus-copper (hard drawn)..	2·5 of P	7·301	7·343	$\lambda=7{\cdot}322-0{\cdot}0034870t+0{\cdot}000001052t^2$
Phosphorus-copper (hard drawn)..	0·95 of P	23·920	23·368	$\lambda=23{\cdot}644-0{\cdot}031238t+0{\cdot}00003882t^2$
Arsenic-copper (hard drawn)	5·4 of As	6·219	6·374	$\lambda=6{\cdot}296-0{\cdot}0023492t+0{\cdot}0000006230t^2$
Arsenic-copper (hard drawn)	2·8 of As	13·356	12·379	$\lambda=12{\cdot}867-0{\cdot}0094757t+0{\cdot}000005743t^2$
Arsenic-copper (hard drawn)	traces of As	60·854	61·255	$\lambda=61{\cdot}055-0{\cdot}16134t+0{\cdot}0002948t^2$

The values in columns A. and B. do not agree in all cases as well as might have been expected. Part of these differences are undoubtedly due to the fact that, the length of all wires made of alloys melting at a low temperature was measured after the determination had been made, as we found very great difficulty in soldering them to the thick copper wires in the trough, for, owing to their low fusing-points, the ends of the wires melted in with the solder. Now they had to be wound round a glass rod, as their length would not permit of their being experimented with in the trough without it; it is therefore probable that, on account of their softness, in unwinding and straightening them they became somewhat lengthened, which will account in a great measure for the differences. The value given for the conducting-power of one alloy (lead-silver, containing 30·64 per cent. of lead, and corresponding to $Pb\,Ag_2$) in the paper already referred to is wrong.

* These values have been altered to the same extent as those given in column B. for the hard-drawn wires, in order that the effect of annealing may remain the same.

† The alloys of these metals formerly tested do not quite correspond in composition to those here given, and therefore the values then found for their conducting-powers are not quoted above. They agree, however, very closely with those in column B.

We not only used part of the same alloy employed for the former determinations, but also made and analysed a fresh quantity, and found the values for the conducting-power in both cases the same; the present value is therefore the correct one for the conducting-power of the alloy. The error made in the former determinations must have been that a wrong normal wire was noted down as the one with which the resistances of the wires were compared; for according to the data from which the conducting-powers were then deduced, those there given are correct.

In order to show in a clear manner the results obtained, and to explain the law which we have arrived at, we will give in the first place the following Tables:—

TABLE XI.

Alloy.	Volumes per cent.	Conducting-power at 100°.		Percentage decrement.	
		Observed.	Calculated.	Observed.	Calculated.
Sn_6 Pb	83·96 of Sn	8·38	8·28	30·18	29·67
Sn_4 Cd	83·10 of Sn	10·35	10·10	28·89	30·03
Sn_2 Zn	77·71 of Sn	11·70	11·37	30·12	30·16
Pb Sn	53·41 of Pb	7·16	7·21	29·41	29·10
Zn Cd_2	26·06 of Zn	17·97	17·75	29·86	29·67
Sn Cd_4	23·50 of Sn	15·36	14·88	29·08	30·25
Cd Pb_6	10·57 of Cd	6·62	7·03	27·74	27·60

TABLE XII.

Alloy.	Volumes per cent.	Conducting-power at 100°.		Percentage decrement.	
		Observed.	Calculated.	Observed.	Calculated.
Lead-silver	94·64 of Pb	6·37	9·35	28·24	19·96
Lead-silver	46·90 of Pb	10·63	40·30	16·53	7·73
Lead-silver	30·64 of Pb	18·08	50·83	17·36	10·42
Tin-gold	90·32 of Sn	6·25	13·23	24·20	13·84
Tin-gold	79·54 of Sn	3·70	18·23	22·90	5·95
Tin-copper (hard drawn)	93·57 of Sn	8·58	12·72	28·71	19·76
Tin-copper (hard drawn)	83·60 of Sn	9·39	18·90	26·24	14·57
Tin-copper (hard drawn)	14·91 of Sn	8·37	61·42	5·18	3·99
Tin-copper (hard drawn)	12·35 of Sn	9·60	63·02	5·48	4·46
Tin-copper (hard drawn)	11·61 of Sn	11·30	63·47	6·60	5·22
Tin-copper (hard drawn)	6·02 of Sn	17·89	66·93	9·25	7·83
Tin-copper (hard drawn)	1·41 of Sn	48·89	69·78	21·74	20·53
Tin-silver	96·52 of Sn	8·67	10·90	30·00	23·31
Tin-silver	75·51 of Sn	9·71	23·91	29·18	11·89
Zinc-copper (hard drawn)	42·06 of Zn	19·09	49·57	12·40	11·29
Zinc-copper (hard drawn)	29·45 of Zn	19·21	55·89	11·49	10·08
Zinc-copper (hard drawn)	23·61 of Zn	24·68	58·82	12·79	12·30
Zinc-copper (hard drawn)	10·88 of Zn	38·76	65·20	17·41	17·42
Zinc-copper (hard drawn)	5·03 of Zn	47·93	68·13	20·61	20·62

Table XIII.

Alloy.	Volumes per cent.	Conducting-power at 100°.		Percentage decrement.	
		Observed.	Calculated.	Observed.	Calculated.
Gold-copper (hard drawn)	98·63 of Au	43·85	55·33	21·87	22·22
Gold-copper (hard drawn)	81·66 of Au	14·89	57·96	7·41	2·53
Gold-silver (hard drawn)	79·86 of Au	19·18	58·25	10·09	9·65
Gold-silver (annealed)	79·86 of Au	19·38	a 58·25 b 60·24	10·21	a 9·75 b 9·43
Gold-silver (hard drawn)	52·08 of Au	14·05	62·58	6·49	6·58
Gold-silver (annealed)	52·08 of Au	14·07	a 62·58 b 65·99	6·71	a 6·59 b 6·25
Gold-silver (hard drawn)	19·86 of Au	19·88	67·60	8·32	8·62
Gold-silver (annealed)	19·86 of Au	19·91	a 67·60 b 72·68	8·44	a 8·63 b 8·03
Gold-copper (hard drawn)	19·17 of Au	18·86	67·68	8·07	8·18
Gold-copper (hard drawn)	0·71 of Au	62·25	70·54	25·90	25·86
Platinum-silver (hard drawn)	19·65 of Pt	6·49	59·31	3·10	3·21
Platinum-silver (hard drawn)	5·05 of Pt	16·75	67·77	7·08	7·25
Platinum-silver (hard drawn)	2·51 of Pt	28·07	69·24	11·29	11·88
Platinum-silver (hard drawn)	23·28 of Pd	8·23	57·27	3·40	4·21
Copper-silver (hard drawn)	98·35 of Cu	65·81	70·66	26·50	27·30
Copper-silver (hard drawn)	95·17 of Cu	61·26	70·66	25·57	25·41
Copper-silver (hard drawn)	77·64 of Cu	52·86	70·66	24·29	21·92
Copper-silver (hard drawn)	46·67 of Cu	57·89	70·68	22·75	24·00
Copper-silver (hard drawn)	8·25 of Cu	61·69	70·69	23·17	25·57
Copper-silver (hard drawn)	1·53 of Cu	71·81	70·69	26·51	29·77
Iron-gold (hard drawn)	27·93 of Fe	1·97	42·62	27·92	1·47
Iron-gold (hard drawn)	21·18 of Fe	1·64	45·64	17·55	1·12
Iron-gold (hard drawn)	10·96 of Fe	2·20	49·68	3·84	1·34
Iron-copper (hard drawn)	0·46 of Fe	33·63	70·34	13·44	14·03

These Tables will require some explanation. Calculated conducting-power means the deduced conducting-power of an alloy, it being assumed that the conducting-power of a wire of any alloy is equal to the sum of the conducting-powers of parallel wires of the metals composing the alloy.

Under the term "calculated percentage decrement between 0° and 100°," we do not mean, as might be supposed, the mean of the percentage decrements which the component metals would suffer in their conducting-powers between 0° and 100°, and which would be, for nearly all the alloys experimented with, 29·307 per cent., inasmuch as it has been shown * that the conducting-power of most of the pure metals decreases between 0° and 100° by 29·307 per cent. (the exceptions to this law, being thallium and iron, the conducting-powers of which decrease between 0° and 100° 31·420 and 38·260 per cent. respectively †). It is therefore clear that the calculated percentage decrement in the conducting-powers between 0° and 100° of most alloys, from the above assumption, must be also 29·307 per cent. It is, however, obvious, on looking at the observed percentage decrements, that the conducting-powers of the alloys, with the exception of those given in Table XI., decrease less than 29·307 per cent. between 0° and 100°. In order to avoid repetitions, instead of the above value (29·307), we have inserted under the heading "calculated percentage decrement" that deduced from the following law:—

The observed percentage decrement in the conducting-power of an alloy between 0° *and* 100° *is to that calculated between* 0° *and* 100° (*viz.* 29·307) *as the observed conducting-power at* 100° *is to that calculated at* 100°.

* *Loc. cit.*

† Philosophical Transactions for 1863.

Or writing the above in symbols,

$$Po : Pc :: \lambda_{100^\circ} : \lambda'_{100^\circ}, \quad \ldots \ldots \ldots \quad (1)$$

where Po and Pc represent the observed and calculated percentage decrements in the conducting-power of the alloy between 0° and 100°, and λ_{100° and λ'_{100° its observed and calculated conducting-power at 100°, Pc is, as just stated, equal to 29·307 in nearly all cases, the exceptions being with the thallium and iron alloys.

If the values so deduced be examined, it will be seen that those given in Table XI. for the observed and calculated percentage decrement agree very closely with each other as well as with the mean value found for the percentage decrement in the conducting-power between 0° and 100° of the pure metals, viz. 29·307. This is just what we expected; for these alloys conduct electricity, as will be seen from the Table, in the ratio of their relative volumes, and therefore their conducting-powers ought to decrease between 0° and 100° in the same percentage amount as that of the mean of their components.

On looking at Table XII., which contains the alloys made of the metals belonging to the two classes, we find that, as long as there is no change in the conducting-power of the metals lead and tin by the addition of another metal, the conducting-power of the alloy decreases between 0° and 100° 29·307 per cent., but the moment the alloys show a greater or smaller conducting-power than that of pure lead or tin, then the percentage decrement is less than 29·307. Again, the alloys of tin or zinc with copper containing small amounts of those metals follow approximatively the above law; and on referring to the curves* which represent the conducting-powers of these alloys, it would appear that, starting with the metal whose conducting-power is greatly altered by a small addition of a foreign metal, the above law, as just stated, is approximatively true for all alloys as far as the turning-point of the curve, and from this point there is no agreement between the observed and calculated values. The difference between these values begins to show itself in some cases much sooner than in others; thus, with tin and copper after the addition of one per cent. of the former; with zinc and copper only after more than ten per cent. of zinc has been added, and from these points it gradually increases with each addition of metal. What the exact law is which these alloys follow with regard to the property under consideration we are unable at present to state, but some of them at least show that the law we have put forth will hold good in their cases. Unfortunately the alloys of this class, containing large percentages of each metal, are exceedingly brittle and unworkable, so that no complete series of determinations with any set of alloys could be made; had we been able to do this with one or two series, we should, in all probability, have found the law which regulates the influence of temperature on the conducting-power of this group of alloys. With regard to those in Table XIII. very little need be said, for the deduced percentage decrements prove that our law holds good for most of the alloys of this group. There are nevertheless a few remarks to be

* *Loc. cit.*

made respecting some of the values given in this Table, namely, on those of the annealed wires. Elsewhere it has been shown that the conducting-power of hard-drawn wires of some metals is greatly altered by annealing them; with the alloys this does not seem to be the case, for the differences here are very small. On account of their smallness we have not thought it worth while to investigate this matter any further at present; for to arrive at such results as might show the connexion between the effect of annealing on the conducting-power of alloys and on that of the metals composing them, would require a long series of experiments. Although the percentage decrements in the conducting-power of these annealed wires are all somewhat higher than those of the hard drawn, yet they may be considered the same, as the percentage decrements in the conducting-power of hard-drawn and annealed wires of the pure metals vary also in a small degree, but not always in the same direction. Thus those found for silver were—

Hard drawn.	Annealed.
28·67	28·82
28·44	28·67
27·82	28·21

We have calculated, as will be seen in the Table, the percentage decrements in two ways:—1st (*a*), using for the calculations the conducting-powers of the hard-drawn, and 2ndly (*b*), those of the annealed metals. The values so obtained for the percentage decrement do not differ very much from one another.

In calculating the results for the iron alloys, Pc has not been taken equal to 29·307, but for each alloy Pc has had to be calculated. Thus for the 1st, iron-gold, which contains 27·93 volumes per cent. iron,

The conducting-power of 1 volume of iron may be said to lose between 0° and 100° 38·260 per cent.; therefore 0·2793 volume will lose	10·686
That of 1 volume of gold may be said to lose between 0° and 100° 29·307 per cent.; therefore 0·7207 volume will lose	21·122
1 volume of iron-gold alloy, containing 27·93 per cent. iron, will therefore lose	31·808

On comparing the values obtained for the conducting-powers, &c. of the iron-gold alloys, the following facts are worth mentioning,—their very low and almost identical conducting-powers, and the high percentage decrements found for the first two and the low one for the third. That there was no error in this value we convinced ourselves by remaking the alloy, which contained, according to analysis, the same percentage amount of iron as that given in the Table, and the percentage decrement in its conducting-power was found equal to 4·04. Again, an alloy, made by a well-known firm*, which gave on analysis 11·94 volumes per cent. iron, conducted at 0° 2·097, and lost between 0° and 100° 4·30 per cent. of conducting-power. Unfortunately experiments with alloys richer

* We are indebted to Messrs. Johnson, Matthey and Co., of Hatton Garden, for many of the alloys experimented with. These were the first two, iron-gold, the platinum-silver, palladium-silver, and aluminium-nickel.

in iron could not be made, owing to the brittleness of the alloys; the high percentage decrement in the conducting-power of the first two indicating something abnormal, which it would have been interesting to have followed out.

On account of the probability of the arsenic and phosphorus being chemically combined with the copper, we have not considered it worth while to calculate the percentage decrements, and therefore no Table corresponding to the last has been made for these alloys.

If the above proportion,

$$\mathrm{Po} : \mathrm{Pc} :: \lambda_{100^\circ} : \lambda'_{100^\circ}, \quad \ldots \ldots \ldots \quad (1)$$

be converted into terms of resistance, the following formula is obtained,

$$r_{100^\circ} - r_{0^\circ} = r'_{100^\circ} - r'_{0^\circ}, \quad \ldots \ldots \ldots \quad (2)$$

where r_{100°, r_{0°, and r'_{100°, r'_{0° represent the observed and calculated resistances at 100° and 0°; for

$$\frac{\mathrm{Po}}{\mathrm{Pc}} = \frac{\lambda_{100^\circ}}{\lambda'_{100^\circ}};$$

but

$$\mathrm{Po} = 100 - \frac{\lambda_{100^\circ}}{\lambda_{0^\circ}} \cdot 100 = \frac{100}{\lambda_{0^\circ}}(\lambda_{0^\circ} - \lambda_{100^\circ}),$$

and

$$\mathrm{Pc} = 100 - \frac{\lambda'_{100^\circ}}{\lambda'_{0^\circ}} \cdot 100 = \frac{100}{\lambda'_{0^\circ}}(\lambda'_{0^\circ} - \lambda'_{100^\circ}).$$

And substituting these values in the above, we have

$$\frac{\lambda'_{0^\circ}}{\lambda_{0^\circ}} \cdot \frac{\lambda_{0^\circ} - \lambda_{100^\circ}}{\lambda'_{0^\circ} - \lambda'_{100^\circ}} = \frac{\lambda_{100^\circ}}{\lambda'_{100^\circ}},$$

or

$$\frac{\lambda_{0^\circ} - \lambda_{100^\circ}}{\lambda_{0^\circ} \cdot \lambda_{100^\circ}} = \frac{\lambda'_{0^\circ} - \lambda'_{100^\circ}}{\lambda'_{0^\circ} \cdot \lambda'_{100^\circ}},$$

or

$$\frac{1}{\lambda_{100^\circ}} - \frac{1}{\lambda_{0^\circ}} = \frac{1}{\lambda'_{100^\circ}} - \frac{1}{\lambda'_{0^\circ}},$$

which is equal to

$$r_{100^\circ} - r_{0^\circ} = r'_{100^\circ} - r'_{0^\circ}, \quad \ldots \ldots \ldots \quad (2)$$

for the reciprocal values of the conducting-powers of bodies are their resistances. The formula (2) expresses the fact that *the absolute difference between* 0° C. *and* 100° C. *in the resistance of an alloy is equal to the absolute difference between* 0° C. *and* 100° C. *in the calculated resistance of the alloy.*

The formula (2) may be written

$$r_{100^\circ} - r'_{100^\circ} = r_{0^\circ} - r'_{0^\circ}, \quad \ldots \ldots \ldots \quad (3)$$

which is equal to saying that *the absolute difference in the observed and calculated resistances at* 100° C. *is equal to the absolute difference between the observed and calculated resistances at* 0° C. Tables XIV., XV., and XVI. contain examples of these deductions taken from the three groups of alloys, taking the resistance of silver at 0° equal to 100.

Table XIV.

Alloy.	Volumes per cent.	$r_{100°}$.	$r_{0°}$.	$r'_{100°}$.	$r'_{0°}$.	$r_{100°}-r_{0°}$.	$r'_{100°}-r'_{0°}$.	$r_{100°}-r'_{100°}$.	$r_{0°}-r'_{0°}$.
Sn_6 Pb	83·96 of Sn	1193·3	833·3	1207·7	853·2	360·0	354·5	19·9	14·4
Sn_4 Cd	83·10 of Sn	966·2	686·8	990·1	699·8	279·4	290·3	23·9	13·0
Sn_2 Zn	77·71 of Sn	854·7	597·0	879·5	621·9	257·7	257·6	24·8	24·9
Pb Sn	53·41 of Pb	1396·6	986·2	1387·0	980·4	410·4	406·6	9·6	5·8
Zn Cd_2	26·06 of Zn	556·5	390·3	563·4	398·4	166·2	165·0	6·9	8·1
Sn Cd_4	23·50 of Sn	651·0	461·7	672·0	474·8	189·3	197·2	21·0	13·1
Cd Pb_6	10·57 of Cd	1510·6	1092·9	1422·2	1005·0	417·7	417·4	88·4	87·9

Table XV.

Alloy.	Volumes per cent.	$r_{100°}$.	$r_{0°}$.	$r'_{100°}$.	$r'_{0°}$.	$r_{100°}-r_{0°}$.	$r'_{100°}-r'_{0°}$.	$r_{100°}-r'_{100°}$.	$r_{0°}-r'_{0°}$
Lead-silver	94·64 of Pb	1569·9	1126·1	1069·5	755·9	443·8	313·6	500·4	370·2
Lead-silver	46·90 of Pb	940·7	785·5	248·1	175·4	155·2	72·7	692·6	610·1
Lead-silver	30·64 of Pb	553·1	457·2	196·7	139·1	95·9	57·6	356·4	318·1
Tin-copper	93·57 of Sn	1165·5	831·3	786·2	555·6	334·2	230·6	379·3	275·7
Tin-copper	83·60 of Sn	1065·0	783·7	529·1	374·1	281·3	155·0	535·9	409·6
Tin-copper	14·91 of Sn	1196·2	1133·8	162·8	115·1	62·4	47·7	1033·4	1018·7
Tin-copper	12·35 of Sn	1041·6	985·2	158·7	112·2	56·4	46·5	882·9	873·0
Tin-copper	11·61 of Sn	885·0	826·4	157·6	111·4	58·6	46·2	727·4	715·0
Tin-copper	6·02 of Sn	559·0	507·1	149·4	105·6	51·9	43·8	409·6	401·5
Tin-copper	1·41 of Sn	204·5	160·1	143·3	101·3	44·4	42·0	61·2	58·8
Zinc-copper	42·06 of Zn	523·8	458·9	201·7	124·6	64·9	59·1	322·1	316·3
Zinc-copper	29·45 of Zn	520·6	460·6	178·9	126·5	60·0	52·4	341·7	334·1
Zinc-copper	23·61 of Zn	405·2	353·4	170·0	120·2	51·8	49·8	235·2	233·2
Zinc-copper	10·88 of Zn	258·0	215·1	153·4	108·4	44·9	45·0	104·6	104·7
Zinc-copper	5·03 of Zn	208·6	165·6	146·8	103·8	43·0	43·0	61·8	61·8

Table XVI.

Alloy.	Volumes per cent.	$r_{100°}$.	$r_{0°}$.	$r'_{100°}$.	$r'_{0°}$.	$r_{100°}-r_{0°}$.	$r'_{100°}-r'_{0°}$.	$r_{100°}-r'_{100°}$.	$r_{0°}-r'_{0°}$.
Gold-copper (hard drawn)	98·63 of Au	228·1	198·2	180·7	127·8	49·9	52·9	47·4	50·4
Gold-copper (hard drawn)	81·66 of Au	671·5	621·9	172·5	122·0	49·6	50·5	499·0	499·9
Gold-silver (hard drawn)	79·86 of Au	521·4	468·8	171·7	121·4	52·6	50·3	349·7	347·4
Gold-silver (hard drawn)	52·08 of Au	711·7	665·3	159·8	113·0	46·4	46·8	551·9	552·3
Gold-silver (hard drawn)	19·86 of Au	503·0	461·2	147·9	104·6	41·8	43·3	355·1	356·6
Gold-copper (hard drawn)	19·17 of Au	530·2	487·6	147·8	104·5	42·6	43·3	382·4	383·1
Gold-copper (hard drawn)	0·71 of Au	160·6	119·0	141·8	100·2	41·6	41·6	18·8	18·8
Platinum-silver (hard drawn)	19·65 of Pt	1540·8	1492·5	168·6	119·2	48·3	49·4	1372·2	1373·3
Platinum-silver (hard drawn)	5·05 of Pt	597·0	554·6	147·6	104·3	42·4	43·3	449·4	450·3
Platinum-silver (hard drawn)	2·51 of Pt	356·3	316·1	144·4	102·1	40·2	42·3	211·9	214·0

What has already been said when speaking of the results contained in Tables XI., XII., and XIII., will of course apply here. In Table XIV., the values in the columns headed $r_{100°}-r'_{100°}$ and $r_{0°}-r'_{0°}$ do not agree in all cases; and at the first glance we should be inclined to suppose that the law was not as correct for these alloys as for those given in Table XVI.; but this is only due to slight errors in the determination of the resistances, &c., for a small percentage difference in these numbers will cause a very marked one in those under the headings $r_{100°}-r'_{100°}$ and $r_{0°}-r'_{0°}$. If, on the contrary, the values in the columns $r_{100°}-r_{0°}$ and $r'_{100°}-r'_{0°}$ in Tables XIV. and XVI. be compared with each other, it will be seen that those in Table XIV. agree together quite as well as those in

Table XVI.; and therefore, if the values in Table XIV. were smaller, those in the columns headed $r_{100°}-r'_{100°}$ and $r_{0°}-r'_{0°}$ would agree much better together.

If

$$r_{100°}-r'_{100°}=r_{0°}-r'_{0°} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

be correct, we may suppose that

$$r_t-r'_t=r_{0°}-r'_{0°};$$

that is, *the absolute difference between the observed and calculated resistances of an alloy at any temperature is equal to the absolute difference between the observed and calculated resistances at* 0° C.; or, in other words,

$$r_t-r'_t=\text{constant.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Table XVII. contains some examples which show this to be the case.

TABLE XVII.

Alloy.	T.	r.	r'.	Difference.
Cd Pb_6	0	1092·9	1005·0	87·9
	20	1171·0	1083·0	88·0
	40	1253·1	1164·9	88·2
	60	1338·7	1249·8	88·9
	80	1424·5	1335·8	88·7
	100	1510·6	1422·2	88·4
Gold-copper, containing 0·71 volume per cent. of gold	0	119·04	100·21	18·83
	20	127·11	107·98	19·13
	40	135·44	116·16	19·28
	60	143·92	124·63	19·29
	80	152·39	133·21	19·18
	100	160·64	141·76	18·88
Gold-silver, containing 79·86 volumes per cent. of gold	0	468·71	121·36	349·35
	20	479·00	130·77	348·23
	40	489·38	140·67	348·71
	60	499·93	150·92	349·01
	80	510·57	161·31	349·26
	100	521·30	171·67	349·63

The values given in the column r were calculated with the help of the formulæ from Tables VII. and IX., those in the column r' with that deduced for the correction of conducting-power for temperature of most of the pure metals, namely,

$$\lambda=100-0·37647t+0·0008340t^2.$$

If, now,

$$r_t-r'_t=\text{constant,} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

it is clear that we may deduce the formula for the correction of resistance or conducting-power for temperature of an alloy as soon as we know its composition and its resistance at any temperature; for, as $r'_{100°}$, $r'_{0°}$, and r'_t may be calculated by means of the formula given for the correction of conducting-power for temperature for most of the pure metals, viz.

$$\lambda=100-0·37647t+0·0008340t^2 *,$$

* *Loc. cit.*

if the constant $r_t - r'_t$ be determined, then

$$r_{100^\circ} = r'_{100^\circ} + \text{constant},$$
$$r_t \;= r'_t \;+ \text{constant},$$
$$r_{0^\circ} \;= r'_{0^\circ} + \text{constant};$$

and from these terms a formula for the correction of resistance or conducting-power for temperature may be calculated, which in most cases will be found very near the truth. Thus, take, for instance, the gold-silver alloy containing 79·86 volumes per cent. gold (hard drawn), and we find

the first observed conducting-power . . . 21·010 at 11°·7,
that calculated . . . . 78·866 at 11°·7,
hence the resistance observed is . . . . 475·96 at 11°·7,
that calculated . . . . 126·80 at 11°·7;
therefore $r_t - r'_t = 349{\cdot}16$.
But the calculated resistance at . . . . . . 0° = 121·36,
" " " . . . . . . 50° = 145·75,
" " " . . . . . . 100° = 171·67,
therefore r, the true resistance, will be at . . 0° = 121·36 + 349·16 = 470·52,
" " " " . . 50° = 145·75 + 349·16 = 494·91,
" " " " . . 100° = 171·67 + 349·16 = 521·83;
or the conducting-powers will be at . . . . 0° = 21·253,
" " " " . . 50° = 20·206,
" " " " . . 100° = 19·200.

The formula deduced from these numbers is

$$\lambda = 21{\cdot}253 - 0{\cdot}021350t + 0{\cdot}000008200t^2.$$

The conducting-power, according to this formula, of the alloy at 11°·45 will be 21·010; but after having kept the alloys at 100° for three days it altered, and was found at that temperature to conduct 21·031. If the above formula be multiplied by $\frac{21{\cdot}031}{21{\cdot}010} = 1{\cdot}001$, we arrive at

$$\lambda = 21{\cdot}274 - 0{\cdot}021372t + 0{\cdot}000008208t^2;$$

and if the conducting-powers be calculated for the different temperatures in the following series, the difference between the observed and calculated values will be found to be very small.

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
11·45	21·031	21·031	0·000
26·04	20·698	20·723	−0·025
40·04	20·391	20·421	−0·030
55·26	20·065	20·118	−0·053
67·73	19·806	19·864	−0·058
84·13	19·463	19·534	−0·071
98·45	19·175	19·250	−0·075

Another example: the gold-copper alloy containing 0·71 volume per cent. gold (hard drawn) conducts 79·884 at 15°·3; the formula deduced in exactly the same manner as the above was

$$\lambda=83{\cdot}843-0{\cdot}26810t+0{\cdot}0005152t^2;$$

and the formula deduced from this, with the help of which the following calculated values were obtained, was

$$\lambda=84{\cdot}204-0{\cdot}26926t+0{\cdot}0005174t^2.$$

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
17·27	79·709	79·708	0·000
28·98	77·952	78·045	−0·093
39·55	74·154	74·364	−0·210
54·26	70·924	71·118	−0·194
69·26	67·920	68·037	−0·117
83·86	65·213	65·263	−0·050
98·78	62·645	62·656	−0·011

Again, let us take another example, the alloy $Sn_4 Cd$, for which the values (Table XIV.) obtained for $r_{100^\circ}-r'_{100^\circ}$ and $r_{0^\circ}-r'_{0^\circ}$ agree worse than any other in that Table; and if the results agree, it will show that the differences in these values are, as before stated, due to errors of observation.

The first observed conducting-power was 14·259 at 6°·8.

The formula deduced, as above, was

$$\lambda=14{\cdot}641-0{\cdot}055250t+0{\cdot}0001158t^2.$$

That deduced to calculate the conducting-powers for comparison with those observed, was

$$\lambda=14{\cdot}455-0{\cdot}054673t+0{\cdot}0001141t^2.$$

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
8·72	13·986	13·968	0·000
25·52	13·086	13·134	−0·045
39·50	12·419	12·473	−0·054
54·96	11·770	11·795	−0·025
69·40	11·218	10·211	+0·007
84·02	10·933	10·666	+0·067
98·85	10·333	10·166	+0·167

These examples are sufficient to prove that the law we have put forth is correct for most of the two metal alloys; we might have experimented with many more alloys whose conducting-power would have followed the above law, but we thought determinations with a few members of each group of alloys would suffice to prove its correctness for most of them. We have endeavoured rather to find the exemptions to the law than to obtain a large number of results which will agree with it.

II. *Experiments on the Influence of Temperature on the Electric Conducting-power of some Alloys composed of three Metals.*

In the course of the foregoing experiments we were induced to try whether the influence of temperature on the conducting-power of the three metal alloys would be regulated by the above law, and Tables XVIII. and XIX. contain the results.

Table XVIII.

1.

Gold-copper-silver alloy, containing 50 volumes per cent. gold, 25 copper, and 25 silver (hard drawn).

Length 341·5 millims.; diameter 0·618 millim.

		Reduced to 0°.
Conducting-power found before heating the wire......	10·6186 at 13°·7	10·6960
Ditto, after being kept at 100° for 1 day	10·6367 at 6·0	10·6681
Ditto, for 2 days	10·5855 at 6·7	10·6232

T.	Conducting-power. Observed.	Calculated.	Difference.
10°·75	10·5637	10·5617	+0·0020
33·52	10·4341	10·4346	−0·0005
55·15	10·3130	10·3148	−0·0018
78·35	10·1846	10·1873	−0·0027
97·52	10·0857	10·0828	+0·0029

$$\lambda = 10{\cdot}6220 - 0{\cdot}0056248t + 0{\cdot}0000009863t^2.$$

2.

Gold-copper-silver alloy, containing 40·67 vols. per cent. gold, 39·81 copper, and 19·52 silver (hard drawn).

Length 532 millims.; diameter 0·625 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	12·007 at 15°·1	12·109
Ditto, after being kept at 100° for 1 day	11·978 at 15·5	12·083
Ditto, for 2 days..................	11·915 at 16·5	12·026
Ditto, for 3 days..................	11·914 at 15·9	12·020

T.	Conducting-power.
8°·0	11·956
54·5	11·647
100·0	11·438

$$\lambda = 12{\cdot}017 - 0{\cdot}0069033t + 0{\cdot}00001111t^2.$$

3.

Gold-copper-silver alloy, containing 3·67 vols. per cent. gold, 83·32 copper, and 13·01 silver (hard drawn).

Length 764 millims.; diameter 0·553 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	44·820 at 18°·4	44·272
Ditto, after being kept at 100° for 1 day	42·994 at 17·1	44·348
Ditto, for 2 days..................	42·983 at 18·2	44·424
Ditto, for 3 days..................	43·047 at 17·0	44·395

T.	Conducting-power.
11°·0	43·591
55·5	40·300
100·0	37·560

$$\lambda = 44{\cdot}472 - 0{\cdot}081525t + 0{\cdot}0001240t^2.$$

Table XIX.

Alloy.	Volumes per cent.	Conducting-power at 100°. Observed.	Calculated.	Percentage decrement. Observed.	Calculated.
Gold-copper-silver (hard drawn).	50 Au, 25 Cu, 25 Ag	10·14	62·89	5·20	4·72
Ditto	40·67 Au, 39·81 Cu, 19·52 Ag	11·52	64·34	4·82	5·25
Ditto	3·67 Au, 83·32 Cu, 13·01 Ag	37·39	70·09	15·54	15·63
Argentan	12·84 Ni*, 36·57 Zn, 50·59 Cu	7·46	44·44	4·39	4·93

* Values found by analysis. Of this wire all our normal wires were made. According to former experiments (Philosophical Transactions, 1862, p. 5), the formula for the correction of conducting-power for temperature of this alloy was

$$\lambda = 7{\cdot}803 - 0{\cdot}0034619t + 0{\cdot}0000003951t^2.$$

The values in Table XIX. indicate that the law will probably hold good for most of the three metal alloys.

There is, however, one of the three metal alloys which we cannot pass over unnoticed, namely, that of copper-nickel-zinc or argentan (german silver). This alloy has long been used, on account of the small effect which temperature has on its conducting-power, for making resistance coils, &c. It is a somewhat curious fact, that the conducting-power of this commercial alloy decreases less between 0° and 100° than almost any other alloy yet known, for in the course of this investigation we have only found the following which show a smaller percentage decrement in their conducting-power than argentan.

The conducting-power of the platinum-silver alloy, containing 19·65 volumes per cent. platinum, decreases between 0° and 100° 3·10 per cent.

The conducting-power of the palladium-silver alloy, containing 23·38 volumes per cent. palladium, decreases between 0° and 100° 3·40 per cent.

The conducting-power of the iron-gold alloy, containing 10·96 volumes per cent. iron, decreases between 0° and 100° 3·84 per cent.

The conducting-power of the argentan decreases between 0° and 100° 4·39 per cent.

III. *On a Method by which the Conducting-power of a Pure Metal may be deduced from that of the Impure one.*

This part of our subject is an important deduction from the law

$$\mathrm{Po} : \mathrm{Pc} :: \lambda_{100^\circ} : \lambda'_{100^\circ}; \qquad (1)$$

for if we consider the two last terms of the proportion, and bear in mind that a small amount of another metal has very little or no effect on λ'_{100°, when it represents the conducting-power of an alloy containing a very small percentage of the one metal, whereas it has a very considerable one on λ_{100°, we may write the proportion

$$\mathrm{P} : \mathrm{P}' :: \mathrm{M}_{100^\circ} : \mathrm{M}'_{100^\circ}, \qquad (5)$$

where P and P′ represent the observed and calculated percentage decrements in the conducting-power of the impure and pure metals between 0° and 100°, and M_{100° and M'_{100° their conducting-powers at 100°. P′ is for most metals 29·307, or we may express it as follows:—

The percentage decrement in the conducting-power of an impure metal between 0° C. *and* 100° C., *is to that of the pure one between* 0° C. *and* 100° C. *as the conducting-power of the impure metal at* 100° C. *is to that of the pure one at* 100° C.

From the results given in Tables XII. and XIII., we have chosen the following alloys to show that a small amount of foreign metal has no influence on the value λ'_{100°, which may therefore be looked upon as equal to M'_{100°.

TABLE XX.

Alloy.	Volumes per cent.	Conducting-power at 100°. Observed.	Conducting-power at 100°. Calculated.	
Tin-copper (hard drawn)	1·41 of Sn	48·89	69·78	Pure copper conducts at 100° 70·27.
Zinc-copper (hard drawn)	5·03 of Zn	47·93	68·13	
Gold-copper (hard drawn)	1·37 of Cu	43·85	55·33	Pure gold conducts at 100° 55·90.
Gold-copper (hard drawn)	0·71 of Au	62·25	70·54	Pure copper conducts at 100° 70·27.
Platinum-silver (hard drawn)	2·51 of Pt	28·07	69·24	Pure silver conducts at 100° 71·53.
Copper-silver (hard drawn)	1·65 of Ag	65·81	70·66	Pure copper conducts at 100° 70·27.

If now, as in the case of most commercial metals, the amount of impurity be much smaller than that in the Table, then of course its influence on the value λ'_{100} is so small that it may be entirely disregarded.

In Tables XXI., XXII., and XXIII., we give some results obtained with impure metals, the conducting-power of the same metal in a pure state having been previously determined.

TABLE XXI.

1.

Gold, containing traces of silver (hard drawn).

Length 1564 millims.; diameter 0·525 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	69·612 at 10°·2	72·056
Ditto, after being kept at 100° for 1 day	70·069 at 10·4	72·578
Ditto, for 2 days	69·274 at 13·8	72·578

T.	Conducting-power.
13°·0	68·969
57·5	60·179
100·0	53·387

$$\lambda = 72{\cdot}548 - 0{\cdot}24692t + 0{\cdot}0005531t^2.$$

2.

Copper, containing traces of tin (hard drawn).

Length 2008 millims.; diameter 0·518 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	88·357 at 12°·8	92·503
Ditto, after being kept at 100° for 1 day	88·690 at 12·6	92·786
Ditto, for 2 days	89·589 at 10·1	92·894

T.	Conducting-power.
11°·0	89·319
55·5	76·619
100·0	66·863

$$\lambda = 92{\cdot}912 - 0{\cdot}33482t + 0{\cdot}0007433t^2.$$

TABLE XXI. (continued).

3.

Copper, containing traces of zinc (hard drawn).

Length 1992 millims.; diameter 0·577 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	81·306 at 13°·2	86·490
Ditto, after being kept at 100° for 1 day	83·185 at 12·8	86·896
Ditto, for 2 days	83·021 at 12·8	86·725

T.	Conducting-power.
13°·0	82·960
56·5	72·071
100·0	63·786

$$\lambda = 86{\cdot}719 - 0{\cdot}29814t + 0{\cdot}0006881t^2.$$

4.

Copper, commercial, containing traces of iron, nickel, lead, and suboxide of copper (hard drawn).

Length 2091 millims.; diameter 0·546 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	74·209 at 18°·6	78·023
Ditto, after being kept at 100° for 1 day	74·610 at 16·2	78·350
Ditto, for 2 days	74·563 at 16·8	78·441
Ditto, for 3 days	74·283 at 18·0	78·427

T.	Conducting-power.
12°·0	75·668
56·0	66·584
100·0	59·351

$$\lambda = 78{\cdot}467 - 0{\cdot}23896t + 0{\cdot}0004780t^2.$$

TABLE XXI. (continued).

5.

Copper, commercial, containing same impurities as No. 3 (hard drawn).

Length 2246 millims.; diameter 0·549 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	74·660 at 18°·8	78·705
Ditto, after being kept at 100° for 1 day	74·958 at 16·4	78·921
Ditto, for 2 days	74·946 at 16·6	78·958
Ditto, for 3 days	74·576 at 18·2	78·958

T.	Conducting-power.
13°·0	75·979
56·5	76·738
100·0	59·633

$\lambda = 79{\cdot}155 - 0{\cdot}25166t + 0{\cdot}0005644t^2.$

6.

Copper, commercial, containing traces of lead, iron, antimony, and suboxide of copper (hard drawn).

Length 3010 millims.; diameter 0·606 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	89·258 at 18°·7	94·896
Ditto, after being kept at 100° for 1 day	89·241 at 17·4	95·118
Ditto, for 2 days	89·524 at 16·5	95·109

T.	Conducting-power.
10°·0	91·849
55·0	78·402
100·0	68·324

$\lambda = 95{\cdot}294 - 0{\cdot}35289t + 0{\cdot}0008309t^2.$

7.

Silver, containing traces of lead (hard drawn).

Length 1473 millims.; diameter 0·513 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	64·909 at 13°·6	66·997
Ditto, after being kept at 100° for 1 day	65·957 at 14·6	68·235
Ditto, for 2 days	66·404 at 13·6	68·539
Ditto, for 3 days	66·801 at 11·4	68·599

T.	Conducting-power.
12°·0	66·543
56·0	60·264
100·0	54·987

$\lambda = 68{\cdot}429 - 0{\cdot}16030t + 0{\cdot}0002588t^2.$

TABLE XXI. (continued).

8.

Silver, containing traces of tin (hard drawn).

Length 2025 millims.; diameter 0·579 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	71·427 at 13°·6	73·964
Ditto, after being kept at 100° for 1 day	72·668 at 13·8	75·287
Ditto, for 2 days	72·735 at 13·7	75·338

T.	Conducting-power.
14°·0	72·696
57·0	65·305
100·0	59·085

$\lambda = 75{\cdot}355 - 0{\cdot}19437t + 0{\cdot}0003167t^2.$

9.

Silver, containing traces of gold (hard drawn).

Length 1780 millims.; diameter 0·648 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	70·847 at 10°·4	72·717
Ditto, after being kept at 100° for 1 day	71·205 at 11·3	73·249
Ditto, for 2 days	70·951 at 13·5	73·389
Ditto, for 3 days	70·929 at 13·5	73·366

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
14°·37	70·763	70·746	+0·017
24·21	69·036	69·044	−0·008
39·25	66·531	66·555	−0·024
54·43	64·172	64·179	−0·007
69·51	61·977	61·954	+0·023
84·30	59·923	59·905	+0·018
98·60	58·028	58·047	−0·019

$\lambda = 73{\cdot}336 - 0{\cdot}18447t + 0{\cdot}0002982t^2.$

10.

Silver, containing minute traces of arsenic (hard drawn).

Length 1298 millims.; diameter 0·376 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	85·119 at 14°·0	88·931
Ditto, after being kept at 100° for 1 day	86·795 at 9·0	89·285
Ditto, for 2 days	87·881 at 7·4	89·949
Ditto, for 3 days	87·091 at 10·0	89·869

T.	Conducting-power.
11°·0	87·029
55·5	76·185
100·0	67·767

$\lambda = 90{\cdot}084 - 0{\cdot}28442t + 0{\cdot}0006125t^2.$

Table XXII.

Metal.	Impurity.	Observed percentage decrement in the conducting-power between 0° and 100°.	Conducting-power.	
			Observed at 0°.	Calculated for the pure metal at 0°.
Lead	Bismuth	27·66*	7·86	8·53
Tin	Copper	28·71†	12·03	12·39
Tin	Silver	30·00†	12·39	11·98
Gold (hard drawn)	Copper	21·87‡	56·12	83·11
Gold (hard drawn)	Silver	26·41	72·06	83·24
Copper (hard drawn)	Tin	28·04	92·50	98·42
Copper (hard drawn)	Zinc	26·44	86·49	99·75
Copper (hard drawn)	Gold	25·90‡	84·01	99·64
Copper (hard drawn)	Silver	26·50‡	89·54	102·95
Copper (hard drawn)	Iron, nickel, lead, and suboxide of copper	24·36	78·02	100·43
Copper (hard drawn)	Ditto	24·66	78·70	99·67
Copper (hard drawn)	Lead, iron, antimony, and suboxide of copper	28·30	94·90	99·67
Silver (hard drawn)	Lead	19·64	67·00	113·64
Silver (hard drawn)	Tin	21·59	73·69	111·33
Silver (hard drawn)	Gold	21·09	72·72	112·79
Silver (hard drawn)	Copper	23·17‡	80·28	110·39
Silver (hard drawn)	Copper	26·51‡	97·71	112·28
Silver (hard drawn)	Minute traces of arsenic	24·77	88·93	111·95

On comparing the values in Table XXII. for the observed and calculated conducting-powers, it will be seen that those calculated for the same metal agree very closely with each other, whereas those observed vary in some cases more than 20 per cent. From Table XXIII. it is evident that the deduced value for the conducting-power of gold and silver is much higher than that found by experiment; on referring, however, to the paper on the influence of temperature on the conducting-power of metals (Table XVI.), it will be found that the percentage decrement in the conducting-power between 0° and 100° of

Silver is	28·44
Copper is	29·69
Gold is	21·30
Tin is	29·89
Lead is	29·61

Let us now recalculate the deduced conducting-powers, using these values instead of the mean of those found for the pure metals (viz. 29·307), and we arrive at much better results. These are shown in Table XXIII.

Table XXIII.

	Deduced from the impure metals.	Conducting-power at 0°.		Deduced from the impure metals, using the observed percentage decrements.
		Observed for hard-drawn wires.	Observed for annealed wires.	
Lead	8·53	8·32	——	8·65
Tin	12·19	12·36	——	12·54
Gold (hard drawn)	83·17	77·96	79·33	79·20
Copper (hard drawn)	100·06	99·95	102·21	101·91
Silver (hard drawn)	112·06	100·00	108·57	107·43

* From Table XXVII. † From Table XII. ‡ From Table XIII.

The values in the last column were obtained as follows: take for instance that of gold. The mean deduced value (column 1) for its conducting-power at 0° was 83·17, under the supposition that the percentage decrement in its conducting-power between 0° and 100° was 29·307; the percentage decrement, however, found for pure gold was 28·30; we must therefore recalculate the deduced value to obtain a more concordant one, and this may be done with the help of the proportion

$$P:P'::M_{100°}:M'_{100°} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

First,

$$M_{100°}=\frac{83·17\times 70·693}{100}=58·80,$$

then

$$29·307:28·30::58·80:M'_{100°};$$

` ence

$$M'_{100°}=56·77,$$

and therefore the deduced value at 0°

$$\frac{56·77\times 100}{71·7}=79·20.$$

Reducing the others in the same manner, we are struck with the coincidence of these values with those really found for the annealed wires by experiment; in fact we must assume that the values deduced for the conducting-power of metals are those of the annealed wires, even when hard-drawn ones are experimented with. What the deduced values for the conducting-power would be when using annealed wires of impure metals we are unable at present to say, for no determinations have been made in this direction. It must be remembered that the effect of annealing on the conducting-power of alloys is very small, so that the deduced values from those found for the annealed wires would not be very different from those deduced from the hard drawn, assuming, as we have done in the former part of this investigation, that the percentage decrement in the conducting-power between 0° and 100° of hard drawn and annealed is the same.

Having thus proved that, by using the expression

$$P:P'::M_{100°}:M'_{100°}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

we may deduce the conducting-power of the pure metal from the impure one, when the observed values do not differ from those calculated by more than 20 to 30 per cent., we will now proceed to give the results of some experiments with impure metals where the conducting-power of the same metals in a pure state has not yet been determined. Tables XXIV. and XXV. contain the results.

Table XXIV.

1.

Platinum, commercial (hard drawn).

Length 371 millims.; diameter 0·614 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	11·209 at 18°·6	11·720
Ditto, after being kept at 100° for 1 day	11·212 at 15·6	11·692
Ditto, for 2 days	11·174 at 16·7	11·687
Ditto, for 3 days	11·159 at 16·8	11·647

T.	Conducting-power.
9°·0	11·427
54·5	10·172
100·0	9·197

$$\lambda=11·708-0·031875t+0·00006762t^2.$$

2.

Platinum, commercial (hard drawn).

Length 209 millims.; diameter 0·243 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	11·039 at 17°·0	11·527
Ditto, after being kept at 100° for 1 day	11·038 at 17·3	11·535
Ditto, for 2 days..................	11·022 at 17·6	11·527

T.	Conducting-power.
11°·0	11·239
55·0	10·072
100·0	9·141

$$\lambda=11·530-0·029721t+0·00005827t^2.$$

3.

Palladium, commercial (hard drawn).

Length 167·5 millims.; diameter 0·379 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	13·230 at 18°·4	13·991
Ditto, after being kept at 100° for 1 day	13·295 at 17·5	14·022
Ditto, for 2 days	13·322 at 16·9	14·025

T.	Conducting-power.
9°·0	13·645
54·5	11·954
100·0	10·658

$$\lambda=14·026-0·043225t+0·00009540t^2.$$

4.

Palladium, commercial (hard drawn).

Length 218 millims.; diameter 0·469 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	12·091 at 17°·2	12·678
Ditto, after being kept at 100° for 1 day	12·087 at 17·6	12·684

T.	Conducting-power.
10°·0	12·357
55·0	10·978
100·0	9·818

$$\lambda=12·704-0·035448t+0·00007383t^2.$$

Table XXIV. (continued).

5.

Magnesium, commercial.

Length 717 millims.; diameter 0·497 millim.

T.	Conducting-power.
15°·0	34·912
57·5	30·312
100·0	26·922

$$\lambda=36·825-0·13252t+0·0003349t^2.$$

6.

Magnesiun (from Mr. E. Sonstadt).

Length 628 millims.; diameter 0·436 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	38·062 at 11°·0	39·662
Ditto, after being kept at 100° for 1 day	37·963 at 12·2	39·735
Ditto, for 2 days..................	37·918 at 12·6	39·747

T.	Conducting-power.
13°·0	37·881
56·5	32·442
100·0	28·347

$$\lambda=39·765-0·14971t+0·0003351t^2.$$

7.

Aluminium, commercial (hard drawn).

Length 1351 millims.; diameter 0·511 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	50·804 at 17°·2	54·073
Ditto, after being kept at 100° for 1 day	51·079 at 16·4	54·210
Ditto, for 2 days	51·146 at 15·7	54·145
Ditto, for 3 days	51·035 at 16·4	54·163

T.	Conducting-power.
12°·0	51·910
56·0	44·542
100·0	38·938

$$\lambda=54·225-0·19843t+0·0004556t^2.$$

8.

Aluminium, alloyed with 0·5 per cent. nickel (hard drawn).

Length 745 millims.; diameter 0·415 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	44·597 at 15°·9	46·950
Ditto, after being kept at 100° for 1 day	44·786 at 15·2	47·043
Ditto, for 2 days..................	45·044 at 13·6	47·071

T.	Conducting-power.
14°·0	44·986
57·0	39·325
100·0	34·785

$$\lambda=47·071-0·15321t+0·0003037t^2.$$

Table XXV.

Metal.	Observed percentage decrement in the conducting-power between 0° and 100°.	Conducting-power at 0°. Observed.	Conducting-power at 0°. Calculated for the pure metal.	Mean.
Platinum (1)	21·45	11·72	17·79	18·03
Platinum (2)	20·73	11·53	18·28	
Palladium (3)	24·01	13·99	18·35	18·44
Palladium (4)	22·09	12·68	18·54	
Magnesium (5)	26·89	36·82	41·50	44·17
Magnesium (6)	28·72	39·66	40·85	
Aluminium (7)	28·19	54·07	57·01	56·06
Aluminium (8)	26·10	46·95	55·12	

[It is scarcely necessary to add, that in the same manner as the formulæ for the correction of conducting-power for temperature may in most cases be deduced where the composition and conducting-power of an alloy at any temperature are known, that for the correction of the conducting-power for temperature of an impure metal may also be calculated, using the conducting-power of the annealed metal for λ'_0, λ'_t, λ'_{100}. This is of practical importance; for in testing copper wire for telegraphic purposes, the formula for the correction of its conducting-power for temperature may be easily deduced, of course only in cases where the conducting-power is within the limits above stated. It has already been elsewhere shown that the conducting-power of commercial metals, copper for instance, varies considerably according to the state of its purity. Thus a specimen of Rio Tinto copper was found to conduct as follows:—

Length 398 millims.; diameter 0·331 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	13·480 at 18°·6	13·622
Ditto, after heating to 100° for 1 day	13·473 at 16·9	13·586
Ditto, for 2 days	13·442 at 14·9	13·573
Ditto, for 3 days	13·420 at 15·7	13·558
Ditto, for 4 days	13·418 at 16·0	13·558

T.	Conducting-power.
14°·67	13·429
57·33	13·064
100·00	12·713

$$\lambda = 13{\cdot}558 - 0{\cdot}0088326t + 0{\cdot}0000038441t^2$$

which corresponds to a percentage decrement of only 6·23, whereas the conducting-power of pure copper decreases between 0° and 100° C. 29·69 per cent.—Feb. 1864.]

Table XXVI. contains a list of the conducting-powers of metals in a pure state. Those marked with a † are those deduced from the impure metals, and they may be called *the probable values for the conducting-powers of annealed wires of the metals.*

TABLE XXVI.

Metal.	Conducting-power at 0°.		
	Hard drawn.	Pressed.	Annealed.
Silver	100·00	....	108·57
Copper	99·95	....	102·21
Gold	77·96	....	79·33
Aluminium	....	....	56·06†
Magnesium	....	....	41·17†
Zinc	....	29·02	
Cadmium	....	23·72	
Palladium	....	....	18·44†
Platinum	....	....	18·03†
Cobalt	....	....	17·22† *
Iron	....	....	16·81† *
Nickel	....	....	13·11† *
Tin	....	12·36	
Thallium	....	9·16	
Lead	....	8·23	
Arsenic	....	4·76	
Antimony	....	4·62	
Bismuth	....	1·245	
Gold-silver alloy	15·03		

IV. *Miscellaneous and general remarks.*

Having thus described the results obtained in this investigation, it only remains for us to make a few general remarks on them.

1. The percentage decrement in the conducting-power of alloys between 0° and 100° is never greater than that of the pure metals composing them; for on looking at Tables XI., XII., and XIII., we only find a few cases where the observed percentage decrement is greater than that of the pure metals composing the alloy, and in these the differences are so small that they are undoubtedly due to small errors in the observations, for the differences between the percentage decrements are not greater than those obtained for different wires of the same metal.

2. The conducting-power of alloys decreases with an increase of temperature. This, however, is not strictly true for all alloys, for we already know of some where this is not the case, viz. a few of the bismuth alloys. The results of our observations are given in the following Table:—

TABLE XXVII.

1.

†Bi Pb_{100}, containing 2·27 volumes per cent. bismuth.

Length 243 millims.; diameter 0·512 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	7·697 at 14°·5	8·090
Ditto, after being kept at 100° for 1 day	7·715 at 14·1	8·099

T.	Conducting-power.
18°·0	7·693
57·5	6·675
100·0	5·860

$$\lambda = 8{\cdot}101 - 0{\cdot}0280217t + 0{\cdot}00005619t^2.$$

		Reduced to 0°.
The conducting-power found in a former research ‡ was	7·03 at 24°·0	7·633

2.

†Bi Pb_{10}, containing 18·85 volumes per cent. bismuth.

Length 122·5 millims.; diameter 0·673 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	4·4167 at 13°·6	4·5799
Ditto, after being kept at 100° for 1 day	4·4479 at 10·6	4·5586
Ditto, for 2 days	4·4378 at 11·5	4·5577
Ditto, for 3 days	4·4285 at 12·5	4·5587

T.	Conducting-power.		Difference.
	Observed.	Calculated.	
12°·97	4·4240	4·4226	+0·0014
24·57	4·3042	4·3064	−0·0022
37·95	4·1769	4·1776	−0·0007
54·40	4·0278	4·0268	+0·0010
69·48	3·8973	3·8961	+0·0012
82·88	3·7864	3·7859	+0·0005
94·43	3·6942	3·6954	−0·0012

$$\lambda = 4{\cdot}5576 - 0{\cdot}010607t + 0{\cdot}00001563t^2.$$

		Reduced to 0°.
The conducting-power found in a former research was	4·35 at 20°·9	4·565

* Philosophical Transactions, 1863.

‡ Ibid. 1860, p. 161.

Table XXVII. (continued).

3.

†Bi Pb_2, containing 53·74 volumes per cent. bismuth.
Length 224 millims.; diameter 0·643 millim.

T.	Conducting-power.	h m	
96°·6	1·8543		1st day.
16·5	2·0385		
12·5	2·0346		2nd day.
12·5	2·0296	>1 10	3rd day.
93·8	1·8539	>4 15	
97·0	1·8708	> 30	
12·8	2·0683		
10·5	2·0277	>1 30	4th day.
97·8	1·8617	>5 10	
93·8	1·8848	> 25	
11·7	2·0831		

The conducting-power found in a former research was 2·09 at 22°·2.

4.

†Bi Sn_8, containing 25·04 volumes per cent. bismuth
Length 194 millims.; diameter 0·713 millim.

T.	Conducting-power.	h m	
94°·8	5·3564	>5 0	1st day.
88·4	5·4696	> 30	
11·6	6·7776		
7·5	7·6698	>1 30	2nd day.
89·5	5·6474	>4 45	
92·9	5·3921	> 30	
12·3	6·7511		
10·3	7·6086		3rd day.

The conducting-power found in a former research was 7·82 at 24°·9.

5.

†Bi_4 Pb, containing 90·28 volumes per cent. bismuth.
Length 90·5 millims.; diameter 0·689 millim.

T.	Conducting-power.	h m	
10°·3	0·5299	>1 50	1st day.
94·4	0·5615	>3 0	
94·1	0·5654	>1 0	
13·3	0·5439		
10·0	0·5402	>2 0	2nd day.
94·6	0·5682	>0 30	
13·6	0·5437		
6·0	0·5413	>1 0	3rd day.
93·8	0·5686	>4 0	
94·0	0·5682	>0 30	
9·6	0·5430		

The conducting-power found in a former research was 0·521 at 20°·0.

These results need a little explanation; on the first two series no remarks are necessary, but on the three last we will say a few words. On experimenting with a wire of Bi Pb_2 we observed nothing remarkable at first, but after making a series of observations at different temperatures up to 100°, on cooling the wire the same conducting-power was not observed for the same temperature as when heating; at first we thought this was due to the wire being badly soldered, but on resoldering it the same results were obtained. In the Table the third series will read thus: at 96°·6 the conducting-power was found 1·8543; on cooling rapidly to 16°·5 it was found equal to 2·0386; on testing it the next morning at 12°·5 it was 2·0346, showing a loss in conducting-power, for it ought to have conducted better, as the temperature is lower; on the third morning we find it still lower; and on the same day, after being kept at 100° for about 4½ hours,

it, on being rapidly cooled, was 2·0683 at 12°·8, showing again an increment. On the fourth morning, at 10°·5, it was 2·0275, and after being kept for 5 hours at 100° and rapidly cooled, it was 2·0837 at 11°·7. There must be, therefore, with some of the bismuth alloys, some disturbing cause, which may act either in the one direction or the other, for on investigating the Bi Sn_8 series the opposite effect is produced. This disturbing cause may be so great that, as in the case of Bi_4 Pb, it appears as if the conducting-power increases with an increase of temperature. Other alloys of bismuth and lead, rich in bismuth, give the same results. As yet, we have not had time to investigate thoroughly this curious property of the bismuth alloys; we hope, however, to be able shortly to do so, as well as explain the reason of these remarkable exceptions to the law, that the conducting-power of alloys decreases with an increase of temperature.

3. Respecting the parts the metals take in the conducting-power of their alloys, we are at present unable to give any definite data; we did hope at one time to have deduced them with the help of the results in this memoir. It is scarcely necessary to point out that in many cases the composition of the alloy may be deduced from its conducting-power in the same manner as it may be from its specific gravity; for as

$$\text{Po} : \text{Pc} :: \lambda_{100^\circ} : \lambda'_{100^\circ}, \quad \ldots \ldots \ldots \ldots \quad (1)$$

then if Po and λ_{100° be determined, Pc being known (=29·307), λ'_{100° can be calculated, and from it the relative amounts of the component metals for

$$\lambda'_{100^\circ} = \frac{xc + (100-x)c'}{100},$$

where x represents the volumes per cent. of the one metal, $(100-x)$ those of the other, and c and c' their conducting-power at 100°.

Thus the observed conducting-power of the gold-silver alloy at 100° is 14·05, and its percentage decrement 6·49,

$$\lambda'_{100^\circ} = \frac{14{\cdot}05 \times 29{\cdot}307}{6{\cdot}49} = 63{\cdot}45,$$

therefore

$$63{\cdot}45 = \frac{71{\cdot}56^* x + 55{\cdot}90^*(100-x)}{100},$$

or

$$755 = 15{\cdot}66x,$$

$$48{\cdot}20 = x.$$

The amount of silver in the alloy was 47·92 volumes per cent. Again, the platinum-silver alloy, containing 19·65 volumes per cent. platinum, conducts at 100° 6·49, and loses in conducting-power between 0° and 100° 3·10 per cent.; calculating in the same manner the percentage amount of silver, we find it equal to 82·67 instead of 80·35. The values deduced for the percentage amounts only agree in a few cases well with those found by analysis, as slight errors in the determinations materially affect them; for instance, if the conducting-power of the gold-silver alloy were equal to 14·20 at 100°

* Observed conducting-power of silver and gold at 100° (Philosophical Transactions, 1862, p. 24).

instead of 14·05, the volumes per cent. of silver deduced from that value would be 52·62 instead of 48·20, the value calculated from the latter number.

4. It may be as well to state in a few words how we determine to which class a metal belongs, whether to the lead, tin, &c., or to the gold-silver, &c. class; to do this it is only necessary to alloy the metal with traces of lead, tin, &c., and if the conducting-power be equal to that of the mean of the components, we say it belongs to the lead class; if, on the contrary, the alloy has a lower conducting-power than the mean of the components, we say it belongs to the gold-silver, &c. class. These are only some of one series of alloys which have a higher conducting-power than the mean of their components, and these are the amalgams.

Table XXVIII. shows that the new metal thallium belongs to the gold-silver, &c. class.

TABLE XXVIII.

1.

Thallium, containing 5 per cent., by weight, tin.

Length 188 millims.; diameter 0·443 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	8·196 at 12°·6	8·522
Ditto, after being kept at 100° for 1 day	8·131 at 12·6	8·455
Ditto, for 2 days	8·097 at 9·8	8·347
Ditto, for 3 days	8·111 at 9·6	8·356

T.	Conducting-power.
10°·0	8·100
55·0	7·093
100·0	6·313

$\lambda = 8{\cdot}355 - 0{\cdot}026075t + 0{\cdot}00005654t^2$.

TABLE XXVIII. (continued).

2.

Thallium, containing 5 per cent., by weight, cadmium.

Length 163 millims.; diameter 0·431 millim.

		Reduced to 0°.
Conducting-power found before heating the wire	8·670 at 14°·4	9·141
Ditto, after being kept at 100° for 1 day	8·744 at 12·8	9·168

T.	Conducting-power.
13°·0	8·737
56·5	7·454
100·0	6·398

$\lambda = 9{\cdot}165 - 0{\cdot}033663t + 0{\cdot}00005998t^2$.

These alloys were not analyzed, the 5 per cent. of foreign metal being added to the thallium fused under cyanide of potassium. From the results it will be seen that they both conduct in a lower degree than the mean of their components; for both cadmium and tin conduct better than thallium, the conducting-power at 0° of cadmium being 23·72, and that of tin being 12·36.

5. In conclusion, we would point out that the law which we have deduced from our experiments only holds good in cases where the alloy may be considered a solution of one metal in the other, the metals belonging to the same class; when the alloy is composed of metals of the two classes, then the law no longer holds good (except for a few of the alloys), even if the alloy be a solution of the one metal in the other. The results which we have obtained and described in this memoir fully bear out the views put forward in a former one regarding the chemical nature of the alloys*.

* Philosophical Transactions, 1860, p. 161.

V. *On the Absorption and Radiation of Heat by Gaseous and Liquid Matter.*—Fourth Memoir.

By JOHN TYNDALL, *F.R.S., Member of the Academies and Societies of Holland, Geneva, Göttingen, Zürich, Halle, Marburg, Breslau, Upsala, la Société Philomathique of Paris, Cam. Phil. Soc. &c.; Professor of Natural Philosophy in the Royal Institution.*

Received June 18,—Read June 18, 1863.

§ 1.

THE Royal Society has already done me the honour of publishing in the Philosophical Transactions three memoirs on the relations of radiant heat to the gaseous form of matter. In the first of these memoirs* it was shown that for heat emanating from the blackened surface of a cube filled with boiling water, a class of bodies which had been previously regarded as equally, and indeed, as far as laboratory experiments went, perfectly diathermic, exhibited vast differences both as regards radiation and absorption. At the common tension of one atmosphere the absorptive energy of olefiant gas, for example, was found to be 290 times that of air, while when lower pressures were employed the ratio was still greater. The reciprocity of absorption and radiation on the part of gases was also experimentally established in this first investigation.

In the second inquiry† I employed a different and more powerful source of heat, my desire being to bring out with still greater decision the differences which revealed themselves in the first investigation. By carefully purifying the transparent elementary gases, and thus reducing the action upon radiant heat, the difference between them and the more strongly acting compound gases was greatly augmented. In this second inquiry, for example, olefiant gas at a pressure of one atmosphere was shown to possess 970 times the absorptive energy of atmospheric air, while it was shown to be probable that when pressures of $\frac{1}{30}$th of an atmosphere were compared, the absorption of olefiant gas was nearly 8000 times that of air. A column of ammoniacal gas, moreover, 3 feet long, was found sensibly impervious to the heat employed in the inquiry, while the vapours of many of the volatile liquids were proved to be still more opaque to radiant heat than even the most powerfully acting permanent gases. In this second investigation, the discovery of dynamic radiation and absorption is also announced and illustrated, and the action of odours and of ozone on radiant heat is made the subject of experiment.

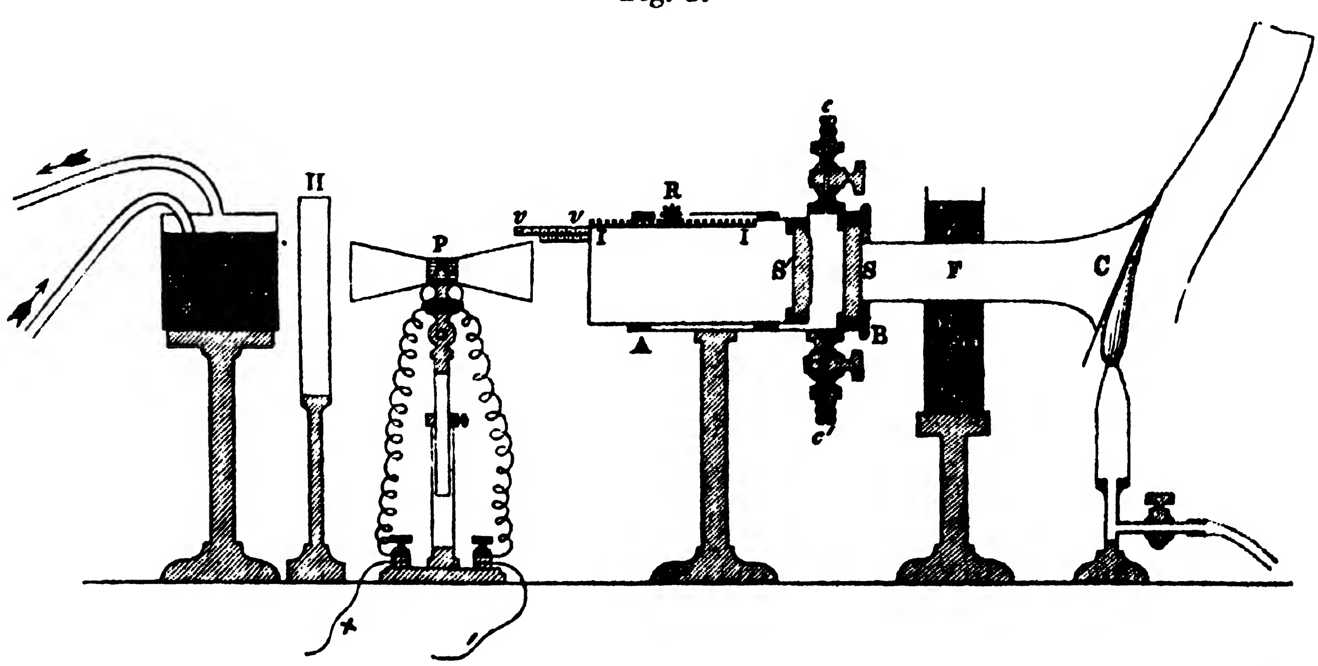
* Philosophical Transactions, February 1861; and Philosophical Magazine, September 1861.

† Philosophical Transactions, January 1862; and Philosophical Magazine, October 1862.

The third paper* of the series to which I have referred was devoted to the examination of one particular vapour, which on account of its universal diffusion possesses an interest of its own—I mean of course the vapour of water. In this paper I considered all the objections which had been urged against my results up to the time when the paper was written; I replied to each of them by definite experiments, removing them one by one, and finally placing, as I believe, beyond the pale of reasonable doubt the action of the aqueous vapour of our atmosphere. In this third paper, moreover, the facts established by experiment are applied to the explanation of various atmospheric phenomena.

I have now the honour to lay before the Royal Society a fourth memoir, containing an account of further researches. Hitherto I have confined myself to experiments on radiation through gases and vapours which were introduced in succession into the same experimental tube, the heat being thus permitted to pass through the same thickness of different gases. A portion of the present inquiry is devoted to the examination of the transmission of radiant heat through different thicknesses of the same gaseous body. The brass tube with which my former experiments were conducted is composed of several pieces, which are screwed together when the tube is to be used as a whole; but the pieces may be dismounted and used separately, a series of lengths being thus attainable, varying from 2·8 inches to 49·4 inches. I wished, however, to operate upon gaseous strata much thinner than the thinnest of these, and for this purpose a special apparatus was devised, and with much time and trouble rendered at length practically effective.

Fig. 1.



The apparatus is sketched in fig. 1. C is the source of heat, which consists of a plate of copper against the back of which a steady sheet of flame is caused to play. The plate of copper forms one end of the chamber F (the "front chamber" of my former memoirs). This chamber, as in my previous investigations, passes through the vessel

* Philosophical Transactions, December 1862; and Philosophical Magazine, July 1863.

V, through which cold water continually circulates, entering at the bottom and escaping at the top. The heat is thus prevented from passing by conduction from the source C to the first plate of rock-salt S. This plate forms the end of the hollow cylinder A B, dividing it from the front chamber F, with which the cylinder A B is connected by suitable screws and washers. Within the cylinder A B moves a second one, I I, as an air-tight piston, and the bottom of the second cylinder is stopped by the plate of rock-salt S′. This plate projects a little beyond the end of its cylinder, and thus can be brought into flat contact with the plate S. Fixed firmly to A B is a graduated strip of brass, while fixed to the piston is a second strip, the two strips forming a vernier, *v v*. By means of the pinion R, which works in a rack, the two plates of salt may be separated, their exact distance apart being given by the vernier. P is the thermo-electric pile with its two conical reflectors; C′ is the compensating cube, employed to neutralize the radiation from the source C. H is an adjusting screen, by the motion of which the neutralization may be rendered perfect, and the needle brought to zero under the influence of the two opposing radiations. The graduation of the vernier was so arranged as to permit of the employment of plates of gas varying from 0·01 to 2·8 inches in thickness. They were afterwards continued with the pieces of the experimental tube, already referred to, and in this way layers of gas were examined which varied in thickness in the ratio of 1 : 4900.

In my former experiments the chamber F was always kept exhausted, so that the rays of heat passed immediately from the source through a vacuum; but in the present instance I feared the strain upon the plate S, and I also feared the possible intrusion of a small quantity of the gas under examination into the front chamber F, if the latter were kept exhausted. Having established the fact that a length of 8 inches of dry air exerts no sensible action on the rays of heat, I had no scruple in filling the chamber F with dry air. Its absorption was *nil*, and it merely had the effect of lowering in an infinitesimal degree the temperature of the source. The two stopcocks *c* and *c′* stand exactly opposite the junction of the two plates of salt S, S′ when they are in contact, and when they are drawn apart these cocks are in communication with the space between the plates.

After many trials, the following mode of experiment was adopted:—The gas-holder containing the gas to be examined was connected by an india-rubber tube with the cock *c′*, the other cock *c* being at the same time left open. The piston was then moved by the screw R until the requisite distance between the plates was obtained. This space being filled with dry air, the radiations on the two faces of the pile were equalized, and the needle brought to zero. The gas-holder was now opened, and by gentle pressure the gas from the holder was forced first through a drying apparatus, and then into the space between the plates of salt. The air was quickly displaced, and a plate of the gas substituted for it. If the layer of gas possessed any sensible absorbing power, the equilibrium of the two sources of heat would be destroyed; the source C′ would triumph, and from the deflection due to its preponderance the exact amount of heat intercepted by the gas could be calculated.

When oxygen, hydrogen, or nitrogen was substituted for atmospheric air, no change in the position of the galvanometer-needle occurred; but when any one of the compound gases was allowed to occupy the space between the plates, a measurable deflection ensued. The plates of rock-salt were not so smooth, nor was their parallelism so perfect as entirely to exclude the gas when they were in contact. The contact was but partial, and hence a stratum of gas sufficient to effect a sensible absorption could find its way between the plates even when they touched each other. On this account the first thickness in the following Tables was really a little more than 0·01 of an inch. The first column in each contains the thickness of the gaseous layer, while the second column contains the absorption expressed in hundredths of the total radiation. The first layer of carbonic oxide, for example, absorbed 0·2, and the second layer 0·5 per cent. of the entire heat.

Table I.—Carbonic Oxide.

Thickness of gas.	Absorption in hundredths of the total radiation.	Thickness of gas.	Absorption in hundredths of the total radiation.
0·01 of an inch	0·2	0·4 of an inch	3·5
0·02 "	0·5	0·5 "	3·8
0·03 "	0·7	0·6 "	4·0
0·04 "	0·9	1·0 "	5·1
0·06 "	1·4	1·5 "	6·1
0·1 "	1·6	2·0 "	6·8
0·3 "	3·0		

Table II.—Carbonic Acid.

Thickness of gas.	Absorption in hundredths of the total radiation.	Thickness of gas.	Absorption in hundredths of the total radiation.
0·01 of an inch	0·86	0·4 of an inch	5·3
0·02 "	1·2	0·5 "	5·7
0·03 "	1·5	0·6 "	5·9
0·04 "	1·9	0·7 "	6·0
0·05 "	2·1	0·8 "	6·1
0·06 "	2·3	0·9 "	6·2
0·1 "	3·3	1·0 "	6·3
0·2 "	4·1	1·5 "	7·0
0·3 "	4·8	2·0 "	7·6

TABLE III.—Nitrous Oxide.

Thickness of gas.	Absorption in hundredths of the total radiation.	Thickness of gas.	Absorption in hundredths of the total radiation.
0·01 of an inch	1·48	0·4 of an inch	10·20
0·02 "	2·33	0·5 "	11·00
0·03 "	3·80	0·6 "	11·70
0·04 "	4·00	0·8 "	12·17
0·05 "	4·20	1·0 "	12·80
0·1 "	6·00	1·5 "	14·20
0·2 "	7·77	2·0 "	15·7

TABLE IV.—Olefiant Gas.

Thickness of gas.	Absorption in hundredths of the total radiation.	Thickness of gas.	Absorption in hundredths of the total radiation.
0·01 of an inch	1·80	0·5 of an inch	23·30
0·02 "	3·08	1·0 "	26·33
0·05 "	5·37	2·0 "	32·80
0·1 "	9·14		

We here find that a layer of olefiant gas only 2 inches in thickness intercepts nearly 33 per cent. of the radiation from our source. Supposing our globe to be encircled by a shell of olefiant gas only 2 inches in thickness, this shell would offer a scarcely sensible obstacle to the passage of the solar rays earthward, but it would cut off at least 33 per cent. of the terrestrial radiation and in great part return it. Under such a canopy, trifling as it may appear, the surface of the earth would be kept at a stifling temperature. The possible influence of an atmospheric envelope on the temperature of a planet is here forcibly illustrated.

The only *vapour* which I have examined with the piston apparatus is that of sulphuric ether. Glass fragments were placed in a U-tube and wetted with the ether. Through this tube dry air was gently forced, whence it passed, vapour-laden, into the space between the rock-salt plates S S'. The following Table contains the results.

TABLE V.—Air saturated with the Vapour of Sulphuric Ether.

Thickness of vapour.	Absorption in hundredths of the total radiation.	Thickness of vapour.	Absorption in hundredths of the total radiation.
0·05 of an inch	2·07	0·8 of an inch	21·0
0·1 "	4·6	1·5 "	34·6
0·2 "	8·7	2·0 "	35·1
0·4 "	14·3		

Comparing these results with those obtained with olefiant gas, we find for thicknesses of 0·05 of an inch and 2 inches respectively the following absorptions:—

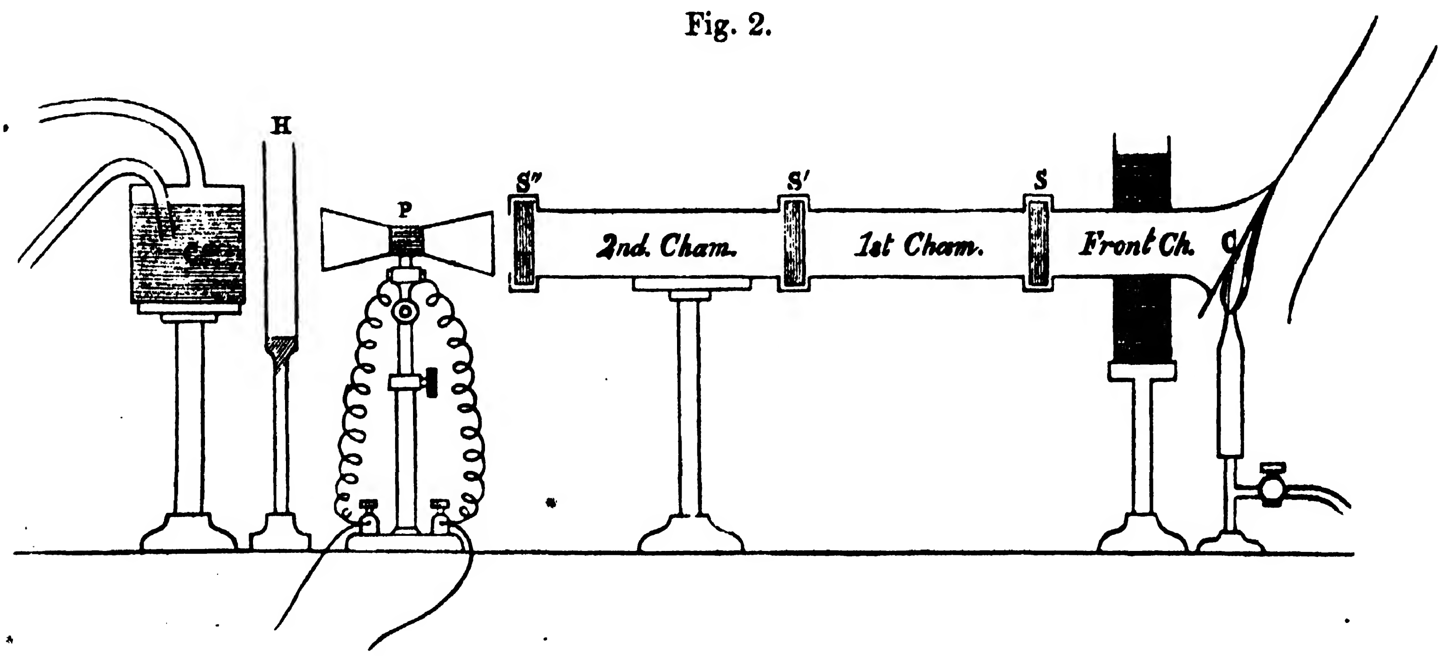
Olefiant gas.		Sulphuric ether.	
Thickness of 0·05 . . .	5·37	Thickness of 0·05 . . .	2·07
Thickness of 2 inches . .	32·80	Thickness of 2 inches . .	35·1

Sulphuric ether vapour, therefore, commences with an absorption much lower than that of olefiant gas, and ends with a higher absorption. This is quite in accordance with the result established in my second memoir*, that in a short tube the absorption effected by the sparsely scattered molecules of a vapour may be less than that of a gas at a tension of an atmosphere, while in a long tube the gas may be exceeded by the vapour. The deportment of sulphuric ether indicates what mighty changes of climate might be brought about by the introduction into the earth's atmosphere of an almost infinitesimal amount of a powerful vapour. And if *aqueous vapour* can be shown to be thus powerful, the effect of its withdrawal from our atmosphere may be inferred.

§ 2.

The experiments with the piston apparatus being completed, greater thicknesses of gas were obtained by means of the composite brass experimental tube already referred to. The arrangement adopted was, however, peculiar, being expressly intended to check the experiments, which were for the most part made by my assistants. The source of heat and the front chamber remained as usual; a plate of rock-salt dividing, as in my previous investigations, the front chamber from the experimental tube. The distant end of the tube was also stopped by a plate of salt; but instead of permitting the tube to remain continuous from beginning to end, it was divided, by a third plate of rock-salt,

Fig. 2.



into two air-tight compartments. Thus the rays of heat from the source had to pass through *three* distinct chambers, and through three plates of salt. The first chamber

* Philosophical Transactions, Part I. 1862; and Philosophical Magazine, vol. xxiv. p. 343.

was always kept filled with perfectly dry air, while either or both of the other chambers could be filled at pleasure with the gas or vapour to be examined. For the sake of convenience I will call the compartment of the tube nearest to the front chamber the *first* chamber, the compartment nearest to the pile the *second* chamber; the term 'front chamber' being, as before, restricted to that nearest to the source. The arrangement is sketched in outline in fig. 2.

The entire length of the tube was 49·4 inches, and this was maintained throughout the whole of the experiments. The only change consisted in the shifting of the plate of salt S′ which formed the partition between the first and second chambers. Commencing with a first chamber of 2·8 inches long, and a second chamber 46·4 inches long, the former was gradually augmented, and the latter equally diminished. The experiments were executed in the following manner:—The first and second chambers were thoroughly cleansed and exhausted, and the needle brought to zero by the equalization of the radiations falling upon the opposite faces of the pile. Into the first chamber the gas or vapour to be examined was introduced, and its absorption determined. The first chamber was then cleansed, and the gas or vapour was introduced into the second chamber, its absorption there being also determined. Finally, the absorption exerted by the two chambers acting together was determined, both of them being occupied by the gas or vapour.

The combination here described enabled me to check the experiments, and also to trace the influence of the first chamber on the quality of the radiation. In it the heat was more or less sifted, and it entered the second chamber deprived of certain constituents which it possessed on its entrance into the first. On this account the quantity absorbed in the second chamber when the first chamber is full of gas, must always be less than it would be if the rays had entered without first traversing the gas of the first chamber. From this it follows that the sum of the absorptions of the two chambers, taken separately, must always exceed the absorption of the tube taken as a whole. This may be briefly and conveniently expressed by saying that the sum of the absorptions exceeds the absorption of the sum.

Table VI.—Carbonic Oxide.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	6·8	12·9	12·9
8·0	41·4	9·6	12·2	12·9
12·2	37·2	10·7	12·2	12·9
15·4	34·0	10·9	12·2	13·4
17·8	31·6	11·1	12·0	13·3
36·3	13·1	12·6	10·3	13·4

TABLE VII.—Carbonic Acid.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	8·6	13·8	13·3
8·0	41·4	9·9	12·7	13·0
12·2	37·2	11·0	11·4	13·0
15·4	34·0	11·8	12·1	13·9
23·8	25·6	11·7	11·4	13·1
23·8	25·6	11·2	11·2	12·6
23·8	25·6	10·4	10·5	12·0
36·3	13·1	11·6	10·0	12·3

Various causes have rendered these experiments exceedingly laborious. Could I have procured a sufficiently large quantity of gas in a single holder for an entire series of experiments it would not have been difficult to obtain concurrent results, but the slight variations in quality of the same gas generated at different times tell upon the results and render perfect uniformity extremely difficult to obtain. The approximate constancy of the numbers in the third column is, however, a guarantee that the determinations are not very wide of the truth. Irregularities, however, are revealed. Some remarkable ones occur in the case of carbonic acid, with the chambers 23·8 and 25·6; the absorptions in the first chamber varying in this instance from 11·7 to 10·4, and in the second chamber from 11·4 to 10·5, and in both chambers from 13·1 to 12·0. The gas which gave the largest of these results was generated from marble and hydrochloric acid; the next was obtained from chalk and sulphuric acid, and the gas which gave the smallest result was obtained from bicarbonate of soda and sulphuric acid. The slight differences accompanying these different modes of generation made themselves felt in the manner recorded in the Table.

TABLE VIII.—Nitrous Oxide.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	16·1	32·9	33·9
12·2	37·2	23·1	30·0	32·0
15·4	34·0	23·6	29·6	32·0
17·8	31·6	26·2	29·6	32·7

The differences arising from different modes of generation are most strikingly illustrated by the powerful gases. My friend Dr. FRANKLAND, for example, was kind enough to superintend for me the formation of a large holder of olefiant gas by the so-called "continuous process," in which the *vapour* of alcohol is led through sulphuric acid diluted with its own volume of water; the following results were obtained:—

TABLE IX.—Olefiant Gas.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	34·6	66·1	67·7
8·0	41·4	44·2	65·3	67·5
15·4	34·0	53·6	62·3	67·0

Considering the difficulty of the experiments, the agreement of the absorption of both chambers, the sum of which was the constant quantity 49·4 inches, must be regarded as satisfactory. This is the general character of the results as long as we adhere to the same gas. Olefiant gas generated by mixing the *liquid* alcohol with sulphuric acid and applying heat to the mixture, gave the results recorded in the following Table:—

TABLE X.—Olefiant Gas.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
12·2	37·2	54·8	70·0	76·3
15·4	34·0	59·1	72·7	77·1
19·8	29·6	67·8	70·4	77·0
23·8	25·6	69·2	70·2	77·6
36·3	13·1	72·8	60·3	78·8

The absorptions of both chambers in this Table are almost exactly 10 per cent. higher than those found with the gas generated under Dr. FRANKLAND's superintendence.

A few remarks on these results may be introduced here. In the case of carbonic oxide (Table VI.), we see that while a length of 2·8 inches of gas is competent, when acting alone, to intercept 6·8 per cent. of the radiant heat, the cutting off of this length from a tube 49·4 inches long, or, what is the same, the addition of this length to a tube 46·6 inches long, makes no sensible change in its absorption. The second chamber absorbs as much as both. The same remark applies to carbonic acid, and it is also true within the limits of error for nitrous oxide and olefiant gas. Indeed it is only when 8 inches or more of the column have been cut away that the difference begins to make itself felt. Thus, in carbonic oxide, the absorption of a length of 41·4 being 12·2, that of a chamber 49·4, or 8 inches longer, is only 12·9, making a difference of only 0·7 per cent., while the same 8 inches acting singly on the gas produces an absorption of 9·6 per cent. So also with regard to carbonic acid; a tube 41·4 absorbing 12·7 per cent., a tube 49·4 absorbs only 13·0 per cent.—making a difference of only 0·3 per cent. As regards olefiant gas (Table IX.), while a distance of 8 inches acting singly effects an absorption of 44 per cent., the addition of 8 inches to a tube already 41·4 inches in length raises the absorption only from 65·3 to 67·5, or 2·2 per cent. The reason is plain. In a length of 41·4 the rays capable of being absorbed by the gas are so much diminished, so few in fact remain to be attacked, that an additional 8 inches of gas produces a scarcely sensible effect. Similar considerations explain the fact, that while by augmenting the length of

the first chamber from 2·8 inches to 15·4 inches we increase the absorption of olefiant gas nearly 20 per cent., the shortening of the second chamber by precisely the same amount effects a diminution of barely 4 per cent. of the absorption. All these results conspire to prove the heterogeneous character of the radiation from a source heated to about 250° C.

The sum of the absorptions placed side by side with the absorption of the sum exhibits the influence of sifting in an instructive manner. Tables VI., VII., VIII., IX., and X., thus treated, give the following comparative numbers:—

TABLE XI.—Carbonic Oxide.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
2·8	46·6	19·7	12·9
8·0	41·1	21·8	12·9
12·2	37·2	22·9	12·9
15·4	34·0	23·1	13·4
17·8	31·6	23·1	13·3
36·3	13·1	22·9	13·4
	Means	22·3	13·1

TABLE XII.—Carbonic Acid.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
2·8	46·6	22·4	13·3
8·0	41·4	22·6	13·0
12·2	37·2	22·4	13·0
15·4	34·0	23·9	13·9
23·8	25·6	23·1	13·1
36·3	13·1	21·6	12·3
	Means	22·6	13·1

TABLE XIII.—Nitrous Oxide.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
2·8	46·6	49·0	33·9
12·2	37·2	53·1	32·0
15·4	34·0	53·2	32·0
17·8	31·6	55·8	32·7
	Means	52·8	32·7

TABLE XIV.—Olefiant Gas.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
2·8	46·6	100·7	67·7
8·0	41·4	109·5	67·5
12·2	37·2	109·4	65·0
15·4	34·0	115·9	67·0
	Means	108·9	66·8

TABLE XV.—Olefiant Gas.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
12·2	37·2	124·8	76·3
15·4	34·0	131·8	77·1
19·8	29·6	138·2	77·0
23·8	25·6	139·4	77·6
36·3	13·1	133·1	78·8
	Means	133·4	77·3

The conclusion that the sum of the absorptions is greater than the absorption of the sum is here amply verified. The Tables also show that the ratio of the sum of the absorptions to the absorption of the sum is practically constant for all the gases. Dividing the first mean by the second in the respective cases, we have the following quotients:—

Carbonic oxide	1·70
Carbonic acid	1·72
Nitrous oxide	1·61
Olefiant gas (mean of both)	1·68

The sum of the absorptions ought to be a maximum when the two chambers are of equal length. Supposing them to be unequal, one being in excess of half the length of the tube, let us consider the action of this excess singly. Placed after the half-length, it receives the rays which have already traversed that half; placed after the shorter length, it receives the rays which have traversed the shorter length. In the former case, therefore, the excess will absorb less than in the latter, because the rays in the former case have been more thoroughly sifted before the heat reaches the excess. From this it is clear that, as regards absorption, more is gained by attaching the excess to the short length of the tube than to the half-length; in other words, the sum of the absorptions, when the tube is divided into two equal parts, is a maximum. This reasoning is approximately verified by the experiments. Supposing, moreover, one of the lengths constantly to diminish, we thus constantly approach the limit when the sum of the absorptions and the absorption of the sum are equal to each other, the former being then a minimum. The effect of proximity to this limit is exhibited in the first experiment in each of the series; here the lengths of the compartments are very unequal, and the sum of the absorptions is, in general, a minimum.

After the absorption by the permanent gases had been in this way examined, I passed on to the examination of vapours. They were all used at a common pressure of 0·5 of an inch of mercury, or about $\frac{1}{60}$th of an atmosphere. The liquid which yielded the vapour was enclosed in the flasks described in my previous memoirs, and the pure vapour was allowed to enter the respective compartments of the experimental tube without the slightest ebullition. The following series of Tables contains the results thus obtained.

TABLE XVI.—Bisulphide of Carbon. Pressure 0·5 of an inch.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	3·6	7·6	7·6
8·0	41·4	4·4	7·3	7·6
15·4	34·0	5·7	6·0	7·5
17·8	31·6	5·8	6·4	7·5
23·8	25·6	6·7	6·0	7·8

TABLE XVII.—Chloroform. Pressure 0·5 of an inch.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	5·5	15·9	16·3
8·0	41·4	9·2	15·6	16·8
12·2	37·2	10·5	14·8	17·1
15·4	34·0	11·6	14·1	16·9
23·8	25·6	15·0	14·0	18·4
36·3	13·1	15·6	10·9	17·2

TABLE XVIII.—Benzol. Pressure 0·5 of an inch.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	4·0	20·0	20·6
8·0	41·4	8·4	17·3	20·4
12·2	37·2	9·8	16·5	19·0
17·8	31·6	11·9	15·7	20·1
23·8	25·6	14·3	15·1	21·0

TABLE XIX.—Iodide of Ethyl. Pressure 0·5 of an inch.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	7·1	23·5	25·4
8·0	41·4	9·1	21·1	23·3
12·2	37·2	12·8	20·5	25·2
15·4	34·0	14·6	20·8	25·2
17·8	31·6	15·8	20·0	25·5

TABLE XX.—Alcohol. Pressure 0·5 of an inch.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	11·7	46·1	46·1
8·0	41·4	18·5	43·6	47·0
12·2	37·2	26·0	44·1	47·5
15·4	34·0	32·1	41·1	47·0
17·8	31·6	32·4	40·0	47·6

TABLE XXI.—Alcohol. Pressure 0·1 of an inch.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
8·0	41·4	8·0	22·2	24·9
15·4	34·0	12·1	20·0	24·7
17·8	31·6	13·1	19·7	25·7
23·8	25·6	14·8	18·4	25·2
36·3	13·1	19·1	13·8	25·1

TABLE XXII.—Sulphuric Ether. Pressure 0·5 of an inch.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	14·8	50·0	51·6
8·0	41·4	23·9	51·0	53·9
12·2	37·2	30·9	48·8	53·6
15·4	34·0	34·0	47·8	53·1

TABLE XXIII.—Acetic Ether. Pressure 0·5 of an inch.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	17·0	60·2	62·9
8·0	41·4	30·7	58·1	64·6
12·2	37·2	41·6	55·1	64·2
15·4	34·0	44·4	55·5	62·4
23·8	25·6	50·9	52·7	64·7
36·3	13·1	58·1	42·6	64·8

TABLE XXIV.—Formic Ether. Pressure 0·5 of an inch.

Length.		Absorption per 100.		
1st Chamber.	2nd Chamber.	1st Chamber.	2nd Chamber.	Both Chambers.
2·8	46·6	17·4	63·0	64·4
8·0	41·4	33·3	59·1	63·4
17·8	31·6	40·0	48·4	60·3
23·8	25·6	45·6	47·2	60·2

I have already compared the sum of the absorptions for gases with the absorption of the sum; in the following Tables the same comparison is made for the vapours.

TABLE XXV.—Bisulphide of Carbon, 0·5 inch.

Length of Chambers.		Sum of Absorptions.	Absorption
2·8	46·6	11·2	7·6
8·0	41·4	11·7	7·6
15·4	34·0	11·7	7·5
17·8	31·6	12·2	7·5
23·8	25·6	12·7	7·8
	Means	11·9	7·6

TABLE XXVI.—Chloroform, 0·5 inch.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
2·8	46·6	21·4	16·3
8·0	41·4	24·8	16·8
12·2	37·2	25·3	17·1
15·4	34·0	25·2	16·9
23·8	25·6	29·0	18·4
36·3	13·1	26·5	17·2
	Means	25·36	17·1

TABLE XXVII.—Benzol, 0·5 inch.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum
2·8	46·6	24·0	20·6
8·0	41·4	25·7	20·4
12·2	37·2	26·3	19·0
17·8	31·6	27·6	20·1
23·8	25·6	29·4	21·0
	Means	26·6	20·2

TABLE XXVIII.—Iodide of Ethyl, 0·5 inch.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
2·8	46·6	30·6	25·4
8·0	41·4	30·2	23·3
12·2	37·2	33·3	25·2
15·4	34·0	35·4	25·2
17·8	31·6	35·8	25·2
	Means	33·1	24·9

TABLE XXIX.—Alcohol, 0·5 inch.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
2·8	46·6	57·8	46·1
8·0	41·4	62·1	47·0
12·2	37·2	70·1	47·5
15·4	34·0	73·2	47·0
17·8	31·6	72·4	47·6
	Means	67·1	47·0

TABLE XXX.—Alcohol, 0·1 inch.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
8·0	41·4	30·2	24·9
15·4	34·0	32·1	24·7
17·8	31·6	32·8	25·7
23·8	25·6	33·2	25·2
36·3	13·1	32·9	25·1
	Means	32·2	25·1

TABLE XXXI.—Sulphuric Ether, 0·5 inch.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
2·8	46·6	64·8	51·6
8·0	41·4	74·9	53·9
12·2	37·2	79·7	53·6
15·4	34·0	81·8	53·1
	Means	75·3	53·05

TABLE XXXII.—Formic Ether, 0·5 inch.

Length of Chambers.		Sum of Absorptions.	Absorption of Sum.
2·8	46·6	80·4	64·4
8·0	41·4	82·4	63·4
17·8	31·6	88·4	60·3
23·8	25·6	92·8	60·2
	Means	86·0	62·07

TABLE XXXIII.—Acetic Ether, 0·5 inch.

Length of Chambers.			Sum of Absorptions.	Absorption of Sum.
2·8	46·6		77·2	62·9
8·0	41·4		88·8	64·6
12·2	37·2		96·7	64·2
15·4	34·0		99·9	62·4
23·8	25·6		103·6	64·7
36·3	13·1		100·7	64·8
		Means	94·5	63·9

An inspection of the foregoing Tables discloses the fact that, in the case of vapours, the difference between the sum of the absorptions and the absorption of the sum is, in general, less than in the case of gases. This resolves itself into the proposition that for equal lengths, within the limits of these experiments, the sifting power of the gas is greater than that of the vapour. The reason of this is that the vapours are examined in a state of tenuity which is only $\frac{1}{60}$th of that possessed by the gases. Thus, no matter how powerful the individual molecules may be, their distance asunder renders a thin layer of them a comparatively open screen.

§ 3.

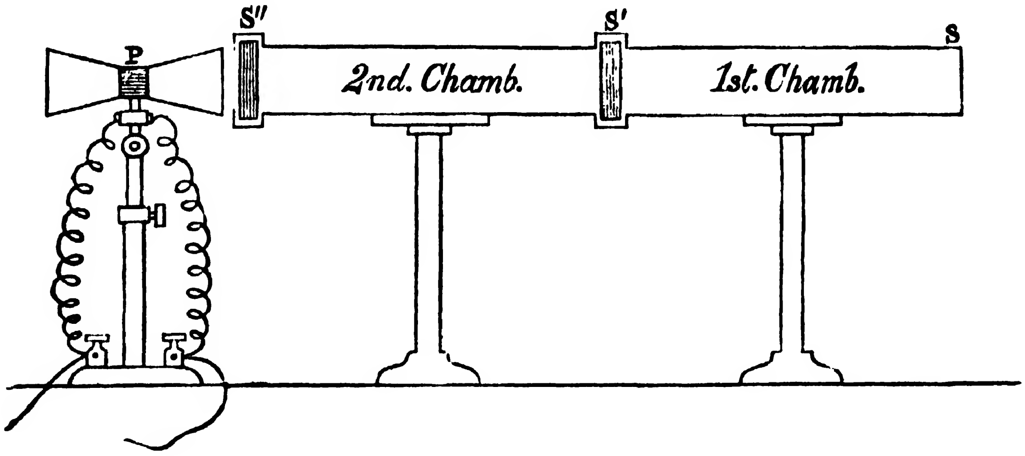
The entrance of a gas into an exhausted vessel is accompanied by the generation of heat; and the gas thus warmed, if a radiator, will emit the heat generated. Conversely, on exhausting a vessel containing any gas, the gas is chilled, and thus an external body, which prior to the act of exhaustion possessed the same temperature as the gas within the vessel, becomes, on the first stroke of the pump, a warm body with reference to the gas remaining in the vessel; and if the external body be separated from the cooled gas by a diathermic partition, it will radiate into the gas and become chilled by this radiation. It was shown in my second memoir* that this mode of warming and of chilling a gas or vapour furnished a practical means of determining, without any source of heat external to the gaseous body itself, both its radiative and absorptive energy. For the sake of convenience I have called the radiation and absorption of a gas or vapour thus dynamically heated and cooled, dynamic radiation and dynamic absorption.

In illustration of the manner in which dynamic radiation may be applied in researches on radiant heat, I have had made, during the last half-year, a considerable number of experiments, some of which I will here describe. In the first place, the experimental tube was divided into two compartments, as in the experiments described in the foregoing section. The source of heat was abolished, and one end of the experimental tube was stopped by a plate of polished metal; the other end was stopped by a transparent plate of rock-salt, while the space between the ends was divided into two compartments

* Philosophical Transactions, Part I. 1862; and Philosophical Magazine, vol. xxiv. p. 337.

by a second plate of rock-salt. The thermo-electric pile occupied its usual position at the end of the tube, the compensating cube, however, being abandoned. For the sake of convenient reference, I will call the compartment of the tube most distant from the pile, the first chamber, and that adjacent to the pile, the second chamber. An outline sketch of the arrangement is given in fig. 3.

Fig. 3.



The experiments were conducted in the following manner:—Both compartments being exhausted and the needle at zero, the gas was allowed to enter the first chamber through a gauge-cock which made its time of entry 40 seconds. The second chamber was preserved a vacuum; the gas on entering the first chamber was dynamically heated, and radiated its heat to the pile through the vacuous second chamber; the needle moved and the limit of its excursion was noted. The first chamber was then exhausted and carefully cleansed with dry air. The second chamber was filled with the same gas, not with a view to determine its dynamic radiation, but to examine its effect upon the heat radiated from the first chamber. The needle being at zero, the gas was again permitted to enter the first chamber exactly as in the first experiment, the only difference between the two experiments being, that in the first the heat passed through a vacuum to the pile, while in the second it had to pass through a column of the same kind of gas as that from which it emanated. In this way the absorption exerted by any gas upon heat, radiated from the same gas, or from any other gas, may be accurately determined. Finally, the apparatus being cleansed and the needle at zero, the gas was permitted to enter the second chamber, and its dynamic radiation from this chamber was determined. The intermediate plate of salt S′ was shifted, as in the former experiments, so as to alter the lengths of the two chambers, but the sum of both lengths remained constant as before.

In the following Tables the three columns bracketed under the head of "Deflection," contain the arcs through which the needle moved in the three cases mentioned; 1°, when the radiation from the gas in the first chamber passed through the empty second chamber; 2°, when the radiation from the first chamber passed through the occupied second chamber; and 3°, when the radiation proceeded from the second chamber.

Dynamic Radiation of Gases.

TABLE XXXIV.—Carbonic Oxide.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Gas in 2nd Chamber.	By 2nd Chamber.
2·8	46·6	1·0°	0·0°	28·0°
15·4	34·0	3·8	2·1	24·4
36·3	13·1	13·7	6·3	16·6

TABLE XXXV.—Carbonic Acid.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Gas in 2nd Chamber.	By 2nd Chamber.
2·8	46·6	1·0°	0·0°	33·6°
15·4	34·0	3·7	1·25	23·3
36·3	13·1	16·8	6·6	17·5

TABLE XXXVI.—Nitrous Oxide.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Gas in 2nd Chamber.	By 2nd Chamber.
2·8	46·6	1·0°	0·2°	44·5°
15·4	34·0	4·3	1·2	31·7
36·3	13·1	19·5	6·2	22·0

TABLE XXXVII.—Olefiant Gas.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Gas in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	11·9°	1·0°	68·0°
23·8	25·6	22·8	3·0	
36·3	13·1	59·0	10·4	65·0

The gases, it will be observed, exhibit a gradually increasing power of dynamic radiation from carbonic oxide up to olefiant gas. This is most clearly illustrated by reference to the results obtained in the respective cases with the first length of the second chamber. They are as follows:—

Carbonic oxide	28°·0
Carbonic acid	33·6
Nitrous oxide	44·5
Olefiant gas	68·0

Its proximity to the pile, and the fact of its having to cross but one plate of salt, makes the action of the second chamber much greater than that of the first.

Each of the Tables exhibits the fact that as the length of the chamber increases the dynamic radiation of the gas contained in it increases, and as the length diminishes the radiation diminishes. We also see how powerfully the gas in the second chamber acts upon the radiation from the first. With carbonic oxide, the presence of the gas in the second chamber reduces the deflection from 13°·7 to 6°·3; with carbonic acid it is reduced from 16·8 to 6·6; with nitrous oxide it is reduced from 19·5 to 6·2. Now this residual deflection, 6°·2, is not entirely due to the transparency of the gas, to heat emitted *by the gas*. No matter how well polished the experimental tube may be, there is always a certain radiation from its interior surface when the gas enters it. With perfectly dry air this radiation amounts to 8 or 9 degrees. Thus the radiation is composite, in part emanating from the molecules in the first chamber, and in part emanating from the surface of the tube. To these latter, the gas in the second chamber would be much more permeable than to the former; and to these latter, I believe, the residual deflection of 6 degrees, or thereabouts, is mainly due. That this number turns up so often, although the radiations from the various gases differ considerably, is in harmony with the supposition just made. In the case of carbonic oxide, for example, the deflection is reduced from 13°·7 to 6°·3, while in the case of nitrous oxide it is reduced from 19°·5 to 6°·2; in the case of olefiant gas it is reduced from 59° to 10°·4, while in other experiments (not here recorded) the deflection by olefiant gas was reduced from 44° to 6°.

As may be expected, this radiation from the interior surface augments with the tarnish of the surface, but the extent to which it may be increased is hardly sufficiently known. Indeed the gravest errors are possible in experiments of this nature if the influence of the interior be overlooked or misunderstood. An experiment or two will illustrate this more forcibly than any words of mine.

A brass tube 3 feet long, and very slightly tarnished within, was used for dynamic radiation. Dry air on entering the tube produced a deflection of 12 degrees. The tube was then polished within, and the experiment repeated; the action of dry air was instantly reduced to 7·5 degrees.

The rock-salt plate at the end of the tube was then removed, and a lining of black paper 2 feet long was introduced within it. The tube was again closed, and the experiment of allowing dry air to enter it repeated. The deflections observed in three successive experiments were

80°, 81°, 80°.

This result might be obtained as long as the lining continued within the tube.

The plate of rock-salt was again removed, and the length of the lining was reduced to a foot; the dynamic radiation on the entrance of dry air in three successive experiments gave the deflections

76°, 74°, 75°.

The plate was again removed and the lining reduced to 3 inches; the deflections obtained in two successive experiments were

66°, 65°.

Finally, the lining was reduced to a ring only $1\frac{1}{2}$ inch in width; the dynamic radiation from this small surface gave in two successive trials the deflections

56°, 56°·5.

The lining was then entirely removed, and the deflection instantly fell to

7°·5.

A coating of lampblack within the tube produced the same effect as the paper lining; common writing-paper was almost equally effective; a coating of varnish also produced large deflections, and the mere oxidation of the interior surface of the tube is also very effective.

In the above experiments the lining was first heated, and it then radiated its heat through a thick plate of rock-salt against the pile. The effect of the heat was enfeebled by distance, by reflexion from the surfaces of the salt, and by partial absorption. Still we see that the radiation thus weakened was competent to drive the needle almost through the quadrant of a circle. If instead of being thus separated from the lining *the face of the pile itself* had formed part of the interior surface of the tube, receiving there the direct impact of the particles of air, of course the deflections would be far greater than the highest of those above recorded. Indeed I do not doubt my ability to cause the needle of my galvanometer to whirl, by the dynamic heating of the surface of my pile, through an arc of 1000 degrees. Assuredly an arrangement subject to disturbances of this character cannot be suitable in experiments in which the greatest delicacy is necessary.

Experiments on dynamic radiation, similar to those executed with gases, were made with vapours. The tube was divided into two compartments as before. Both compartments being exhausted, vapour was permitted to enter the first chamber. Dry air was afterwards permitted to enter the same chamber; the air was heated, it warmed the vapour, and the vapour radiated its heat against the pile. The heat passed in the first experiment through a vacuous second chamber, and in the second experiment through the same chamber when it contained 0·5 of an inch of the same vapour as that from which the rays issued. A third experiment was made to determine the dynamic radiation from the second chamber. The following Tables contain the results:—

Dynamic Radiation of Vapours.

Table XXXVIII.—Bisulphide of Carbon, 0·5 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	2°·4	1°·6	14°·2
36·3	13·1	9·75	5·5	9·0

Table XXXIX.—Benzol, 0·5 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	3°·0	1°·1	34°·0
36·3	13·1	21·6	11·9	15·1

Table XL.—Iodide of Ethyl, 0·5 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	3°·4	2°·7	38°·8
36·3	13·1	25·4	13·8	19·0

Table XLI.—Chloroform, 0·5 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	4°·5	2°·1	41°·0
36·3	13·1	22·3	10·0	19·0

Table XLII.—Alcohol, 0·5 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	4°·9	2°·0	53°·8
36·3	13·1	33·8	16·9	34·9

Table XLIII.—Alcohol, 0·1 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	2°·0	1°·3	35°·7
36·3	13·1	21·8	16·2	11·5

Table XLIV.—Boracic Ether, 0·1 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	6°·3	2°·1	61°·0
36·3	13·1	29·1	15·7	31·6

Table XLV.—Formic Ether, 0·5 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	6°·3	2°·5	68°·0
36·3	13·1	46·0	23·8	41·0

Table XLVI.—Sulphuric Ether, 0·5 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	5°·6	2°·5	68°·0
36·3	13·1	45·3	22·4	36·5

Table XLVII.—Acetic Ether, 0·5 inch.

Length.		Deflection.		
1st Chamber.	2nd Chamber.	By 1st Chamber. 2nd Chamber empty.	By 1st Chamber. Vapour in 2nd Chamber.	By 2nd Chamber.
15·4	34·0	5°·7	1°·0	73°·9
36·3	13·1	49·1	22·0	41·0

Collecting the radiations from the second chamber for the lengths 34 inches and 13·1 inches together in a single Table, we see at a glance how the radiation is affected by varying the length.

TABLE XLVIII.

	Dynamic radiation of various vapours at 0·5 inch pressure and a common thickness of	
	34 inches.	13·1 inches.
Bisulphide of carbon	$14^{\circ}·2$	$9^{\circ}·0$
Benzol	34·0	15·1
Iodide of ethyl	38·8	19·0
Chloroform	41·0	19·0
Alcohol	53·8	34·9
Sulphuric ether	68·0	36·5
Formic ether	68·0	41·0
Acetic ether	73·9	41·0
	At a pressure of 0·1 of an inch.	
Alcohol	$35^{\circ}·7$	$11^{\circ}·5$
Boracic ether	61·0	31·6

The extraordinary energy of boracic ether as a radiant may be inferred from the last experiment. Although attenuated to $\frac{1}{300}$th of an atmosphere, its thinly scattered molecules are able to urge the needle through an arc of 61 degrees, and this merely by the warmth generated on the entrance of dry air into a vacuum.

Arranging the gases in the same manner, we have the following results:—

TABLE XLIX.

	Dynamic radiation of gases at 1 at. pressure and a common thickness of	
	34 inches.	13·1 inches.
Carbonic oxide	$24^{\circ}·4$	$16^{\circ}·6$
Carbonic acid	23·3	17·5
Nitrous oxide	31·7	22·0
Olefiant gas	68·0	65·0

The influence of tenuity which renders the vapour at 0·5 of an inch a more open screen than the gas at 30 inches is here exhibited. In the case of the vapour, a greater length is available for radiation than in the case of the gas, because the radiation from the hinder portion of the column of vapour is less interfered with by the molecules in front of it than is the case with the gas. By shortening the column we therefore do more injury to the vapour than to the gas; by lengthening it we promote the radiation from the vapour more than that from the gas. Thus while a shortening of the gaseous

column from 34 inches to 13·1 causes a fall in the case of nitrous oxide only from 23°·3 to 17°·5, the same amount of shortening causes benzol vapour to fall from 34° to 15°·1—a much greater diminution. So also as regards olefiant gas, a shortening of the radiating column from 34 inches to 13·1 inches causes a fall in the deflection only from 68° to 65°; the same diminution produces with sulphuric ether a fall from 68° to 36°·5; and with acetic ether from 73°·9 to 41°. In the long column acetic ether vapour beats olefiant gas, but in the short column the gas beats the vapour.

One of the earliest series of experiments of this nature which were executed last autumn, though not free from irregularities, is nevertheless worth recording. The experiments were made with a brass tube, slightly tarnished within, the tube being 49·4 inches long, and divided into two equal compartments, each 24·7 inches in length, by a partition of rock-salt placed at the centre of the tube.

TABLE L.—Dynamic radiation of Vapours.

	Deflection.		
	By 1st Chamber. 2nd Cham. empty.	By 1st Chamber. Vapour in 2nd Cham.	By 2nd Chamber.
Bisulphide of carbon	8°·2	5°·8	21°·2
Benzol	20·0	12·4	45·9
Chloroform	24·3	10·9	55·2
Iodide of ethyl	27·5	14·7	55·3
Alcohol	42·7	22·3	69·0
Sulphuric ether	46·3	21·7	80·5
Formic ether	47·5	19·8	79·5
Propionate of ethyl	49·8	25·0	82·3
Acetic ether	53·3	30·0	82·1

To ascertain whether the absorption by the vapours bears any significant relation to the absorption by the liquids from which these vapours were derived, the transmission of radiant heat through those liquids was examined. The open flame of an oil-lamp was used, and the liquids were enclosed in rock-salt cells. Thus the total radiation from the lamp, with the exception of the minute fraction absorbed by the rock-salt, was brought to bear upon the liquid.

In the following Table the liquids are arranged in the order of their powers of transmission.

TABLE LI.

Name of liquid.	Transmission in hundredths of the radiation.
Bisulphide of carbon	83
„ „ saturated with sulphur	82
„ „ „ with iodine	81
Bromine	77
Chloroform	73
Iodide of methyl	69
Benzole	60
Iodide of ethyl	57
Amylene	50
Sulphuric ether	41
Acetic ether	34
Formic ether	33
Alcohol	30
Water saturated with rock-salt	26

These results are but approximate, but they are not very far from the truth; and it is impossible to regard them without feeling how purely the act of absorption is a *molecular* act, and that when a liquid is a powerful absorber the vapour of that liquid is sure also to be a powerful absorber.

To experiment with water, it was necessary to saturate it with the salt of which the cell was formed, but the absorptive energy is due solely to the water. We might infer from this alone, were no experiments made on the aqueous vapour of the atmosphere, that that vapour must exert a powerful action upon terrestrial radiation. In fact, in all the statements that I have hitherto made I have underrated its action.

The deportment of the elements sulphur and iodine, dissolved in bisulphide of carbon, is in striking harmony with all that we have hitherto discovered regarding the action of elementary bodies. The saturation of the bisulphide by sulphur scarcely affects the transmission, while a quantity of iodine sufficient to convert the liquid from one of perfect transparency to one of almost perfect opacity to light, produces a diminution of only two per cent. of the radiation. This shows that the heat really used in these experiments consists almost wholly of the obscure rays of the lamp. It is worth remarking that the obscure rays of a luminous source have a much greater power of penetration in the case of the liquids here examined than the rays from an obscure source, however close to incandescence. The deportment of bromine is also very instructive. The liquid is very dense, and so opaque as to cut off the luminous rays of the lamps, till it transmits 77 per cent. of the total radiation. It stands in point of diathermancy above every compound liquid in the list except bisulphide of carbon. This latter substance is the rock-salt of liquids.

Before a strict comparison can be made between vapours and liquids, they must be examined by heat of the same quality, and I have already made arrangements with which I hope to obtain more complete and accurate results than those above recorded.

VI. *A Comparison of the most notable Disturbances of the Magnetic Declination in* 1858 *and* 1859 *at Kew and at Nertschinsk; preceded by a brief Retrospective View of the Progress of the Investigation into the Laws and Causes of the Magnetic Disturbances.*

By Major-General EDWARD SABINE, *R.A., President of the Royal Society.*

Received April 28,—Read May 26, 1864.

BEFORE I proceed to the particular subject of this paper as noticed in its title, it may perhaps be desirable to take a brief retrospective view of the advances which have been made from time to time in our knowledge of the phenomena of the magnetic disturbances since they became the subjects of systematic investigation; and more especially since the publication of the Report of the Royal Society in 1840, and the establishment of magnetic observatories adopting and pursuing the methods of inquiry founded upon the instructions contained in that Report.

The observations of the German Magnetical Association, conducted by MM. GAUSS and WEBER, which was the immediate precursor of the British observatories, commenced in 1834 and terminated in 1841, the first year of the British Observatories. It was itself preceded by an earlier German Association, formed in 1828 under the auspices of Baron ALEXANDER VON HUMBOLDT, having for its object to make a series of strictly synchronous observations of the magnetic declination at concerted times at widely separated localities, for the purpose of inquiring into the nature and investigating the laws, if laws should be found to reveal themselves, of the apparently casual and irregular fluctuations of the magnetic needle which had then recently begun to attract the notice of scientific men, as natural phenomena proceeding from and indicating some hitherto unknown agency, and as such well meriting systematic investigation.

Berlin was the centre of the first German Association, as Göttingen was of the second. In 1829 and 1830 the Berlin Association had correspondents in very distant parts of the European continent, such, for example, as St. Petersburg, Kasan, and Nicolaieff, by whom the direction of the declination magnet was observed with great care and precision at hourly intervals of absolute time for forty-four successive hours at eight concerted periods of the year; and to the continuance of these *term-observations*, as they were called, there were added in March 1834 similar observations at Göttingen, but made with greater frequency, viz. at intervals of ten minutes. The intercomparison of the *hourly* observations revealed the general fact that very considerable fluctuations, still happening however on days that were apparently casual and irregular, were synchronous at all the stations of observation; whilst the *ten-minutely* observations at Göttingen, which had no parallels elsewhere, showed numerous intermediate fluctuations of similar

character. In order to bring to the test of positive evidence the question whether these intermediate fluctuations were also general, or were of merely local origin, *five-minutely* observations were now appointed at all the stations; and the result of four such term-days, in May, June, August, and September 1834, was to establish conclusively, that almost all the numerous and apparently irregular movements observed at Göttingen occurred also at the other places; and although with varied relative magnitudes, yet with an agreement which did not admit of mistake. The Göttingen Association now took the lead in the inquiry, the number of terms in the year being fixed at six, each of twenty-four hours' duration, with intervals of five minutes between the observations. The number of associated stations appears to have been about twenty, distributed generally over the continent of Europe. Besides the increased frequency of the observations, improvements were introduced in the apparatus used in the observations of the declination; and the bifilar magnetometer, devised by M. GAUSS for a corresponding record of the variations in the intensity of the horizontal component of the magnetic force, was employed at a few stations where the activity was greatest. The Göttingen Association continued its terms with regularity until 1841, stimulated by the great advantage which it possessed in the discussion of the results from time to time by MM. GAUSS and WEBER in the well-known publication entitled "Resultate aus der Beobachtungen des magnetischen Vereins." The conclusions already noticed as having been obtained in 1834 were confirmed by the careful examination and discussion to which the observations of each recurring term-day were subjected. The disturbing action was found to be frequently so considerable in amount, that partial and even total obliteration of the regular diurnal movements was a very common occurrence; and to be of such general prevalence, not only in the larger but also in most of the smaller oscillations, over the greater part of Europe, as to cause it to be viewed as in a very high degree improbable that the disturbances could have either a local or an atmospherical origin. No connexion or correspondence whatsoever was discoverable between the indications of the magnetical and meteorological instruments; nor had the state of the weather any perceptible influence. It happened very frequently that either an extremely quiescent state of the needle or a very regular and uniform progress was preserved during the prevalence of the most violent atmospherical storm; and as with wind-storms, so with thunder-storms, even when close at hand they exercised no perceptible influence on the magnetic instruments*.

* As a *magnetical* question, the supposition of an atmospherical origin of the disturbances may be considered to have been disposed of by the conclusions of the Göttingen Association. There remained, however, a problem which might be interesting to *meteorologists*. It was possible to suppose that, although the magnetic disturbances did not originate in the atmosphere, their presence, or possibly that of their producing cause, might occasion some atmospherical condition (which might be indicated either by the meteorological instruments or by some peculiar state of the weather), affecting simultaneously all parts of the globe on the particular days when the magnetic instruments were disturbed. The simultaneous observations of both classes of phenomena at the widely distributed stations of the British Colonial Observatories were well calculated to bring into view any such general atmospheric condition or affection if it existed; but the most careful collation of the simultaneous records of many years has failed to reveal any such correspondence.

The variations in the proportional magnitude of the disturbances in different localities, even when the similarity was otherwise unequivocal, had in one respect the appearance of a systematic indication, a decrease being shown in the energy of the disturbing force as its action was traced and followed from north to south. Hence the probability was inferred (so far as it might be safe to draw such conclusions from experiments which embraced comparatively but a small portion of the earth's surface) that the great focus or foci from whence the most powerful disturbances in the northern hemisphere emanated might be situated, and might possibly be sought with success, in parts of the globe to the north, or to the north-west, of the European continent. But even admitting this supposition to be well founded, so many of the phenomena still remained unexplained, that in the 'Resultate' for 1836, p. 99, M. GAUSS took occasion to express his matured conviction that "we are compelled to admit that on the same day and at the same hour various forces are contemporaneously in action, which are probably quite independent of one another and have very different sources, and that the effects of these various forces are intermixed in very dissimilar proportions at various places of observation relatively to the position and distance of these latter; or these effects may pass one into the other, one beginning to act before the other has ceased. The disentanglement of the complications which thus occur in the phenomena at every individual station will undoubtedly prove very difficult. Nevertheless we may confidently hope that these difficulties will not always remain insuperable, when the simultaneous observations shall be much more widely extended. It will be a triumph of science should we at some future time succeed in arranging the manifold intricacies of the phenomena, in separating the individual forces of which they are the compound result, and in assigning the source and measure of each."

The term-days of the Göttingen Association were limited to the observation of a single element, viz. the declination, with the exception of a few stations at which the bifilar magnetometer was occasionally employed. Instrumental means had not as yet been devised for observing the disturbances of the inclination and the total magnetic force, either directly, or by means of their theoretical equivalents, the horizontal and vertical components of the force. We find it indeed expressly admitted by M. GAUSS that it could not be doubted that the Inclination and Force are subject to disturbances similar to those observed in the Declination, but that the time had not yet arrived for including the three elements in the circle of combined inquiry: adding, "that as soon as the means of observation should be so far perfected that we could recognize with certainty, follow with ease, and measure with accuracy the variations, and especially the rapidly varying changes of the dip and total force, these variations would have the same claim on the united activity of inquirers, as the variations of the declination possessed during the period of the Göttingen Association."

We come now to the epoch when the inquiry was taken up and its further prosecution carried on by our own country. The two German Associations had prepared the way for the more extended and more complete organization which, on the recommenda-

tion of the British Association for the Advancement of Science, assembled at Newcastle in September 1838, and concurred in by the President and Council of the Royal Society in the spring of 1839, the inquiry subsequently received under the sanction and with the warm support of the Ministry of which Lord MELBOURNE was the principal member, and the succeeding Administration of which Sir ROBERT PEEL was the first minister. The field of research was no longer limited to a single continent, but included the most widely separated localities on the globe. Stations were selected in both hemispheres, and in the tropics, on continents and on islands, the selection being guided either by diversity of geographical circumstances, or by magnetical relations of prominent interest. The objects of investigation were also enlarged, so as to include not alone the transient and irregular fluctuations which had occupied the chief attention of the German Associations, but also "the actual distribution of the magnetic influence over the globe at the present epoch in its mean or average state, together with all that is not permanent in the phenomena, whether it appear in the form of momentary, daily, monthly, semiannual or annual change and restoration, or in progressive changes, possibly not compensated by counter-changes, or possibly receiving compensation, either in whole or in part, in cycles of unknown relation and unknown period." Suitable instruments, which in many respects were novel in construction, were provided for the observation of each of the three magnetic elements in this scheme of comprehensive research; and a report, prepared with much deliberation and care by a special committee of the Royal Society, was printed for the instruction and guidance of those who should be employed in conducting the magnetic surveys by sea and land, and of those who should direct or superintend the investigations to be carried out at the stationary magnetic establishments.

The present communication having reference to one branch only of one department of this extensive inquiry, viz. to that which relates to the *magnetic disturbances*, its notices are strictly limited to what may be necessary for placing before the Society as briefly as possible the successive steps which have advanced our knowledge of these phenomena, in respect to their diversities and mutual relations, their connexion with the general phenomena of terrestrial magnetism, and their probable cosmical origin.

The simultaneity of the days on which magnetic disturbances take place had already been shown by the term-days of the Göttingen Association to be coextensive with its sphere of operation, viz. the greater part of the continent of Europe. The wider extension of the British system, embracing stations in all quarters of the globe, now caused the fact of the simultaneity of disturbance to be recognized as a general feature common to the whole of our planet; whilst the evidence of diversity in the action of individual forces, even in the most clear cases of synchronous disturbance, was even more distinctly manifested than in the previous more limited experience. Thus the comparison of the term-days in 1840, 1841, and 1842 observed at different stations on the continents of Europe and America, and collated in the first volume of the Observations at the Toronto Observatory, published in 1845, gave occasion to the following general conclusion:— "The correspondence so strikingly manifested in the fluctuations in America, and which

has its counterpart in the correspondence shown by the term-observations at the different stations in Europe, is not found to prevail in anything like the same degree between the curves of the two continents when they are exhibited in comparison. Nevertheless indications are not wanting of participation in disturbances having a common cause..... The character of the term-day, in respect to the degree of disturbance by which the magnetometers are affected, may always be derived alike, whether we view the European or the American curves; and instances are not infrequent of individual perturbations common to both continents, having their culminating points at the same individual instant. There are sometimes disturbances in the same direction in both continents and sometimes in opposite directions. On the other hand, there are perturbations, and occasionally of considerable magnitude, on the one continent, of which no trace is visible in the observations on the other."

These facts were in full accordance with the conclusions which had been derived by the eminent geometrician of Göttingen from the observations of the Association formed under his auspices. They were further confirmed by a still more extensive and searching comparison, the means for which were furnished by a practice adopted at the British Colonial Magnetic Observatories shortly after their operations had commenced, of summoning the whole observing staff of the Observatory whenever in the course of the hourly observations of the magnetometers (maintained without intermission except on Sundays) they were perceived to be under the influence of an unusual disturbance; and thus the movements of each of the magnetometers were recorded at as short intervals as circumstances would permit, until the disturbance appeared to have subsided. These records were received at the Headquarter Office at Woolwich from Toronto, St. Helena, the Cape of Good Hope, and Van Diemen Island, as well as from the Expedition employed under Sir James Ross in the Magnetic Survey of the Antarctic regions, whenever the ships were sufficiently long in port to admit of the magnetometers being established and observed. The comparison of the records showed that magnetic disturbances prevailed, almost invariably, on the same days and at the same hours, in all these very various parts of the globe. The observations themselves were subsequently published in two parts; Part I. in 1843, containing the observations in 1840 and 1841, and Part II. in 1851, containing those in 1842, 1843, and 1844; together with the corresponding values of the declination and of the horizontal force in Part I., and of the declination, horizontal and vertical forces, and of their theoretical equivalents, the Inclination and the Total Force, in Part II.; accompanied by the normal values of the elements at the different stations in the months in which the disturbances occurred, and their absolute values at each of the stations. Abundant evidence is to be found in these publications that fluctuations of the most marked character are strictly synchronous in the northern and southern hemispheres, as well as in Europe and America; whilst at stations remote from each other the disturbance of the one element may differ widely in amount, and occasionally may be even reversed in direction. Not unfrequently also a disturbance showing itself at the same instant at distant stations is found to affect one

element at one station and another element at another station,—all confirmatory of the conclusions arrived at by M. GAUSS, and of the opinions of those who, antecedently to the establishment of the British Colonial Observatories, had anticipated that *the distinctive characters of the disturbances at individual stations would require to be studied, as the first step in a systematic inquiry into their causes, sources, and mutual relations.*

The hourly observations made at the Colonial observatories were received at Woolwich in the form of monthly tables, in which the days of the month were arranged in successive horizontal lines, and the hourly observations in twenty-four vertical columns; an additional column at the side showed the mean of each day, and an additional line at the bottom of the Table the mean of each hour in the month. Even a very superficial examination of these Tables at any one station sufficed to show that certain hours were more affected by disturbance than others. These hours were not the same at different stations; and no distinct relation could be traced at any station between the hours of principal disturbance and those of the well-recognized horary fluctuation due to the regular solar-diurnal variation. It was obvious therefore that the horizontal line at the bottom of each monthly table, which showed the mean values at the several hours (or what might be termed the diurnal inequality), did in fact represent two variations, viz. 1st, the regular solar-diurnal variation, and 2nd, a diurnal variation due to the disturbances; the two having every appearance of proceeding either from distinct causes or from distinct actions of the same original cause. The means of separating them *perfectly* from each other did not readily present themselves, but to do so *approximately*, and with an approximation quite sufficient for many practical purposes, was merely a work of labour. The very feature which marked certain of the observations as disturbed, viz. the magnitude of their discordance with the other records standing in the same column with themselves, or (as more readily seen) the magnitude of their differences from the mean value at the same month and hour at the foot of the page, appeared to supply a ready means (in the absence of any more exact criterion) of distinguishing the observations which were most affected by disturbance. It was soon found that by assuming for each element and for each station a certain amount of difference from the monthly mean at the same hour as the *indication of disturbance*, the records in each month might be separated into two portions, of which the smaller, containing the disturbed observations, might be set apart for an examination of the laws of disturbance; whilst the larger portion, from which the disturbances had been thus eliminated, would become more available for obtaining a correct knowledge and analysis of the progressive and regular variations than when they were mixed up with the casual and transitory affections.

By maintaining these assumed discriminating or separating values (forming the criteria whereby each observation was assigned either to the disturbed or to the undisturbed category), *constant at each station*, the laws of disturbance in different months and different years, if such laws existed, might be studied with convenience and security; and by so adjusting the values adopted at the different stations as to cause the number of the disturbed observations at each station to bear nearly an equal proportion to the

whole body, an advance might be made towards an approximate estimate of the degree in which the disturbing action prevailed in different parts of the globe. In the practical application of this scheme of first or primary analysis it was found that, provided the selected separating value at each station were such as to place in the category of disturbed observations a proportion equivalent to between one and two tenths of the whole body of the observations, small alterations within these limits occasioned no significant alterations in the derived diurnal progression either of the disturbed or of the (for the most part) undisturbed observations.

The monthly records of a single year at any one of the observatories sufficed to manifest an order and sequence in the ratios of the aggregate amounts of disturbance in each of the twenty-four hours to the mean amount in the twenty-four hours taken as unity, which placed beyond a doubt the fact that, casual and irregular as the disturbances might appear in respect to the particular times of their occurrence when viewed in single days, they were in their *mean effects* strictly periodical phenomena; exhibiting, by the character of their periodical variations, *a dependence on the sun as their primary source.* To this important fact the disturbances of each of the magnetic elements, the Declination, the Inclination, and the Intensity of the magnetic force, bore concurrent testimony, although the hours of maximum and minimum of their respective diurnal progressions were dissimilar; confirming in that particular the inference of the existence of distinct periodical laws in the disturbances of each of the elements.

The bearing of this result upon the methods by which magnetical investigations could most successfully be prosecuted was important. It had been remarked at a very early date, viz. in the 1st volume of the Toronto Observations, published in 1845, p. xv, that "if the disturbances took place without any systematic prevalence at certain hours rather than at others, and with no systematic inequality in regard to direction and amount, their influence would be limited to a lengthening of the time required for obtaining accurate mean values of the solar-diurnal variation; but that if systematic inequalities were found to prevail in those respects, it was obvious that no duration of the observations would eliminate their influence; and the diurnal inequality obtained from the whole body of the observations, whatever might be the duration it represented, must include the effects of two distinct phenomena, viz. of the disturbances, and of the diurnal variation properly so called; these two phenomena having possibly distinct causes, or at least distinct laws." The conclusion could no longer be doubted, therefore, that the first step in the systematic treatment of a body of observations, whether for the purpose of studying the laws of the disturbances, or for obtaining a correct knowledge of the more regular periodical variations, must be to separate the observations into two portions, one of which should include the more significant disturbances, and the other should contain the remainder of the observations, from which the disturbances had been for the most part eliminated. Our present concern is with the treatment of the disturbed portion only; the periodical variations of more regular occurrence are discussed elsewhere.

And here it becomes proper to recall the instructions regarding the casual and transitory variations contained in the Report of the Royal Society referred to in page 230, in which we find this conclusion to have been in great measure anticipated,—the importance of treating the laws and mutual relations of the disturbances as a distinct subject of investigation clearly recognized,—and the probable results of such investigation not obscurely indicated. In pages 2 and 3 of that Report it is stated that "the investigation of the laws, extent, and mutual relations of the casual and transitory variations is become essential to the successful prosecution of magnetic discovery.... because the theory of those transitory changes is in itself one of the most interesting and important points to which the attention of magnetic observers can be turned, *as they are no doubt intimately connected with the general causes of terrestrial magnetism, and will probably lead us to a much more perfect knowledge of those causes than we now possess.*" In the opinion thus expressed, being myself one of the Committee by whom the Report was drawn up, I fully concurred; and having been appointed by Her Majesty's Government to superintend the observations made at the British Colonial Observatories, and to coordinate and publish the results, it is alike my duty and my desire to show that the methods pursued have been in strict conformity with the spirit of those instructions, whilst the conclusions derived will be seen to be in full accordance with the anticipations expressed therein*.

* The importance which M. GAUSS attached to the further and full investigation of the magnetic disturbances was not less than that expressed in the Report of the Royal Society. Having had occasion, at the request of the President and Council of the Royal Society, to visit Berlin and Göttingen in conjunction with Dr. LLOYD in the autumn of 1839, when the British Colonial Observatories were in contemplation, I transcribe the following notice of M. GAUSS's opinions from a copy which I have retained of a letter to Baron ALEXANDER VON HUMBOLDT, written from Elberfeld on the 24th of October 1839, since it is more to the purpose than anything which I could now write from recollection:—"The conferences with M. GAUSS did not close till late on Monday night: we left Göttingen early on Tuesday morning, and this is our first stoppage. We found M. GAUSS's attention resting principally on that part of our proposed system of observation which is directed to the determination of the laws of the periodical fluctuations, and of the mode of action of the causes which produce them. Fully satisfied with the hourly observations as an almost certain means of attaining these objects, he was only desirous, for the full solution of the problem, that the number of stations should be increased so as to comprise the greatest practicable extent of latitude; care being also taken that there should be one or two parallels in which there should be stations in meridians widely apart. The relative importance of different localities in reference to the *secular changes* does not yet appear to have received M. GAUSS's attention. The bearing of the stations on the periodical fluctuations was the chief and almost the only consideration on which he *dwelt*. We may hope that, in respect to the secular changes, the results obtained at the nineteen contemplated stations, so extensively distributed on the surface of the globe, will at least serve to test the validity of physical theories, though they may not include those points which a more advanced knowledge might indicate as most suitable for suggesting the true theory. Barnaoul and Yakutsk appear well situated to throw light on the easterly progression of the maximum of force in the Siberian quarter, which is by some believed to be more rapid than the progression, also easterly, of the maximum in the American quarter; forming in their combined effect a double system of translation in the lines to which the changes of Declination and Inclination in the northern hemisphere, ever since they were observed, appear to have been conformable. Our solicitude was strongly expressed to learn from M. GAUSS if there were any stations, exclusive of those chosen for the fixed observatories, at which a new determination of the three

The Report anticipates, as the probable result of the researches then about to be instituted, the establishment of an intimate connexion between the casual and transitory variations and the "general causes of terrestrial magnetism." Whatever these may be, our best inferences in regard to them must be based upon the knowledge we possess of the actual distribution of the magnetic influence upon the surface of the globe. In regard to this distribution, the Report refers throughout to two works as containing the embodiment of the totality of the known phenomena, viz. 1, a memoir, published two years antecedently (1838) in the Transactions of the British Association for the Advancement of Science, entitled "On the Variations of the Magnetic Intensity in different parts of the Earth's Surface," in which the results of recent researches in almost all the accessible parts of the globe were brought together and coordinated, and their bearing on earlier systematic views discussed; and 2, M. GAUSS's 'Allgemeine Theorie des Erdmagnetismus,' published in 1839, being the year preceding that in which the Report of the Royal Society was published *. These two works are referred to throughout the Report as supplying, the first the observational, and the second the theoretical bases of the Instructions drawn up for the guidance of those who were to conduct, and of those who were willing to take part in the proposed magnetic researches by sea and by land. In both works, the facts which had been ascertained were found to be in accordance with (and so far confirmatory of) the theory which we owe to the combined industry and sagacity of our illustrious countryman and Fellow, HALLEY, of the existence of a *double* system of magnetic attraction on the surface of the globe, the direction and intensity of the magnetic force being at all points the resultant effects of the two separate systems. In both works, the localities to which the resultant Poles, or Points of greatest force (in the northern hemisphere), were traced, were nearly the same, viz. one in the northern part of the American continent, and the other in the northern part of the Europæo-Asiatic continent. To have determined their *precise* geographical positions, it would have been requisite that the observations from which they were derived should have corresponded, or nearly so, to one and the same epoch, inasmuch as one of the magnetic systems is regarded as subject to a movement of translation in a geographical sense, giving rise to the phenomena of secular change. But the approximation in the conclusions from two such extensive and laborious coordinations as those which have been named, was fully sufficient to establish that the general causes of terrestrial magnetism referred to must be such as would produce the phenomena of a double system. Now, combining the expectation expressed in the Report,

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magnetic elements was particularly desirable towards a revision of his theory. It appears that of the seven parallels of latitude which he has employed to give the basis of his numerical calculation, the most southern is in 20° S. latitude. Observations carried round a parallel in a high southern latitude are consequently the principal desideratum. This is precisely what we have reason to hope will be accomplished by the Antarctic Expedition."

* An English translation by Mrs. SABINE of M. GAUSS's "Allgemeine Theorie" was published in 1839 in Taylor's Scientific Memoirs, vol. ii. Art. V.

of the probability of a connexion subsisting between the magnetic disturbances and the more general phenomena of the earth's magnetism, with M. GAUSS's inference from the Göttingen researches, that the source or sources ("point or points of apparent origin") from whence the disturbing action in the northern hemisphere proceeds must necessarily be sought in the north, or in the north-west, of the European continent, it seemed reasonable to infer hypothetically that a connexion might be found between the "points of origin" of the disturbances,—if these could be more precisely ascertained and their separate effects distinguished apart,—and the poles or points of the two magnetic systems, of which we have the resultants in the centres of the two isodynamic lemniscate-loops. The *first* analysis of the disturbances had shown the disturbances to be strictly periodical phenomena in their mean effects, and had traced them directly to the sun as their *primary source*, inasmuch as they were found to be governed everywhere by laws depending upon the solar hours. Those who are familiar with the theory by which the transmission of light from the sun to the earth is explained, will have little difficulty in admitting a similar explanation of the mode by which magnetic influences may be conveyed from the sun to the earth. The analogy has been directly recognized and reasoned upon in the explanation of magnetic phenomena by Professor CHALLIS in recent papers. It is when the influences reach the earth that the modes of their reception, distribution, and transmission may be less clearly apprehended; but these are within our own proper terrestrial domain and sphere of research, and are therefore more particularly the subjects to which our investigations may be most usefully directed. We have here to guide us the simple analogy of a magnetic impulse imparted to a bar already magnetized; the impulse is at once distributed throughout the bar; the poles or points of greatest force being affected in the greatest degree, and the effects diminishing as the middle of the bar is approached. We may conceive that in like manner a magnetic impulse communicated from without might, in either hemisphere or in both simultaneously, be received by and produce its principal effect on the poles or points of greatest force belonging to the hemisphere, either augmenting or diminishing, as the case may be, the mean or ordinary magnetism of each, and thenceforward acting generally and conjointly throughout the hemisphere according to laws which are or may be capable of determination by suitable means. The possibility of tracing a certain locality, or localities, on the globe as a "point or points of origin" where the magnetic influences being received might thenceforward distribute themselves according to the laws of magnetic propagation, had already been entertained by M. GAUSS. In the first analysis of the disturbances at the British Colonial Observatories, referred to in p. 233, those of each element were treated simply in their *aggregate* effects, as might be conceived to be suitable on the supposition of their proceeding from a *single source* only. The result was sufficient to manifest their strictly periodical character, and to refer them to the sun as their *primary* source; but it was at the same time obvious that this first analysis could by no means be regarded as a final one, inasmuch as in every case there was exhibited a plurality of maxima and minima in the diurnal progression;

giving reason to infer that, by subjecting the disturbances to a more searching analysis, systematic progressions indicative of two or more distinct sources of disturbance in each hemisphere might be made to disclose themselves.

It had been found, moreover, that at every station where the examination had been made the disturbances of the declination were occasionally deflections to the East, and occasionally deflections to the West, from the mean position of the magnet; and those of the Dip, and of the total Force, occasionally increasing and occasionally decreasing the mean values. The aggregate amounts of disturbance in each element were now therefore separated into distinct categories, and the ratios of disturbance at the several hours in each category to the mean hourly ratios were determined by a process similar to that adopted in the analysis of the aggregate values. The results fully justified the labour expended in this proceeding; each category presented progressions still more systematic and of much greater simplicity than had appeared in the preceding investigation previous to which the categories had not been separated; giving great probability to the inference that at every station a similar process would manifest that there were at least two, and probably only two, distinct sources in each hemisphere, to which disturbances occurring simultaneously might be ascribed; and that by an increase in the number of stations, particularly if they were judiciously selected, the geographical localities in which the greater part at least of the disturbances originated, might be approximately traced. Confining ourselves, for brevity, to the illustration afforded by a single element, viz. the Declination, it was found that at all stations, in all parts of the globe, the disturbances of the declination resolved themselves into two distinct and dissimilar categories; the same two distinct and dissimilar forms of diurnal progression being everywhere reproduced with little other variation than that of the particular hours of maxima and minima; but having this additional important peculiarity, that the particular form of the curve of the diurnal progression which characterized the Easterly Deflection at certain stations marked the Westerly Deflection at certain other stations, and *vice versâ*. It was also found that at some stations the Easterly Deflections greatly preponderated over the Westerly, whilst at other stations the Westerly were predominant. An attentive consideration of the facts elicited by this extensive though somewhat laborious investigation strengthened the previously prevailing impression, that the progressive increase of our knowledge of these remarkable phenomena would lead, in both hemispheres, to the establishment of a connexion—if not to the identification—of the terrestrial sources of the casual and transitory disturbances with the foci, as they are sometimes called, of the two magnetic systems of the globe.

Proceeding from these premises, it appeared desirable to examine whether, if two stations were taken in a suitable and nearly similar latitude, one of which might be on the eastern and the other on the western side of one of the supposed points of terrestrial origin, and if a sufficient comparison were made of the disturbances simultaneously observed at the two stations, the category of easterly deflections at the one station might not be found to correspond in the form of the curve, and possibly also in the hours of

maxima and minima taken in absolute time, with the category of westerly disturbances at the other station. To test this by experiment, it was desirable to select stations, in suitable localities, where trustworthy observations could be relied on, inasmuch as the experiment would be somewhat of a crucial nature. The Russian stations on the eastern side of Siberia and at Pekin, where hourly observations of the declination had been made for some years, seemed the most favourably situated for supplying a station on the eastern side of the Europæo-Asiatic focus, whilst Kew might furnish a corresponding station on the western side, as soon as its photographic records should be sufficiently advanced. For the Asiatic station Pekin was selected in the first instance, although its latitude being about 12° south of Kew, might seem to render it a rather less eligible station of comparison than one of the eastern-Siberian stations; but there was at that time an idea, originated by Sir CHARLES TREVELYAN at the Treasury, that Pekin might become a station of a British magnetic observatory, and in that view it was desirable to know what had already been accomplished there. The first thing to be done was to ascertain by a careful scrutiny the degree of reliance to be placed on the observations, these having been made, under the Russian superintendent of the Pekin Observatory, by Chinese observers; and a decisive test was at once adopted. It consisted in rewriting in *lunar* hours the monthly Tables which record the observations taken at *solar* hours, and deriving from the Tables so rewritten the lunar-diurnal variation. If this very small variation be shown consistently in different years by the observations thus transposed from the original record, the observations are entitled to be regarded as good. The Pekin hourly observations, from 1852 to 1855 inclusive, as printed in the volumes of the 'Observatoire Physique Central de Russie,' having been thus tested, were found to be quite trustworthy. The lunar-diurnal variation derived from them in each of the four years is shown in Table CXX., p. cxiv of the second volume of the St. Helena Observations, having been included in that volume for reasons stated in page cxxxvi. The aggregate values of the easterly and of the westerly portions of the disturbance-diurnal variation at Pekin, as well as the ratios of disturbance at the several hours, are printed in Table CXVIII. (p. cxi) of the same volume. The corresponding results obtained by the Kew photograms between January 1858 and December 1862 are given in a paper in the Philosophical Transactions for 1863, Art. XII., Table II., and in the same paper (Philosophical Transactions, 1863, Art. XII., p. 282) the comparison is made of the Kew and Pekin disturbance-deflections, showing that the *conical form and single maximum* which characterize the *easterly* deflections at Kew, characterize the *westerly* deflections at Pekin at approximately the same hours of absolute time.

In confirmation of this result a second comparison was made between the results at Kew and those obtained from the hourly observations at Nertschinsk in Eastern Siberia from 1851 to 1857, printed also in the 'Annales de l'Observatoire Physique Central de Russie.' Nertschinsk is almost identically in the same latitude as Kew, whilst in longitude it scarcely differs from Pekin. Here also the observations, having been submitted to the same test in respect of accuracy, were found to be equally trustworthy:

and the comparison of the disturbance-deflections showed a still more perfect accord between the curve representing the easterly deflections at Kew and the westerly at Nertschinsk at approximately the same absolute hours.

To this it should be added that at each of these stations, as at all others, the forms of the easterly and westerly deflection-curves are so distinct that they cannot be mistaken for one another: the difference is well shown in figs. 1 and 2 of plate 1 in the "Reade Lecture delivered in the Senate House of the University of Cambridge in May 1862:" the curves there represented are those of the east and of the west deflections at Kew and at Hobarton (in Tasmania); and on the same page the westerly curve at Nertschinsk, shown in fig. 3, is seen to accord with the easterly curve at Kew, fig. 1. In Plate XIII. accompanying the discussion of the Kew observations (Phil. Trans. 1863, Art. XII.), the easterly curve at Kew and the westerly at Nertschinsk are also shown in figs. 1 and 5; these figures represent the ratios derived from the aggregate values of the respective disturbance-deflections at Kew from 1858 to 1862, five years, and at Nertschinsk from 1851 to 1857, seven years. My purpose on the present occasion is to show the correspondence between these deflections (the easterly at Kew and the westerly at Nertschinsk) in what may appear to some a more impressive manner, viz. a direct comparison of nearly synchronous disturbances in absolute time in the easterly and westerly disturbances at the two stations, Kew and Nertschinsk, on the most notable occasions of disturbance in the years 1858 and 1859. I am limited to these two years because the photographic record at Kew did not commence until January 1858, whilst the hourly observations at Nertschinsk for 1860 and the succeeding years have not yet reached England.

I have adopted the same characteristic at both stations for the days of most notable disturbance, viz. all those days in which twelve at least of the twenty-four equidistant epochs were disturbed to an amount equalling or exceeding "the separating value," viz. 3′·3 at Kew, and 3′·5 at Nertschinsk; the differences from the normals of the same month and hour at Nertschinsk being entered in the Table at the close of this paper, as those at Kew were in the Table in the Philosophical Transactions for 1863, Art. XII., p. 274, with which it may be compared. The number of days so characterized in 1858 and 1859 are at Kew forty-two, and at Nertschinsk forty-four; a great part of the disturbances being on the same days at both stations, but not invariably so, since, as is known, "a disturbance affecting one *element* at one station does not always affect the same *element* at another station." In inspecting the Summary at the close of the Table, it must be borne in mind, on the one hand, that a *very regular* progression can scarcely be looked for from disturbances occurring in the very limited space of two years; but, on the other hand, that 1858 and 1859 were years of maximum disturbance in the decennial period, and are therefore years of peculiar suitability in the case of a very limited comparison. The aggregate value of the disturbances at Nertschinsk in 1854 was 3497 minutes of arc, and in 1859 5602 minutes*.

* An inquiry into the years corresponding to the epochs of minimum of the decennial variation from 1823–

The comparison of the contemporaneous disturbances at Kew and Nertschinsk in 1858 and 1859, which are given in detail in the Table at the close, may perhaps be facilitated by the subjoined Tables I. and II., in both of which the hours are those of absolute solar time at Kew, whilst the deflections are *easterly* at Kew and *westerly* at Nertschinsk.

TABLE I.

Stations.	Kew Astronomical Hours.											
	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.
Kew	67′	93′	75′	144′	187′	214′	183′	232′	206′	131′	77′	25′
Nertschinsk	36	47	40	69	106	93	146	204	198	159	103	75

TABLE II.

Stations.	Kew Astronomical Hours.											
	18.	19.	20.	21.	22.	23.	0.	1.	2.	3.	4.	5.
Kew	27′	34′	33′	8′	13′	12′	59′	40′	46′	33′	46′	27′
Nertschinsk	75	79	154	88	72	63	32	46	54	20	25	16

It is seen that much the larger proportion of the disturbances at both stations occur between the hours of 6 and 17, Kew time. They exhibit a generally progressive increase of disturbance, easterly at Kew and westerly at Nertschinsk, from 6 to 13 hours, and a progressive decrease from 13 to 17 hours, also easterly at Kew and westerly at Nertschinsk. It is at these hours, viz. the hours contained in Table I., that the disturbances which produce opposite deflections at the two stations, and may therefore be supposed to proceed from a source intermediate between the stations, have their principal preponderance. In Table II. containing the hours, also of Kew time, from 18 to 5, and the deflections still easterly at Kew and westerly at Nertschinsk, we find the disturbances at both stations generally lessened in their aggregate amount, as we may suppose might be occasioned by the interference of disturbances of an opposite character proceeding from another and a more distant source. Admitting this supposition, the principal operation of the interfering cause does not take effect at the same hours of absolute time at the two stations; it appears to be chiefly influential at Kew from 18 to 23 hours, and at Nertschinsk from 0 to 5 hours.

I have thus endeavoured to trace consecutively the steps by which the probability of

1824 to 1853–1854 is to be found in the second volume of the St. Helena Observations, published in 1860, pages cxxi–cxxxvi. Assuming the period to be approximately decennial, we should now (1863–1864) be arrived at the fourth recurrence of an epoch of minimum in forty years. Appearances, as yet, seem to favour the recurrence of the minimum at the expected epoch. In 1865 and 1866 and succeeding years the disturbances should be expected to be on the increase.

a connexion subsisting between the points of terrestrial origin of the disturbances, and the Poles or Points of maximum force of the two systems which conjointly determine the distribution of the magnetic influence on the globe, has been examined, and to some extent strengthened. We have now to await the concurrent evidence which may result from a similar examination of the disturbances of the Dip and of the Total Force, which it is hoped may appear in a continuation of the papers on the results obtained at the Kew Observatory. But for the completion of the retrospective view of the progress which has been made in developing the theory of the magnetic disturbances, and in conducting us possibly to a more perfect knowledge of the general causes of terrestrial magnetism than we previously possessed, I must revert to the remark occurring in the earlier part of this paper (page 232), that the value adopted for each element and at each station to characterize what should be regarded as a disturbed observation, was purposely made a *constant amount*, with a view to an examination of the relative amount of disturbance in different months and in different years. It was in this way learnt, as is stated in the second volume of the Toronto Observations, pp. xxii and xxiii, that "1843, 1844, and 1845 were years in which the proportion of observations affected by a certain constant amount of disturbance was much smaller than the preceding years 1841 and 1842, or the following years, 1846, 1847, and 1848;" presenting thus the aspect of a *periodical variation* of which the epoch of minimum might be assigned to the years 1843 and 1844, but of which the period or cycle had yet to be learnt. The phenomena were not peculiar to a single station, but were found to correspond in localities most distant from each other: nor were they confined to one only of the magnetic elements, but were exhibited by all, each element having its own distinct instrumental means of measurement. They were therefore recognized as the indication of a magnetic affection common to the whole of our globe, constituting a periodical variation in the amount of disturbance in different years. In 1851 and 1852 the annual ratios of disturbance were found to be everywhere decidedly on the *decrease*, the epoch of maximum appearing to have taken place in 1848–1849. The evidence of the existence of a *decennial* variation appearing to be thus complete, its announcement, as a fact of which the knowledge was acquired by a process of investigation specially designed for the discovery of any such periodical variation, if one should exist, was on the point of taking place, when a fortunate incident (the receipt from M. DE HUMBOLDT of a proof-sheet of his 'Kosmos,' containing the first publication of HOFRATH SCHWABE's Table of the variations of the solar spots from 1826 to 1850) brought to my knowledge the existence of a corresponding variation in the physical aspect of the sun, precisely similar in period and epochs to the terrestrial magnetic variation. The importance of a revelation which gave a present apparent connexion, and presented the promise of establishing a permanent connexion, between the previously isolated terrestrial magnetic phenomena and the physical affections of the central body of our system, could not well be overrated. It was not alone the cosmical character which it imparted to a single terrestrial magnetic variation otherwise unconnected and inexplicable,—but there could scarcely fail to be impressed on the

mind of every reflecting magnetician the possibility, almost amounting to probability, that the second system of the terrestrial magnetism, which by the change in its relations to geographical space seemed to be distinct and dissevered from the magnetism of the earth properly so called (*i. e.* the collective action of all the permanent magnetic particles of the earth's mass, having its seat in the earth itself), might, like the decennial variation, be in truth assignable to a cosmical origin. The movement of translation on the earth's surface of the second system, and with it the whole phenomena of the secular change, would thus be regarded as belonging to, or being part of, a cosmical variation. It has, indeed, all the characters befitting such a relation, besides appearing inexplicable on any other hypothesis: we do not, indeed, yet know the duration of this far longer period, nor are we able to trace its course by visible signs on any of the heavenly bodies, as we trace the decennial period by the changes in the magnitude and frequency of the sun-spots. We infer its existence only from the terrestrial manifestation afforded by the secular change in the magnetic elements.

The "Terrella," by which HALLEY figured to himself a cause capable of producing phenomena of the order and regularity of those which his laborious and extensive generalization had disclosed to him, has never, I imagine, found favour as a probable physical reality. Viewed simply as an illustration of the systematic arrangement, symmetrical progression, and exceeding regularity of the *effects*, and the consequent necessity for the admission of qualities of the same order in the *causes*, of the terrestrial magnetism and its secular changes, HALLEY'S Terrella had its proper value; and it would have been well if the lesson which it inculcated had received more consideration than it has done from those who, more than a century after his publications, have attempted to explain the phenomena of the progressive magnetic change by accidental or adventitious variations in the superficial temperature of the globe or of its atmosphere, or in the occasional development or protrusion of magnetically attractive or repulsive rocks beneath its surface. The order and harmony of the facts manifested by the researches of a much earlier date had already effectually removed them from the category of partial or accidental occurrences. The symmetry of their general distribution, the counterpart to each other presented by the phenomena of the northern and southern terrestrial hemispheres, and the regularity with which the periodical changes take place, indicated a systematic causation which, obscure as it might be, was obviously anything but fortuitous. And when to the increased knowledge of the general phenomena acquired in the last and present centuries, confirming and extending the previous conclusions, was added the evidence obtained by the observations of the British Colonial Observatories, that the secular change is progressive in the *extremest sense*, that *each week* shows (and that if the means of observation were sufficiently refined it is more than probable that *each day* would show) an exact aliquot part of the annual change, the conviction became almost irresistible, that the causes which produce such remarkable effects can only have a cosmical origin.

The objections that might have impeded the reception of such an hypothesis before

we had learnt to recognize in the sun itself a source of magnetic energy,—before we had been informed by the sun-spots of the existence of periodical variations in the physical aspect, and consequently in the physical condition of that luminary,—and before we had succeeded in connecting these by their identity in period and epoch with the magnetic variations of our terrestrial sphere,—are no longer tenable. The solar origin of the variations in the magnetic phenomena of the earth's surface is indeed legitimately inferrible from their correspondence to solar hours; but in the decennial cycle, discovered in the solar spots and in the terrestrial magnetic disturbances, we have the *absolute* evidence and the *ocular* demonstration of a periodical variation common to the sun and to the earth, which in the sun is cognizable by our visual organs, and which, in the case of the earth, we know to be a magnetic variation.

We do not, as yet at least, possess a similar ocular demonstration of a connexion between the sun and the earth in the cycle of longer duration corresponding to the earth's secular magnetic change. But careful observations of the variable phenomena of the solar disk can only be said to be in their commencement; and it would be premature to assume that no visible phenomena will ever be discovered in the sun which will render the evidence of connexion as complete in the one case as in the other. But such evidence is not a necessary condition of an existing connexion; the decennial period would have been equally true (though not so readily perceived by us) if the sun-spots had been less conspicuous.

In the cosmical hypothesis here imagined, the north "pole or point of greatest attraction" (adopting HALLEY'S phraseology) of the *induced* terrestrial system at this epoch is in the north of the Europæo-Asiatic continent, whilst that of the *magnetism proper* of the globe is in the north of the American continent; the direction of the magnet "in those parts which lie adjacent to either being governed thereby, the nearest pole being always predominant over the more remote"*.

In the references made in this paper to the existence of a Theory of Terrestrial Magnetism, and to the advantage which I have myself endeavoured to derive from it in guiding experimental inquiry, I wish it to be understood that I employ the term "Theory," and regard its office in the work of inductive research, in the same light in which both were viewed by the late Professor PLAYFAIR. "In physical inquiries the work of theory and observation must go hand in hand, and ought to be carried on at the same time; more especially if the matter is very complicated, for then the clue of

* I have recalled these words of HALLEY in the text, because they show that he already recognized what has since been dwelt on by other magneticians, viz., that we must discriminate between the *true* poles or points of greatest force of the terrestrial and induced systems, and the *apparent* poles or centres of the isodynamic loops, which are the *resultants* of the double system. It is not improbable that the further observation and study of the magnetic disturbances, when those of the three elements are brought to bear together on the question, may guide us directly to a knowledge of the geographical positions of the true foci, as distinguished from the resultant foci. We have now learnt experimentally, *i. e.* by the observations of Captains MAGUIRE and M'CLINTOCK (Phil. Trans. 1863, p. 657), that the resultant foci are not themselves the points of origin of the disturbances.

theory is necessary to direct the observer. Though a man may begin to observe without any hypothesis, he cannot continue long without seeing some general conclusion arise; and to the nascent theory it is his business to attend, because by seeking either to verify or to disprove it, he is led to new experiments and new observations. He is led also to the very experiments and observations that are of the greatest importance; namely, to those '*Instantiæ Crucis*' that naturally present themselves for the test of every hypothesis. By the correction of his first opinion a new approximation is made to the truth, and by the repetition of the same process certainty is finally obtained. Thus Theory and Observation mutually assist one another; and the spirit of system, against which there are so many and so just complaints, appears nevertheless as the animating principle of inductive investigation. The business of sound Philosophy is, not to extinguish this spirit, but to restrain and direct its efforts."

TABLE III.—Synopsis of Forty-four days of most notable Disturbance of the Magnetic Declination at Nertschinsk in the years 1858 and 1859, to be compared with the Synopsis of the most notable disturbances of the same element in the same years at Kew (Philosophical Transactions, 1863, page 276).

	Astronomical Hours.																								
Kew	h 0·2	h 1·2	h 2·2	h 3·2	h 4·2	h 5·2	h 6·2	h 7·2	h 8·2	h 9·2	h 10·2	h 11·2	h 12·2	h 13·2	h 14·2	h 15·2	......	h 16·2	h 17·2	h 18·2	h 19·2	h 20·2	h 21·2	h 22·2	h 23·2
Nertschinsk	8·0	9·0	10·0	11·0	12·0	13·0	14·0	15·0	16·0	17·0	18·0	19·0	20·0	21·0	22·0	23·0	......	0·0	1·0	2·0	3·0	4·0	5·0	6·0	7·0
1858.	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	1858.	′	′	′	′	′	′	′	′
February 16......	......	E. 3·9	......	E. 2·1	E. 3·1	E. 3·1	......	E. 3·7	......	......	......	......	W. 5·1	W. 11·3	W. 9·2	W. 7·6	Feb. 17	W. 3·2	W. 2·9	W. 2·4	......	......	......	......	......
17......	......	......	......	......	......	......	W. 8·0	W. 5·3	W. 4·2	W. 2·7	......	......	......	W. 2·0	W. 3·5	W. 2·5	18	W. 6·9	W. 5·6	W. 4·6	W. 2·2	......	W. 2·0	W. 2·4	......
March 13......	W. 2·6	E. 7·1	E. 2·3	E. 2·3	......	......	E. 3·9	E. 4·4	......	E. 3·3	......	E. 2·7	W. 2·1	W. 3·1	W. 4·7	W. 6·7	Mar. 14	W. 3·9	W. 3·7	W. 4·2	W. 6·5	W. 2·6	E. 2·2	W. 6·9	W. 5·8
14......	......	E. 3·2	E. 4·0	E. 2·0	E. 3·7	E. 3·6	......	W. 2·2	W. 3·6	W. 3·7	......	W. 5·7	W. 7·8	W. 6·0	W. 3·9	W. 2·0	15	W. 7·0	W. 9·2	W. 3·8	W. 2·2	W. 3·0	W. 6·2	W. 3·9	......
15......	......	......	......	......	......	E. 4·2	......	W. 4·6	W. 4·8	W. 6·5	W. 7·4	W. 4·7	W. 3·8	W. 3·1	W. 4·3	W. 5·1	16	......	......	......	W. 1·9	......	E. 10·6	E. 3·2	......
April 9......	E. 3·1	E. 2·7	E. 9·5	E. 5·8	E. 11·7	E. 11·9	E. 15·5	E. 20·0	E. 19·1	E. 10·2	E. 2·6	W. 2·1	W. 3·3	W. 5·1	W. 7·0	W. 6·0	April 10	W. 5·3	......	......	......	......	W. 3·1	W. 3·9	W. 2·0
10......	W. 4·2	......	E. 3·3	E. 3·3	E. 3·5	E. 4·1	......	W. 4·0	E. 2·6	......	W. 6·4	W. 7·2	W. 8·4	W. 5·6	W. 10·6	W. 7·0	11	W. 5·8	W. 4·6	W. 3·4	W. 4·7	W. 8·8	......	......	......
June 24......	......	E. 8·6	......	......	......	......	......	W. 6·9	......	W. 4·4	W. 4·2	W. 10·9	W. 5·8	W. 10·3	W. 8·1	W. 5·9	June 25	......	W. 2·0	......	......	W. 3·1	W. 2·2	W. 4·3	......
July 1......	......	E. 3·7	......	......	E. 2·0	......	......	......	E. 2·0	E. 2·0	E. 3·0	E. 2·8	......	......	......	E. 2·6	July 2	......	......	W. 2·2	W. 2·3	W. 3·0	W. 2·3	W. 2·8	W. 3·1
2......	......	E. 2·3	E. 2·3	......	......	......	......	......	......	W. 2·2	W. 3·5	W. 2·8	W. 3·3	W. 2·6	......	......	3	E. 3·1	......	......	W. 2·5	W. 3·2	W. 2·2	......	E. 2·0
September 20......	E. 4·8	W. 2·4	......	......	......	E. 2·8	E. 4·2	E. 2·5	E. 3·0	E. 2·2	......	W. 4·4	W. 6·4	W. 11·0	W. 6·7	W. 6·0	Sept. 21	......	......	......	......	......	W. 2·7	W. 3·6	W. 2·0
22......	......	......	......	E. 2·8	E. 3·1	E. 4·4	E. 3·1	E. 2·4	......	......	W. 6·8	W. 2·9	W. 2·9	W. 3·1	W. 2·6	W. 2·2	23	W. 2·7	W. 3·1	......	......	......	......	......	......
24......	W. 2·4	......	E. 3·0	E. 4·0	W. 2·9	......	E. 2·0	E. 1·9	......	W. 2·5	W. 4·6	W. 3·0	W. 3·9	W. 4·6	W. 2·2	......	25	......	W. 2·9	W. 3·9	W. 2·4	W. 3·8	E. 2·3	......	W. 4·2
October 9......	......	E. 2·6	E. 2·5	......	......	......	E. 3·9	E. 4·1	E. 3·4	......	......	W. 2·6	W. 4·3	W. 6·0	W. 3·9	W. 2·8	Oct. 10	W. 3·8	W. 2·5	W. 2·7	W. 2·8	......	......	......	......
1859.																	1859.								
January 15......	W. 2·0	......	......	E. 6·8	E. 9·8	E. 5·8	......	W. 3·8	W. 3·0	......	W. 2·2	W. 3·8	W. 7·3	W. 6·5	W. 5·9	W. 4·5	Jan. 16	......	......	......	......	......	......	W. 2·8	E. 2·1
February 9......	......	E. 2·2	E. 3·1	E. 2·5	E. 2·1	E. 5·7	E. 6·6	E. 4·7	......	E. 2·1	W. 2·5	......	W. 2·1	W. 17·1	W. 6·2	W. 2·2	Feb. 10	......	......	......	......	......	......	......	......
23......	......	......	......	E. 5·2	E. 2·1	......	W. 2·5	W. 3·4	W. 6·4	W. 2·3	W. 4·7	......	E. 1·9	W. 2·0	W. 2·5	W. 3·9	24	......	......	W. 3·1	W. 2·2	......	......	......	......
24......	E. 2·4	E. 13·6	E. 6·2	E. 3·5	E. 5·5	E. 10·0	E. 3·8	E. 3·6	E. 3·2	......	......	......	......	......	......	W. 2·7	25	W. 1·8	W. 2·9	W. 5·0	W. 3·6	W. 2·3	W. 5·5	W. 3·3	W. 2·9

TABLE III. (continued).

Astronomical Hours.

Kew …………… / Nertschinsk ……	h 0·2 / 8·0	h 1·2 / 9·0	h 2·2 / 10·0	h 3·2 / 11·0	h 4·2 / 12·0	h 5·2 / 13·0	h 6·2 / 14·0	h 7·2 / 15·0	h 8·2 / 16·0	h 9·2 / 17·0	h 10·2 / 18·0	h 11·2 / 19·0	h 12·2 / 20·0	h 13·2 / 21·0	h 14·2 / 22·0	h 15·2 / 23·0	…… / ……	h 16·2 / 0·0	h 17·2 / 1·0	h 18·2 / 2·0	h 19·2 / 3·0	h 20·2 / 4·0	h 21·2 / 5·0	h 22·2 / 6·0	h 23·2 / 7·0
1859.	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	1859.	′	′	′	′	′	′	′	′
March 26……	……	……	……	……	……	E. 3·4	E. 2·4	E. 4·8	E. 6·2	E. 5·0	……	……	E. 2·7	W. 3·3	W. 2·9	W. 2·9	Mar. 27	……	W. 3·4	W. 2·7	W. 2·6	……	……	……	……
30……	……	……	……	E. 4·2	E. 3·1	E. 5·2	W. 2·1	E. 2·0	……	……	E. 2·6	……	……	E. 1·9	……	W. 2·3	31	W. 2·9	W. 3·4	W. 3·4	W. 2·7	W. 2·7	W. 2·3	……	……
April 21……	……	……	W. 2·2	……	E. 7·9	E. 3·6	E. 9·3	E. 3·2	E. 10·5	W. 6·0	W. 5·1	W. 3·9	……	W. 12·1	W. 10·5	W. 8·1	April 22	……	……	W. 2·4	……	……	……	……	……
22……	……	……	……	……	……	……	……	……	……	W. 6·1	W. 3·4	……	W. 3·1	……	W. 2·9	……	23	W. 7·1	W. 4·2	W. 3·3	W. 2·9	W. 2·7	W. 2·9	W. 3·4	W. 2·5
23……	……	E. 3·5	E. 3·7	E. 4·6	E. 3·2	……	……	……	……	……	……	……	W. 6·4	W. 5·6	W. 4·1	W. 2·1	24	W. 3·4	W. 2·6	……	W. 2·9	W. 2·5	……	……	……
29……	……	E. 2·6	……	……	E. 3·8	E. 6·0	E. 10·7	E. 6·9	……	……	W. 7·4	W. 3·1	W. 14·7	W. 6·4	W. 11·8	W. 7·9	30	W. 2·3	……	W. 2·4	W. 2·3	W. 3·5	W. 3·3	W. 2·4	E. 2·7
May 4……	E. 2·2	E. 3·2	E. 4·7	E. 3·4	E. 3·0	E. 2·9	E. 2·9	E. 2·2	……	……	……	……	E. 3·4	E. 4·2	E. 5·8	E. 2·1	May 5	……	E. 2·1	E. 2·0	……	……	……	……	E. 1·9
19……	……	W. 5·7	W. 3·5	……	E. 3·1	E. 2·4	E. 8·7	E. 3·0	……	……	……	……	W. 2·9	W. 3·3	W. 2·9	W. 3·1	20	……	……	……	……	……	W. 2·0	……	W. 2·7
June 8……	W. 3·1	W. 2·3	……	W. 6·1	E. 2·2	E. 2·5	E. 2·5	E. 4·8	E. 2·5	……	E. 4·4	……	W. 2·6	W. 3·9	W. 3·9	W. 2·6	June 9	……	……	……	……	……	……	W. 5·5	W. 5·1
August 15……	……	……	……	……	……	E. 3·1	E. 3·0	E. 3·1	E. 2·6	E. 2·0	……	……	E. 2·7	E. 2·9	E. 3·0	E. 4·5	Aug. 16	E. 3·2	E. 2·8	E. 3·2	E. 2·4	……	……	……	……
28……	W. 2·8	W. 3·8	……	……	E. 2·8	……	……	……	E. 2·8	E. 5·0	E. 7·0	……	E. 15·0	W. 3·6	W. 10·5	W. 9·0	29	……	……	……	……	E. 2·3	E. 2·3	W. 3·3	W. 5·0
29……	E. 14·0	E. 2·3	……	W. 8·6	W. 2·4	……	W. 4·7	W. 6·6	W. 3·4	……	……	……	……	W. 2·7	……	W. 2·3	30	W. 10·1	E. 15·4	W. 8·4	W. 8·8	W. 22·9	W. 7·1	W. 2·4	……
September 2……	W. 9·3	W. 22·6	W. 23·0	E. 54·1	W. 12·0	W. 5·3	W. 6·5	W. 9·8	W. 6·6	W. 16·5	……	W. 4·0	W. 5·0	W. 8·8	W. 11·5	W. 5·9	Sept. 3	……	E. 21·4	W. 4·6	E. 5·7	W. 74·8	W. 31·0	W. 13·4	W. 6·5
3……	E. 13·2	E. 8·1	E. 3·5	E. 16·9	W. 3·1	W. 3·2	……	……	……	……	E. 4·3	E. 2·3	W. 6·2	W. 5·8	W. 4·2	W. 5·1	4	W. 8·2	W. 6·0	W. 3·7	W. 3·5	……	E. 3·0	E. 10·7	E. 7·2
4……	E. 5·6	E. 4·3	E. 4·0	E. 2·9	……	……	……	……	……	……	……	W. 3·3	……	E. 2·5	……	W. 2·4	5	W. 4·8	W. 5·9	W. 3·1	W. 5·2	W. 6·7	W. 4·0	……	E. 2·4
5……	E. 6·7	E. 8·1	E. 6·2	E. 4·6	E. 2·6	W. 7·2	W. 7·2	……	……	……	……	E. 2·3	……	W. 2·8	W. 3·3	W. 5·1	6	……	……	……	……	E. 2·4	……	E. 2·4	E. 7·3
6……	……	……	W. 11·5	……	……	……	E. 2·0	E. 2·5	E. 3·5	E. 4·1	E. 4·9	E. 4·4	E. 2·9	……	W. 4·2	W. 2·9	7	W. 5·4	W. 3·8	……	……	……	W. 2·9	E. 7·9	E. 5·0
24……	……	……	E. 2·4	……	……	E. 2·9	E. 6·6	E. 7·7	E. 7·6	E. 5·2	E. 4·3	……	E. 3·3	W. 8·0	W. 3·8	……	25	E. 3·4	E. 2·9	E. 3·1	……	……	……	……	……
October 12……	E. 5·2	E. 6·9	……	E. 12·2	E. 11·5	E. 36·0	E. 23·5	E. 15·7	……	W. 6·0	W. 15·5	W. 2·9	W. 2·4	W. 5·2	W. 6·7	W. 7·0	Oct. 13	W. 4·6	……	……	W. 3·3	W. 2·8	W. 3·0	……	……
18……	E. 15·6	W. 2·1	W. 14·2	E. 3·0	E. 2·5	E. 2·1	……	……	……	W. 8·3	W. 9·6	W. 5·7	W. 5·7	W. 6·9	W. 9·6	W. 7·3	19	W. 4·6	……	W. 3·5	W. 3·9	W. 5·4	……	W. 5·2	W. 8·7
19……	……	……	……	……	……	……	……	……	W. 5·0	W. 2·7	W. 3·9	W. 3·5	W. 5·0	W. 4·7	W. 3·5	……	20	W. 6·5	W. 3·0	W. 2·2	……	……	E. 2·2	E. 2·7	……
20……	E. 4·6	E. 3·9	……	E. 4·3	E. 3·7	E. 7·6	……	……	……	……	W. 6·0	W. 5·5	W. 4·1	W. 5·2	W. 5·1	W. 2·4	21	……	……	……	……	……	……	……	W. 3·4
November 12……	……	……	……	E. 8·8	E. 3·0	E. 2·3	E. 2·3	E. 2·9	……	E. 3·1	W. 4·1	W. 4·4	W. 4·2	……	W. 4·8	W. 3·2	Nov. 13	……	……	……	W. 2·7	……	……	……	……
13……	E. 9·0	W. 3·4	E. 2·9	……	E. 4·1	……	……	E. 5·2	……	……	W. 5·3	E. 3·5	W. 8·6	W. 3·9	W. 4·7	W. 3·1	14	W. 2·5	W. 2·8	……	W. 2·3	……	W. 3·7	E. 2·1	W. 6·4
December 11……	W. 5·9	W. 3·8	E. 3·3	E. 3·5	E. 2·5	E. 3·1	……	E. 7·8	E. 6·0	……	……	W. 4·1	W. 4·1	W. 5·5	W. 2·5	W. 4·0	Dec. 12	……	……	……	W. 2·9	……	……	W. 2·0	W. 2·4
13……	……	E. 20·7	……	W. 5·2	W. 4·8	……	W. 5·1	E. 2·2	W. 3·4	……	W. 3·2	W. 4·8	W. 4·8	W. 3·8	W. 2·9	W. 3·4	14	……	……	……	……	……	……	……	E. 4·7

Summary.

Easterly Deflections 1858	7·9	34·1	26·9	22·3	27·1	34·1	32·6	39·0	30·1	17·7	5·6	5·5	……	……	……	2·6	1858	3·1	……	……	……	……	15·1	3·2	2·0
1859	78·5	79·4	40·0	140·5	78·5	104·6	84·3	82·3	44·9	26·5	27·5	12·5	31·9	11·5	8·8	6·6	1859	6·6	44·6	8·3	8·1	4·7	7·5	25·8	33·3
Sums ……	86·4	113·5	66·9	162·8	105·6	138·7	116·9	121·3	75·0	44·2	33·1	18·0	31·9	11·5	8·8	9·2	Sums…	9·7	44·6	8·3	8·1	4·7	22·6	29·0	35·3
Westerly Deflections 1858	9·2	2·4	……	……	2·9	……	8·0	23·0	12·6	22·0	32·9	43·4	57·1	73·8	66·7	53·8	1858	38·6	36·5	27·2	27·5	27·5	20·7	27·8	17·1
1859	23·1	43·7	54·4	19·9	22·3	15·7	28·1	23·6	27·8	47·9	72·9	49·0	89·1	127·1	130·9	105·4	1859	64·2	38·0	47·8	51·8	126·3	67·7	43·7	45·6
Sums ……	32·3	46·1	54·4	19·9	25·2	15·7	36·1	46·6	40·4	69·9	105·8	92·4	146·2	200·9	197·6	159·2	Sums…	102·8	74·5	75·0	79·3	153·8	88·4	71·5	62·7
Astronomical Hours.																									
Kew …………… / Nertschinsk ……	h 0·2 / 8·0	h 1·2 / 9·0	h 2·2 / 10·0	h 3·2 / 11·0	h 4·2 / 12·0	h 5·2 / 13·0	h 6·2 / 14·0	h 7·2 / 15·0	h 8·2 / 16·0	h 9·2 / 17·0	h 10·2 / 18·0	h 11·2 / 19·0	h 12·2 / 20·0	h 13·2 / 21·0	h 14·2 / 22·0	h 15·2 / 23·0	…… / ……	h 16·2 / 0·0	h 17·2 / 1·0	h 18·2 / 2·0	h 19·2 / 3·0	h 20·2 / 4·0	h 21·2 / 5·0	h 22·2 / 6·0	h 23·2 / 7·0

Note.—No observations were made at Nertschinsk in July 1859.

VII. *Theoretical Considerations on the Conditions under which the (Drift)* Deposits containing the Remains of Extinct Mammalia and Flint Implements were accumulated, and on their Geological Age.* By JOSEPH PRESTWICH, *Esq.*, *F.R.S.*, *F.G.S.*

Received March 20,—Read March 27, 1862.

On the Loess of the Valleys of the South of England, and of the Somme and the Seine. By JOSEPH PRESTWICH, *Esq.*, *F.R.S.*, *F.G.S.*

Received May 15,—Read June 19, 1862.

[NOTE.—By permission of the Council of the Royal Society these two papers have been incorporated. At the time of reading the first paper, the author felt difficulties respecting the origin of the Loess, which led him to defer the consideration of the subject. When he afterwards brought forward the second paper, it proved so clearly complemental to the first, that the rearrangement of the two became desirable and almost necessary. This has also enabled the author to shorten both papers. The main portion of the second now appears in § 4. The bracketed remarks in the Introduction are inserted in consequence of a suggestion made to the author that it would be desirable to state in what respect the views advocated by him differ from those previously brought forward. Returned May 21, 1863.]

§ 1. INTRODUCTORY REMARKS.

IN the paper I had the honour to lay before the Royal Society in May 1859†, on the occurrence of Flint Implements in France and in England associated with the remains of extinct mammalia, I postponed the consideration of the theoretical questions involved, to allow time for a more complete investigation of the physical phenomena. The facts I sought on that occasion to establish were,—1, the artificial nature of the Flint-implements; 2, their occurrence in undisturbed ground; 3, their contemporaneity with the extinct animals; and 4, their postglacial origin. Subsequent researches by myself‡ and other geologists have confirmed my views upon these several points§.

When I first visited Amiens in 1859, the opinion I formed was that the St. Acheul gravel-beds were deposited before those of St. Roch, and that the excavation of the

* The term "Drift" has been hitherto used as a convenient expression for the superficial beds generally; but as the relative positions of these beds are becoming better determined, we shall now be able to drop this term and introduce others of greater precision.

† Philosophical Transactions, vol. cl. p. 277. See also Mr. EVANS's paper, Archæologia, vol. xxxviii. p. 280.

‡ Journ. Geol. Soc. vol. xvii. p. 362, where the various localities are mentioned.

§ FLOWER, Quart. Journ. Geol. Soc. vol. xvi. p. 190, June 1859. GAUDRY, Comptes Rendus, Oct. 1859, p. 465. G. POUCHET, Actes du Mus. d'Hist. Nat. de Rouen, 1860, p. 33. L'Abbé COCHET, Mém. de la Soc. d'Emulation d'Abbeville, 1858–61, p. 607. EVANS, Archæologia, vol. xxxix. p. 5, 1861. See also Sir CHARLES LYELL's Address at the Aberdeen Meeting of the British Association, Sept. 1859; Mr. LEONARD HÔRNER's Anniversary Address to the Geol. Soc. Feb. 1861; Sir RODERICK MURCHISON's Address, Brit. Assoc. Sept. 1861.

Somme valley was of intermediate date; but I hesitated to adopt this view until facts could be obtained for a surer decision. The upper section at Montiers, however, which I discovered in 1861, was conclusive as to the relative ages of the two gravels. I had further considered that, supposing even this relation to be established, it was possible for the excavation of the valley to have been partly the result of some exceptional agencies, by which the interval of time between the formation of the beds of St. Acheul and those of St. Roch might have been shortened. But after repeated visits to the several districts during the last three years, and looking at the question from every point of view, I find myself unable to discover a sufficient explanation in the direction in which I first sought for one, and have been led to form conclusions respecting the causes in operation differing considerably on some points from those I at one time thought to be the more probable.

[A few very brief remarks on the opinions hitherto held respecting the position and age of the deposits of this class may here not be out of place. In my former paper I showed that the flint-implement-bearing beds were of later date than the Boulder Clay, and that at Abbeville the latest of them passed directly under the recent alluvium of the valley of the Somme. They thus occupy a definite geological period, which yet remained to be studied as a whole. The quaternary deposits in general, of which these beds form part, had long been the subject of my special investigation. The various drift-gravels had been regarded,—1, as being of marine origin; 2, as due to cataclysmic action; and 3, as of fluviatile origin. In one place we had marine shells, at others freshwater shells. But as the greater number of the gravel-beds were without fossils and occurred at very different levels, it was a long-debated question how they should be correlated. Palæontologists too were of opinion that the fossils indicated different ages, so that the freshwater deposits in a single valley, like that of the Thames, were held to be of independent and not synchronous formation*. On the palæontological evidence, the beds of Grays were generally supposed to be pliocene or preglacial, a view maintained by Sir CHARLES LYELL† until 1857, when he expressed uncertainty as to their age. Other beds in the same valley, as those at Brentford, were considered by Sir CHARLES to be newer than those of Grays. On physical grounds I had long been satisfied of the contemporaneity of these deposits, and contended for their posteriority to the Boulder Clay. Professor MORRIS and Mr. TRIMMER had also arrived at very similar conclusions, and were both in advance of me in attributing the phenomena to old river-action, but neither they, nor, as far as I am aware, any other geologists had attempted to make the rule general; nor had they taken in the high-level gravels, or the Loess, as belonging to the same series and as part of the same phenomena. When, further, I found similar land and freshwater shells at Hurley Bottom and other places in the Thames valley, the different deposits, showing the same conditions, became readily correlated.

* FORBES, Mem. Geol. Survey, vol. i. pp. 393, 395; SEARLES WOOD, Trans. Palæont. Soc. for 1848, p. vi. and 1856, p. 304; WOODWARD, Manual of the Mollusca, p. 298; and others.

† Manual of Elementary Geology, 5th edit. 1855, pp. 153, 154, and Supplement, 1857, p. 5.

But although it was evident that there were old land surfaces and possibly old rivers, there was no evidence that any supply of water could have existed to fill such large valleys; and the present streams seemed totally inadequate to have spread out such vast beds of gravel and sand, by far the greater part of which are also without organic remains to indicate their origin. Sir CHARLES LYELL, who advocated the fluviatile origin of these lower valley-gravels, considered that it would be "a rash inference" to conclude "that rivers in general have grown smaller, or become less liable to be flooded than formerly"*. This view, more or less modified, was held also by many other distinguished geologists. I could not accept it, because it seemed to me that to form such beds of gravel some greater water force must have been in operation than that which now obtains; at the same time, the hydrographical basins and the watersheds being the same as they were at that Quaternary period, I did not see whence the larger supply of water, which seemed to me indispensable, could have been obtained.

That valleys have been excavated by rivers was the hypothesis brought forward by HUTTON and PLAYFAIR†. It has been frequently advocated since; but the opinion has made little progress, owing to the absence of proof of how such an operation could have been effected, and to the insufficient physical and palæontological evidence. The subject, as far as regards Auvergne, was ably touched upon by Mr. POULETT SCROPE‡ in 1827, and discussed and argued more fully by CROIZET and JOBERT§ in 1828. Some remarkable cases were described by these geologists, to show, from the position of old shingle-beds preserved under masses of basalt, high above the present rivers, that the rivers in that part of France had excavated the valleys in which they now flow; but the cause of such phenomena remained unexplained, and the date undetermined.

Mr. GODWIN-AUSTEN showed, so early as 1837‖, that in Devonshire there were terraces of gravel fringing the valleys; and in 1851 and 1855¶, in correlating these and other quaternary deposits, he considered that the ancient low-level alluvia of the Thames and Seine valleys, and the old beach and the Elephant-beds of Brighton, were anterior to the Boulder Clay, and he was further of opinion that river- and ice-action had played an important part in producing these valley deposits. Sir CHARLES LYELL also discussed with his usual ability the question of the origin of valleys, and of ancient river-alluvia and river-terraces, both in his 'Principles' and in his 'Elements'**, but without attaching to the phenomena the importance I would show them to possess in such valleys as those of the Thames and the Seine. He was rather disposed to attribute the erosion of some lower parts of the valley of the Seine to sea-action††. Mr. TRIMMER‡‡

* *Op. cit.* pp. 70 & 84.

† Theory of the Earth, vol. ii. p. 401.

‡ Memoir on the Geology of Central France, 1st edit. pp. 163–4.

§ Ossemens Fossiles du dépt. du Puy-de-Dome, pp. 66–88.

‖ Trans. Geol. Soc. 2nd series, vol. vi. p. 439.

¶ Q. Journ. Geol. Soc. vol. vii. p. 136, and vol. xiii. p. 40.

** *Op. cit.* p. 85, and Principles of Geology, 9th edit. pp. 219 & 484.

†† *Op. cit.* p. 269 & 271.

‡‡ Quart. Journ. Geol. Soc. vol. ix. p. 286.

noticed the existence of terrace-gravels in the Thames valley, but explained them by alternate movements of depression and elevation*.

Sir R. MURCHISON also described at length some of the Drift phenomena of the South-east of England, more especially of the Wealden area. This distinguished geologist arrived at the conclusion that the heaps of detritus and angular débris following certain lines on the borders of the Wealden area, and found also in the Thames valley, result from the action of waves of translation passing from west to east†, and that the fossil mammalia (at Folkestone) were destroyed "by violent oscillations of the land, and were swept by currents of water from their feeding-places into the hollows where we now find them"‡. Mr. HOPKINS, in reviewing the question of the Drift, agreed with Sir RODERICK in supposing that the Wealden area has been traversed by waves of translation§, and in attributing to such agencies much of the Drift phenomena.

The observations of the distinguished naturalist the late Professor E. FORBES, recorded in his Anniversary Address, in 1854, to the Geological Society, express the then unsettled state of the question relating to the Drift ||; whilst the opinion hitherto commonly held with regard to the range in time of the large mammalia is manifested by Professor PHILLIPS ¶ and so many other eminent writers on the subject having restricted them to the preglacial period.

In France similar differences of opinion have prevailed respecting these particular quaternary deposits. The views generally adopted, however, with regard to the valley-gravels have been that they are the result of diluvial action, caused by waves of translation, or by cataclysms arising from the bursting of lakes, or by the sudden melting of the snow on mountain-chains. The deposits of this age in the valley of the Seine and other rivers in the North of France are usually classed under four divisions, viz. *Loess*, *Diluvium rouge* (part), *Sables lacustres*, and *Diluvium gris*, each being regarded as of separate and distinct origin, and the two diluviums referred to cataclysmic origin**.

Thus there were two extremes; I have been led to adopt an intermediate course. I could not admit the possibility of river-action, as it now exists, having in any length of time excavated the present valleys and spread out the old alluvia; neither was it possible to admit purely cataclysmic action in cases where the evidences of contemporaneous old land-surfaces and of fluviatile beds were so common. But with river-action of greater intensity, and periodical floods imparting a torrential character to the rivers, the consequences of the joint operation are obtained, and the phenomena admit of

* The occurrence of old river-terraces along narrow valleys is one of the features earliest noticed by geologists, but these are quite distinct from the great and isolated beds of gravel capping the adjacent hills.

† Quart. Journ. Geol. Soc. vol. vii. p. 361. ‡ Ibid. p. 386. § Ibid. vol. viii. p. li. || Ibid. vol. x. p. xliii.

¶ Manual of Geology, edit. 1855, p. 408. This opinion held good till 1859 and 1860.

** An excellent résumé of this subject is given in M. D'ARCHIAC's 'Progrès de la Géologie,' vol. ii. pp. 1–4, 154–221, 421–433. See also ANSTED's 'Elem. Course of Geol.' 2nd edit. 1856, p. 416 *et seq.*; D'OMALIUS D'HALLOY's 'Abrégé de Géol.' 7th edit. 1862, pp. 228–38, 449, 478; and a paper by M. CH. D'ORBIGNY (followed by one by M. LEYMERIE) in Bull. Soc. Géol. 2e sér. vol. xii. p. 1297–1304, with observations by M. HÉBERT and others; and another by M. BUTEUX in vol. xvii. p. 72.

more ready explanation. I long since had proposed the separation of the gravels into the high-level gravels and low-level gravels, and shown that the former were older than the latter. I was, however, at one time disposed to adopt in part some of the views of M. Elie de Beaumont with respect to the operation of cataclysmic action in preference to the slower action of rivers; but further research, and the discovery of land and freshwater shells in so great a number of low-level gravels, and in some of the high-level gravels, and especially the striking evidence eventually afforded by the beds of St. Acheul, and by the higher-level gravels around Paris*, satisfied me that river-action peculiar to each valley commenced with the high-level gravels, while the mass of débris and the large blocks present in the beds indicate the action of a large volume of water and of ice-transport. Further, I was ultimately led to connect the Loess with both series of valley-gravels, and the frequent independence of the former, which at first seemed an irreconcileable difficulty, finally proved an important auxiliary fact; for the separate range of this deposit now serves as a measure of the old flood waters, and of the extent of the river inundations during this quaternary period.

I conceive that the hypothesis brought forward in this paper gives consistency to the whole subject. It brings down the large mammalia to a period subsequent to that when the extreme glacial conditions prevailed, and closer to our own times; it places all the old river alluvia in the same period, and groups together the previously isolated fluviatile beds of Grays, Brentford, and other places in England, together with the Loess and various "Sables lacustres" and "diluviums" (part) of the French authors; it connects the great platform terraces of gravel skirting so many of our river-valleys with the same period, and makes the connexion between these, and the excavation of the valleys themselves and the formation of the Loess, dependent upon one prolonged and uniform set of operations, in accordance with the climatal conditions and necessarily resulting from them.—May 1863.]

§ 2. GEOLOGICAL POSITION OF THE FLINT-IMPLEMENT-BEARING BEDS.

In almost every instance the flint implements have been found in beds of sand and gravel along the line of existing river-valleys,—in some cases but little above the level of the rivers, in others on adjacent hills at heights of from 30 to 100 feet above the river. In these valleys one series of gravel-beds is spread over more or less of their breadth, rising occasionally on their flanks to a height of 10 to 30 feet, and ranging throughout their length, though constantly obscured and hidden by recent alluvial deposits. The lower ranges of hills which flank these valleys on either side are occasionally capped by other similar gravels, but which, so far from being continuous like the lower-level beds, occur only at intervals, and there are long tracts without any such drift. The higher gravels are also generally separated from the lower gravels by a bare sloping surface, whilst they rarely extend far from the valley and never reach the tops

* This evidence is, with a few rare exceptions, wanting in the high-level gravels of England.

of the higher hills. The two series bear nevertheless a definite relation one to the other. They both consist of débris derived from rocks in the valleys through which the present rivers or their tributaries flow, and they both occasionally contain organic remains, of which the greater number of species are common to the two. Both series may be considered as "valley-gravels;" but for the sake of distinction I purpose calling that which occupies the bottom of the valleys, and reaches to a comparatively small height above the river-level, the "low-level valley-gravels," whilst to that found on the adjacent hills, I would apply the term "high-level valley-gravels." The height of the latter above the valley is variable; and though generally limited to one main platform, it is not always on the same level, and there are cases of minor intermediate terraces between the extreme levels. The heights are of course relative one to the other, and not directly to the sea-level.

Valley of the Waveney.—The levels recorded in my former paper establish the fact that the flint-implement-bearing deposit of Hoxne is at a height of 40 feet above the Gold Stream, and 50 feet above the Waveney, of which the Gold Stream is a tributary. Mr. Evans and I found a very similar deposit, also overlying the Boulder Clay, at Athelington, a few miles higher up the valley of the former stream. In following the course of the Waveney, from above Diss to the sea at Lowestoft, terraces of gravel are found at distant intervals on the adjacent hills. They never extend far from the valley, and the intermediate higher but flat ground between the river-valleys invariably presents bare tracts of Boulder Clay. It is particularly between Diss and Harleston that these terrace-gravels are best exposed, and where I have determined with some care their extent and development (Plate V. fig. 3). I have found them on the right bank of the Waveney, at Stutston Common, Oakley, and Shotford Heath, lying upon the Boulder Clay; and on the left bank at Scole, Billingford, Thorpe Abbots, and Needham. Thence to the sea they may be traced at intervals on both sides of the valley; but they seem gradually to fall to a lower relative level. At the places above mentioned, on the contrary, they occupy a tolerably regular level of from 40 to 60 feet above the valley, are from 5 to 12 feet thick, and rarely exceed a quarter of a mile in length, or more than 200 to 400 feet in width. They consist chiefly of a mass of subangular flints, with pebbles of siliceous sandstones and of the older rocks, in a matrix of ochreous sand and clay. No organic remains have been found in them. The low-level gravel is not often exposed, being generally covered by alluvial deposits. It may, however, be seen at Oakley and at Needham, and has been reached at various places under the recent alluvium of silt and peat. Care must be taken to distinguish these gravels from those which underlie the Boulder Clay in this district. The latter are more sandy and far more distinctly stratified*. They may be seen in superposition in a pit on the hill above Oakley Street; in contact with the high-level gravel at Needham near Harleston, and at Moor Bridge near Hoxne; and lying on the Boulder Clay at Thorpe Abbots. The following section, taken across the valley

* The great number of pebbles of white quartz is one of the chief features which serves to distinguish the gravel under the Boulder Clay from the valley-gravels.

of the Waveney between Scole and Hoxne, shows the relation of these gravels to each other, to the Boulder Clay, and to the surrounding district*.

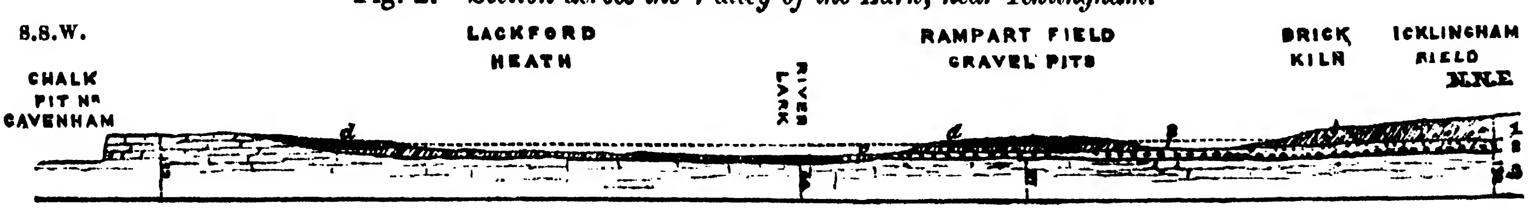
Fig. 1.—*Section across the Valley of the Waveney, near Hoxne.*

a. Alluvial deposits. *c.* Low-level valley-gravel. *d.* High-level valley-gravel.
Traces, but somewhat indistinct, of Loess exist on the gravel *d*, and on the slopes of the valley.
1. Boulder Clay. 2. Sands and gravels under the Boulder Clay. 3. Chalk.
The dotted lines here and in the other sections show the presumed old surfaces.

The higher valley deposits assume in the tributary valley of the Gold Stream at Hoxne a more lacustrine character. The lower-level gravels may be seen in the field opposite the Swan Inn, overlying the sand and gravel of the Boulder Clay series.

Valleys of the Ouse and Lark†.—In the main valley of the Ouse at Bedford eight flint implements have been found under circumstances which admit of no doubt of their geological position; and at Icklingham in the valley of the Lark, a tributary of the same river, two specimens have been met with, which, although not discovered *in situ*, there is good reason to believe came from the high-level gravel‡. In descending the valley of the Lark, from Bury St. Edmunds to Icklingham, Mr. Evans and I found it flanked by low ridges of gravel rising from 20 to 30 feet above the valley, and these again commanded by higher ground formed of the Boulder Clay. In a pit at Flempton I found in the gravel a fragment of bone, and remains of the Elephant have been met with at various places near Bury St. Edmunds; but we saw no traces of shells in any of the drift-gravels or sands of this district. The section of the valley is as follows:—

Fig. 2.—*Section across the Valley of the Lark, near Icklingham.*



a. Recent alluvial deposits. *c.* Low-level gravel. *d.* High-level gravel. (Only traces of Loess.)
1. Boulder Clay. 2. Sands and gravel under the Boulder Clay. 3. Chalk.

In the valley of the Ouse at Bedford (see fig. 3) I have been unable to detect at

* The scale of height in these and all the following river-valley sections is 1 inch to 400 feet. In the horizontal scale 1 inch equals about ½ a mile (except figs. 4 & 5). The base-line gives the sea-level approximately.

† I originally made a section of the valley of the Ouse at Bedford in 1854, and then determined the relation which the well-known mammaliferous gravel of this valley bore to the adjacent Boulder Clay. The number of fossil bones subsequently discovered in the cutting of the Great Northern Railway, led Mr. Evans and myself at once to direct our attention to this valley on our return from the valley of the Somme in 1859. The discovery, by Mr. Evans, of fluviatile shells in the Biddenham gravel confirmed the analogy we suspected, and we directed Mr. Wyatt's attention to it and to the probable occurrence of flint implements, in the search for which this gentleman has since been so successful. 27 specimens are now recorded by Mr. Wyatt.—Feb. 1864.

‡ The author in Quart. Journ. Geol. Soc. vol. xvii. p. 363. A number more have been since found (1864).

present any distinct high-level gravel. I am rather disposed, however, to consider it possible that the gravel at the Biddenham pit, where flint implements have been discovered, may belong to that series, although not to the highest level; for although the ground slopes very gently towards Bedford, and I could not mark any break in the continuity or much change in the character of the gravel, still, in the distance of 1½ mile, there is a difference of level of some 20 to 30 feet between the Biddenham pit and the gravel adjoining the railway and under the town*. There are also apparently certain slight differences in the fauna. At Biddenham the remains of the *Hippopotamus* have not yet been met with, whereas in the railway-cutting near the town they were very abundant. The profusion also of the other mammalian remains at the latter place is in marked contrast to their rarity at the former.

I have traced the gravels with similar conditions of structure above Bedford, as far as Olney and Wolverton, and in descending the river I have followed them at intervals as far as Lynn. At Offord, 3 miles S. of Huntingdon, there is a well-marked low terrace of gravel, in which a large number of mammalian remains were found during the making of the Great Northern Railway; and at Hemingford, 2½ miles E. of Huntingdon, a freshwater deposit with mammalian remains has been described by the Rev. Mr. De la Condamine†, which I consider belongs to the low-level valley series.

Herne Bay.—I am unable to offer a sufficiently satisfactory account and explanation of the position of the flint implements found on the shore near Herne Bay. Whether derived from a clay drift or from the gravel which caps the cliffs is uncertain. With respect to that at Swalecliffe, between Herne Bay and Whitstable, it would seem to have been derived from a freshwater and mammaliferous clay and gravelly sand, belonging to a low-level valley deposit and abutting against a cliff 54 feet high, on the top of which is a bed of high-level gravel. The same gravel may be traced on the other side of the valley at

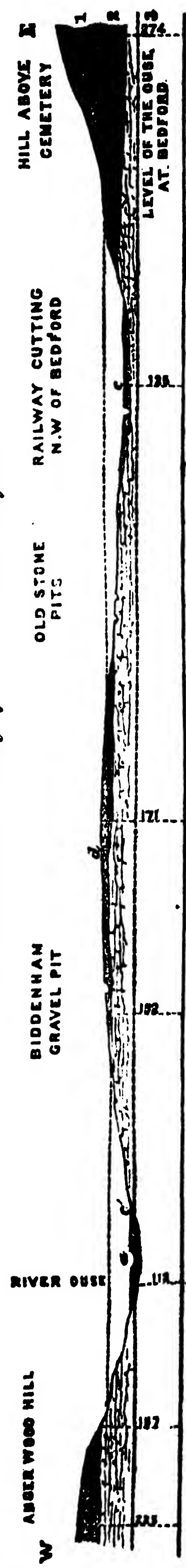


Fig. 3.—*Section across the Valley of the Ouse near Bedford.*

a. Recent alluvium. *c.* Low-level valley-gravel. *d.* High-level valley-gravel. 1. Boulder Clay. 2. Oxford Clay. 3. Oolitic series. (*Cornbrash*).

Traces of Loess, in the form of a thin bed of brown sandy clay with angular fragments of flint, overlie both the gravels.

* A recent visit to Bedford, and fresh levels taken with the aneroid barometer, have confirmed this distinction. By taking another line of section, I traced the Biddenham gravel to a height of 60 feet above the river, and found it separated from the gravel at Bedford by a bare tract of oolitic strata. The section given above (fig. 3) embodies these last observations. Mr. Wyatt and Mr. Evans have lately found two flint implements of the ovoid or Menchecourt type in the low-level gravel of Summerhouse Hill.—Feb. 1864.

† Quart. Journ. Geol. Soc. vol. ix. p. 271 (1853).

Studhill, and in it Mr. Evans found part of an Elephant's tooth. I am inclined to believe that the features observable at Swalecliffe are partly dependent upon the small lateral valley which runs a few miles up the country, and are only indirectly connected with the more general phenomena of the main valley of the Thames (fig. 4).

Fig. 4.—*Section along the coast east of Whitstable—1½ mile in length.*

W SWALECLIFFE 54 FT

COAST GUARD STATION

STUD HILL CLIFF 56 FT E

b. Loess with shells and bones. *c.* Low-level gravel. *d.* High-level gravel.

With regard to the valley of the Thames* the structure is far more complicated, from the circumstance of there being in this district, in addition to the high- and low-level valley-gravels, a wide-spread set of higher or hill-gravels, of marine origin, presenting a very close similarity to some of the valley-gravels, and covering large tracts of country. As this district will form the subject of a separate communication from me elsewhere, I here merely allude to it for the purpose of remarking that, after eliminating the foreign element, there remains a set of valley- and terrace-gravels which, though not so marked or well characterized as in the Seine valley, are nevertheless of nearly similar order and age. The same remarks apply to most of the valleys of the South of England, including the valley of the Severn. In the latter there are fossiliferous terrace-gravels skirting the Severn and the tributary Avon†, with valley deposits corresponding to those at Grays and Menchecourt, whilst, as in the Thames district, there is a set of higher-level hill-gravels more wide-spread, and probably of marine formation.

These cases all point to some common origin, and concur in showing that the flint implements hitherto met with have been found in beds holding like positions. The only exceptions are the discovery by Mr. Evans of a flint implement of the Amiens type upon the chalk hills of Hertfordshire, 200 feet above the valley, and of one of the Abbeville type on the chalk hills three miles from Dartford in Kent, by Mr. Whitaker. Both these are considerably above the valley and the valley-gravels, and the conditions are such as do not admit of exact correlation with any of the other cases.

The confusion just alluded to arising from the occurrence, on levels often not far apart, of gravels and drifts of different ages and different origin, is common through a great part of England, while also questions connected with the direction of the transporting currents are obscured in consequence of the materials of the high- and low-level gravels having been formed in large part out of higher-level hill- and marine gravels. Owing to the

* The flint implement found so many years since in the Gray's Inn Road still remains, with respect to the London district, a unique and remarkable case. Several specimens, however, have been found by Mr. Hughes near Sittingbourne and Faversham, lower down the Thames valley and its tributaries.—Feb. 1864.

† There are also in the valley of the Severn low-level gravels of marine origin. See Strickland's paper in Geol. Trans. 2nd ser. vol. vi. p. 552–5, and various papers by the Rev. W. S. Symonds.

absence of the marine pliocene and post-pliocene beds in the North of France, we there obtain clearer evidence relating to the valley-gravels, as they are free from rock-fragments and boulders foreign to their own origin and area. From causes to which we shall presently allude, the whole class of these phenomena is also more marked and on a larger scale in France than in England, and has attracted more attention and been more fully investigated. It is sometimes not easy to determine the high-level gravels in our valleys, whereas in France they are generally on a scale of height and extent which prevents any doubtful interpretation of their relative position.

Valley of the Somme.—M. BUTEUX * gives a number of localities at which the beds he terms Diluvium occur in the valley of the Somme, between Amiens and St. Quentin, but without going into structural details and levels. On my first visit to St. Acheul and St. Roch I suggested the possibility of the beds at the former place † being a stage older than the latter. The distance between them being $1\frac{1}{2}$ mile, it seemed quite possible that the difference of level might have arisen from other causes than a further excavation of the valley subsequent to the deposition of the St. Acheul beds, and anterior to that of the St. Roch beds, but the difficulty of proof was considerable; the existence of faults, the slope of the ground, or an unequal extent of elevation might have been the cause of the difference. The point being one of importance in its bearing on the question of the age and antiquity of the St. Acheul beds ‡, I reserved my opinion in the hope of finding better evidence elsewhere in this valley.

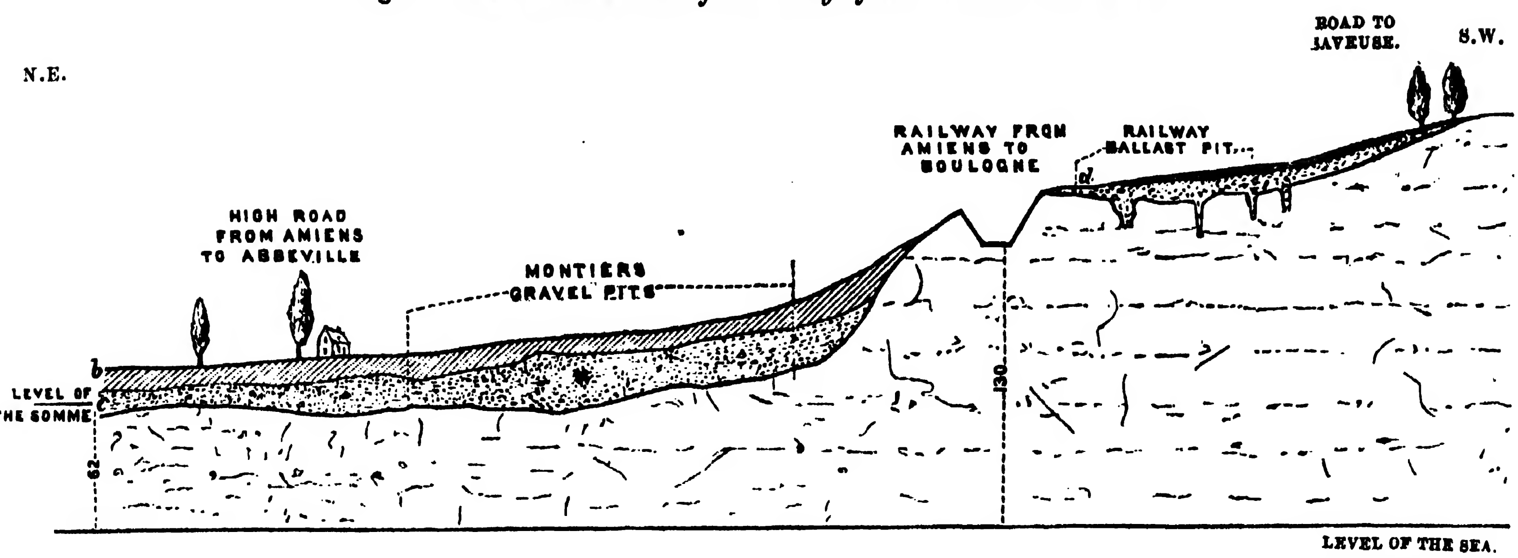
At a distance of from one to two miles N.W. from Amiens, and parallel with the river, is a series of large pits extending past Montiers along the road to Abbeville. The gravel is on the same level and of the same character as that of St. Roch. Few mammalian remains, and no shells or flint implements, had been found there up to the period of my first visit in 1859. As at St. Roch, the gravel abuts against a low chalk hill, and no fossiliferous gravel had been met with on the hill above. The railway runs just at the back of the pits, and at a height of some 30 feet above them. In the spring of 1860 I found a pit just opened immediately above the slope of chalk through which the line passed, and in such a position that the gravel could be wheeled on a level from this new pit into the railway trucks in the cutting. In position and level this last deposit is related to the beds of St. Acheul as the beds in the old pits are to that of St. Roch; but here the two beds were in close and determinable proximity. The section from one to the other is fortunately perfectly clear, and I could detect no fault in the separating chalk ledge cut through by the railway.

* Esquisse Géologique du département de la Somme, Paris, 1849, and two supplements; and 1862; also a paper by the same author in Mém. de la Soc. d'Emul. d'Abbeville for 1857, p. 561.

† Phil. Trans. 1860, p. 303.

‡ I have since been enabled to trace, by means of some trial pits near the Amiens railway station, the low-level gravel with remains of *Rhinoceros tichorhinus* close up to the base of the hill on the top of which the St. Acheul gravels extend, forming a section therefore very analogous to that at Montiers, and establishing directly the relation of the St. Acheul beds to the lower-level gravels, which continue uninterruptedly from the railway station to St. Roch.—Feb. 1864.

Fig. 5.—Section on the side of the valley of the Somme at Montiers.



b. Brick-earth (*Loess*) .... 3 to 12 feet

c. Irregularly stratified sands and gravels (low-level valley-gravel). Remains of Horse, Ox, Elephant, &c. Flint implements of the flake type not rare; one discovered at ✠ ...... 20 to 25 feet.

d. Rude mass of coarse gravel (high-level valley-gravel). A few fossil bones; numerous shells 10 to 12 feet.

1. Chalk. (The height at the railway cutting is only approximate.)

The gravel at the new ballast-pit is much mixed with chalk débris, and is less regular in its structure than the gravel at St. Acheul. It contains similar boulders of sandstone, and the identity of the two deposits was further confirmed by the discovery in the upper and more sandy part of the gravel bed of an abundance of land and freshwater shells, more numerous as to individuals than at St. Acheul, but of fewer species.

Shells from the new ballast-pit south of the railway at Montiers.

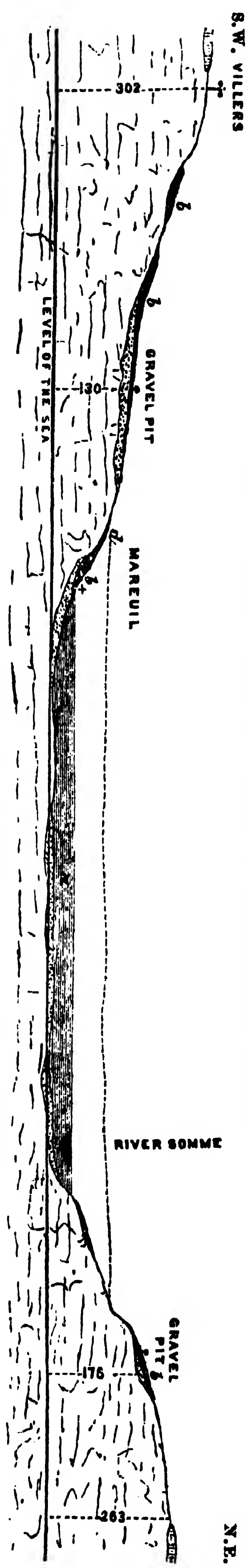
Helix concinna.	*Pupa marginata.*	*Ancylus fluviatilis.*	*Pisidium fontinale.*
—— *pulchella.*	*Succinea elegans.*	*Limnæa palustris.*	*Planorbis spirorbis.*
—— *hispida.*	—— *putris.*	—— *truncatula.*	*Valvata piscinalis.*

I also found a few fragments of bone, but not determinable. Of flint implements I could discover none, nor have any been yet found by the men. In the lower pits on the other side of the line the gravel is spread out in great horizontal beds, or rather lenticular masses, and is interstratified with some very sandy beds; the beds vary more in colour, and no shells have been found. Their thickness amounts to about 25 feet, whereas the higher gravel (*d*) is only about 12 to 15 feet thick. In all respects the lower gravel (*c*) resembles that of St. Roch. On the occasion of a former visit I had shown the men a flint implement from St. Acheul, and requested them to look for and keep any specimens. On my second visit, accompanied by Sir Charles Lyell, to whom I had mentioned the interest of the section, the men showed us a flint implement which they had just discovered at a depth of 17 feet from the surface, and at a point marked ✠ in the section*. This specimen is quite white, has dendritic markings, and is of the simple broad flake type common in the low-level gravel at Mautort†. On my last visit I obtained five more specimens, all of the narrow flake type known as flint knives, except

* Whilst in the pit we employed a man to work at a heap of the weathered gravel. A small flint knife was the result of an hour's search.

† See fig. 2, Plate XII. Phil. Trans. 1860.

one, which more resembles the St. Acheul spear-head type. The deep discoloration and curious dendritic markings on these flints, their form (not upon the neighbouring St. Acheul type, but upon one before unknown to the men), and their small number preclude the idea of any collusion or deception, and substantiates the statement of Dr. RIGOLLOT, that such implements have been found in the like gravel at St. Roch*. On the hill rather higher than the new ballast-pit is another pit, where the gravel is shallower, contains no organic remains, and is probably of rather older date.



a. Recent alluvium. *b.* Loess. *c.* Low-level valley-gravels. *d.* High-level valley-gravels. 1. Chalk.

Fig. 6.—*Section across the Valley of the Somme near Abbeville.*

The more general examination I have made of the Somme valley, from about six miles above Amiens, to the sea at St. Valery, a distance of forty-seven miles, has been sufficient to show the persistence of the same structural features. (Pl. V. fig. 2.) In descending the valley below Amiens, after passing the gravel-pits of Montiers and Breilly, the chalk hills rise abruptly from the valley, covered only with more or less loam or brick-earth, except near Picquigny, where a few gravel pipes in the escarpment attest the former presence of a high-level gravel apparently 60 or 80 feet above the valley. On the hill which projects into the valley between Le Gard and Crouy is a capping of flint gravel.

At Condé the valley-gravel again occurs, but thence by Fontaine and Liercourt the hills present a bare surface of chalk, or else a capping and skirting of brick-earth only. At Mareuil, three miles south from Abbeville, and still on the left bank of the river, we find the gravel, capping, to a thickness of 6 to 8 feet, the hills which rise behind the village to a height of 110 to 130 feet. I could not discover any organic remains or flint implements†. From this spot to Mautort the chalk hills rise abruptly from the valley, either bare or more or less covered by brick-earth, but still showing here and there traces of gravel pipes, indicating the former existence of gravel beds at about the level of 80 to 100 feet above the river. Here also, as at other spots where the hills advance upon the valley, the depth of the latter seems to be greater, so that all the valley-gravel and Loess is covered up and hidden by the recent alluvial deposits which come close to the foot of the chalk escarpment. (Pl. V. fig. 1.)

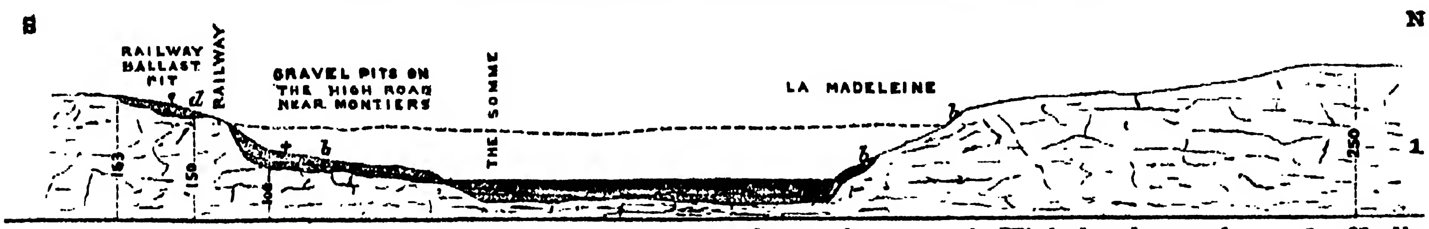
* I have since had the opportunity of confirming the statement of Dr. RIGOLLOT, the men having found four rude but undoubted flint-flakes at this spot (April 1863).

† M. BOUCHER DE PERTHES has two specimens in his collection labelled as from this locality.

Four miles lower down, and immediately opposite Menchecourt, are the pits of Mautort, described in my former paper. A bed of gravel there occurs on the slope of the hill at a height of 80 feet* above the river, and lower down another bed forms a bank on which the village stands. At a pit belonging to M. Ducastel, these lower gravels are very sandy and distinctly bedded, and there is good evidence for believing that marine shells (*Cardium edule* and *Littorina littorea*) occur at their base. I found no flint implements, but Sir Charles Lyell obtained, in the gravel-pit at the further end of the village, two indisputable specimens of the low-level ovoid form. There is a patch of high-level gravel near Saigneville, whilst at the mouth of the Somme a considerable width of ochreous flint gravel caps the hills near St. Valery, at a height of about 100 feet above the sea. I could discover no organic remains in any of the pits†.

The right bank of the Somme, between Amiens and Abbeville, shows a much greater amount of denudation. The hills are steeper and present generally bare surfaces of chalk, with more or less brick-earth on their summits and flanks.

Fig. 7.—*Section across the valley of the Somme near Amiens.*



a. Recent alluvium. *b.* Loess. *c.* Low-level gravel. *d.* High-level gravel. 1. Chalk.

Near Amiens I found nothing to correspond with the opposite St. Acheul high-level valley-gravel, except some scattered sandstone boulders at Longpré. From this point to Pont Remy‡, the ground requires further examination. Thence to Abbeville the hills, at a height of from 100 to 150 feet, are capped occasionally by flint gravel, in which no remains of any sort are met with until we reach the gravel-pits of St. Gilles and Moulin Quignon, described in my former paper.

In all the foregoing cases the sandy and ochreous subangular flint gravel nowhere occurs on hills higher than about 150 feet above the level of the Somme, and flint implements have not been found in beds more than 100 feet above that level; these gravel beds range parallel to the Somme, and in no case, except at the embouchure of the river, extend more than half a mile from the side of the valley.

* There was a discrepancy between my estimated height of the upper pits on the road to Moyenville, near Mautort, and the measurement obtained for me by M. Boucher de Perthes, for which I could not account in 1860. A well has been since sunk adjoining the old pit (now filled up), and the water-level in the chalk reached at a depth of 81 feet. Allowing 5 feet for the fall to the river, the height of the ground at that spot would be 86 feet.

† A raised beach containing *Cardium edule* has been stated to occur at the top of the hill near the old castle; but after a careful search I could not find a trace of any such bed. Sir Charles Lyell came independently to the same conclusion. On a more recent visit there with Mr. Evans and Mr. Lubbock, we merely found numbers of weathered valves of recent *Cardium edule* &c., and fragments of pottery in a black soil—a sort of Kjökkenmödding.

‡ M. de Perthes has a tooth of the *Elephas primigenius* from this place, but the exact position is not recorded.

The same features are repeated on a smaller scale in some of the lateral valleys. Near Amiens, for example, in the tributary valley of the Arve at Boves, four miles from its junction with the Somme, are large beds of gravel like those of St. Roch, abutting against the side of the chalk hills on the right bank of the stream. Remains of the *Elephas primigenius*, *Rhinoceros tichorhinus*, Horse, and Deer have been found there, together, it is reported, with a few flint implements.

At Abbeville the Escardon joins the Somme. At Oneux, eight miles up the valley of this tributary, a bed of flint gravel was formerly worked, in which, it is said, mammalian remains and flint implements were met with; whilst at Drucat, on another branch of this stream, a singular isolated patch of gravel has been found high up on the hills, at an elevation of about 100 feet above the valley, and 150 feet above the Somme at Abbeville. The sand and gravel are of great thickness, owing to their being in a depression of the chalk, from which they further descend in huge cylindrical holes or pipes to a depth of more than 100 feet. On the top of the light-coloured sands and gravels are other beds of reddish gravel and thin brick-earth. In M. BOUCHER DE PERTHES's collection there are two flint implements which are said to have come from these upper beds.

The Valley of the Seine.—Another important discovery of flint implements was made early in 1860, in the valley of the Seine at Paris, by M. GOSSE*. The specimens are ruder and less abundant than those of the Somme valley. They were found in the pits of the Allée de la Motte Piquet†, near the Ecole Militaire, at a depth of 16 feet in a gravel composed chiefly of subangular tertiary and cretaceous débris about 20 feet thick, roughly bedded, and containing remains of extinct mammalia. No shells are recorded. Large blocks of *Meulière*, little worn, are dispersed through the gravel. At this spot, the height of which above the Seine is 36 feet, the relation of the beds to any of the "higher-level" quaternary deposits of the neighbourhood of Paris is not seen, but at the Gare d'Ivry, on the other side of Paris, and at a similar level with regard to the river, the same bed of gravel, only more sandy and containing more large granite boulders, is again largely worked. It here abuts against the calcaire grossier, which forms, immediately behind, the heights of Gentilly, where these tertiary strata come to the surface and are extensively quarried. Following the plain in a direction at right angles to the river, some coarse gravels of moderate thickness set in, and are worked near the Barrière de Vitry, at a height of about 130 feet above the Seine. Further on a cutting, on the side of the road leading from the Barrière d'Italie to Gentilly, exposes a section described in 1840 by M. DUVAL‡ and singularly like that at St. Acheul§. The lower bed is a white sandy gravel of subangular chalk flints

* Comptes Rendus, 1860, vol. l. p. 812.

† M. BOUCHER DE PERTHES had in 1847 expressed an opinion that flint implements would be found in these pits: 'Antiquités Celtiques et Antédiluviennes,' vol. ii. pp. 123, 494, and 501.

‡ Bull. de la Soc. Géol. vol. xi. p. 302, and vol. xiii. p. 297.

§ It is, however, not improbable that the deposits both of Gentilly and Charonne may be older than that of St. Acheul, although belonging to the series of the high-level valley-gravels.

mixed with much-worn tertiary débris with a few older rock pebbles, and above which is a sandy marl. In these beds a considerable number of land and freshwater shells and mammalian remains were found. The whole is capped by a variable thickness of Loess. The slopes of the adjoining higher ground of Bicêtre are bare, with the exception of occasional patches of Loess.

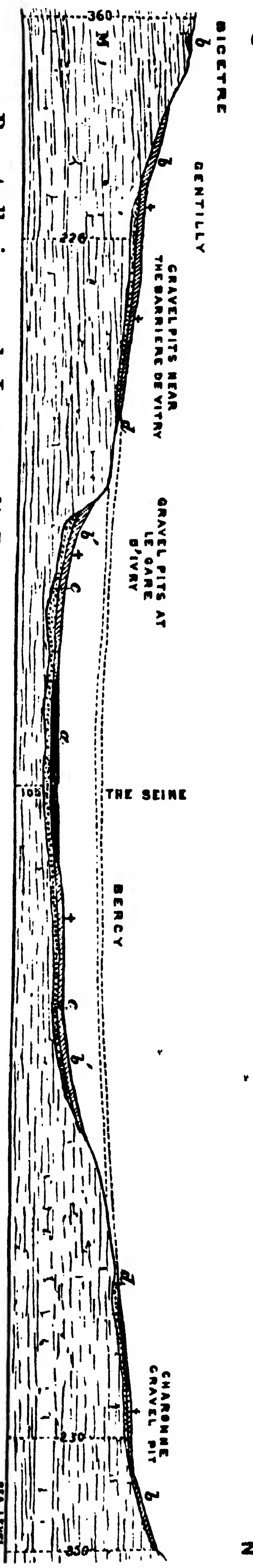


Fig. 8.—*Section across the Valley of the Seine East of Paris.*

a. Recent alluvium. *b.* Loess. *b'.* Loess of a lower level. *c.* Low-level valley-gravels. *d.* High-level valley-gravels.
M. Freshwater marls and limestones (gypseous series), overlying the Calcaire grossier and Lower tertiary strata.

No freshwater shells have hitherto been recorded from the low-level valley-gravels of the Seine; but in April 1862 I found at Sotteville* near Rouen, intercalated in the middle of thick beds of gravel, about 10 to 20 feet above the river-level, a seam of marly sand, in which I obtained a few specimens of the *Limnæa peregra* and opercula of the *Bythinia tentaculata*; and at Paris I discovered in the hard concreted gravel of the Petite Rue de Reuilly, and about 30 to 40 feet above the Seine,

Limnæa peregra, *L. truncatula*, *Valvata piscinalis*, *Zua lubrica*;

whilst at Clichy I found, low down in the gravel containing, I am informed by M. Lartet, the *Elephas primigenius* and *E. antiquus*, three specimens of the *Limnæa peregra*. All these specimens are entire and uninjured. In this latter locality M. Lartet had also recently found a very perfect ovoid flint implement of the Abbeville type.

In the high-level gravel of Gentilly have been found†—

Pisidium amnicum.	*Bulimus.*	*Hydrobia.*
Valvata piscinalis.	*Limnæa.*	*Helix.*

On the north side of the Seine, thick gravel-beds extend from the river to the base of the hills, and some distance up their slopes, as shown at the canal de l'Ourcq and in the pit in the Petite Rue de Reuilly. As the tertiary strata again come to the surface a short distance further on, it is probable that this bed of gravel, like that on the south of the Seine, abuts or slopes rapidly up against their escarped edges. At Charonne, about a mile beyond the last-named pit, and near the Barrière du Trône, there are several pits of sand and gravel, containing numerous land

* M. l'Abbé Cochet states that there are in the Museum at Rouen two flint implements of the St Acheul type said to have been found in these pits, *op, cit.* p. 8.

† *Op. cit.* p. 297. M. Duval does not give the specific names in his list. Few specimens are now to be found.

and freshwater shells together with the remains of mammalia. This deposit is identical with that at Gentilly*. The section of this part of the valley of the Seine may therefore be represented as in fig. 8†.

M. CHARLES D'ORBIGNY has traced the high-level gravel of Charonne, with few interruptions, as far as Joinville five miles east of Paris. It there caps a hill rising abruptly 80 to 100 feet above the river, and contains a large and interesting series of land and freshwater shells‡, associated with the remains of the *Elephas primigenius* and *Rhinoceros tichorhinus*. No flint implements are recorded from these gravels.

The same features hold good throughout the course of the Seine and its tributaries. BRONGNIART, in his description of the Paris basin, observes that the drift (Terrain de transport) occupies two positions—one in the valleys, and another on plains considerably raised above the actual rivers. He seems to suggest for them different origins§.

Higher up the course of the Seine this structure has been more specially noticed. M. LEYMERIE, in describing the country traversed by the upper part of this river and its tributaries, remarks that at the commencement of the valleys there is but little drift (Diluvium), but that some way down them "the beds of drift (Terrain diluvien) exhibit a great extension, both horizontally (maximum four leagues), and also in a vertical direction (maximum sixty metres) above the ordinary level of the valley-waters"||. In the neighbourhood of Troyes they form a plain eight miles in length and of five miles average breadth, and attain in places a height of from 120 to 180 feet or even more above the river, with a thickness of 10 to 12 feet, and contain teeth of Elephant, Deer, Horse, &c. At Nogent there is another expanse of gravel 10 feet thick, and 200 feet above the river. In the tributary valley of the Aube, M. LEYMERIE describes at Brienne a similar expanse of gravel five leagues long by three broad, and 130 feet above the river¶.

Of the lower part of the valley of the Seine we possess but few details**. The plain through which the river winds is covered irregularly with a coarse sandy gravel, abounding especially in large blocks of Meulière, Calcaire grossier, and of other tertiary rocks;

* M. CHARLES D'ORBIGNY in Bull. de la Soc. Géol. 2e sér. vol. xii. p. 1295 (1855). The pit he described is now filled up, but several others are open beyond the outer Boulevards. My sections are taken from these.

† One of the best plans of valley-gravels, well and carefully worked out, that I have seen, is the one executed by M. TRIGER for the Department of the Sarthe. The river Sarthe, which flows through palæozoic rocks, has a gravel abounding with blocks of granite &c., whilst the Huisne, which joins the former river at Le Mans, contains nothing but tertiary débris. The extent of these old alluvia are well shown on this fine map.

‡ For a list of these and a description of the section, see Bull. de la Soc. Géol. 2e sér. vol. xvii. p. 66 (1859). (I have recently had reason to believe that these fossils may be from old pits on a lower level than those now worked. Owing to this uncertainty I omit the list of fossils I had at first given.—March 1864.)

§ Déscription Géol. des Environs de Paris, édit. 1835, pp. 118 & 569.

|| Statistique Géol. et Minér. du dép. de l'Aube, p. 99 (1846). M. CLÉMENT MULLET had also remarked that at Fresnoy a gravel from 22 to 26 feet thick occurred at a height of 100 to 130 feet above the river; and a like arrangement exists at Pougy in the valley of the Aube. Bull. de la Soc. Géol. de France, vol. xii. p. 116.

¶ Ibid. pp. 88, 90 to 92, 94.

** See DE SENARMONT'S 'Déscrip. Géol. du dépt. de Seine et Oise.'

and I found gravel rising to a height of about 100 feet at Mantes, of 140 feet at Pont de l'Arche, and of above 100 feet near Rouen.

In the valley of the Yonne the gravel attains a height of 66 feet at Pontaubert*; but exact particulars are wanting of the terrace-gravels in this valley. Of the valley of the Marne, M. MICHELIN† observes that "on the heights around St.-Menehould there is a gravel containing the remains of the Elephant, Horse, and some other mammalia." M. CORNUEL states that the same valley higher up and that of the Blaise are covered with an oolitic gravel containing some remains of the Elephant, and names two small hills (buttes) near St. Dizier, and two near Vassy, which are capped by a similar gravel‡.

Valley of the Oise.—In this large tributary valley of the Seine, M. DE VERNEUIL§ has reported the discovery, at Précy near Creil, of a flint implement in beds of gravel containing the remains of the Elephant, Deer, &c. In this instance the specimen was not found *in situ*, but was picked up amongst the gravel thrown on the adjacent line of railway. In April 1861 Sir CHARLES LYELL and I visited the ballast-pit whence the gravel was extracted. It is situated half a mile north from the Précy station, is about 25 feet above the river, and presents a fine section of light-coloured subangular gravel, roughly stratified, 12 to 15 feet thick, and overlain by 5 to 10 feet of Loess. Some beds of the gravel are very coarse, and amongst the flint and tertiary débris are fragments from the oolitic strata and older rocks. The valley at this spot is about half a mile wide, and the hills on either side rise to a height of from 150 to 250 feet above the valley. In going over this ground some years since with the deeply lamented EDWARD FORBES, we found the rocks almost everywhere bare of drift, but I have a recollection of having met with traces of gravel with old-rock pebbles on a hill about 80 feet high on the right bank of the river between Auvers and Beaumont. M. GRAVES, in speaking of the drift (Diluvium des vallées) of the Oise, observes that "it ascends the slope of the hills to some height" .... "some incontestable traces of it are sometimes found on surfaces of high platforms" ||, but he gives no exact position or levels.

M. D'ARCHIAC, in describing the valley of the Aisne, the upper portions of the valley of the Oise, and their tributaries, states that the gravel (dépôt de cailloux roulés, avec des blocs erratiques et des ossements nombreux des grands mammifères) "occupies the floor of the principal valleys, rises sometimes up their sides to a certain height, but very rarely extends over the adjacent high-ground plateau"¶. Further on he mentions that between Menneville and Neufchatel (valley of the Aisne) the gravel rises 65 feet above the river, and is about 16 feet thick, that between Voyenne and Marle (valley of the Serre) it is more than 130 feet** above the river—whilst at Guise (valley of the Oise) it rises 184 feet above the river, is 16 feet thick, and extends $1\frac{1}{4}$ mile

* Bull. de la Soc. Géol. 2e sér. vol. ii. p. 683 (1845).

† Ibid. vol. vii. p. 83 (1836).

‡ Mém. de la Soc. Géol. de France, vol. iv. p. 270 (1841).

§ Bull. de la Soc. Géol. de France, 2e sér. vol. xvii. p. 555.

|| Topographie Géognostique du dépt. de l'Oise, pp. 529, 530 (1847).

¶ Mém. de la Soc. Géol. de France, vol. v. p. 188 (1842).

** English feet are always given.

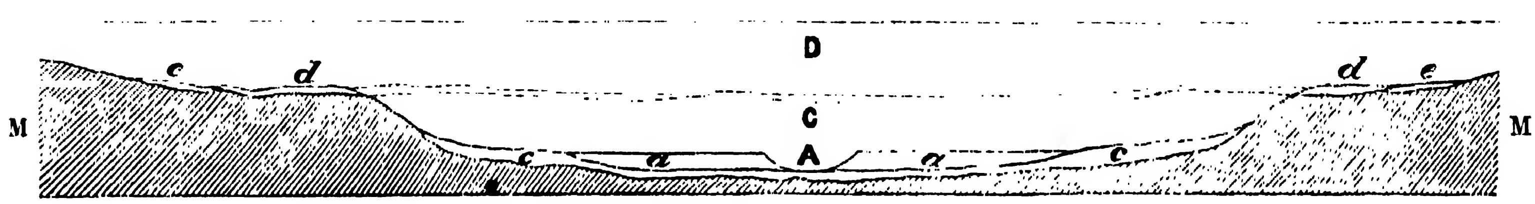
from the river*. Respecting the upper part of these valleys, MM. Sauvage and Buvignier remark that beds of sand and gravel cover the hills which border the valleys, at levels far above the present rivers†; and M. Buvignier has described beds of gravel 4 to 6 feet thick, with remains of Elephants and other animals, at heights of from 100 to 130 feet above the Aire, and 160 to 200 feet above the Chée, the Ornain, and the Saulx‡.

It is therefore certain that at points more or less distant along the course of all the rivers in the South of England and the North of France§, detached and isolated beds of sand and gravel occur on terraces, or on the top or shoulders of the low hills or platforms bordering the valleys, at various and often considerable heights above the present river-levels. Another and more continuous stream of gravel stretches along the bottom of the valleys. Beds of Loess cover both gravels irrespectively, and extend beyond them. It is in these deposits only (caves excepted) that the flint implements have hitherto been found.

§ 3. HIGH- AND LOW-LEVEL VALLEY-GRAVELS.

From the constancy of the phenomena described, we may arrive at a general proposition which can be expressed by the following diagram (fig. 9) and terms:—

Fig. 9.



M. General section of the ground.
D. Major valley of denudation anterior to the excavation of the valley C.
C. Minor valley of river-excavation.
A. Present river-channel.
e. Non-fossiliferous drift on the slopes and base of the major valley D.
d. High-level valley deposits } with or without fossils.
c. Low-level valley deposits } with or without fossils.
a. Recent alluvium.

This diagram represents the conditions of the case on the supposition that all the parts are complete. But this rarely happens. The gravel along the base of the major valley (D) is spread out or contracted according to the width, depth, and shape of its channel. If the secondary valley C should extend on either side beyond the limit of the beds *e* or *d* deposited along that first depression, the section would be represented by diagram fig. 10, where, as at Amiens, the hills on one side are completely denuded, whilst on the other side a portion of *d* remains. Or it might happen, as is the case in many parts of the valley between Amiens and Abbeville, also in the valley of the Oise near Creil, again in the valley of the Seine near Paris, and commonly elsewhere, that the width of C exceeds that of the first-formed channel characterized by the drift *e* and *d*, in which case the river-valley would pass through bare hills denuded of all traces of *e* and *d*, as in the

* Mém. de la Soc. Géol. de France, vol. v. pp. 190, 193 (1842).
† Statistique Minér. et Géol. du dépt. des Ardennes, p. 58 (1842).
‡ Géologie du dépt. de la Meuse, pp. 95, 96 (1852).
§ I restrict my remarks to this area; but the fact has a much wider application.

Fig. 10.

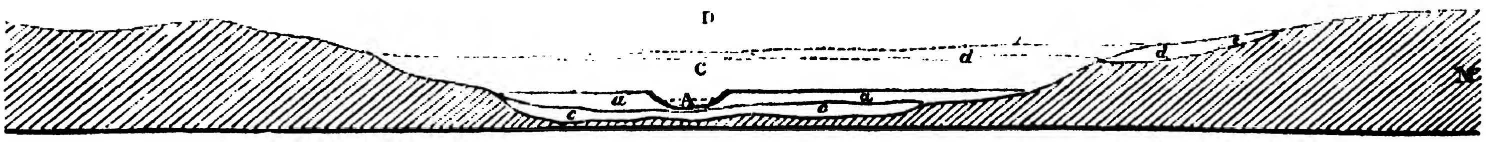
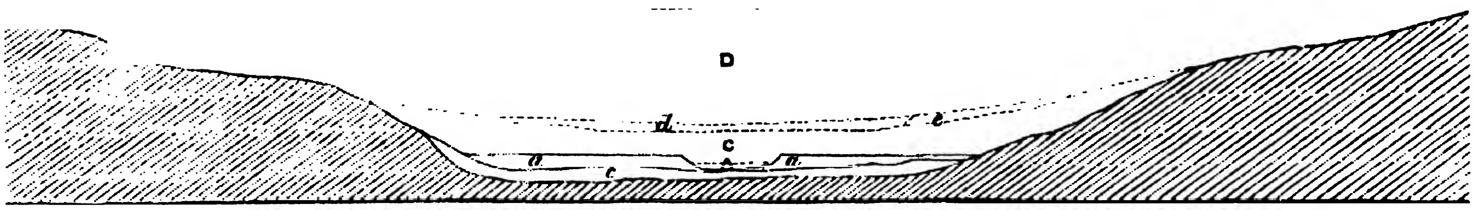


diagram fig. 11. Or the various gravels may also have been wanting originally, as there must have been parts of the old river-bed always left bare.

Fig. 11.



It is therefore not essential to the case that the high-level valley-gravels *d* should be permanent or continuous in the direction of their length. We have seen that they are generally found at heights above the river of from 50 to 150 feet, though occasionally more or less. The low-level gravels, on the contrary, often have even their upper terraces so little above the level of present inundations that they might be attributed to such recent causes. The coincidence, however, arises from merely local conditions, and we can hardly apply to them a different rule, the more especially as, although the level of the terraces may be at places so low as to merge in the gravel at the bottom of the valley, still they even then form a breadth and mass of deposit which indicate a very different power to that now in operation in the existing rivers.

That the formation of the higher terraces can be owing to the action of the present rivers is clearly impossible under existing conditions; for they are far above the level reached by the rivers at the highest floods.

Taking the mean depth of the Waveney near Hoxne at 3 feet and its width at 33 feet, whilst the depth of the valley between the two gravel terraces is about 40 feet and the width 2000 feet, the sectional area of the former will be 100 square feet nearly, whereas the latter will be represented by about 80,000 feet—in round numbers as 1 : 800. The Ouse at Bedford has an average depth of 5 feet and a width of 200 feet, or a sectional area of 1000 feet. The valley at the same point gives us about 12,000 feet—or as 1 : 600. The rise of these rivers above their ordinary level never exceeds a few feet. Very little is known of the discharge of either river during floods*.

If we take the Somme at Amiens at 4×200, the sectional area is 800 feet, whereas the valley at the level of the St. Acheul gravel gives 80×5000, equal to a sectional area of 400,000, or as 1 : 500. The floods of the Somme rarely exceed 5 feet above its mean level. Taking the Seine at Paris at a mean width of 600 feet and a depth of 4 feet, the sectional area is 2400 feet. The valley between the heights of Gentilly and of Charonne

* The numbers in this and the next paragraph are only approximately correct; exact data are wanting.

has a depth of 120 feet; and taking as a mean a width of 8000 feet, we have a sectional area of 960,000 feet, or as 1 : 400. The greatest flood of the Seine on record is that of the year 1658, when it rose to a height of 29 feet. Even in this case a flood of nearly 60 times that magnitude would be required merely to fill the valley to the level of the high-level gravels, without taking into consideration the more rapid discharge. But neither in this nor in the other cases of modern times, are we aware of an increase in the volume of water, during floods in these regions, to many times the ordinary mean average, whereas we see that in a case such as is presented at Amiens a flood having a volume five hundred times that mean would be required to reach the beds of St. Acheul.

This I conceive is sufficient to prove that the high-level valley gravels cannot be ascribed to floods of the present rivers, as has been, even of late, suggested. The only means adequate to fill, under existing conditions, the river-valleys of the Waveney, the Ouse, the Somme, and still more of the Seine, would be the ingress of the sea; but such a supposition is at once refuted by the fact of the prevalence of land and freshwater shells in both the high- and low-level gravels, and the absence of marine remains unless immediately adjacent to the present coast. If therefore neither the supposition of river-floods, nor of a different relative level of land and sea allowing the latter to penetrate up the valleys, be admissible, how far are the facts in accordance with the hypothesis of these deposits being the alluvia of old rivers, and the valleys their excavated channels?

Let us in the first instance trace the direction whence the materials have come.

The valley of the Waveney traverses a district formed of Boulder Clay, with underlying sands and shingle, reposing on Chalk, which latter comes to the surface in the upper part of the river's course, and is just visible at Scole. There is little in the valley-gravels to indicate a distant origin, as most of the débris composing them might generally have been derived from the surrounding hills. The only material in excess is the mass of subangular flint-fragments, derived probably in part from the more distant chalk area. Although not quite conclusive, still the evidence affords fair presumptive proof of the transport of the gravel from the watershed of the Waveney to the sea along the breadth of surface indicated by the valley-gravels (see Map, Plate IV.).

A nearly similar uniformity exists in the case of the valley of the Ouse, which traverses a district of Boulder Clay overlying various members of the oolitic series. It is not, however, easy to determine in what proportion and to what distances the different materials composing the gravel of the valley of the Ouse have been transported from their parent rocks. They exhibit, as in the case of the Waveney, a local origin in connexion with the existing valley. All the materials found in the gravels can be referred to rocks or to older drift deposits traversed by these valleys and their tributaries, and in no instance do we find the introduction directly of any foreign element. Thus, besides the flint débris derived from the chalk in the valley of the Waveney, and the oolitic limestone and sandstone débris derived from the Oolitic series in the valley of the Ouse, all the superadded pebbles and boulders of the older rocks, as well as a

certain proportion of the chalk-flints found in the gravels of both valleys, can be traced to the adjacent Boulder Clay.

I have noticed the same facts in almost all the river-valleys of the South-east of England where the valley passes through belts of different formations, as, for example, the occurrence of Wealden and Greensand débris on the chalk hills in the valley of the Medway, between Maidstone and Rochester; and in the same manner the occurrence of Purbeck and Greensand débris on terraces in the valley of the Wiley between Wilton and Salisbury. Traces of Greensand and Oolitic débris are to be detected in the valley of the Thames, also mixed with débris of the Boulder Clay.

With reference to the French area, the phenomena have been well studied by many French geologists*, to whose works, before cited, I beg leave to refer for fuller particulars than I can here introduce. I will say a few words, however, of the valleys of the Somme, the Oise, and the Seine,—more especially of such parts as I have myself visited.

The Somme flows through a chalk district with a few tertiary outliers, while the tributary valley of the Arve penetrates to the main body of the tertiaries. At St. Acheul the quantity of sandstone blocks and pebbles, and of flint pebbles, derived from tertiary strata, is very considerable, and the presence of specimens of *Nummularia lævigata* and of the *Venericardia planicosta* shows the upland direction of their origin, as no beds containing these fossils exist below Amiens. In the several patches of high-level gravels (chiefly of subangular chalk-flints) between Amiens and Abbeville, tertiary débris continue to be found, and at Moulin Quignon sandstone-fragments of a large size are still numerous, although they are fewer and smaller than at St. Acheul†.

The occurrence of rolled fragments and boulders of granite and other old rocks in the valley of the Seine at Paris, is a fact which has long been well known. The presence of similar materials of distant origin has also been proved to hold good in the higher-level gravels of Gentilly and Charonne. M. Duval described the section at the first-named place (which applies to the other), as consisting of sands passing downwards into a gravel composed of pebbles and fragments of cretaceous and tertiary rocks, together with others of porphyry, and "a prodigious quantity of grains and pebbles of red granite"‡, some of which weighed from eleven to thirty-four pounds. Land and freshwater shells and small reptilian bones were common, with some bones of the large mammalia.

The valley-gravels at Frénoy, Nogent-sur-Seine, and Troyes consist chiefly of oolitic

* The case is very clearly put by M. d'Archiac in his admirable work the 'Histoire du Progrès de la Géologie,' vol. ii. p. 139, where he observes, "On reconnaîtra que les matériaux des dépôts meubles sont distribués ou répartis exclusivement soit sur les flancs de certaines gibbosités principales et dans les vallées qui y prennent naissance, soit dans de larges dépressions ou bassins hydrographiques qui ne sont limités à leur pourtour par aucun relief bien prononcé. Dans l'un ni dans l'autre cas il n'y a mélange des matériaux transportés."... ..."Cette distribution nous démontrera que l'orographie générale était au commencement de la période diluvienne à très peu près ce qu'elle est aujourd'hui, et nous reconnaîtrons que les lignes de partage entre certains bassins hydrographiques et dont l'élévation devait être bien faible alors ont pu cependant restreindre les effets du phénomène erratique aux mêmes limites que les eaux qui se rendent actuellement dans chacun d'eux." He excepts Great Britain and the North of Europe from the exhibition of these conditions.

† See sect. figs. 3, 5, 6 in Phil. Trans. for 1860, pp. 287–290.

‡ *Op. cit.* p. 304.

pebbles*. At none of these places is there any admixture of old-rock pebbles. These latter all come into the valley of the Seine through its tributary valley of the Yonne, which, with that of the Cure, originate in the Morvan—a district consisting of granitic, porphyritic, and slate rocks, forming ranges of hills from 800 to 1200 feet high.

The valley of the Aube, which joins that of the Seine between Nogent and Troyes, contains only oolitic and cretaceous débris; so also, judging from the incidental notices of M. Cornuel, the upper parts of the valley of the Marne and its tributaries. These valleys traverse Cretaceous and Jurassic formations only (see Map, Plate IV.).

Speaking of the gravel of the valley of the Oise, M. d'Archiac remarks that it "is composed of the débris of tertiary, secondary, and transition rocks, always rolled, and that the characters of the deposit vary according to the region from which its constituent elements have been brought, and consequently according to the valleys where these are found"†. The slate rocks of the Ardennes, from which the Oise flows, form a range of hills on the frontiers of Belgium, from 1400 to 1600 feet high.

The next feature we have to notice in the high-level valley-gravels is the presence of large boulders, and the irregularity, confusion, and general want of stratification of the beds, which are, further, frequently contorted.

In the valley of the Waveney there are no hard rocks to furnish boulders; the few therefore that are found are derived from the Boulder Clay; but very large masses of flint with sharp and intact angles are common. In the valley of the Lark the features are precisely analogous. At Flempton we found, mixed up in the flint gravel, large blocks of half a ton weight of basalt and hard sandstones derived from the Boulder Clay. At Biddenham in the valley of the Ouse the lower part of the gravel is full of small blocks of oolitic limestones and sandstones very little worn, and there are none of a large size. In the valley of the Thames, boulders of large size are rare; I have not met with ten in the course of as many years.

In the valley of the Aire, M. Sauvage and Buvignier remark that the gravel capping the hills has an irregular surface and is waved (ondulé). In the valleys of the Seine and Aube, M. Leymerie describes the base of the gravel capping the hills at Frénoy as irregular, and at Troyes as containing blocks and angular pieces of shelly neocomian strata, together with a few unworn flints. At Brienne the gravel also wants regularity. In the valley of the Oise at Guise, M. d'Archiac states that this gravel contains some fragments of quartz "the size of the fist," and that it is "without distinct stratification"‡. In the valley of the Seine, M. Charles d'Orbigny, in speaking of the lower bed of the gravel at Joinville, remarks that it forms "a tumultuous deposit, at the base of which are found large erratic blocks§. One mass of tertiary sandstone (B, fig. 12) measured by us was 8½ feet long by 3 feet 4 inches thick, and was on the top of the light-coloured lower gravel (diluvium gris), or rather at the base of the red gravel (diluvium rouge).

* Leymerie, *op. cit.* p. 92.

† Mém. de la Soc. Géol. de France, vol. v. p. 188; and Graves, *op. cit.* pp. 535, 542.

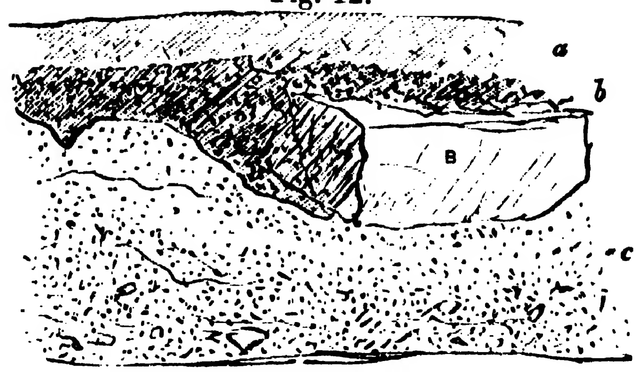
‡ Ibid. p. 193.

§ *Op. cit.* p. 68.

A pit near the Barrière de Vincennes (fig. 13) afforded me an excellent example of contorted bedding.

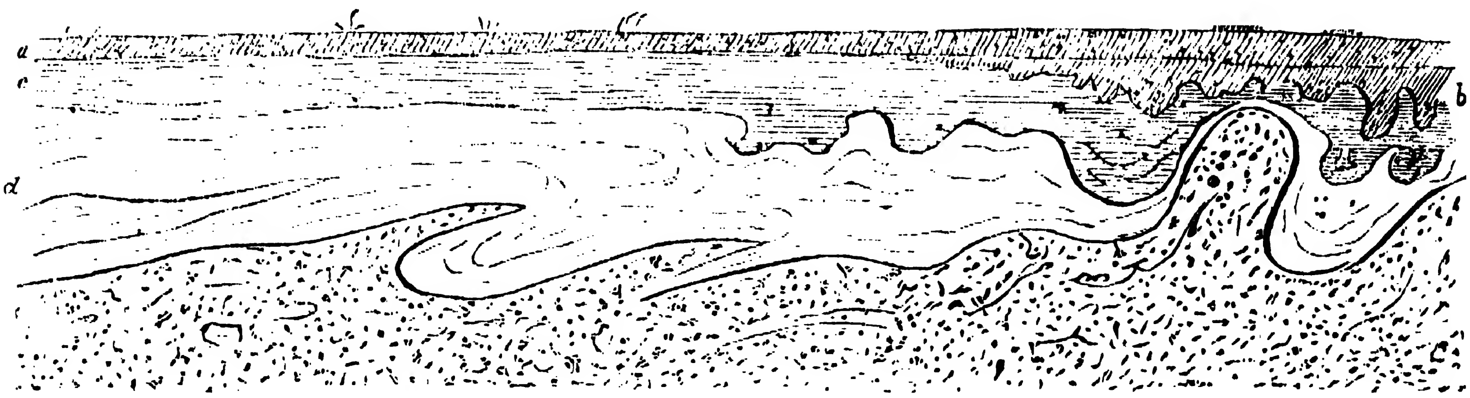
In the valley of the Somme this class of phenomena is particularly well marked and decisive. I know of no place where they are better shown than in the pits at St. Acheul (figs. 14, 15). The two following sections taken from my former paper will illustrate these peculiarities. There are two points to be noted,—the one the pre-

Fig. 12.



a. Surface soil and reddish loam.
b. Red argillaceous gravel (diluvium rouge).
c. Light-coloured sandy fossiliferous gravel.

Fig. 13.



a. Surface soil and made ground.
b, *c*. Loam and red gravel, 2½ feet.
d. Sand and fine gravel with a few shells, 3 to 4 feet.
e. Coarse light-coloured gravel, with mammalian remains, 5 to 8 feet.

Sections in the gravel-pits of St. Acheul.

Fig. 14.

Fig. 15.

a, *b*. Soil and brick-earth and subordinate beds of gravel—with Gallo-Roman graves, *a'*.

c'. Whitish marly sands with land and freshwater shells, containing here and there a few patches of gravel and blocks of sandstone, and showing in places fine lamination.

c. Subangular gravel, in places level, and in other places disturbed, containing numerous blocks of sandstone, and large flints not worn: *flint implements, mammalian remains*, and a few *shells*.

sence of large blocks of sandstone, some weighing as much as four to five tons, dispersed irregularly through the lower bed of gravel,—and the other the disturbed and contorted condition of the surface of that gravel, and of the bed of sand overlying it. These two features are perfectly independent. It is not the presence of the blocks which

gives rise to the heaping of the gravel, nor are the depressions in its surface marked by the presence of any such interfering masses; yet the disturbance has been from above, and sometimes before the deposition of the brick-earth; for the upper part of the gravel (*c*) and the overlying sand-bed (*c*′) are often affected independently, without there being any corresponding disturbances in the lower gravel or overlying brick-earth. The irregularities in the bedding of the gravel do not depend upon the presence of blocks, which occur in all levels through it; and the lamination of the marly sand (*c*′) is continued through the curved planes of contortion in a manner and at angles which such a sedimentary bed could never have assumed in process of deposition by the mere action of water. The tertiary blocks come from a distance of twenty to forty miles above Amiens.

Low-level Valley-Gravels.—It may not be possible to draw an exact line of demarcation between those gravels which I have designated as the high-level valley-gravels and these low-level gravels. They are the extremes of a series, marking a long period of time and probably formed under analogous but not identical conditions. The higher-level terraces are generally, however, so distinct, so broad, and so clearly separated from the low-level gravels, by bare slopes of the underlying rock formations, that, although they may differ amongst themselves to the extent of several feet, the space between the two groups is usually sufficiently distinct to make it not difficult to refer each bed to its right relative position. In places there are passage beds following the more gentle slopes, and at other places there are intermediate terraces, though commonly of little importance and trifling width. The broad distinction consists in the one being on hills of various heights flanking the valley, while the other occupies the immediate river-valley, always following its main channel and constantly rising on its sides to the height of several feet, and, where the valley is broad, forming low terrace platforms on its sides. Unlike the high-level gravels, of which the interrupted and local occurrence, comparative isolation, and unequal levels render the course and connexion indistinct, these low-level gravels, from their general continuity and their slight difference in level, leave us in no doubt as to their relation to the existing valleys.

The main points of difference to note are the greater thickness of the low-level gravels, their more uniform bedding, the more common presence of beds of sand and fine gravel with oblique lamination, and the absence generally of contorted strata. When we speak of bedding or stratification in these deposits, it is not continuous and persistent seams that we refer to, but to the greater or lesser extension of lenticular masses of sands and gravel of various thickness, which gives to a small section the appearance of bedding; but none of the beds are persistent. Large blocks are often common in these lower gravels. They are generally more worn than in the upper gravels, and the question arises to what extent they may be derived from the former. A great number occur in the lower gravels of the valley of the Somme, though they are not so numerous as in the upper gravels. In the valley of the Seine, on the contrary, they are extremely numerous in the lower gravels at the Gare d'Ivry, at Grenelle, and in various pits between Paris and Rouen. In the valley of the Thames I know of but very few. On the whole these

lower gravels have a more washed, sorted, and worn character than the upper gravels, although there is in these also a certain proportion of angular materials.

As the various Eocene, Miocene, or Pliocene strata spread over the South of England and North of France prove that, at some comparatively late geological period, the sea or large lakes extended over that area, it follows as an almost necessary consequence, that, when the land rose from beneath those seas or those lakes, a mass of débris, in quantity and length of transport proportionate to the greater or less rate of elevation and the depth of the superincumbent waters, must have been spread over the bottom of the channels along which the water flowed off; and assuming the rise to have been tolerably uniform over considerable districts, the course of the currents would be influenced by the form the land assumed during its emergence, or in fact by the present watersheds, which either result from or else bore part in that operation. In either case the ultimate effect, so far as our position is concerned, would be the same. That this must have been a *vera causa* to a certain extent is manifest, inasmuch as that which was sea or lake is now dry land; and although it will not explain the origin of all the high-level gravels, there is a certain proportion of these beds which may nevertheless have this older and independent origin*: it is to be distinguished by rising higher up the hills, and not being restricted to so definite a level as some of the other beds. I have reason to believe that some of the gravels in the South of England and the higher levels of the Somme valley may be of this age,—such as those above Epagnette, and the higher levels of those above Breilly and Montiers. In proof of the existence of some gravels older than those of St. Acheul and yet belonging to the Somme valley, I would mention the not uncommon occurrence at that place of subangular flints, with a deep brown staining, imbedded in layers of perfectly white gravel. As this discoloration can only arise from the flint having been imbedded at some time in an ochreous or ferruginous matrix, I infer that such specimens are derived from beds of gravel of that character, and older at all events than the St. Acheul gravels. As a proof of their staining being clearly unconnected with any colouring-matter of the existing matrix and being of older origin, I noticed that they have all had a second rolling, and that where their first angles have been worn off and the outer brown coat removed, the eroded surfaces often show, first an original white crust, and then the black core of the flint, without any change of colour on these fresher surfaces due to their present position.

The theories that have been formed to account for the entire series of these valley-gravels by the bursting of lakes, by the sudden melting of the glaciers and snow of mountain-regions, or by the transitory passage of any body of water over the land, are open to objection, because the débris would have been swept along in fewer and more definite directions, or would have held its course more irrespective of the existing watersheds, and would have shown an amount of wear in all cases proportionate to the distance travelled. Not only is this not the case, but the condition of the fragile shells

* Another portion of the high-level gravels must have been formed before the country became inhabited, and would therefore also be unfossiliferous.

shows the Testacea to have generally lived on the spot; and the sharp and entire state of the greater part of the fossil bones shows that they have rarely been rolled far or subjected to much violence*. The deposits we have noticed might be, it is true, lake deposits so far as these organic remains are concerned; but the continuity and the distant transport of the gravel, and its uniform relative wear, point to water with a course, in each case, in one direction of commensurate length; while the absence of the remains of any independent basins, depressions, or barriers in the high-level valley, referable to old lakes, indicate that even the oldest beds are due rather to early river-action—the rivers, however, having been necessarily broader at some places than at others.

§ 4. THE BRICK-EARTH OR LOESS.

All the valleys of the South-east of England contain brick-earth forming banks on their sides and often beds at various heights on their slopes. This is particularly apparent in the valleys of the Stour, the Medway, and the Thames. In France it is seen on a larger scale in the valleys of the Somme and the Seine and of their tributaries. This deposit contains occasionally some land-, and rarely a few freshwater shells, and it is in almost every respect identical with the Loess of the valley of the Rhine. This well-known formation has been described as an independent deposit by various geologists, amongst others by Sir CHARLES LYELL†, who concluded that it was a fluviatile silt deposited by that river after its hydrographical basin had acquired very nearly its present outline of hill and valley; and he explained the height (near Basle 1200 feet above the sea) to which it rises above the valley by supposing a filling up of the main and lateral valleys by river silt during a period of subsidence, and a re-excavation in part during a subsequent upheaval. In many respects the analogy established by this distinguished authority between this ancient river-deposit and that of the plain of the Mississippi, and his argument in favour of its fluviatile character, are satisfactory and conclusive.

The difficulty I have felt in applying this hypothesis in its entirety to certain valleys in the South of England, and more especially to the valleys of the Somme and the Seine, has been the great height to which the Loess rises above the levels of these rivers, and the impossibility, for reasons assigned at p. 265, of admitting such a deposition of silt to have taken place since their present basins had been formed; nor would it be easy to conceive pre-existing depressions, of the depth we shall have to speak of, to have been filled up without an influx of the sea further inland than at present, and a conversion of the river-valleys into narrow estuaries far beyond existing tidal influences. But the evidence is rather in a contrary direction. The quaternary deposits adjoining the estuary of the Thames show conditions more purely freshwater than those which now obtain in the adjoining waters. The Loess at Menchecourt contains no trace of marine organisms; and in the valley of the Seine we have recently found freshwater shells in the low-level valley-gravels of Rouen. At the same time a subsidence so regulated as to be

* There are, however, a sufficient number of rolled fragments to show wear in river-shingle.

† Manual of Elem. Geol., 5th edit. p. 122–5. By some it has been considered a glacier mud.

exactly proportioned to the accumulation of river sediment and to keep out the sea is very difficult to imagine, and is certainly not supported by the facts.

What I propose to show is, on the contrary, that the brick-earth is intimately associated with all the valley-gravels and is contemporaneous with and dependent upon them from the beginning to the end of the series, the higher deposits having been formed before the excavation of the valleys, and those on the lower terraces being of later date.

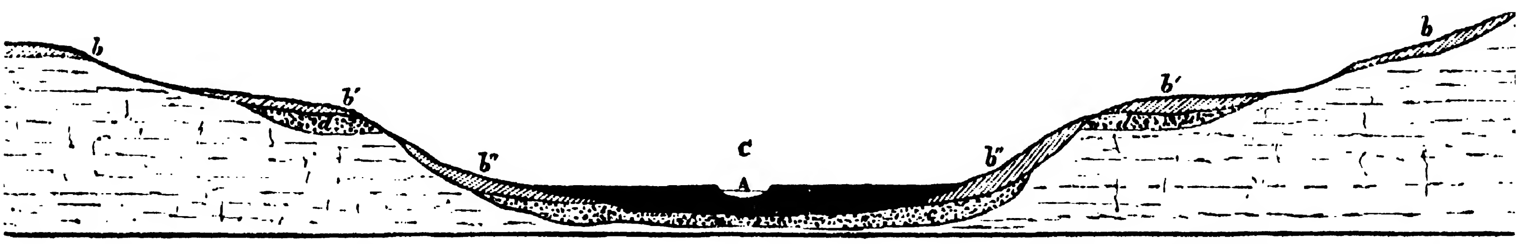
I have said that in the valley of the Somme the high-level gravels rise to the height of 100 feet above the river; and from some recent observations I conclude that there are some beds which attain a height of 150 feet. Now, if we take this latter level as having been the floor of the valley through which the river flowed at the period when these gravels were deposited, we shall find the Loess still extending to about 60 or 80 feet above such a plane. At the same time we know that the greater portion of it is accumulated in irregular masses on the slopes of the hills at a still lower level, where it either reposes immediately upon the chalk substratum or else covers the low-level gravels. In the adjoining valley of La Bresle I found traces of the higher gravels 40 feet above the river, and the Loess, with *Pupa* &c., rising to 110 feet above this level. In the valley of the Oise, near Creil, I noticed traces of gravel on hills 65 feet above the river, whilst the Loess rises to a height of 115 feet. In the valley of the Seine I observed the high-level gravel at the Pont de l'Arche rising 140 feet above that river, and again the Loess 50 feet higher. In the neighbourhood of Mantes and of Rouen we found analogous phenomena. The great height to which the Loess rises in the neighbourhood of Paris has often been remarked upon by the French geologists, and its occurrence on the hills above Meudon has long presented a difficulty*. Its height there is about 320 feet measured by an aneroid barometer. It is perfectly well developed, is several feet thick, contains a few land shells, and presents all the characters of the same material on the lower levels. M. Charles d'Orbigny has shown me the Loess rising up to the top of the hill above Ivry, and between the fort and Villejuif. It there attains a height of 305 feet above the river. These levels give a height above the high-level gravels of 150 to 200 feet, for at the Butte-aux-Cailles these are not more than 130 feet above the river; but some gravels I observed on a hill-terrace near Mont Valérien may be as much as 150 to 160 feet above the Seine. Further search may also show the existence of higher beds; or such beds may have been denuded.

However much, in fact, the Loess may extend beyond the limits of the present river-valleys, it is always bounded by higher hills flanking the plains and the lower ranges. Thus, though it spreads over the low plains and hills of Belgium, it does not rise more than two-thirds up the flanks of Mont St. Pierre at Maestricht. In Kent it extends far up the slopes of the hills flanking the river-valleys, but it is in all cases bounded

* This outlier of Loess has been represented as capping those hills. This would greatly increase the difficulty of connecting it with any old river-action, as it would extend the river boundaries to a comparatively indefinite distance. Such, however, is not the case. High as it is, it is still at least 100 feet below the summit of the hill.

by the higher chalk hills*. So again in the valley of the Somme, both the flanks of the valleys and the lower hills adjoining the valley are covered, but the higher water-sheds between the different valleys are free from it. A general section across the Somme valley may be represented thus (fig. 16).

Fig. 16.—*Theoretical section across the valley of the Somme.*



A. River. *a*. Recent alluvium. *b*, *b'*, *b''*. Loess on different levels and of different ages. *c*. Low-level gravels. *d*. High-level gravels.

The Loess is therefore, like the high- and low-level valley-gravels, connected with existing river-valleys, although the connexion is, owing to its irregularity and wider extension, not so apparent; and it becomes a question whether they may be related to the same common cause, presenting two phases connected with temporary changed conditions. In the first place, such organic remains as are found in the Loess are all common to the fluviatile gravels; but in the latter they are coordinate with a fauna of a more freshwater character. Thus there exists in the lower marl, gravels, and sands at Menchecourt the same species of *Helix*, *Pupa*, and *Clausilia* as occur in the overlying Loess. In the former, however, they are intermingled with *Limnæa*, *Bythinia*, *Planorbis*, and other freshwater shells, whereas in the Loess these shells are the exception (see p. 285). With the shells in the Loess are also associated the remains of the same species of Elephant, Horse, Deer, &c. as are found in the underlying series.

It is well known that in all rivers subject to floods and carrying down much sediment, as, for example, the Severn in its lower course, three forms of sediment will be deposited: 1st, coarse gravel and shingle in the more direct channel through which the waters flow with the greatest velocity; 2nd, sand and fine gravel in those portions of the more direct channel where the velocity of the stream is checked from any cause; and 3rd, fine silt and sediment in those parts where the flood-waters out of the direct channel remain for a time in a state of comparative repose; such places are the lee-side of the hills, lateral valleys, and plains, and any local depressions or hollows: none or little would accumulate in the main channel, as the scour of the retiring waters would there prevent its deposition.

In like manner I conceive the Loess to be the result of river-floods commencing at the period of the highest valley-gravels†, and continued down to the end of that

* In the valley of the Thames the Loess covers the low-level gravels in thick masses, but is very scantily spread over any part of the high-level gravels, and is rarely to be traced much above them.—March 1864.

† The Loess may have begun to form even before the fossiliferous high-level gravels, and therefore probably when the floor of the original valley was yet higher than these beds indicate.

of the lowest valley-gravel; that the higher beds (*b*) were formed at the time the higher-level gravels were being accumulated in the bed of the old river; the beds (*b'*) after the gravel (*d*) was left dry; whilst the lower beds (*b''*) result from inundations of the river after the excavation of the valley C, and when the higher levels were beyond the reach of floods. The same characters prevail throughout all the levels, except that the lower masses usually contain more shells, and show more distinct traces of bedding and lamination. That the deposit of Loess was out of the reach of the ordinary current of the river is evident from the circumstance of its containing no rolled pebbles or fluviatile shells—except a few of either sometimes swept in from the ordinary river-channel, and serving to show the connexion of the two deposits,—from the uniformity of its composition, and from the local nature and angularity of its débris. It presents precisely those characters which would result from fine deposition from turbid waters, with the occasional presence of a few angular fragments of the adjacent rocks. As the waters subsided, this silt would be left in sheltered positions, would be added to from year to year, until, as the river wore a deeper channel, the first-formed sediment would be left permanently dry, only to be disturbed by meteorological agencies, which would from time to time carry down portions of it into the river, now at a lower level, there to be re-deposited at such lower levels.

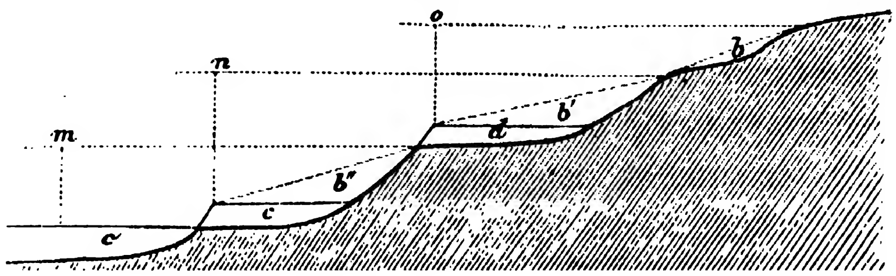
Another proof of its limited extent and of its not having at any time filled the valleys, is that in the lower levels, when it abuts against the hill-sides, it always slopes up towards the hill, following in fact nearly the present shape of the ground, and does not form horizontal beds cut off and truncated in the way which so generally occurs with the higher-level gravels, or when sedimentary strata have been excavated.

On this view of the subject the difficulty otherwise experienced in conceiving floods of a magnitude such that the waters would rise to the brim of these deep valleys, or in supposing these valleys to be filled up with silt and afterwards re-excavated, is avoided, for it would reduce the rise to be accounted for to limits more compatible with our experience of certain existing rivers. It is possible also that variations in the extent of elevation of the land may even cause the difference often to appear greater than it is.

There is one seeming difficulty to this hypothesis, which must occur to every field geologist: it is, the apparently distinct separation of the Loess and the underlying gravels wherever they occur in association. But a close examination will sometimes show thin seams of loam, not to be distinguished from the Loess, intercalated in the gravels, especially in the higher levels, though as a rule the masses of this material invariably overlie the gravels. The explanation of this fact is, I apprehend, to be found in the circumstance that it is only when the shingle or gravel ceased to be deposited, owing to the changes in the level or course of the river, that the Loess could accumulate. So long as the place was in the direct line and on the level of the stream, and subject to its scouring action, no such deposit could take place. Consequently, so long as the bed of the old river was on the level of *d*, diag. fig. 17, so long would the accumulation of the Loess from the rise of the flood-waters to *o* be restricted to *b*. When the valley

became deepened to *c*, the gravels *d*, though raised above the river-channel, would still be subject to be covered on the occasion of the flood-waters rising to *n*, and the Loess would accumulate at *b'*, covering the gravel *d* and sloping up against the side of the hill to the level of the line *n*. So again when further deepened to *c'*, the Loess would be deposited at *b''* so long as the flood-waters rose to *m*, or to any point between *c* and *m*.

Fig. 17.—*Diagram representing one side of a valley with a series of gravel and Loess beds.*



The Loess occasionally contains thin seams of gravel, derived sometimes from the underlying gravels *d* and *c*, but more frequently from the adjacent land-surface. The extreme sharpness, as a rule, of the débris from the latter, leads me to believe that the transport of such angular fragments, and the common irregularity of their bedding, arose from drifting shore-ice. The usually small size of the fragments and the general absence of subangular gravel is also compatible only with the thin ice that might be formed at night during the spring floods, and away from the shingle-strewed river-channel.

The valley-gravels by themselves give no direct evidence of the extent of the floods. They merely show, in the quantity of débris, the coarse shingle, and the worn blocks, the results of torrential action. If, however, we admit the flood-water origin of the Loess, it necessarily follows that, as we find this deposit on ground 50 (if not 100) feet above the highest beds of the valley-gravels (which fix approximately the position of the main channels of the old rivers), it gives a measure of the floods of that period, and shows them to have exceeded even those of arctic rivers at the present day, for the waters of these rivers rarely rise more than 40 to 50 feet above their low summer level. This fact furnishes, therefore, strong corroborative evidence of the scouring and erosive energy of these old rivers, and tends to strengthen the opinion, before expressed, of their power to excavate, when taken in conjunction with the other agencies before described, the large valleys through which the rivers now flow in such dwarfed volumes. I would observe that in each valley the height to which the Loess rises above the high-level valley-gravels is proportionate to the length of the valley and the area drained, showing therefore its dependence upon them, and that it is not referable to any uniform level or to any general cause.

Another character common to the Loess, and pointing, I conceive, to the same flood-water origin, is the presence of numerous small worm-like cavities penetrating the mass. These appear to me to be likely to have arisen in most instances from the presence of the vegetable and animal matter with which flood-waters are always more or less charged, and which, deposited with the mud, decomposed and gave rise to gases that, as they

escaped through the soft and pasty silt, leavened, as it were, the mass, and produced these innumerable tubular cavities.

Sometimes the Loess puts on a local character, derived from either adjacent chalk, sand, or clay beds. It then becomes so modified as to render its true character not easily distinguishable. In some cases it becomes very argillaceous, in others very sandy or very chalky. Occasionally it is so full of angular rock- or flint-fragments as to pass into an angular gravel. It is the presence of a local bed of this nature, made up in part of the angular fragments so peculiar to the Loess, and partly from the subangular gravel taken up from the underlying white gravel (Diluvium gris), and generally deeply coloured, which has given rise, in a number of cases, to the Diluvium rouge (part) of the French geologists, as exhibited in the sections at Charonne, Gentilly and Joinville, as well as at Abbeville and Amiens. Therefore, so far from being a separate deposit, this bed at these places is, I consider, merely an accident of the Loess, which again is merely a condition of a river-deposit of the period of the valley-gravels; so that, instead of the four separate deposits of 'Loess,' 'Diluvium rouge,' 'Sable lacustre,' and 'Diluvium gris,' into which some able French geologists would divide the deposits at Gentilly, Joinville, St. Acheul, &c. *, I would divide them, on lithological characters alone, into two groups—the Loess and the Valley-gravels; whilst so far as age is concerned I should consider them as one, representing phases of like causes under different conditions.

§ 5. ORGANIC REMAINS OF THE VALLEY-GRAVELS.

As it is very desirable to determine whether any changes in the fauna occurred during the period which elapsed between our earliest fossiliferous high-level valley-gravels and the latest low-level gravels, I have attempted to form separate lists of the organic remains of each series. I feel, however, that we are not yet in a position to do so with certainty or completeness. The difficulties in the way are, 1st, the uncertainty as to which series the beds may in some cases belong to; 2nd, the frequent absence of record as to the level at which the organic remains have been found; 3rd, the incompleteness of the search. I therefore submit these results merely as an approximation and a commencement. I am indebted to Dr. Falconer and M. Lartet for much valuable information respecting the mammalian remains, and to Mr. Gwyn Jeffreys for the correction of my former lists of the Mollusca, and for his kind assistance in determining additional species. As a term of comparison for the Mollusca, I have taken the group of land- and freshwater shells now inhabiting the districts† where the fossil species are found, adopting, generally, Mr. Gwyn Jeffreys's nomenclature and distribution for England, and taking for their range in France M. Picard's "shells of the department of the Somme"‡, M. Bouchard-Chanteroux for those of the "Pas de Calais," and M. Baudon §

* Bull. de la Soc. Géol. de France, 2e sér. vol. xii. pp. 1277, 1297, 1298; vol. xvii. pp. 18, 19, 67–78, 103. (With reference to M. Hébert's observations, 2e sér. vol. xvii. p. 18, see his "rectification," vol. xii. p. 255.) It was not until towards the end of 1859 that this able geologist expressed belief in the discoveries at St. Acheul.

† Forbes and Hanley's 'British Mollusca,' and Mr. Gwyn Jeffreys's 'British Conchology,' vol. i.

‡ Journ. de la Soc. Linnéenne du Nord de la France, vol. i. p. 149.

§ Soc. Acad. de l'Oise, 1855.

for those of the Oise. I have given in the Appendix the results of this examination of the range of the quaternary species of the beds under review.

HIGH LEVELS.—The high-level gravel of St. Acheul has furnished a distinct and tolerably complete list of Mollusca; but in England the evidence is more imperfect. Hoxne supplies but few species, and some uncertainty attaches to the level of Biddenham*.

Out of a total of 109 land- and freshwater shells now inhabiting the South of England and North of France, 36 species have been found in the flint-implement-bearing high-level gravels; and of the 12 freshwater genera 8 are represented in these old alluvia.

There is a singular scarcity of Unionidæ and Paludinidæ. The Neritinidæ are not present at all. The Limnæadæ, of species inhabiting marshes and pools, on the contrary, abound, together with, in places, the fluviatile species of *Ancylus*; the Helicidæ are, *Succinea* excepted, poorly represented, compared to their numbers in the low-level gravels. A variety of species are common at St. Acheul, while at Montiers one species of *Pupa* (*P. marginata*) occurs in thousands with but few other shells, and in the various other isolated patches of high-level gravels between Amiens and Abbeville, including the beds at Moulin Quignon, I have not been able to discover any traces of shells. They are equally rare in the high-level gravels of other valleys, except at a very few places. Combined with this scarcity, there is also an absence of such shells as would mark full deep rivers or lakes, whilst such as might be found in broad and shallow rivers, that, like many of those of Northern America at the present day, are flooded at one period of the year and nearly dry at others, occur at long intervals. In most of these northern rivers shells are also extremely scarce. Mr. BELL mentions an instance where he travelled along the banks of the river Magdalen for four weeks, during which the only shell he met with was a species of *Limnæa*†. This scarcity of shells is common in most of the rivers in these high latitudes. Along such rivers, however, there are often quiet pools, where shells are more numerous. If to the limited fauna found under these circumstances we add the land-testacea carried down by freshets and by small tributary streams, the shells so brought together would form a collection very similar in character to that which we find in these post-pliocene deposits.

At St. Acheul Mr. GODWIN-AUSTEN called my attention to a large sandstone block having on its surface numerous rudely worm-shaped lines of concreted sand, bearing a very close resemblance to those made by the sand-covered gelatinous attached polyzoa so common in our clear stony-bedded streams, as a proof that it had lain in a running stream. Such specimens are not rare, and taken with the absence of mud or clay beds in these gravels, and the general character of the shells, lead also to the inference that the old rivers were usually clear and limpid.

With respect to the species of Mollusca, they show, with few exceptions, a near

* I give the list of shells I have found at Biddenham, together with the list of those found by Mr. WYATT at Summer-house Hill in beds which certainly belong to the low-level series. I do not, however, feel sure that both Biddenham and Joinville, though distinctly above the low-levels, should not be placed on an intermediate level.—Feb. 1864.

† Geological Survey of Canada, 1857 & 1858.

identity with those now living in the same area. There are rarely sufficient modifications in size or form to lead us *à priori* to infer any material differences in the conditions under which they lived. Certain species of *Pisidium*, *Cyclas*, *Limnæa*, *Succinea*, *Planorbis*, *Helix*, &c., were then, as now, among the most abundant shells. Some varieties, such as one of *Pisidium fontinale*, and that of *Helix pulchella*, are, I am informed by Mr. JEFFREYS, forms more peculiar to the north of Europe. The *Hydrobia marginata*, though not known in northern Europe, ranges high in the subalpine tracts of the Jura and Alps.

But though there is nothing sufficiently specific in the individual species to indicate a climate different from that of the present day, there is at the same time nothing to require restriction to an identical climate. If, further, we look at the group as a whole, we shall find it to have not only a very wide range, but one more in a northern than in a southern direction. Of the 36 species found in these high-level gravels, 29 are found in the plains or on the hills of Lombardy*, whereas 34 of them range to Sweden and 31 to Finland; amongst these are the common *Succinea putris*, *S. elegans*, *Helix hispida*, *H. nemoralis*, *H. pulchella*, *Zonites radiatulus*, *Pupa marginata*, *Limnæa peregra*, *L. palustris*, *L. truncatula*, *Planorbis vortex*, *P. complanatus*, *P. albus*, *P. spirorbis*, *Bythinia tentaculata*, *Ancylus fluviatilis*, *Valvata piscinalis*, *Cyclas cornea*, *Pisidium amnicum*, and *P. fontinale*. Many range further north in Europe, but there is no list of the land and freshwater shells of any more northern district so complete as that of Finland†. It is known, however, that a great number of these same species are found in Siberia. The annual freezing of the rivers, even in less northern latitudes, proves at all events the power of these mollusks to endure a great winter cold‡. This capability of enduring severe cold, and their northern range, show that they could readily have adapted themselves to great changes of climate in the region where they are now found. The general absence of southern species is also not without its significance; for while out of a total of 77 Finland species 31 have been found at the few places named in our list (see Appendix), only 29 out of the 193 Lombardy species have been met with in these beds.

Our knowledge of the fossil Mammalia of the high-level gravel-beds is very incomplete. I might, it is true, include those I have reason to believe are of the same age and which would furnish us with more complete and positive data—such, for instance, as a great number of the cave-deposits; but, from the circumstances I have before alluded to, I deem it best to confine myself to these particular beds and to the localities where flint implements have been found. Few of our own high-level gravels contain any organic remains at all: in the greater part they are entirely absent.

* Catalogo dei Molluschi della Lombardia, by A. & G. B. VILLA. Milan, 1844 & 1853.

† Finland Mollusker, af Nordenskiöld och Hylander, Helsingfors, 1856.

‡ PICARD mentions an instance where some specimens of *Limnæa auricularia* he had placed in a vase, froze in the ice, which was exposed to a temperature of $-2°$ Fahr. When the thaw came and they were released they soon recovered and became as active as ever.—*Op. cit.* p. 278.

The mammalian remains at present recorded from the High-level Flint-implement-bearing Gravels are:—

	Hoxne.	Bedford. *Biddenham.*	Amiens. *St. Acheul.*
Elephas primigenius	* (sp.?)	*	*
—— *antiquus*	....	*	*
Rhinoceros tichorhinus	....	*	....
Equus (the common fossil species)..	*	*	*
Bos (*primigenius?*)	*	*	*
Cervus	*	*	*
—— *tarandus*	....	* ?	....

This list is very small, and would not of itself suffice to prove the character of the climate. We know only that the *Elephas primigenius* and *Rhinoceros tichorhinus* are considered, from their association and their structure, to have inhabited countries possessing cold climates; and that their remains are found chiefly in the subarctic and north temperate zone, and are not known south of the temperate zone. We know also that the Horse and the Ox brave the winters of Siberia and Northern America. The Reindeer is essentially a northern animal.

Of the flora of this period we have very limited means of judging. Generally the high-level gravels contain no vegetable remains, with the rare exceptions of mere fragments of decayed wood. Occasionally, where lodgements have been effected out of the direct course of the main stream, plant-remains are more abundant, and the wood larger and more perfect—so much so as to lead to a hope that ultimately we may find a favoured spot containing the requisite evidence respecting the vegetation of the period. In the mean time there have been found at Hoxne large pieces of the stem and branches of the Oak, Yew, and Fir, and traces also of leaves and seeds *. These trees, common in our latitudes, have at the same time a considerable northern range: and, as with the Testacea, there is thus far an absence of southern forms. If we may consider the Oak as defining an extreme of climate, we shall find ourselves restricted in a northern direction to some moderate degree of cold, for the Oak in Europe does not extend beyond 58° to 63° lat., with a winter temperature of from 15° to 20°. In America it reaches, and that only in a stunted form, no further north than the basin of the Saskatchewan, within a short distance of Cumberland House, in 54° lat., where, however, the winter temperature is $-3°\cdot7$ †. It may be a question whether in those latitudes where the sun would have more power than in the higher latitudes first named, and where consequently the ground in the summer would thaw to a greater depth, the Oak may not have thriven under a winter temperature more nearly like that of the American region.

* I since found, in the autumn of 1863, a few impressions of a leaf, apparently of a Bilberry, in bed *d*, at Hoxne. With these there were also numerous valves of *Cyprides* and some calcareous grains of *Arion ater*.—Feb. 1864.

† Arctic Searching Expedition, by Sir JOHN RICHARDSON, London, 1851; The Vegetation of Europe, by A. HENFREY, London, 1852; Géographie Botanique, par A. DE CANDOLLE, Paris, 1855.

It is evident that if we had to depend only upon the organic remains for decisive evidence of the nature of the climate of the period under inquiry, we might fail to arrive, on the present data, at any exact or positive conclusion. All the recent species are such as are now to be found within the limits of the temperate zone, but they appear to agree better with the fauna and flora of its northern than of its southern provinces; the Fossil Mammalia may also, from their general association and distribution, be considered to have inhabited cold countries; so that there is a balance in favour of the probability of a severer, but not of an extreme climate. On the one hand, we are restricted in the degree of mean winter cold by the presence of trees, and more especially of the Oak; on the other, we are restricted in the degree of heat by the range of the Reindeer and the absence of southern forms.

If, further, we take these indications in conjunction with the physical features before described, the conjoint evidence has more weight and preciseness. Limited as the evidence of the organic remains is, it is at all events in accordance with the physical evidence in favour of a considerable winter cold. It is possible even to attempt some approximate limitation. Thus a climate where the Oak, the Yew, and the Fir (and the Bilberry) throve, where Reindeer lived, where Deer, Horse, and Ox abounded, and where the rivers were subject to periodical floods, and froze so as to transport large boulders for considerable distances, presents conditions which would probably accord with a mean winter cold of not less than 20°, while it may have been as low as 10° or even lower. This would be from 19° to 29° under that which now obtains in these regions, taking the winter temperature of the S.E. of England and N.W. of France at 39° —a difference, under normal conditions, equal to that of from 10° to 20° of latitude on one meridian.

Low Levels.—The organic remains of this series are more numerous and afford better evidence of climatal conditions.

Mollusca.—Of the 36 species of shells enumerated in the high-level gravels, 31 occur also in our lower gravels, together with 20 species which have not yet been found in the higher series of these districts, making a total of 51 species belonging to the beds now in question*. In the present state of the inquiry I attach little weight to the presumed absence of 5 high-level gravel species; while it remains to be seen how many of the additional 20 species may be found in higher beds elsewhere. Amongst the additional species are the *Carychium minimum* and *Helix fruticum*. The former has a wide range in England and over every part of the Continent; the latter, on the contrary, is a temperate and north European species, but does not live in England. The *Helix arbustorum* is also of the small variety characteristic of cold or alpine regions, while the *H. pygmea* is likewise a northern species. On the other hand, several other species of *Helix* (see Appendix) found in these beds range through central and southern Europe only. The other species to be noticed in these newer beds, are the *Pomatias obscurus*,

* The lists from the low levels of Abbeville and Bedford are given in the Appendix. The latter are from Mr. Wyatt's descriptions of the sections at Harrowden and Summerhouse Hill.

Clausilia plicatula, and *Vitrina diaphana*, shells now living in France but not in England, and all of which Mr. GWYN JEFFREYS has recognized in my collection from Menchecourt*. The last-named shell is more especially alpine; whereas the *Cyclostoma elegans*, which abounds at this place, and has not been found at either St. Acheul or Biddenham, has a range essentially southern, not being now found north of central Germany†.

Besides the land and freshwater shells, the following species of marine and estuarine shells have been met with at Menchecourt:—

Buccinum undatum.	*Littorina squalida.*	*Cardium edule.*
Fusus.	*Nassa reticulata.*	*Ostrea edulis.*
Littorina littorea.	*Purpura lapillus.*	*Tellina Balthica.*

These, like the freshwater shells, present, with two exceptions, no differences from the recent species. These exceptions are, according to Mr. GWYN JEFFREYS, *L. squalida*, which is now absent from our shores, but lives on the coast of Norway; and the *T. Balthica*, which is of the northern variety. Both these shells are found in other postpliocene deposits of this country, and range back as far as the Crag.

But the most important shell at Menchecourt is the *Cyrena fluminalis*, of which I have found three specimens‡. This mollusk, as is well known, now lives in the Nile and in mountainous streams of central Asia—a range presenting great extremes of climate.

It is evident that at Menchecourt the waters were saline or brackish. As in the estuary of the Thames at present, the *Cardium edule* and the *Littorina littorea* were then common shells. The *Buccinum undatum* was far from rare. Of the *Ostrea* I have only found one dwarfed specimen. Associated with these are numbers of *Limnæa*, *Helix*, and other freshwater and land shells, washed down into the estuary by freshets of the old streams or rivers. On some of the large flints in bed *e* (former paper, p. 284), which are as fresh and perfect as though just taken from the chalk, I have found attached the opercula of *Bythinia*. These flints were probably carried down undisturbed with the *Bythinia* itself attached, from the bed of some small tributary stream, by the agency of ice.

That shells should be found at Menchecourt and Mautort and not at Montiers and St. Roch, arises, I imagine, from the overwhelming floods, and from the large mass of constantly shifting shingle along the bed of the old river, which would be influenced thereby throughout the length of its main channel above tidal influence. But where the tide met the current, near Abbeville, the force of the latter being much checked—especially where any projecting land sheltered portions of the river from its full effect,

* A curious feature of the period is the abundance of two species of slugs. In some of these beds, and more especially in the Loess, at Menchecourt I had found numbers of small oval calcareous bodies, for the origin of which I was at a loss to account until Mr. GWYN JEFFREYS recognized them as the calcareous grains found in the *Arion ater*. The small shelly shield of the *Limax agrestis* is also found, but less abundantly.—April 1864.

† GWYN JEFFREYS, 'British Conchology,' vol. i. p. 305.

‡ It is singular that in the greater number of places where the *Cyrena fluminalis* occurs in a fossil state the waters have been brackish or estuarine, as at Grays, Chislet, Abbeville, and in Norfolk (the Crag) &c.

as the hill of St. Gilles with respect to Menchecourt—there we might expect a quieter deposition of sedimentary matter; and this we find to be the case at the latter place.

These causes, as in all rivers of the present day subject to heavy periodical floods, must have operated very generally during this quaternary period, and have rendered the remains of freshwater mollusca in these fluviatile deposits the exception and not the rule. As a proof of this, I may remark that I am not aware that land and freshwater shells had been noticed in the lower gravels of the Seine valley until I discovered the few species mentioned at page 261*. Neither the valley of the Lark nor the valley of the Waveney have yet yielded any. In the valley of the Ouse they are more common, though still comparatively scarce.

As with the pulmoniferous Testacea of the high-level gravels, although there is nothing in the individual species found in the low-level series to give a definite clue to the character of the climate of the period, still the group maintains its northern tendencies, there being, out of a total of 52 species, 42 now living in Sweden and 37 in Finland (or nearly one-half of the Finnish species), whereas only one-fifth, or 38, of the Lombardy species occur in these quaternary beds. The marine shells are also common temperate and cold-climate species. The *Littorina squalida* alone slightly weighs the balance in a more northern direction; but, on the other side, we have to notice the introduction of a few more southern land and freshwater species, such as the *Unio littoralis*, of the *Helix caperata*, *Pomatias obscurus*, and the abundance of *Cyclostoma elegans*, which may be taken as some evidence of a more genial climate than that of the period preceding it. The profusion also of the land and freshwater testacea, and the greater abundance and variety of animal life, support this latter view. The great bulk of the species, both of Mollusca and Mammalia, being common to both levels of gravel, we may presume that no very important change in the mean temperature then took place, and that any transition was gradual, although it is possible that the winter climate of the one period may have differed to some extent from that of the other.

The following species of Mammalia have been found in these low-level gravels. I am indebted to Dr. Falconer for the determination of all the Bedford specimens, and to M. Lartet for the Paris list. The species from the Somme valley are the same as given in my former paper.

This list, although more complete than that of the other series, must be taken only as a very partial representation of the fossil Mammalia of the period. There are other localities beyond our inquiry which, as with the Mollusca, would afford a larger and more varied list, like, for example, the valley of the Thames†; but, as I purpose to show their synchronism at length, I consider it better in this more special inquiry to confine myself to the above species. There are enough, independently of other localities, for the general argument, which, so far as regards the Elephant and Rhinoceros, follows the same line as that relating to the high-level gravels.

* Besides these (*ante*, p. 261) I found in a marl in a higher level and corresponding with the Loess, *Helix hispida*, *H. pulchella*, *H. nemoralis*, *Pupa marginata*, *Succinea putris*, *Arion ater*, and *Bythinia tentaculata*.

† And the valley of the Wiley at Salisbury (Dr. Blackmore's remarkable collection).—March 1864.

	Bedford, *Great Northern Railway, or Summer-house Hill.*	Abbeville, *Menchecourt.*	Amiens, *St. Roch.*	Paris, *Grenelle, Ivry, Clichy, or the Rue de Reuilly.*
Elephas primigenius, *Blum.* . .	*	*	*	*
—— antiquus, *Falc.*	* *s*	. . . .	*	* *r*
Rhinoceros tichorhinus, *Cuv.*	* *r*	*	*	*
—— megarhinus, *Christol.*	* *r*	. . . .	. . . .	* ?[1] *r*
Ursus spelæus, *Blum.*	* *s*	*	. . . .	. . . .
Hyæna spelæa, *Gold.*	. . . .	*	. . . .	* ? *g*
Felis spelæa, *Gold.*	. . . .	*	. . . .	*
Bos primigenius, *Boj.*	* ?	*	*	*
Bison priscus, *Boj.*	* *r*	*	. . . .	* *r*
Equus (possibly two species)	*	*	*	*
Cervus[2] euryceros, *Aldr.* . .	* *r*	. . . .	. . . .	. . . .
—— elaphus, *Linn.*	*	*	*	*
—— tarandus, *Linn.*	*	*	. . . .	* *r*
Hippopotamus major, *Nesti.*	*	. . . .	*	* *g*
Sus	* *r*	. . . .	. . . .?	*

[1] This determination is by Dr. FALCONER, but rests upon a single fragment of a tooth I found in the pit at the Rue de Reuilly. I have a molar tooth of probably another species of *Rhinoceros* (*R. hemitœchus*, Falc.) from Bedford, but it is too low worn for a confident determination. [2] I omit the *Cervus Somonensis*, as Dr. FALCONER and M. LARTET consider its specific distinctness very doubtful. The specimens from the railway cutting are in my collection; those from Summerhouse Hill are in Mr. WYATT's. Where, in columns 1 and 4, the specimens are known to me only from one of the localities, that place is designated by its initial letter.

There is no doubt that the Reindeer lived during the period of formation of the St. Acheul beds, although not found at that spot; in the lower valley series it is a common species. We know that this creature now ranges from the shores of the Arctic Sea to about 46° N. lat. in Asia, to 47° in America, and 60° in Europe *. This corresponds, in the first instance, with a winter temperature of about 19°; in the second with one of 16°; and in the third with 23° Fahr. These parallels, however, give only the line of winter migration of the Reindeer. Their chief home is in the more arctic districts, of which these latitudes are merely the southern boundaries. The Aurochs is now restricted to a region of which the winter climate is 25°.

The Musk Ox, which is found fossil in the low-level gravels of the valley of the Oise† (as well as in the Thames valley), is more essentially an animal of cold countries, ranging now only from the extreme polar regions to lat. 64° N. in arctic America.

On the other hand, there are two animals which might be considered to militate against this northern tendency. The one is a large *Felis*. It is, however, well known that a species of Tiger is common on the Lower Amoor, where the river is frozen for five months in the year. A Tiger also lives constantly in the severe climate of the district around the sea of Aral, where the shore-waters and rivers freeze every winter. In his survey of that district, Commander BUTAKOFF‡ remarks that "Tigers roam constantly in

* RICHARDSON, 'Fauna Boreali-Americana:' DESMAREST, in D'ORBIGNY's 'Dictionnaire d'Histoire Naturelle,' art. "Renne:" LOGAN, 'Geological Survey of Canada for 1857,' Appendix, pp. 227, 244.

† Flint implements have been found in these gravels.

‡ Journal of the Geographical Society, vol. xxiii. p. 95 (1852).

the vicinity of Aralsk, and particularly in the winter, notwithstanding the frost." The January temperature of this part of Asia falls as low as between 14° and 10° Fahr.

The other is the Hippopotamus. Remains of this latter creature are met with at St. Roch, but none are yet recorded from Menchecourt. It is found also in abundance at Bedford. In the various flint-implement-bearing localities it is confined to the low-level gravels. Should this prove to be the rule, which I am not prepared yet to assert, it will be one of some interest. The difficulty felt about the possibility of the Hippopotamus living in a severe climate, arises from the habits of this creature leading it to pass so much of its time in the water. But if the possibility, so far as regards the supply of food and protection by special covering against the cold, of the other large pachyderms living in such a climate be admitted, then why should not the Hippopotamus also have been fitted for a cold climate, provided it partook of the same special conditions. Like its congeners the Elephant and Rhinoceros, this Hippopotamus belongs to an extinct species, and it becomes a question whether, like them, it may not have been adapted to endure the rigours of a severer climate than the living species of these genera can now endure.

Plants.—On this point our ground is almost barren. A few traces of decomposed wood, and one solitary small specimen, apparently of a branch of the common *Chara*, from Menchecourt, are all we possess from the places under review. This plant is found in almost all the rivers of Europe, extending as far as the Volga in lat. 56° and 60° N.

After examining the Fauna and Flora of the low-level gravels, we cannot but feel that the premises from which we have to draw our conclusions respecting the climate of the period are still limited. The physical features show an absence of those marked indications of ice-action we detect in the high-level gravels, but point to the presence of ice in quantity sufficient to transport large boulders. The shells throw a little more light on the question, showing the continued prevalence of a northern group, into which, however, several southern forms have been introduced. The Mammalia continue, with few exceptions, to give evidence of the persistence of a rigorous climate. On the whole, although the climate may have been less severe than that of the previous period, it is probable that the winter temperature was not higher than some point between 15° and 25°. The circumstance that the old valleys differ from the excavations made by existing rivers—which cut deep gorges rather than broad valleys with sloping sides—rather confirms the opinion that the winter cold and spring floods may have diminished from year to year throughout the period of the valley-gravels, the result having been to cause the channels made by these old rivers to be of gradually contracting dimensions: hence possibly the difference in width between the top and the base of C (fig. 19, p. 298), and hence in part the sloping sides of the valleys.

LOESS, LOW-LEVEL.—I have found in this deposit at Menchecourt, besides the remains of Mammalia common to the underlying sands and gravel,—

Arion ater.	—— *hispida.*	*Pupa marginata.*	*Pisidium amnicum.*
Limax agrestis.	—— *nemoralis.*	*Vitrina diaphana.*	—— *fontinale.*
Clausilia (rugosa?).	*Helix arbustorum,* var.	*Zonites radiatulus.*	Pupa, Helix, and Arion are abundant; Pisidium rare.
Succinea elegans.	—— *pulchella*	*Zua lubrica.*	

Loess, high-level.—There is a portion of the horns of *Cervus elaphus* in M. Boucher de Perthes' collection, from a bed of high-level clay near St. Riquier; exact level not known. Mammalian remains are reported to have been found in a clay-pit on the plateau between Treport and Abbeville, but I have myself never found either shells or bones in such positions.

Note.—In the original paper read in March 1862, I had introduced a discussion on the uses of the flint implements, treating them as fossils of this period. The subject, however, is too long and too hypothetical to enter upon here. I would merely remark that these rude implements may almost all be referred to flint-flakes for cutting and flaying, and to pointed weapons of offence and defence. There are ovoid forms to which it is difficult to assign a use. Some of the more spatula-shaped implements I suggested might have been used as ice-chisels: in arctic regions the inhabitants never travel in winter without some such instrument attached to a stick, for the purpose of obtaining water when required, or for making holes in the ice for fishing. I may also remark that in the high-level gravels the lance- or spear-headed instruments predominate, whilst in the lower-level gravels the simple flakes of various shapes are the commoner forms.

§ 6. CLIMATAL CONDITIONS. EXCAVATION OF THE VALLEYS.

I have shown, on the authority both of Continental and English geologists, as well as by the evidence brought forward by myself in this or in my former paper,—

1. That certain beds of gravel, at various levels, follow the course of the present valleys, and have a direction of transport coincident with that of the present rivers.

2. That these beds contain, in places, land and freshwater shells in a perfect and uninjured condition, and also the remains, sometimes entire, of land animals of various ages.

3. That the extent and situation of some of these beds of gravel so much above the existing valleys and river-channels, combined with their organic remains, point to a former condition of things when such levels constituted the lowest ground over which the waters passed.

4. That the size and quantity of the débris afford evidence of great transporting power; whilst the presence of fine silt, with land shells, covering all the different gravel beds, and running up the combes and capping the summits of some of the adjacent hills to far above the level of the highest of these beds, points to floods of extraordinary magnitude.

These conditions, taken as a whole, are compatible only with the action of rivers flowing in the direction of the present rivers, and in operation before the existing valleys were excavated through the higher plains, of power and volume far greater than the present rivers, and dependent upon climatal causes distinct from those now prevailing in these latitudes. The size, power, and width of the old rivers is clearly evinced by the breadth of their channel, and the coarseness and mass of their shingle beds; whilst the volume and power of the periodical inundations are proved by the great height to which the flood-silt has been carried above the ordinary old river-levels—floods which swept down the land and marsh shells, together with the remains of animals of the adjacent shores, and entombed them either in the coarser shingle of the main channel, or else in the finer sediment deposited by the subsiding waters in the more sheltered positions. As the main channel was deepened from year to year by the scouring action of the rivers, the older shingle banks were after a time left dry, except during floods,

when they became covered up with the flood-silt, which, extending also over the adjacent land and shores, was there deposited directly upon the rocky substratum. As the channel became deeper, and the tributary valleys partook of the same erosion, they, being out of the main river-current, tended especially to receive thick deposits of the flood-silt (Loess), while the higher grounds were left permanently dry.

Rivers subject to periodical floods are extremely variable in their course and direction, flowing first on one side and then on the other side of the valley, shifting the shoals and gravel banks, and distributing them in a very irregular manner. Consequently it is by no means necessary to suppose that a bed of gravel like that at Oakley and Abbots Thorpe extended across the valley of the Waveney, or that the gravel bed of St. Acheul extended the whole width of the valley of the Somme, but we may rather infer that local conditions led to the great accumulations of gravel at certain spots, especially on the sides, whilst others would be left more or less bare. The subsequent denudation may therefore have been comparatively slight, and the present outliers of high-level gravels may yet represent a not inconsiderable portion of the alluvium of the old rivers. That these rivers had at times a torrential character, is evident from the nature of their transporting power, as indicated by the prevalence and coarseness of the gravels, by the absence of mud-sediment, and by the rough and irregular lines of bedding. But not only have we these exhibitions of the power of the old rivers; it is further evident, from the presence in the terrace-gravels of large blocks, often but little worn and transported from considerable distances, together with much sharp and angular smaller débris, that there was some other power in operation besides the ordinary transporting power of water, great though that be. For the blocks in the one case would have shown an amount of wear in proportion to the length of transport, and the smaller débris would have been separated from the larger; whereas the blocks are always more or less angular, they are scattered indiscriminately through the gravel, are often associated with the most delicate and fragile shells, and with bones of Mammalia but little or not at all worn. The only cause adequate to produce these results is, I conceive, the action of river-ice, whereby these blocks and a portion of the débris were carried down and deposited along the river-channel, more especially in those parts where the currents may have been checked either by a widening of the river or by the influx of a tributary stream. The recent phenomena, with reference to the transport of blocks by ice on the St. Lawrence at its breaking up in the spring, have been so well proved by Captain Bayfield* and Sir W. Logan†, and illustrated by Sir Charles Lyell and other geologists, that it is unnecessary to enlarge upon them here. I may, however, mention that more lately in sinking the caissons for the Victoria Bridge at Montreal, the bed of the river, through its width of about two miles, was found to be strewed with large rock boulders.

The remarkable contortions in the clay cliffs of Norfolk have been attributed by

* Proc. Geol. Soc., vol. ii. p. 223 (1836).

† Ibid. vol. iii. p. 766, and Canadian Journal.

Sir CHARLES LYELL to the grounding of icebergs on the soft sea-bed, and I am disposed to attribute to a somewhat like action, on a small scale, of the river-ice, the analogous structure exhibited in the St. Acheul and other high-level gravels (figs. 13, 14, 15, p. 269). Mr. A. MURRAY, in exploring the shores of the Mississagui river, noticed instances of similar recent effects of ice, "where the coarse shingle was loosely piled up in great conical heaps. The accumulations were usually at a turn in the river where there was a strong current above. The ice, brought down with violence and impinging on the side at the turn, appeared to have ploughed up the shingle and pushed it forward on to the bank. One of the heaps was estimated to be 10 feet high at the apex, with a diameter at the base of 40 to 50 feet; it rested on closer packed materials of the same kind, which also formed the bed and the margin of the stream in the neighbourhood"*.

These results agree with and confirm the indications furnished by the organic remains, viz. that at the period of the high-level gravels the winter cold, which so froze large rivers as to furnish ice-rafts capable of transporting innumerable boulders, many of 5 to 10 tons weight or more, for great distances, was not less than that of Moscow or Quebec at the present day, and that it may have been even lower. It is generally admitted that previous to this time, in the pliocene or early post-pliocene period, the cold was still more severe. Then the greater part of England was under the sea, whereas Switzerland and the greater part of France had emerged from the sea at an earlier, or Miocene period, and there is no proof of their having been subsequently submerged.

It was during this previous period of intense cold that the wonderful extension of the Alpine glaciers took place, and that many minor chains, such as the Jura and the Vosges, had also their glaciers. On the north of the Alps these old glaciers descended to within 1200 to 1000 feet of the present sea-level, whilst those now existing in Switzerland do not come lower than within 3400 feet of that level. M. LEBLANC† has calculated that such a difference of level might be accounted for by a reduction in the mean annual temperature of 12½° Fahr. But although that might give the limits to which glaciers could descend in these latitudes under ordinary circumstances and like conditions, it by no means proves that a greatly lower temperature may not have accompanied and hastened their enormous growth; nor, when we look at the length and extent of the valleys, of which the fall is but small, over which the old glaciers passed, can their progress along surfaces so slightly inclined be compared with that where the inclination, as usually in their present beds, is steeper and the channel narrower‡. I do not believe, therefore, that this estimate of M. LEBLANC furnishes us with even an approximation to the extreme cold of that glacial period. If, however, we were to assume that,

* Geological Survey of Canada, for 1858, by Sir W. LOGAN, p. 103.

† Bull. de la Soc. Géol. de France, vol. xii. p. 132 (1841).

‡ Thus in the valley of the Aar, where the inclination of the surface is about 2½°, the glacier of the Aar does not come down lower than within 6000 feet of the sea-line; whereas the glaciers Du Bois and des Bossons, with beds inclined at 8° to 10°, descend to within 3500 feet of the sea. On the Italian side of the Alps the old glaciers descended lower and nearer to the sea-level than those on the Swiss side.

at a certain time towards the end of the glacial period, a temperature of 12½° below that of the present day had supervened, the further extension of the glaciers may have been thereby checked; and as the present mean annual temperature of the S.E. of England and N.W. of France may be taken at 50°, this would have made it equal to 37½°—the mean annual temperature of the two stations before named, Moscow and Quebec, being respectively 40°·02 and 41°·85 *, and that of Cumberland House (Northern America) being 30°. This mean annual temperature would agree with the conclusions at which we had arrived with respect to a mean winter temperature below 20° and above 5°, or possibly between 10° and 15°, having prevailed during our high-level gravel period.

If there had been no amelioration in the climate at the period of the high-level gravels, the permanent ice and snow accumulated during the preceding long-continued and severe cold on the hills and mountain-chains of Europe would have remained without change, and the discharge of the rivers would only have been in proportion to the annual rainfall, whatever that was; and if that fall were not excessive, we should have no extraordinary agents in operation beyond the winter frosts and snow and the attendant spring floods. But if we suppose that (as must necessarily have happened at some time between the glacial period and the recent period), owing to a further improvement in the climate, the winter temperature became permanently and most probably gradually higher, then it would follow that during each recurring spring the rivers would have had their former ordinary discharge increased by the addition to the annual rainfall of a certain proportion of the snow and ice stored up during the former cold period. This quantity might have been equal to the accumulation of one, two, or more winters, according as the rate of elevation of the mean annual temperature was slow or rapid. In all valleys connected with mountain-chains the result of these climatal changes must have greatly increased the power of the annual floods—whence the greater excavation of the valleys connected with such regions. In our case, however, the extreme conditions do not generally apply, though I believe the foregoing general cause influenced the results. Most of the valleys we have to investigate are not connected with areas of old glaciers. The Waveney, Ouse, and Somme are not so connected. No traces of old glaciers are recorded even in the Ardennes, and those of the Morvan have been contested †, although I think without sufficient reason. Nevertheless with the degree of cold we suppose to have existed at this subglacial time, the mere winter accumulation of ice and snow on the higher ranges of hills must have been large. The effects of the greater water-power observable in the valley of the Oise, and of the Seine more especially, may be due not only to the height of the ranges of hills in which they take their source, but also to the larger areas of drainage.

Starting from the point that the high-level gravels of our district are of an age subsequent to the maximum period of cold, that they mark a period during which the winter temperature was gradually becoming less rigorous, and that the excavation of

* The various mean temperatures are from Dove's valuable Tables, Reports of Brit. Assoc. for 1847.

† Bull. de la Soc. Géol. 2nd series, vol. ii. p. 683 (1845).

the valleys proceeded with greater energy in consequence of successive increments in the mean annual temperature of each succeeding year, let us consider what other effects might have resulted from the operation of these causes.

The mean annual rainfall of the South-east of England and the North of France is 24 inches. The chief fall is in autumn, and the greater portion of it is carried off as it falls; and there is rarely any large accumulation of winter snow. This fall is so small that it requires but a moderate excess in the fall of any one period of the year to produce floods which cover the whole breadth of the present valley-channels. It was a fall of only $3\frac{3}{4}$ inches* in the twenty-four hours that caused the disastrous floods of Morayshire and Aberdeenshire in 1829. Amongst the other remarkable facts connected with that event, Sir T. Dick Lauder† states that at Invercauld the small river Dee rose $14\frac{1}{2}$ feet above the usual level, and spread 400 yards wide. A tributary stream cut away 6000 square yards of gravel, and spread the débris over thirty acres of land. Lower down, at Maryfield, the Dee rose 25 feet above its ordinary level. At Park the rise was 13 feet, and the breadth of the inundation not less than half a mile‡. At Murtle the river changed its channel from one side of the valley to the other, and acres of land were covered with gravel brought down from the upper parts of the river. The Findhorn rose in one place 50 feet, and in another place cut "a new channel for itself for at least a quarter of a mile"§. Although, owing to the difference in the geological nature of the ground, the effects of such an exceptional rainfall in the south of England would be less than in Morayshire, it would not require any extravagant addition to the small rainfall of the present day to increase both the permanent volume and the floods of our rivers to the extent even of producing inundations more of the character of those indicated (at page 276) by the position of the brick-earth, or of those of arctic regions. Such a result might have been formerly obtained, 1st, by a direct increase in the rainfall; 2ndly, by the accumulation and rapid melting of the winter snow; or by the two causes combined; and 3rdly, by the fall of rain in the spring while the ground was in a frozen state ‖.

The line of 35 inches rainfall now touches the north-western point of France, the western point of England, and the south-western part of Ireland. An advance inland of this line, arising from the greater precipitation determined by the low temperature of the land surface, would result from a general winter covering of snow—the accompaniment of a climate of the character we have inferred. It may be objected that, judging from the fact of the decrease generally observed in the rainfall in proceeding

* Sir Dick Lauder considers, however, that the fall may have been greater amongst the hills at a distance of twenty to fifty miles, but we are without information on this point. The rain for the month was 7·36 inches at Huntley, and $8\frac{1}{2}$ inches at Inverness.

† An Account of the great floods of August 1829, in the province of Moray, &c., 2nd edit. p. 372.

‡ Ibid. pp. 390, 391.

§ Ibid. pp. 33 and 104.

‖ This is of rare occurrence in this country, but when it does happen it leads to disastrous floods. Mr. Evans informs me that the only occasion on which the valley of the Gade is flooded is when rain falls after a evere winter before the ground is thawed.

from the tropics to the Arctic zone (it being but 17 inches at St. Petersburg*, and 76 inches at the tropics), we might expect the rainfall to have been less, rather than greater than at present, in the subglacial period. But in cases, whatever the latitude, where we have cold surfaces presented to vapour-laden sea-winds, as in the mountainous districts of the north-west of Spain, in our own lake districts, and in Scandinavia, we find a very heavy rainfall, it being 82 inches at Bergen and 104 inches in Westmoreland. At Sitka also, in lat. 57° N., the rainfall is almost constant.

But even if a greater rainfall be problematical, a greater concentration of it cannot be so considered: it would follow as a necessary consequence of the low winter temperature. Sir R. MURCHISON†, speaking of the appearance of part of Russia in the spring time, makes the following apposite remark: "The enormous volume of water, by which large portions of the surface are still covered at every annual melting of the snows, can scarcely be imagined except by those who have travelled (we may say sailed) over some of the central and southern countries in the spring season, when to the eye of the geologist the lands seem to be emerging like isles and promontories on all sides from beneath the waters. It is then that each broad valley is, for six weeks or more, in a condition similar to that which we can imagine to have been the state of England, France, and other countries, when their streams, instead of occupying their present beds, were lake rivers or estuaries of great width, wherein many of the old gravel and sand banks of geologists were accumulated, and in which the bones of extinct mammals are found. The height of the waters during this annual inundation can indeed be exactly read off wherever any great stream has rocky banks. In gorges we have occasionally noted the spring high-water mark as having been 40 feet above the dry summer level."

A very similar observation is made by Baron WRANGELL‡, who says, "the overflowing of many of the rivers on either side of the Ural Chain impeded our journey, but made us amends by the variety which was thus given to the landscape—the valleys being all changed into lakes, and the rising grounds forming green islands." This happens in a country where the rainfall is very small. It is still less in Siberia. Many cases in point are mentioned by the same author in speaking of the rivers of the latter country, and he remarks that the "overflowings of the rivers take place more or less every year;" that "on the 22nd of May the ice, which had covered the river for 259 days, broke up. On the 26th the usual inundation followed, forcing us to take refuge with all our goods on the flat roofs of the houses, there to await the termination of the flood."

Travellers in the Arctic regions of America make the same remarks; but I need not here multiply cases, as the fact is well known, and can be readily observed in most

* The Scandinavian range of mountains diminish the rainfall over a considerable part of north-western Europe by freeing the warm and damp westerly winds of their moisture.

† The Geology of Russia in Europe and the Ural Mountains: London, 1846, p. 572.

‡ Narrative of an Expedition to the Polar Sea; edited by General SABINE: 2nd edit. pp. 5, 63, & 258.

mountain districts. What I wish to point out is the probability of the continuance of severe cold during the period when the high-level gravels were in course of formation, with, at the same time, a concurrent gradual amelioration of the climate, accompanied possibly by a greater rainfall, and certainly by great spring floods.

I have before shown how impossible it would be for the present rivers, even during their greatest floods, to attain a height at all approaching to the level of the high-level gravels; but, taking the additional discharge resulting only from this melting of the snow, independently of any larger rainfall, the floods must formerly have been far greater than those of the same districts in the present day, and have given to the rivers for a portion of the year a torrential character. That the water-supply was adequate to fill at times the broad and shallow old channels is evident from the facts and is borne out by calculation. The Waveney waters may, even now, when the valley is flooded, give a sectional area of, say 1400 square feet. To fill the channel of the old river, supposing it to be on the level and of the width of the high-level gravels, would have only required a volume of water of a sectional area not exceeding 7000 feet, or five times as large. So with the Ouse, the measure, with the valley flooded, may be 4000 feet for the present river, and 20,000 feet for the old postpliocene river; for the Somme of to-day 3000 feet, and for the old river 16,000 feet; and for the Seine 8000 feet now, and 36,000 feet formerly. These are merely rough approximate estimates. They will serve, however, to show that to fill the old channels, before the excavation of the existing valleys, to their entire breadth, would not have required more than, if so much as, four or five times as much water as now flows during floods; but it must be remembered that the normal condition of these quaternary rivers would be like that of rivers of the present day that are subject to heavy periodical floods and have large and wide channels, small portions only of which are filled by the river during a great part of the year,—dry sand and shingle banks then occupying the larger portion of the area. A supply in fact very little if any larger than that drained off by the existing rivers might have occupied the comparatively dry channels during the dry season, whilst these old channels would be filled to overflowing during the melting of the snow in spring, independently of any excess of rainfall, and be subject to periodical floods and inundations, such as now are of annual occurrence in Arctic countries, when the waters rise 40 to 50 feet or more above their ordinary level*.

Although I can conceive that, granting an indefinite length of time, the wearing power of torrential rivers might effect considerable erosions, we shall find that other causes have assisted to produce the immense valley-excavations we are now contemplating. For, if the period is assumed to have been one of severe winter cold, we must follow out the consequences of that assumption, not only with regard to the floods following upon the winter snows, but in all its collateral bearings.

The effect of the freezing of the rivers and the transport by ice of the boulders, gravel, and organic remains lying on the shores, has already been discussed. In addi-

* The periodical rains of tropical countries produce a somewhat similar but smaller result.

tion, however, to the ice so formed, observations of late years have shown that a very considerable formation of ice takes place along the beds of certain rivers, especially when those beds are stony and gravelly. In these climates we rarely have the opportunity of observing this phenomenon on a large scale, although, from a few facts noticed, it appears even here to be far more common than has been supposed.

That this agent is one of considerable power in producing changes of the character we are referring to, is evident from the facts recorded by ARAGO*, and the experiments made by M. LECLERCQ† at Liége, and by Colonel JACKSON in Russia. These observers show that most running streams give rise, under certain conditions, on the setting in of winter, to the formation of ground-ice. In the first place the whole body of water becomes reduced, by intermixture caused by the flow of the river, to a uniform temperature of 32°. Any pointed surfaces in the bed of the river then determine, as is the case with a saturated saline solution, a sort of crystallization, needles of ice being formed, which gradually extend from point to point and envelope the substances with which they are in contact. By this means the whole surface of a gravelly river-bed may become coated with ice, which on a change of temperature, or of atmospheric pressure, or on acquiring certain dimensions, rises to the surface, bringing with it all the loose materials to which it adhered.

According to M. LECLERCQ, whose observations were made in the winters of 1840 and 1841, when the mean temperature of the end of December was 12° Fahr., ground-ice is formed in a current of 3·60 feet per second on the fifth day; and with a current of 9·52 feet to 11·58 feet, on the ninth to the eleventh day. The greatest depth at which he observed the formation of ground-ice was not quite 4 feet, and the greatest thickness the ice attained was 2·63 feet. At one time he found the river-bed, for a length of a mile, covered with lumps of ice, "which became detached from time to time, in angular masses of a metre square, and carried away pebbles and stones, which after a time became detached and fell on the beds over which they were carried." The conclusions at which M. LECLERCQ arrived were—

"1st. That the ice is formed under water so much the more as the cold is the more intense and the sky is the clearer." "2nd. That the ice under water gains in thickness so much the more as the current is less swift."

He also observed that a bed of fine clay and gravel gave rise to no ice, and that "the bed best suited to produce it was one formed of pebbles of considerable size."

Colonel JACKSON‡ experimented on the Neva, which at St. Petersburg is about 1500 feet broad and in places 50 feet deep, and moves with a velocity of about 2½ miles per hour. It is frozen during five months in the winter, and the surface-ice attains a thick-

* "Sur les glaçons que les rivières charrient en hiver," Annuaire du Bureau des Longitudes pour 1833, p. 244.

† "Sur la formation de la glace dans les eaux courantes," Mém. couronnés par l'Acad. de Bruxelles, t. xviii. 1845.

‡ "On the Congelation of the Neva at St. Petersburg, and Temperature of its waters when covered by ice," Journal of the Royal Geographical Society, pp. 2, 7 & 13, vol. v. 1835.

ness of 3½ feet. He found the temperature at top and bottom not to vary one-sixth of a degree*, and that a "flaky congelation" forms in immediate contact with the bed of the river and "becomes gradually transformed into solid ice, which, if not thawed in the spring at the bottom itself, gets detached and rises to the surface." In noticing a paper by Dr. PLOTT, he observes "that the flakes of ice which rise from the bottom of the Angara (in Siberia) often bring up in like manner large stones."

In a subsequent memoir† Colonel JACKSON translates some interesting observations made in Siberia by M. WEITZ, superior officer of the Mining Corps. They are not so exact as the observations of M. LECLERCQ, but they are so important, as showing the effects of such an agent under more favourable conditions of temperature, that I give the greater part of the extract. M. WEITZ remarks that "the great transparency of these rivers (of the North of Siberia) enables us to see clearly what is at the bottom. At a depth of 14 feet and more one might see the ice formed at the bottom, whose greenish tinge gave it an appearance somewhat similar to that of patches of the confervoideæ..... It frequently happens that these pieces in rising from the bottom bring up with them sand and stones, which are thus transported by the current...... When the thaw sets in, the ice becoming rotten, lets fall the gravel and stones in places far distant from those whence they came...... So long as the congealed masses continue small with regard to the volume of the water immediately above them, they adhere as if rooted to the bottom; but when by degrees they increase in bulk, the difference in their specific gravity operates to overcome their adhesion to the bottom, and they rise, bringing with them, as we have said, such gravel and stones as we find attached to them, whence we may conclude that not only does the current occasion a change in the bed of the river by its erosion of the looser soil which it carries from one place to depose in another, but that the ice which forms at the bottom of rapid rivers in very cold countries, tends also to effect a change in the beds of those rivers."

Colonel JACKSON, it is true, thinks that M. WEITZ attributes too much influence to the bottom-ice in effecting changes in the beds of the rivers; but the Neva, where Colonel JACKSON's own observations were made, is a deep muddy-bedded river, offering precisely the least favourable conditions for the formation of ground-ice.

The interesting narrative of Baron WRANGELL contains amongst much important scientific observation the following remarks:—"In the Anini, as well as in all the more rapid and rocky streams of this district, the formation of ice takes place in two different manners: a thin crust spreads itself along the banks and over the smaller bays where the current is least rapid; but the greater part is formed in the bed of the river, in the hollows amongst the stones, where the weeds give it the appearance of a greenish mud.

* Colonel JACKSON found that the water at the bottom of the river was generally a fraction of a degree ($\frac{1}{8}$) above freezing-point when congelation commenced, an observation since confirmed by Mr. ADIE in a recent communication to the Chemical Society (Proc. vol. xv. p. 90).

† "On Ground-ice in the Siberian Rivers," Journal of the Royal Geographical Society, p. 417–18, vol. vi. 1836.

As soon as a piece of ice of this kind attains a certain size, it is detached from the ground and raised to the surface by the greater specific gravity of the water; these masses, containing a quantity of gravel and weeds, unite and consolidate, and in a few hours the river becomes passable in sledges instead of in boats" (p. 202).

These, and similar observations made in northern America, establish the efficacy of ice in transporting no inconsiderable quantity of shingle along the beds of rivers, and show that it tends both to shift the shoals and to deepen the channels. The conditions of the old postpliocene rivers were precisely such as to favour this formation of ground-ice; for, without exception, the old alluviums are composed of coarse subangular shingle with but little sand, and very rarely with any subordinate seams of clay.

These two agents, floods and ground-ice, would affect chiefly the river-channel. There is another agency which would co-operate in that direction, but would affect more especially the banks and shores of the river; that is, the freezing of the ground and the rending of rocks by great cold. The power of this agent is well known; I will therefore confine myself to a few observations bearing upon our particular case. CRANTZ speaks of the talus of débris at the foot of the hills in Greenland as looking "like a demolished city," and says that some of "the lesser hills or ledges of rock are still more subject to breaking, and many of them grow so rotten and brittle with age that they are pulverized by the air"*. Sir JOHN RICHARDSON† says that near Cape Krusenstern "the whole surface is covered with thin pieces of limestone." "I should infer that the frost splits off the layers and breaks them up more effectively than any agent to which rocks are exposed in warmer climates." The same thing occurs at Point Keats; whilst of the limestone cliffs on Lake Winipeg he says, "Under the action of frost the thin horizontal beds of this stone split up, crevices are formed perpendicularly, large blocks are detached, and the cliff is rapidly overthrown, soon becoming masked by its own ruins. In a season or two the slabs break into small fragments," which go to form the beach.

Dr. SUTHERLAND‡, in describing the effects of a still colder climate, with reference to the great talus generally found at the base of the cliffs in the Arctic regions says, "Strong and bold as this coast may appear to be, and bidding defiance to assault in all directions, time, with its invisible agent, heat alternating with cold, assisted only by water, saps its foundations, and runs mines into its lofty citadels; and the result of this action is an increasing heap of rubbish, upon which the same agents are still exerting their irresistible power, reducing to splinters and small fragments, and ultimately to a fine powder, liable to be washed or blown into the sea, what had been set free in masses of more than a ton weight."

In Siberia the same phenomenon is often alluded to by Baron WRANGELL§.

* History of Greenland, vol. i. p. 53: London, 1767.

† Searching Expedition, vol. i. pp. 295, 281, & 65.

‡ Journal of a Voyage in Baffin's Bay and Barrow Straits, vol. i. p. 286: London, 1852.

§ *Op. cit.* pp. 193, 374.

In our country the effect of frost on freshly exposed perpendicular surfaces of chalk, sandstone, and oolites is very marked. The former especially disintegrates very rapidly. I have seen a low cliff of chalk 15 feet high form a talus at its foot, in the course of one ordinary winter, 6 feet broad by 5 feet high. The wetter the ground the greater, necessarily, are the effects of the frost; so, as we assume in our hypothesis that the excavation of C (fig. 18, p. 298) was effected by the removal of successive layers (c^1–c^4), commencing with the one at the base of D, each layer, when first uncovered, having been at or near the level occupied by the river, must have been at or near the line of general water-level or of springs, and therefore more largely and constantly charged with moisture than the same strata on higher ground, and such surfaces consequently presented conditions the most favourable for the operation of frost.

Sir R. Murchison gives some very illustrative instances of what he appropriately terms recent "fluvio-glacial action." Amongst others, in speaking of the Dwina, about sixty-four miles above Archangel, where it flows over a white limestone in horizontal layers, he remarks, "About 30 feet above the summer level of the stream, the terrace on the river-side is covered for two or three versts by a band of irregularly piled loose and large angular blocks of the same limestone, arranged in a long uniform ledge...... In other words, these materials (all purely local) constitute a broken ridge of stones between the road and high-water mark...... When the Dwina is at its maximum height, the water, which then covers the edges of the thin beds of horizontal limestone, penetrates into its chinks, and when frozen and expanded, causes considerable disruptions of the rock, and the consequent entanglement of stony fragments in the ice. In the spring the fresh swollen stream inundates its banks (here very shelving), and upon occasions of remarkable floods so expands that in bursting it throws up its icy fragments 15 or 20 feet above the highest level of the stream. The waters subsiding, these lateral ice-heaps melt away and leave upon the bank the rifted and angular blocks as evidence of the highest ice-mark"*. Dr. Bigsby also gives a section in illustration of a like case on the banks of the Ottawa river†.

Besides the ordinary eroding power of running water, we have had therefore three main causes in operation in those regions at the period under consideration: viz., 1. the taking-up of the shingle and boulders along the sides of the rivers by the shore-ice, and its transport thereby to points lower down the river; 2. the action of the ice forming on the bottom of the rivers, and lifting, as it rose to the surface, shingle and boulders from the river-bed and carrying them also to a distance down the stream; and 3. the rending and disintegration of the rocks by frost. The districts traversed by the rivers whose courses have been described are peculiarly favourable for the operation of these causes, being formed essentially of sands, chalk, soft sandstones, and fissile limestones; not that the harder rocks do not yield to the influence of the same causes, but that the others are more readily and quickly affected.

The combined operation of these causes is visible in many of the rivers of Russia at

* The Geology of Russia, &c., pp. 566, 567.

† Quart. Journ. Geol. Soc. vol. vii. p. 235.

the present day; but it is proceeding on a grander and larger scale in the vast regions of Northern America, where the streams, flowing through extensive champaign countries, have furrowed the land with deep and precipitous channels, generally much below the level of the great plains they traverse*. In Europe, however, the connexion of cause and effect is by no means so apparent. The regularity so common in the former case is generally wanting in the latter, where the weathering is more excessive.

I doubt also whether, without a change in the general level of the land, the full effects of the changes we are contemplating could have been produced. The excavating power of the rivers would, in a measure, depend upon the adjustment to be made between the inclination of the valley along which they flowed and the sea-level.

The coasts of these opposite shores of England and France are fringed at places by a raised beach. Of this we have evidence at Sangatte near Calais†, Brighton‡, the Sussex coast§, and possibly at Havre||. In all these places it is about 5 to 10 feet above the level of high water. With this beach I would correlate the estuarine bed connected with the low-level valley-gravels at Menchecourt. But besides this zone we have in each district the higher-level gravels fringing the river-estuaries, and sometimes the coast, at an elevation of from 40 to 100 feet above the raised beaches. These mark the relative difference of water-level at our two valley-gravel periods. It is immaterial to our inquiry whether that difference resulted solely from an elevation of the land or partly from the encroachment of the sea on the coast. Probably both have contributed to the result. A slow elevation of the land may have commenced at the high-level gravel period, leading to an increase in the velocity and erosive power of the rivers until a state of repose again obtained in the low-level gravel period. During such a change of level the causes which we have above alluded to, acting upon the portions of the substrata successively subjected to the action of the maintained water- and ice-power, gradually effected the excavation of those deep and broad channels forming the valleys through which the present comparatively insignificant rivers of these districts now find their way. The sharper angles produced on the river-banks by the erosion of the stream have been rounded off and in great measure obliterated by the action of the severe cold combined with the periodical floods,—operations by which the exposed rock-surfaces were alternately disintegrated and denuded; while at the same time the flood-waters in retreating from the higher platforms, before falling in with the main current, further grooved and furrowed the sides of the valleys, and, breaking the continuity of the river-terraces, helped to give our valley-sides their peculiar and varied outlines.

The following diagram will serve to illustrate my meaning:—

* The geological structure in both instances often greatly facilitates the operation, the country being constantly thickly covered by loose sands and gravels, offering little resistance to the erosive power of the rivers.

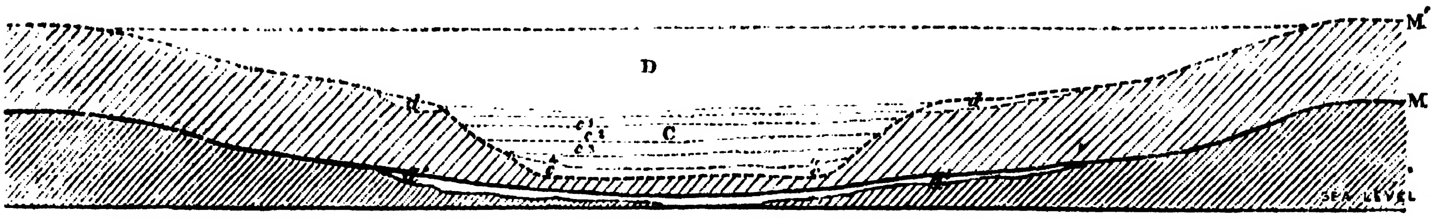
† The author in Quart. Journ. Geol. Soc. vol. vii. p. 274.

‡ Mantell, 'Fossils of the South Downs,' p. 277.

§ Godwin-Austen in Quart. Journ. Geol. Soc. vol. xiii. p. 61.

|| Passy, Desc. Géol. de la Seine Inférieure, p. 84.

Fig. 18.



M. Original level of the country at the time of the formation of the high-level gravel $d' d'$.

M'. Present level of the country, with the remaining portions of the high-level gravel $d\,d$.

M M'. Supposed extent of elevation between the two periods. The dotted lines mark portions of the substrata successively raised to the level $c^1 c^2 c^3 c^4$, and consecutively removed by the denuding action, the total amount of denudation being represented by C (or the space embraced between d and c^4). c^1 to c^4 may represent any thickness of strata; the rate of elevation from M to M' may have been continuous or interrupted or partial, and the extent of elevation variable in different districts. According to any variability in the rate of elevation, to intervals of repose, or to deflections in the flow and velocity of the river, so there may exist intermediate terraces or levels, sudden variations in the slopes, and gravels lodged on different levels. As these not unfrequently occur, they often add much to the complexity of the problem.

§ 7. THE QUESTION OF TIME AND SUCCESSION.

In looking back at the subjects we have discussed, we are forcibly reminded of our dependence on the value of probabilities. On various points geology has not at present, and probably never will have, any other means of inference. All that can be done to give weight to our argument is to multiply probabilities, and by attending to the general concordance to reduce to the minimum the chances of error. The difficulty of one branch of the inquiry is considerably increased by the circumstance that the recent researches of naturalists tend for the present to give less security to any argument founded upon analogy of past with recent life. The case of the adaptation of the large extinct Pachyderms to a rigorous northern climate has long since deprived the remains of such genera of any weight with reference to the climatal conditions of past periods. It has now further become a question with some distinguished naturalists whether even the distribution of recent species is originally dependent on the influence of climate—whether the existence of certain kinds of food, the presence or absence of certain other animals, may not have been amongst the causes regulating the range of the animals. It is certain that the experience gained of late years of the facility of acclimatization indicates the necessity of caution. Nevertheless, in the absence at present of sufficient data with regard to this power of adaptation, we can only in the mean time rely on the evidence furnished by recent life so far as it regards species of known habits and range, provided especially it be supported by independent collateral proof.

I will now proceed to make a few remarks on the question of time. We have to look at it both with reference to geological time, or the order of succession, which is merely relative, and to that which in the present instance more concerns this particular inquiry—the actual date of the existences and changes under consideration. With regard to geological time, I have before shown that the period is subsequent to that of the Boulder Clay—consequently to that of the great extension of the European glaciers

—and that it may be brought down to the time when our valleys and plains began to receive their tranquil and inequality-levelling deposits of silt and peat, and the modern order of things commenced.

To estimate the time to which we have to carry back the high-level gravels, we have to consider what may have been the duration of their accumulation, and that of the subsequent excavation of the valleys with the resulting low-level gravels*. A difficulty here meets us at the onset. The accumulation of sand, gravel, and shingle along the course of rivers is so irregular (sometimes very rapid, at other times slow, what is done one year being undone another) that we are entirely without even the few data by which we are approximately guided in ordinary sedimentary strata. The thickness of the deposits affords no criterion of the time required for their accumulation. They rarely exceed 20 feet, and are more frequently not above 10 to 12 feet thick. It is well known that recent inundations have covered valleys with sand and gravel to the depth in places of 4, 6, or even 10 feet in the course of a few days; and therefore there are no high-level gravels which, so far as thickness is concerned, might not have been deposited in the course of a few weeks or even a few days. But the evidence of time lies in their length, breadth, and extent,—in the life existences of the period,—and in the physical changes in progress, such, for example, as the subsequent valley-excavation, and the wide distribution of the resulting débris. There is also another phenomenon connected with this period which I would point out as containing some elements for an approximate estimate of the duration of time. We have as yet no data to judge of the rate of progress of the operation; but it is one which admits, to a certain extent, of time-measurement, and may hereafter, perhaps, be employed with some chance of success. At present it will merely serve to give us some idea of the time employed; but that even is a step gained. I allude to those cylindrical perforations in calcareous strata filled by the sand and gravel beds overlying or formerly overlying them. These are of various dates, but a large proportion of them commenced, I believe, with the high-level gravel period, or with that of the gravel which immediately preceded it.

These cylindrical and funnel-shaped holes, or gravel and sand pipes as they are termed, vary usually from 5 to 50 feet in depth and 1 to 10 feet in width, though they are sometimes much larger. I have seen traces of them in the chalk skirting the S.E. side of the hill of St. Acheul. Near Picquigny, on the road from Amiens to Abbeville, there is an escarpment of the chalk in which there are the remains of several gravel (high-level) pipes from 10 to 20 feet in depth. Others are to be seen near Mareuil, and again near Yonval; the gravel itself in most of these cases has been denuded from the surface, and remnants only preserved in these natural funnels. The outlier of high-level gravel on the hill above Mautort presents a section of one measuring 15 feet in diameter, but the depth is not shown. The most remarkable instance that has come under my

* I am speaking now of the postpliocene valleys. Where the land, as in Auvergne, was earlier raised above the sea, we may have valley gravels going back to pliocene, or miocene periods, and continued in uninterrupted succession to the recent period.

notice is at Drucat, near Abbeville, and which I take to belong to the high-level gravel period. This outlying mass of sand and gravel is in a depression of the chalk, which probably accounts for its preservation; at the bottom of the depression there was exposed on the occasion of my first visit a deep circular shaft in the underlying chalk 22 feet in diameter at the top and 18 feet at the depth of 30 feet, to which extent the sand and gravel had been cleared out. The prolongation of this great natural excavation in the chalk probably reaches a depth of at least 100 feet. A number of these sand pipes underlie this quaternary outlier, but I saw no others of the dimensions of this one.

These pipes are not filled up indiscriminately, as if they had been formed first and subsequently filled up, but they show, as usual, a succession of concentric and continuous vertical layers, following the encircling surface of chalk and enclosing a core distinct from the outer coats. Further, where the beds of sand and gravel are undisturbed and in their horizontal position, it is found that the core of the pipe always subtends from the uppermost bed or seam of gravel or sand, or not unfrequently from the superincumbent Loess, which proves that the superincumbent beds were deposited before the excavation of the pipes, and that they were lowered into them by the gradual removal of the chalk. These excavations have been referred to various causes, of which I consider the action of carbonic acid held in water as the only one possible*. It is evident that to have an excavation of this sort we must have the slow and constant passage of water. If the line of water-level in the chalk had remained permanently near the level of the high-level gravels, this prolonged downward action could not have occurred. The water-line, although at first necessarily on that level, must, as the excavation of the valley proceeded, have gradually been lowered 50 to 100 feet or more; so that the surface-water collected in these beds of sand and gravel, left standing above the base of the gradually deepening valleys, would, in draining off, have to pass down, along the lines of least resistance, through a successively increasing depth of chalk, before it met with the line of permanent water-level into which it would merge. The gradual and constant operation of this percolation of water through definite lines in the chalk, from the first emergence of the high-level gravels above the old river-bed and continued in the same channels down to the time of the lower valley-gravel, resulted in eroding these perpendicular shafts or funnels, into which, as the excavation proceeded, the overlying gravel and sand coordinately subsided, while the Loess of the periodical floods continuously tended to level the resulting inequalities of surface. The process must necessarily have been extremely slow. That these pipes are connected with a former state of things and not with the present, is shown by there being now no indication of their presence on the surface of the ground †.

* See a paper by Sir Charles Lyell in Phil. Mag. 3rd ser. vol. xv. p. 257, and another by the author in Quart. Journ. Geol. Soc. vol. xi. p. 64.

† This in some cases may arise from the cultivation of the surface. In a few favourable localities a slight action of this sort would still, however, appear to be going on, if we may judge from an occasional sinking or giving way of the ground.

The next possible standard of measure is the time required for the excavation of the valleys themselves. I have already described the agents which probably cooperated in this gigantic operation. That it must have been one of great time there can be no doubt; but the like operations at present in progress by no means furnish us with the gauge of the rate of the denuding action. In considering this point there is, besides the greater floods and severer cold, another element which must not be overlooked. This is the varying solvent power of spring- and river-waters. This there is reason to suppose may be greater in cold than in temperate climates, for AGASSIZ has shown that fallen snow holds excessive proportions of air in combination; so that, during inundations resulting from the melting of snow in spring, the flood-waters becoming loaded with soil and vegetable matter must necessarily have presented conditions favourable for generating carbonic acid in large quantities, with which the ice-cold waters would become highly charged. Thus the solution, both of the calcareous beds forming the river-channel and of the strata perforated by the gravel and sand pipes just alluded to, may have been accelerated much beyond any effects now observable in these districts from the present action of ordinary spring- and river-waters.

An indication of time-measurement, which has been often referred to in relation both to the lapse of time and its late date, is the formation or excavation of the British Channel between the South-east of England and the opposite coast of France. The grounds on which it has been inferred are, the identity of the strata on the two sides of the channel, and the community of the fauna and flora. This to a certain and great extent is true, and there can be no doubt that the severance of the two countries took place at a comparatively late geological period; but that it was the last change of all I am not prepared to admit. In fact the question has been treated in its immediate application in a manner purely hypothetical. The geological evidence of the substrata has been constantly had in view, whilst that of the superficial postpliocene beds, which relate directly to the period under consideration, has not been attended to.

Whether or not there may have been a break between the two countries at the high-level valley-gravel period I could not say with certainty. We have evidence of these beds occupying, on or near the coast-line, a level of from 50 to 100 feet above the sea on both sides of the channel. This may arise from the sea encroaching on the land and so intersecting, at varying distances from the old line of coast, the planes of the old river-channels—which, like the present river-beds, necessarily slope from certain heights inland to the sea-level,—or from an elevation of the land. The evidence, probably, is in favour of the operation of both causes. The difference of height between the fossiliferous high-level gravels at Amiens and at Abbeville is 60 feet. This is in a distance of 28 miles. If we prolonged these beds at Abbeville, where they are 96 feet above the sea, on the same plane sea-ward, they would reach the level of the sea at a distance of 45 miles below Abbeville, or 29 miles beyond the present coast-line. The same measurements applied to the high-level gravels of our own coast give nearly similar results. If this be correct, a sea-channel, although very contracted, may then possibly

have existed between the two countries; but as the raised beaches of later date prove an elevation of the land subsequent to the period of the high-level gravels, the old channel must have been larger in the proportion of the difference of level so produced to the difference which would have resulted from the wear of the land alone. That the elevation of the Wealden area had taken place before this period is proved by the occurrence of high-level gravels within its limits (see Map, Plate IV.), whilst the hydrographical conditions of the whole area show that those deposits hold the same relative position to the adjacent coast in one part as in another; whence I should infer both a widening of the sea-channel, and a former somewhat greater extent of land.

With reference to the condition of things at the time of the low-level gravels the evidence is more positive. We have old cliffs running nearly parallel with the present line of coast, and estuarine deposits in position nearly coincident with the like modern deposits. There are the old cliffs and raised beach at Brighton on the one side of the channel, and those at Sangatte, near Calais, on the other, while the deposits near Havre, Abbeville, and on the Stour, near Canterbury, furnish us with examples of estuarine beds of this late postpliocene age. On a coast so exposed to the action of the sea, and with cliffs constantly though slowly yielding to its incessant action, it is not to be expected that traces of old raised beaches should be preserved, except at a few sheltered spots. These we have at places so closely allied to the present contour of coast—showing, too, old cliffs forming, like the present range of cliffs, bold escarpments to an old sea—that although I conceive the channel to have been considerably widened since then, I am satisfied that it existed at the time of the low-level gravels, whatever doubt there may be of its prior existence. There is no palæontological objection to this view, inasmuch as the land and freshwater Mollusca had spread over this country at an anterior period; the greater bulk of them had in fact made their appearance in this country previous to the Boulder Clay, and many at the period of the Crag. A nearly similar observation applies, with few exceptions, to the Mammalia. With a climate, however, such as we have inferred, and with a channel of less breadth than the present one, the sea between the two lands might have been frozen every winter and have allowed of the passage of man and large animals, as happens at this day in latitude 52° at the island of Saghaleen, where the strait between it and the adjacent mainland is frozen every winter for a period of some months*.

Nor are we entirely without evidence, although very slight, derived from the land Mollusca, of the existence at this latter period of a barrier impassable to them. There are two species, the *Pomatias obscurus* and *Clausilia plicatula*, Drap., living French shells, both of which I have found fossil at Menchecourt, but which are not known either living or fossil in England.

All these phenomena indicate long periods of time. I do not, however, feel that we are yet in a position to measure that time, or even to make an approximate estimate

* On the east coast of Saghaleen the sea freezes every winter as far as the eye can reach. Occasionally the Tiger crosses over to that island.—Ravenstein's 'Amur,' pp. 284 & 320.

respecting it. That we must greatly extend our present chronology with respect to the first existence of man appears inevitable; but that we should count by hundreds of thousands of years is, I am convinced, in the present state of the inquiry, unsafe and premature.

Nevertheless, just as, though ignorant of the precise height and size of a mountain-range seen in the distance, we need not wait for trigonometrical measurements to feel satisfied in our own minds of the magnitude of the distant peaks, so with this geological epoch, we see and know enough of it to feel how distant it is from our time, and yet we are not in a position at present to solve with accuracy the curious and interesting problem of its precise age.

Before leaving this subject I would direct attention to one other condition connected with this later division of the glacial period, which possibly may eventually afford an additional clue towards the solution of this important time-question. Here, again, we have not at present all the data we require, but we have enough to show the possibility of obtaining from this source some elements for more exact calculations.

In conducting experiments upon the temperature of the crust of the earth, it is well known that, after passing the limits of the line of mean annual temperature, there is a gradual increase of 1° Fahr. for every 60 feet, nearly, of additional depth. But the rule is by no means constant, the rate of increase being subject to fluctuations and variations for which no sufficient reason has been assigned. Is it not possible that these disturbances may arise from differences between the former (glacial) and the present (temperate) temperature of the place, combined with the variable conductivity of the strata? Let us, for example, take a place like Yakutsk in Siberia, where the ground is perpetually frozen to a depth of 382 feet—the depth, therefore, at that place of the line of 32°. To reach a heat of 53°, the invariable constant under the Observatory of Paris at a depth of 90 feet, we should have to sink at Yakutsk (taking, as a mean, an increase of 1° for every 60 feet) to a depth of $382+(21\times60)$, or 1642 feet, before reaching the same isothermal plane. If, from any circumstances connected with geological changes, we could suppose the mean temperature of Yakutsk to be raised to that of this part of Europe, the isothermal plane of 53° would tend to take a vertical range upwards of $1642-90=1552$ feet. In a perfectly homogeneous mass, and all conditions equal, this plane would travel at all parts in equal times, or would move in lines parallel with the original position it held; but as such uniformity over large areas never obtains, and the strata which it would have to pass through must differ materially in conductivity of heat, it follows that the isothermal planes would, in different places, travel with different velocities, and, until adjusted by lapse of time, aberrations in the increment of heat at different depths must exist. I apprehend that a very long period of time would also elapse before an equilibrium in accordance with the changed mean temperature of the place could be established in each successive zone of depth.

Now if we apply this hypothetical case to these parts of Europe, the question I would suggest is if it might not be possible to determine, by calculations founded upon suffi-

cient data, whether any of the perturbations in the increment of heat at different places and depths within this area may not be due to the circumstance of a very much lower temperature having prevailed here at an antecedent period*; and whether, if so, the date of that period (taking that of the extreme glacial cold) could not be fixed within a certain limit, by the application to this investigation of the known laws regulating the transmission of heat through solid bodies.

The uninterrupted succession of life from this postpliocene period to our own time cannot fail to have been noticed in the course of this inquiry—a succession so large and so important, that it is not possible to contemplate the occurrence of any intervening catastrophe of such a nature as to destroy the life of the period, and seek for an explanation of its return by immigration from adjoining districts. Apparent even as the connexion is in the limited ground we have studied, it is infinitely stronger when the whole series of pliocene and postpliocene deposits comes under review. Even in the aspect here presented the conclusion is inevitable, that no general cause has led to the extinction of life over this part of Europe at any recent geological period. There have been great river-floods and great changes, but no interruption in the succession of life from the time of the great extinct mammals to our own times. There are still serious difficulties in the way of explaining the cause of the disappearance of so many of these large Mammalia; but a sufficient number remain to attest the direct descent of a portion of the old fauna to our day. The Reindeer, the *Bos primigenius*, the Aurochs, are amongst those which survived all the successive changes†. Why the larger Pachyderms should not also have survived we cannot explain, we can only admit the fact, which is the more remarkable from the non-extinction of other classes. The change of climatal conditions could scarcely have been the sole cause, as that would affect one class equally with the other; and besides, as the climate at this time presented no extreme character, they could, as the changes progressed, have found, by migration or limitation, as with the other animals, places still adapted to their former condition. But by far the most remarkable and convincing feature in the case is the transmission from the quaternary period of so large a proportion of the small and delicate land and freshwater shells. Not only are they found inhabiting the same land as formerly, but their distribution follows very much the same law. More than two-thirds of our recent species are found in a fossil state; and when we consider that the list of living species is the result of close examination of numerous observers for a series of years over a large extent of country, whereas that of the fossils is the result of a necessarily limited search at very few places, where they are buried in the ground and rendered fragile by age, it is rather a matter of surprise that the collection should be already so large. Many of these mollusks will no doubt live for a time out of their element, and they might survive

* I apprehend that some of the calculations that have been made on the earth's temperature and refrigeration may also be affected by this disturbing cause.

† On this subject M. Pictet of Geneva has made some interesting observations in a paper published in the 'Archives des Sciences de la Bibliothèque Universelle' for August 1860.

floods and inundations which would destroy their large contemporaries, but there is a limit to this power. Some land mollusks are not destroyed by immersion in water for days, and freshwater mollusks will revive after immersion in salt water; but this applies to some species only, and with these, even, their mode of protection, although it might suffice for days, would not avail for a lengthened period.

Although I may be quitting the strict limits of induction, I cannot conclude this paper without mentioning one impression which a review of the circumstances connected with the subject has made upon my mind. There is no doubt that great vicissitudes in the climate of any particular region may be caused by fluctuations in the isothermal lines resulting from changes in the relative distribution of land and water. But these fluctuations have a limit, which limit seems to me to have been greatly exceeded during the height of the glacial period. Looking at the special nature of such a remarkable reduction of temperature, closing as it were a vast cycle of anterior geological changes, and seeing its exceptional nature with reference to the general indications of higher temperatures which previously prevailed, I confess I feel deeply and strongly impressed with the probability that in this unexpected succession of changes we may trace evidence of great and all-wise design. If the cause were general (and there are strong reasons to believe that such was the case), the fact of the earth having been subjected to the severe and rigorous temperature of the glacial period must have led to a more rapid abstraction of heat from the surface than would have occurred without the intervention of a cold period, establishing, as it were in anticipation, a state of equilibrium which might otherwise have been indefinitely deferred had the refrigeration been gradual and uninterrupted; for on the removal or cessation of the refrigerating cause, the surface would be left in a condition to suffer for a certain period little or no further loss by radiation and no further contraction. The state of repose thus effected may have helped to impart to the earth's crust that stability and immobility which render it fit and suitable for the habitation of civilized man.

APPENDIX.—List of the Testaceous Mollusca now living in the South of England and North of France, showing those which are found fossil in the High- and Low-level Valley-gravels, and their range to certain points northwards and southwards in Europe.

Species living in the South of England (excepting those marked in italics, which are found in France but not in England). This list (of 110 species) embraces nearly all the freshwater species and the greater part of the land shells of the North of France.	Species occurring in the Valley-gravels.				Species living in Finland and Lombardy (the Alpine districts excepted). In the former country there are 77 species of Testaceous Mollusca, and in the latter 193 species.	
	England.		France.			
	High-level?	Low-level.	Low-level.	High-level.		
	Bedford, *Biddenham.*	Bedford, *Harrowden &c.*	Abbeville, *Menchecourt.*	Amiens, *St. Acheul.*	Finland.	Lombardy.
FRESHWATER BIVALVES.	1.	2.	3.	4.	5.	6.
Anodonta anatina, *Linn.*	*?	......	......	......	———	———
—— cygnea, *Linn.*	......	......	......	......	———	———
Cyclas (*Sphærium*) cornea, *Linn.*	*	*	*	*	———	———
—— lacustris, *Müll.*	......	......	*	......	———	———
—— ovalis, *Fér.*	......	......	......	......	......	......
—— rivicola, *Leach*	......	......	......	......	......	......
Pisidium amnicum, *Müll.*	*	*	*	*	———	———
—— fontinale, *Drap.*	*	*	*	*	———	......
—— nitidum, *Jen.*	*	*	*	......	......	......
—— pusillum, *Gmel.*	......	......	......	*	———	———
—— roseum, *Scholtz*	......	......	......	......	......	......
Unio *littoralis*	......	*	......	......	......	......
—— margaritifer, *Linn.*	......	......	......	......	———	......
—— pictorum, *Linn.*	......	......	......	......	———	———
—— tumidus, *Phil.*	......	......	......	......	———	......
FRESHWATER UNIVALVES.						
Ancylus lacustris, *Linn.*	*	......	......	......	———	———
—— fluviatilis, *Müll.*	*	*	*	*	———	———
Bythinia Leachii, *Shep.*	......	......	......	......	......	......
—— tentaculata, *Linn.*	*	*	*	*	———	———
Hydrobia marginata, Mich.	*	......	*	......	......	......
Limnæa auricularia, *Linn.*	*	*	*	......	———	———
—— glabra, *Müll.*	......	......	......	*	———	———
—— glutinosa, *Müll.*	......	......	......	......	———	......
—— palustris, *Drap.*	......	*	*	*	———	———
—— peregra, *Müll.*	*	*	*	*	———	———
—— stagnalis, *Linn.*	*	*	*	*	———	———
—— truncatula, *Müll.*	*	*	*	*	———	———
Neritina fluviatilis, *Linn.*	......	......	......	......	———	———
Paludina contecta, *Müll.*	......	......	......	......	———	......
—— vivipara, *Linn.*	......	......	......	......	———	———
Planorbis albus, *Müll.*	......	*	*	*	———	———
—— carinatus, *Müll.*	......	......	*	......	———	———
—— complanatus, *Linn.*	*	......	*	*	———	———
—— contortus, *Linn.*	......	......	......	......	———	———
—— corneus, *Linn.*	......	......	*	......	———	———
—— glaber, *Jeffr.*	*	*	......	......	......	———?
—— lineatus, *Walk.*	......	......	......	......	......	......
—— Nautileus, *Linn.*	*	......	*	......	———	———
—— nitidus, *Müll.*	......	......	......	*	———	......
—— spirorbis, *Linn.*	*	*	*	*	———	———
—— vortex, *Linn.*	*	*	*	*	———	———
Physa fontinalis, *Linn.*	......	......	......	......	———	———
—— hypnorum, *Linn.*	......	......	......	......	———	———
Valvata cristata, *Müll.*	*	......	*	......	———	———
—— piscinalis, *Müll.*	*	*	*	*	———	———
LAND SHELLS.						
Achatina acicula, *Müll.*	......	......	......	......	———	———
Acme lineata, *Drap.*	......	......	......	......	......	———
Azeca (*Cochlicopa*) tridens, *Pult.*	......	......	......	......	......	......
Balea perversa, *Linn.*	......	......	......	......	———	———
Bulimus acutus, *Müll.*	......	......	......	......	......	......
—— montanus, *Drap.*	......	......	......	......	......	......
—— obscurus, *Müll.*	......	......	......	......	———	———

APPENDIX (continued).

Species living in the South of England (excepting those marked in italics, which are found in France but not in England). This list (of 110 species) embraces nearly all the fresh-water species and the greater part of the land shells of the North of France.	Species occurring in the Valley-gravels.				Species living in Finland and Lombardy (the Alpine districts excepted). In the former country there are 77 species of Testaceous Mollusca, and in the latter 193 species.	
	England.		France.			
	High-level ?	Low-level.	Low-level.	High-level.		
	Bedford, *Biddenham.*	Bedford, *Harrowden &c.*	Abbeville, *Menchecourt.*	Amiens, *St. Acheul.*	Finland.	Lombardy.
LAND SHELLS (continued).	1.	2.	3.	4.	5.	6.
Carychium minimum, *Müll.*	......	......	*	......	———	———
Clausilia biplicata, *Mont.*	......	......	......	......	......	......
—— laminata, *Mont.*	......	......	......	......	———	......
—— *plicatula*, Drap.	......	......	*	......	———	———
—— Rolphii, *Gray*	......	......	*	......	......	......
—— rugosa, *Drap.*	......	......	*	......	———	———
Cyclostoma elegans, *Müll.*	......	......	*	......	......	———
Helix aculeata, *Müll.*	......	......	......	......	———	......
—— *apicina*,	......	......	* ?	......	......	......
—— arbustorum, *Linn.*	......	......	*	......	———	......
—— aspersa, *Müll.*	......	......	......	......	......	———
—— Cantiana, *Mont.*	......	......	*	......	......	———
—— caperata, *Mont.*	......	......	*	......	......	———
—— Carthusiana, *Müll.*	......	......	*	......	......	———
—— concinna, *Jeffr.*	*	......	*	*	......	......
—— ericetorum, *Müll.*	......	......	......	......	......	———
—— *fruticum*, Müll.	......	......	*	......	———	......
—— fulva, *Müll.*	......	......	......	......	———	———
—— fusca, *Mont.*	......	......	......	......	......	......
—— hispida, *Linn.*	*	*	*	*	———	———
—— lapicida, *Linn.*	......	......	......	......	———	......
—— nemoralis, *Linn.*	......	......	*	*	———	———
—— obvoluta, *Müll.*	......	......	......	......	......	———
—— pomatia, *Linn.*	......	......	......	......	———	———
—— pulchella, *Müll.*	*	*	*	*	———	———
—— pygmæa, *Drap.*	......	......	*	......	———	......
—— revelata, *Mich.*	......	......	......	......	......	......
—— rotundata, *Müll.*	......	......	*	*	———	———
—— rufescens, *Pennant*	......	......	......	......	......	———
—— rupestris, *Stud.*	......	......	......	......	......	———
—— sericea, *Müll.*	......	......	......	......	———	......
—— virgata, *Da Costa*	......	......	......	......	......	......
Pomatias obscurus	......	......	*	......	......	......
Pupa marginata, *Drap.*	*	*	*	*	———	———
—— ringens, *Jeffr.*	......	......	......	......	......	......
—— secale, *Drap.*	......	......	......	......	......	......
—— umbilicata, *Drap.*	......	......	......	......	———	......
Succinea elegans, *Ris.*	......	*	*	*	———	......
—— oblonga, *Drap.*	* ?	* ?	*	*	———	———
—— putris, *Linn.*	*	......	*	*	———	———
Vertigo angustior, *Jeffr.*	......	......	......	......	......	......
—— antivertigo, *Drap.*	......	......	......	......	———	———
—— edentula, *Drap.*	......	......	......	......	———	———
—— minutissima, *Hartm.*	......	......	......	......	———	———
—— pusilla, *Müll.*	......	......	......	......	———	———
—— pygmæa, *Drap.*	......	......	......	......	———	———
—— substriata, *Jeffr.*	......	......	......	......	———	......
Vitrina pellucida, *Müll.*	......	......	......	......	———	———
—— *diaphana*, *Drap.*	......	......	*	......	......	———
Zonites alliarius, *Müll.*	......	......	......	......	......	......
—— cellarius, *Müll.*	......	......	......	......	———	———
—— crystallinus, *Müll.*	......	......	*	......	———	———
—— excavatus, *Bean*	......	......	......	......	......	......
—— nitidulus, *Drap.*	......	......	*	......	———	———
—— nitidus, *Müll.*	......	......	......	......	———	———
—— purus, *Ald.*	......	......	*	......	———	......
—— radiatulus, *Ald.*	*	......	*	*	......	......
Zua lubrica, *Müll.*	......	*	*	*	———	———

Explanation of the Plates.

PLATE IV.

The uncoloured ground-plan shows the chief geological divisions of the country. The coloured lines indicate the course of the débris derived from the several principal formations. The lines are carried through from the parent rocks to the sea; but it is to be observed that the débris of each of these lines becomes less and less abundant as they recede from their source, so that all traces of some of them sometimes nearly disappear before the end of the river-valley is reached. In places, also, the gravels formed by these débris meet with long interruptions, those on the higher levels especially; the lower-level gravels are more continuous, though they are constantly hidden by recent alluvia. They also vary materially in width. These more minute details are represented for small portions of two river-valleys in Plate V. The scale of the Map does not admit of the delineation of the lines of débris of each valley. The chief river-valleys and their principal tributaries are therefore selected, but the same law of the occurrence of high- and low-level gravels, and of the local limitation of the various rock débris to the several river-basins, is applicable to all the river-valleys in the area comprised in the Map. The authorities for France are the several authors mentioned in the text, with a few observations of my own. The English part of the Map is given from my own personal observations. The commencement of some lines of débris is not unfrequently higher up the valleys than marked on the Map, or from rocks beyond the present range of the river. This arises from the presence of outliers of certain formations more or less beyond the limits of the main mass, which outliers are not represented on the Map: in other cases the débris are derived from secondary sources, like the palæozoic, oolitic, and cretaceous débris of the Boulder Clay; and in a few cases, especially in the Wealden area, they arise from the originally greater length of the rivers.

PLATE V.

Figs. 1 & 2 show the distribution of the high- and low-level valley-gravels, and of the Loess in parts of the valley of the Somme adjacent to Abbeville and Amiens. The relative position of the high-level gravels and of the plateau Loess is not always quite clear. In many cases I consider the latter to be newer than some of the former; but, at the same time, there is a quaternary argillaceous deposit on parts of the chalk hills which I believe to be older than any of the valley-gravels. This is generally more removed from the river-valleys than shown in these plans.

The valley-gravels are here divided into two stages only. Each stage, however, must be considered to represent not one exact level, but the several nearly allied terraces formed during a particular time. It may be assumed, for example, that the high-level gravels include all the terraces at heights

of from 90 to 150 feet above the river, and the low-level gravels those up to 30 or 40 feet above the river. In fact, no definite line can properly be drawn, as all the terraces are members of one series: nevertheless it is not only for the sake of convenience that this division is adopted; it is to a great extent conformable to the phenomena as they exist; for the great bulk of these quaternary gravels occur, one portion on terraces at or near 100 feet above the Somme, and another portion at or near 30 feet. The gravels on both levels, especially the lower one, are often covered by Loess—the low-level gravel being constantly buried under it and hidden, whilst both Loess and gravel disappear under the recent alluvium. The heights are taken in part from the French Ordnance Maps; the others are from observations I have taken with the aneroid barometer. The general topographical outlines of the districts are also taken from the French Ordnance Maps.

Fig. 3 shows the valley-gravels along part of the valley of the Waveney. The scale is that of the Ordnance Map, from which the topographical outlines are taken. The same observations apply to these gravels as to those of the Somme; only the series is more limited, and the heights to which they rise is not so great; 50 feet is about the extreme height. The Loess is here in so rudimentary a state that I have not laid it down. The heights at Hoxne are from the levellings given in my former paper, or are taken with an aneroid.

Not to interfere with the details, the roads, and all except a few chief places, are omitted, but it is easy to find any particular spot by transfer to the French and English Ordnance Maps, on the scales of which these plans are made.

The dotted lines across the valleys refer to the lines of section given in woodcuts pp. 253, 258, and 259.

Note.—There is some obscurity in the shading, arising from a mistake of the artist in taking my rough MS. sketch instead of the Ordnance Maps which accompanied it for his guide in these topographical details. I must refer to the Ordnance Maps therefore for the more correct delineations of the surface. Some errors of geographical detail have from the same cause crept into the Map.

INDEX OF CONTENTS.

VIII. *Experiments to determine the effect of Impact, Vibratory Action, and long-continued Changes of Load on Wrought-Iron Girders.*

By W. FAIRBAIRN, *LL.D., F.R.S.*

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A QUESTION of great importance to science and the security of life and property has been left in abeyance for a number of years,—namely, to determine by direct experiment to what extent vibratory action, accompanied by alternate severe strains, affects the cohesive force of bodies. It is immaterial whether the body be crystalline, homogeneous, or elongated into fibre, such as cast or wrought iron; the question to be solved is, how long will a body of this description sustain a series of strains produced by impact (or the repeated application of a given force) before it breaks? In the case of bridges and girders, this is a subject on which no reliable information has yet been given which may be considered as a safe measure of strength for the guidance of the architect and engineer. It is true that regulations have been established by the Lords Commissioners for Trade; but they appear to have had their origin on limited data, and in cases where the material and workmanship are good they may be relied upon as sufficient for the public safety. What, however, is wanted is experimental data to enable the engineer to comply satisfactorily with the conditions of the Board of Trade, and cordially to unite with the Government in affording ample security to constructions in cases where the lives of the public are at stake.

To remove all doubts on this question, I have been enabled, through the liberality and at the request of the Board of Trade, to undertake a series of experiments to determine, or to endeavour to ascertain, whether a continuous change of load, and the strains produced by those changes, have any effect (and to what extent) upon the ultimate strength of the structure,—or, in other words, to ascertain the rate of endurance the material is able to sustain under these trials.

To comply with this request, a wrought-iron beam was constructed, representing the girders of a bridge of questionable strength, to be employed to determine, experimentally, the strength and durability of such a structure. This beam was made of the ordinary construction, of moderately good, but not the best quality of iron, and subjected to vibration and a perpetual change of load until the cohesive powers of the material were destroyed.

Of the resisting-powers of material under the severe treatment of a continuous change of strain, such as that which the axles of carriages and locomotive engines undergo when rolling over iron-jointed rails and rough roads, we are very imperfectly informed.

Few facts are known, and very few experiments have been made bearing directly on the solution of this question. It has been assumed, probably not without reason, that wrought iron of the best and toughest quality assumes a crystalline structure when subjected to long and continuous vibration—that its cohesive powers are much deteriorated, and it becomes brittle, and liable to break with a force considerably less than that to which it had been previously subjected. This is not improbable; but we are apparently yet ignorant of the causes of this change, and the precise conditions under which it occurs.

In the year 1837 I instituted a long series of experiments to determine an important quality in the strength of materials, viz. the powers of crystalline bodies to sustain pressure for an indefinite period of time, and to ascertain whether cast iron, when subjected by a given weight to long-continued transverse strain, would or would not be subject to fracture.

It appears that former writers on the transverse strength of materials had come to the conclusion that the bearing-powers of cast iron were confined within the limits of that force which would produce a permanent set, and that it would be unsafe to load this material with more than one-third of the weight necessary to break it. This assumption is incorrect, as in the experiments to which we refer some of the bars, six in number, were loaded within one-tenth of the weight that would break them.

From these experiments it was ascertained that cast iron, when sound, is more to be depended upon, and exhibits greater tenacity in resisting long-continued heavy strains, than is generally admitted, and its bearing-powers have deserved a much higher reputation than has at any former period been given to them. This is even more apparent with wrought iron, as it is safer, being more tenacious and ductile, and less liable to flaws and imperfections, which, too, should they exist, are much more easily detected than in cast iron.

The experiments, as respects the effects of time, on loaded cast-iron bars 1 inch square and 4 feet 6 inches between the supports, were exceedingly curious and interesting. They embraced a period of seven years, from 1837 to 1844, when they were discontinued,—the heaviest-loaded bars continuing to sustain their load without any apparent increase in the deflection. The deflections were taken monthly and carefully recorded, and the following Table exhibits the changes that took place in both the hot- and cold-blast iron bars from June 1838 to June 1842. It is satisfactory to observe that during the whole time of the experiments the bars, whether loaded with the lighter or heavier weights, exhibited little or no change beyond what may be traced to the variations of temperature. One of the bars was, however, found broken, but whether from accident or the effects of continued strain I am unable to determine. I am inclined to believe that the former was the case, as the corresponding bars retained their position, indicating changes so exceedingly small as to be scarcely perceptible, even when examined by the microscope and our best instruments.

Deflections produced with permanent weights on hot- and cold-blast cast-iron bars 4 feet 6 inches between the supports.

Cold-blast, Weight in lbs.	Deflection, in inches.	Date of observation.	Temperature, Fahr.	Hot-blast, Weight in lbs.	Deflection, in inches.
336	1·316	June 23rd, 1838.	78°	336	1·538
336	1·308	April 19th, 1842.	58°	336	1·620
392	1·824	June 23rd, 1838.	78°	392	1·803
392	1·828	April 19th, 1842.	58°	392	1·812
448	1·457	June 23rd, 1838.	78°	448	
448	1·449	April 19th, 1842.	58°	448	

From the above it will be seen that there is no increase in the deflection of the cold-blast bar with the 336 lb. load, but a slight increase of ·082 of an inch in the hot-blast. With the 392 lbs. there is a slight and progressive increase in both bars, and in those with a load of 448 lbs. there is no change but what is due to the difference of 20° of temperature between the month of June and that of April. As respects the load of 448 lbs., it is proper here to observe that the hot-blast bars broke at once with that weight, and one of the cold-blast bars also broke after sustaining the load 37 days, but whether by accident or from vibration is not determined. It is, however, evident from the breaking of the hot-blast bars, and one of the cold-blast, that the load of 448 lbs. approximated very close on the point of fracture, and that the slightest vibration of the floor would break the bar.

Viewing the subject in this light, it would appear from these experiments that time is an element which in a greater or less degree affects the security of materials when subjected to long-continued pressure. It may at first sight appear that the cohesive powers and the resistance may be so nicely balanced as to neutralize each other, and in this state would continue to sustain the load in that condition *ad infinitum*, provided there be no disturbing force to produce derangement of the parts, and thus destroy the equilibrium of the opposing forces. This cannot, however, be expected, and I think we may reasonably, under ordinary conditions of disturbance, conclude that long-continued strain will tend to lessen the cohesive force which unites the particles of matter together, and ultimately destroy that power of resistance so strongly exemplified in the above experiments. (Vide Report, Transactions of the British Association for 1842.)

As the object of this inquiry is to ascertain the limit of safety in structures, such as railway bridges, subjected to vibration and impact from a rolling load, it may be necessary, for the purpose of illustration, to refer to experiments made by the Commission appointed in 1848 to inquire into the application of iron to railway structures. In these inquiries the late Professor Hodgkinson and Professor Willis entered elaborately into the experimental as well as the mathematical investigation; but the experiments which bear more directly upon the present inquiry are those of Captain Henry (now Sir Henry) James and Captain Galton, for determining the effects pro-

duced by passing weights over bars with different velocities, and subjecting others to reiterated strain corresponding to loads equal to some fractional part of the breaking-weight. The latter experiments were made with cams, caused to revolve by steam machinery, which depressed the bars and allowed them to resume their natural position for a large number of times. Two cams were used; one communicated a highly vibratory motion to the bar during the deflection, and the other greatly depressed the bar subjected to it, and released it suddenly when the ultimate deflection due to the load had been obtained, the rate of deflections being from four to seven per minute. Three bars, subjected by the first-mentioned cam to a deflection equal to what would have been produced by one-third of the statical breaking-weight obtained from similar bars, received 10,000 successive depressions, and when afterwards broken by statical pressure, bore as much as similar bars subjected to dead weight only. Of two bars subjected to a deflection equal to what would have been caused by half the statical breaking-weight, one broke with 28,602 depressions, the other withstood 30,000, and did not appear weakened to resist statical pressure.

Of the bars subjected to the second cam, three bore 10,000 depressions, each giving it a deflection equal to what would be produced by one-third of the statical breaking-weight, without having their strength to resist statical pressure apparently at all impaired; one broke with 51,538 such depressions, and one bore 100,000 without any apparent diminution of strength; whilst three bars, subjected by the same cam to a deflection equal to what would be produced by half the statical breaking-weight, broke with 490, 617 and 900 depressions respectively. It must therefore be concluded that iron bars will scarcely bear the reiterated application of one-third their breaking-weight without injury.

A bar of wrought iron 2 inches square in section and 9 feet long between the supports, was subjected to 100,000 depressions, by means of the first-mentioned or rough cam, each depression producing a strain corresponding to about $\frac{5}{9}$ths of the strain that permanently injured a similar bar. These depressions only produced a permanent set of ·015 inch.

Three wrought-iron bars were subjected to 10,000 depressions each from the step-cam, depressing them through $\frac{1}{3}$ inch, $\frac{2}{3}$ inch, and $\frac{5}{6}$ inch respectively, without producing any perceptible permanent set. A bar depressed through 1 inch obtained a set of ·06 inch, and one depressed 300 times through 2 inches acquired a set of 1·08 inch. The largest deflection which did not produce any permanent set appears, by an experiment on a similar bar, to be that due to rather more than half the statical weight which permanently injured it.

A small box girder of boiler-plate riveted, 6 in. by 6 in. in section and 9 ft. long, was also subjected to depressions by means of the rough cam, principally with the view of ascertaining whether any effect would be produced on the rivets by the repeated strain; but a strain corresponding to 3752 lbs. repeated 43,370 times did not produce any appreciable effect.

From the experiments made by the Commissioners it may be inferred—

1st. That cast-iron bars or girders are not safe when subjected to a series of deflections due to one-half the load that would break them.

2nd. That they are perfectly secure in sustaining a dead weight not exceeding one-third of the weight that would break them; and

3rd. That these reiterated deflections appear to have no injurious effect upon the metal from which the bars were cast.

As respects wrought iron, it appeared from the experiments that a progressive increase in the deflections and permanent set was observable during every depression produced by the same cam as that employed on the cast-iron bars, exhibiting great deficiency in its elastic powers. Where the bar retained its power of restoration up to 30,000 deflections, with 10,000 more changes it took a set of ·06 inch, and from that number, with 810 additional depressions, the set increased to 1·84 inch, evidently showing that it would have continued still further to increase until the bar was rendered useless.

Comparing these experiments with those obtained from the riveted wrought-iron beam in the following experiments, it will be found that a load equivalent to one-fourth the breaking-weight produced no visible change nor any permanent set after being subjected to 1,000,000 depressions of ·17 and ·22 inch. By increasing the load from one-fourth to two-fifths, it sustained 5175 additional deflections of ·22 inch, when it broke. The difference between the experiments on the wrought-iron bar and the wrought-iron manufactured girder consists in the greater rigidity of the latter, and in its increased power of resistance to vibration and the force of impact, the weight on the girder descending upon it by the force of gravity.

The institution of experiments for the purpose of ascertaining the value of wrought-iron riveted plates, in the form of tubes, through which a railway train should pass, was a conception which led to a new era in the history of bridges, and ultimately effected the passage of the estuary of the Conway and the Menai Straits. These experiments not only gave the form and strengths required for the construction of these colossal structures, but they developed an entirely novel system of constructive art, and established the principle on which wrought-iron bridges should in future be made. Since then some thousands of bridges, many of them of great span, have been constructed, composed entirely of wrought iron, and are now in existence supporting railways and common roads to an extent hitherto unknown in the history of bridge-building, and such as could not have been accomplished by any other description of material than malleable iron or steel.

The construction of the Britannia and Conway bridges in the tubular form led to other constructions, such as the tubular girder, the plate and lattice girder, and other forms, all founded on the principle developed in the construction of the large tubes as they now span the Conway and the Menai Straits. In the tubular bridges, it was first designed that their ultimate strength should be six times the heaviest load that could ever be laid upon them, after deducting half the weight of the tube. This was considered a

fair margin of strength; but subsequent considerations, such as generally attend a new principle of construction with an untried material, induced an increase of strength, and, instead of the ultimate strength being six times, it was increased to eight times the weight of the greatest load.

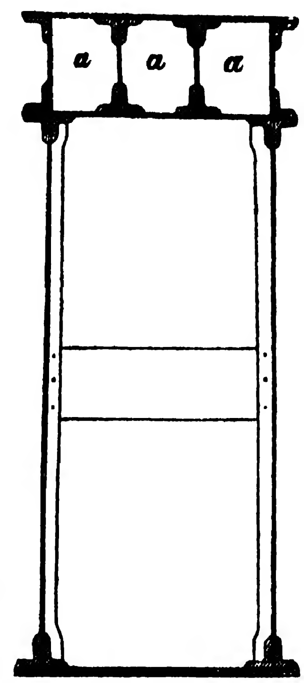
The stability and great success of these bridges gave increased confidence to the engineer and the public, and for several years the resistance of six times the heaviest load was considered an amply sufficient margin of strength.

Owing to the success of these undertakings, there was a general demand for wrought-iron bridges in every direction, and numbers were made without any regard to first principles, or to the law of proportion necessary to be observed in the sectional areas of the top and bottom flanges, so clearly and satisfactorily shown in the early experiments. The result of this was a number of weak bridges, many of them so disproportioned in the distribution of the material as to be almost at the point of rupture with little more than double the permanent load. These discrepancies, and the erroneous system of contractors tendering by weight, led not only to defects in the principle of construction, but the introduction of bad iron and, in many cases, equally bad workmanship. Now there is no construction which requires greater care and more minute attention to sound principles than *wrought-iron girders*, whether employed for bridges of large or small span or for buildings. The lives of the public entirely depend upon the knowledge and skill of the engineer, and the selection of the material which he employs.

The defects and break-downs which followed the first successful application of wrought iron to bridge-building led to doubts and fears on the part of engineers; and many of them contended for eight, and even ten times the heaviest load as the safe margin of strength. Others, and amongst them the late Mr. BRUNEL, fixed a lower standard; and I believe that gentleman was prepared in practice to work up to one-third or two-fifths of the ultimate strength of the weight that would break the bridge. Ultimately it was decided by the authorities of the Board of Trade, but from what data I am not informed, that no wrought-iron bridge should with the heaviest load exceed a strain of 5 tons per square inch. Now on what principle this standard was established does not appear; and on application to the Board of Trade the answer is, that "The Lords Commissioners of Trade require that all future bridges for railway traffic shall not exceed a strain of 5 tons per square inch."

The requirement of 5 tons per square inch on the part of the Board of Trade is not sufficiently definite to secure in all cases the best form of construction. It is well known that the powers of resistance to strain are widely different with wrought iron, according as we apply a force of tension or compression; it is even possible so to disproportion the top and bottom areas of a wrought-iron girder calculated to support six times the rolling load, as to cause it to yield with little more than half the ultimate strain or 10 tons on the square inch. For example, in wrought-iron girders with solid tops it requires the sectional area in the top to be nearly double that of the bottom, to

equalize the two forces of tension and compression; and unless these proportions are strictly adhered to in the construction, the 5-ton strain per square inch is an error which may lead to dangerous results. Again, it was ascertained from direct experiment that double the quantity of material in the top of a wrought-iron girder was not the most effective form for resisting compression. On the contrary, it was found that little more than half the sectional area was required, and, when converted into rectangular cells similar to *a*, *a*, *a*, was in its powers of resistance equivalent to double the area when formed of a solid plate. This discovery was of great value in the construction of tubes and girders of wide span, as the weight of the structure itself (which increases as the cubes, and the strength only as the squares) forms an important part of the load to which it is subjected. On this question it is evident that the requirement of the Board of Trade cannot be applied in both cases, and is therefore ambiguous as regards its application to different forms of structure. In the 5-ton per square inch strain there is not a word said about the dead weight of the bridge; and we are not informed whether the breaking-weight was to be so many times the applied weight plus the multiple of the load, or, in other words, whether it included or deducted the weight of the bridge itself.



These data are wanting in the railway instructions; and until some fixed principle of construction is determined upon, accompanied by a standard measure of strength, it is in vain to look for any satisfactory result in the erection of road and railway bridges composed entirely of wrought iron.

I have been led to inquire into this subject with more than ordinary care, not only on account of the imperfect state of our knowledge, but from the want of definite instructions from the authorities whose duty it is to secure the safety of bridges and to protect the public from malconstructions. To accomplish this, I have in the following experimental researches endeavoured to arrive at the extent to which a bridge or girder of wrought iron may be strained without injury to its ultimate powers of resistance. I have endeavoured to ascertain the exact amount of load to which a bridge may be subjected without endangering its safety, or, in other words, to determine the fractional strain of its estimated powers of resistance.

To arrive at correct results and to imitate as nearly as possible the strain to which bridges are subjected by the passage of heavy railway trains, the apparatus specially adapted for that purpose was designed to lower the load quickly upon the beam in the first instance, and subsequently to produce a considerable amount of vibration, as the large lever with its load and shackle was left suspended upon it in the second. The apparatus was sufficiently elastic for that purpose, as may be seen on reference to the drawings.

The beam A, Plates VI. & VII., is composed of an iron plate riveted with angle-irons 22 feet long, $\frac{1}{8}$ of an inch thick, and 16 inches deep.

It was supported on two brick piers 20 feet apart. Beneath the bottom flange is fixed the lever B, which, by means of the link and shackle C, grasps the lower web of the beam close to the fulcrum D. This fulcrum, on which the lever oscillates, is formed of a vertical bar E, which acts as a standard, and has screw-nuts to regulate the height from the cast-iron plate F to the fulcrum D. The machinery for lifting the lever and scale at H consists of the shaft and pulley I, driven by a water-wheel; and from this shaft the apparatus for lifting the load is worked by a strap from the pulley on the pinion-shaft K, which drives the shaft and spur-wheel L, giving motion to the connecting rod M. This rod has an oblong slot at its lower end, in which the pin at the end of the lever works. From this description it will be seen that, in turning the spur-wheel L, the weight is not raised until the bottom of the slot comes in contact with the pin of the lever, when the load is taken entirely off the beam. That being accomplished, the connecting rod descends, when the load is again laid upon the beam and left suspended with a vibratory motion for some seconds, until the remainder of the stroke is completed, when the connecting rod again rises for the succeeding lift. In this way the weights are lifted off and replaced alternately upon the beam at the rate of seven to eight strokes per minute. The apparatus is worked night and day by a water-wheel, and the number of changes are registered by the counter attached to the vertical post at G.

The girder subjected to vibration in these experiments is a wrought-iron plate beam of 20 feet clear span, and of the following dimensions:—

	inches.	
Area of top, 1 plate 4 inches $\times \frac{1}{2}$ inch	2·00	
„ 2 angle-irons $2 \times 2 \times \frac{5}{16}$	2·30	
	——	4·30
Area of bottom, 1 plate 4 inches $\times \frac{1}{4}$ inch	1·00	
„ 2 angle-irons $2 \times 2 \times \frac{3}{16}$ inch	1·40	
	——	2·40
Web, 1 plate $15\frac{1}{4} \times \frac{1}{8}$		1·90
Total sectional area		8·60

Depth	16 inches.
Weight	7 cwt. 3 qrs.
Breaking-weight (calculated)	12 tons.

The beam having been loaded with 6643 lbs., equivalent to one-fourth of the ultimate breaking-weight, the experiment commenced as follows:—

TABLE I.—Experiment on wrought-iron beam with a changing load equivalent to one-fourth of the breaking-weight.

Date, 1860.	Number of changes of load.	Deflection produced by load.	Remarks.
March 21.	0	0·17	
„ 22.	10,540	0·18	
„ 23.	15,610	0·16	
„ 24.	27,840	....	Strap loose, and failed to lift the weight.
„ 26.	46,100	0·16	
„ 27.	57,790	0·17	
„ 28.	72,440	0·17	
„ 29.	85,960	0·17	
„ 30.	97,420	0·17	
„ 31.	112,810	0·17	
April 2.	144,350	0·16	
„ 4.	165,710	0·18	
„ 7.	202,890	0·17	
„ 10.	235,811	0·17	
„ 13.	268,328	0·17	
„ 14.	281,210	0·17	
„ 17.	321,015	0·17	
„ 20.	343,880	0·17	The strap broke.
„ 25.	390,430	0·17	
„ 27.	408,264	0·16	
„ 28.	417,940	0·16	
May 1.	449,280	0·16	
„ 3.	468,600	0·16	
„ 6.	489,769	0·16	
„ 7.	512,181	0·16	
„ 9.	536,355	0·16	
„ 11.	560,529	0·16	
„ 14.	596,790	0·16	

The beam having undergone above half a million changes of load, by working continuously for two months, night and day, at the rate of about eight changes per minute, without producing any visible alteration, the load was increased from one-fourth to two-sevenths of the statical breaking-weight, and the experiment proceeded with till the number of changes of load reached a million.

Table II.—Experiment on the same beam with a load equivalent to two-sevenths of the breaking-weight, or nearly 3½ tons.

Date, 1860.	Number of changes of load.	Deflection, in inches.	Remarks.
May 14.	0	0·22	In this Table the number of changes of load is counted from 0, although the beam had already undergone 596,790 changes, as shown in the preceding Table.
15.	12,623	0·22	
17.	36,417	0·22	
19.	53,770	0·21	
22.	85,820	0·22	
26.	128,300	0·22	
29.	161,500	0·22	
31.	177,000	0·22	
June 4.	194,500	0·21	
7.	217,300	0·21	
9.	236,460	0·21	
12.	264,220	0·21	
16.	292,600	0·22	At this point the operations were suspended, the beam having suffered a million changes of load.
26.	403,210	0·23	

The beam had now sustained one million changes of load without any apparent injury; it was then considered necessary to increase the load to 10,486 lbs., or two-fifths of the breaking-weight, when the machinery was again put in motion. With this additional weight the deflections were increased, with a permanent set of ·05 inch, from ·23 to ·35, and after sustaining 5175 changes, the beam broke by tension a short distance from the middle. It is satisfactory here to observe that during the whole of the 1,005,175 changes none of the rivets were loosened or broke.

Table III.—Beam repaired.

The beam fractured in the preceding experiment was repaired by replacing the broken angle-irons on each side, and putting a patch over the broken plate equal in area to the plate itself. Thus repaired, a weight of three tons was placed on the beam, equivalent to one-fourth of the breaking-weight, when the experiments were again continued as before.

Date.	Number of changes of load.	Deflection, in inches.	Permanent set, in inches.	Remarks.
1860. August 9.	158	....	....	The load during these changes was equivalent to 10,500 lbs., or 4·6875 tons at the centre. With this weight the beam took a large but unmeasured set.
Aug. 11.	12,950	....	....	During these changes the load on the beam was 8025 lbs., or 3·58 tons.
„ 12.	25,742	0·22	?	
Aug. 13.	25,900	0·18	0	Load reduced to 2·96 tons, or ¼th of the breaking-weight.
„ 16.	46,326	0·18	0	
„ 20.	71,000	0·18	0	
„ 24.	101,760	0·18	0	
„ 25.	107,000	0·18	0	
„ 31.	135,260	0·18	0·01	
Sept. 1.	140,500	0·18	0·01	
„ 8.	189,500	0·18	0·01	
„ 15.	242,860	0·18	0·01	
„ 22.	277,000	0·18	0·01	
„ 30.	320,000	0·18	0·01	
Oct. 6.	375,000	0·18	0·01	
„ 13.	429,000	0·18	0·01	
„ 20.	484,000	0·18	0·01	
„ 27.	538,000	0·18	0·01	
Nov. 3.	577,800	0·18	0·01	
„ 10.	617,800	0·18	0·01	
„ 17.	657,500	0·18	0·01	
„ 23.	712,300	0·18	0·01	
Dec. 1.	768,100	0·18	0·01	
„ 8.	821,970	0·18	0·01	
„ 15.	875,000	0·18	0·01	
„ 22.	929,470	0·18	0·01	
„ 29.	1,024,500	0·18	0·01	
1861.				
Jan. 9.	1,121,100	0·18	0·01	
„ 19.	1,214,000	0·18	0·01	
„ 26.	1,278,000	0·18	0·01	
Feb. 2.	1,342,800	0·18	0·01	
„ 11.	1,426,000	0·18	0·01	
„ 16.	1,485,000	0·18	0·01	
„ 23.	1,543,000	0·18	0·01	
March 2.	1,602,000	0·18	0·01	
„ 9.	1,661,000	0·18	0·01	
„ 16.	1,720,000	0·17	0·01	
„ 23.	1,779,000	0·17	0·01	
„ 30.	1,829,000	0·17	0·01	
April 6.	1,885,000	0·17	0·01	
„ 13.	1,945,000	0·17	0·01	
„ 20.	2,000,000	0·17	0·01	
„ 27.	2,059,000	0·17	0·01	
May 4.	2,110,000	0·17	0·01	
„ 11.	2,165,000	0·17	0·01	
„ 20.	2,250,000	0·17	0·01	
Sept. 4.	2,727,754	0·17	0·01	
Oct. 16.	3,150,000	0·17	0·01	

At this point, the beam having sustained upwards of three million changes of load without any increase of the permanent set, it was assumed that it might have continued to bear alternate changes to any extent with the same tenacity of purpose as exhibited in the foregoing Table. It was then concluded to increase the load from one-fourth to one-third of the breaking-weight; and having laid on 4 tons, which increased the deflection to ·20 inch, the work was proceeded with in the same order as in the previous experiments.

TABLE IV.

Date.	Changes of load.	Deflection, in inches.	Permanent set, in inches.	Remarks.
1861.				
Oct. 18.	0	0·20		
19.	4000	0·20	0·	
Nov. 18.	126,000	0·20		
Dec. 18.	237,000	0·20		
1862.				
Jan. 9.	313,000	....	....	Broke by tension across the bottom web.

From these experiments it is evident that wrought-iron girders of ordinary construction are not safe when submitted to violent disturbances with a load equivalent to one-third the weight that would break them. They, however, exhibit wonderful tenacity when subjected to the same treatment with one-fourth the load; and assuming that an iron girder bridge will bear with this load 12,000,000 changes without injury, it is clear that it would require 328 years, at the rate of 100 changes per day, before its security was affected. It would, however, be dangerous to risk a load of one-third the breaking-weight upon bridges of this description, as according to the last experiments the beam broke with 313,000 changes; or a period of eight years, at the same rate as before, would be sufficient to break it. It is more than probable that the beam might have been injured by the previous three million changes to which it had been subjected; and assuming this to be true, it would then follow that the beam was progressing to destruction, and must of necessity at some time, however remote, have terminated in fracture.

The experiments throw considerable light on this very intricate and very important subject. They are probably carried sufficiently far to enable us to state with certainty what is the safe measure of strength; and as much depends upon the quality of the material and the skill with which the girders are put together, it becomes necessary for the public safety that a measure of strength should be established without encumbering the structures with unnecessary weight. On this question it must be borne in mind that every additional ton that is not required beyond the limits of safety, is an evil that operates as a constant quantity tending to produce rupture; and hence follows the necessity of a careful distribution of the material, in order that the tube or girder shall be duly proportioned to the strains it has to bear, and that every part of the structure shall have its due proportion of work to perform.

I have assumed, for the sake of illustration, that every description of material, as regards its cohesive properties, follows the same law as that which we have experimented upon, or, in other words, in the ratio of its physical powers of resistance, that is to say, any beam will follow the same law in regard to its ultimate powers of resistance, when operated upon by a corresponding load due to that power. If this be true, we have only to follow the same rule as observed in the experiments, by loading cast-iron or wooden beams in the ratio of their cohesive powers of resistance, and their breaking-weights respectively. This has not been proved experimentally, but I hope at some future time to have an opportunity of extending the experiments, in order to determine to what extent these views are correct.

The Lords Commissioners of Trade, in the exercise of their functions as conservators of the public safety, have adopted the rule that no railway bridge composed of wrought iron shall exceed with its heaviest rolling-load a strain of five tons per square inch of section upon any part of the bridge. The formula for this maximum of strain upon the material has been deduced from my own experiments on the model tube at Millwall.

Assuming the top of the girder to be sufficiently rigid to prevent buckling by compression, the formula for the strength of the bottom section derived from these experiments is

$$W=\frac{adc}{l},$$

where the constant $c=80$.

Applying this formula to the beam experimented upon, we have

a, the area of the bottom $=2{\cdot}4$ inches,
d, the depth of the beam $=16$ inches,
c, the constant deduced from the model tube $=80$,
l, the span or distance between the supports $=240$ inches.

Hence $$W=\frac{2{\cdot}4\times16\times80}{240}=12{\cdot}8 \text{ tons},$$

the ultimate strength of the beam.

In order to determine the strain per square inch in these experiments, we find

$$S=\frac{lw}{4ad},$$

where S represents the strain per square inch upon the section a, produced by the greatest load w, laid upon the middle of the girder.

It is necessary to observe that in a girder properly proportioned, the greatest strain per square inch will take place upon the bottom section; so that if the strain upon the bottom section of such a girder be within the Government Commissioner's condition of safety, the strain upon the top section will necessarily be within that limit also. In a girder having the cellular structure at its top section, the area of the top section should be very nearly once and a quarter that of the bottom section, or the areas of their

sections should be respectively as 5 : 4; and the strain per square inch upon these parts will be respectively inversely as their areas; that is, the strain per square inch upon the top section will be $\frac{4}{5}$ths of the strain per square inch upon the bottom section. In one of the foregoing experiments, we have

l, the length of the girder=240 inches,
w, the weight laid on the middle=2·96 tons,
a, the area of the bottom section=2·4 inches,
d, the depth of the girder=16 inches;

therefore
$$S=\frac{240\times 2{\cdot}96}{4\times 2{\cdot}4\times 16}=4{\cdot}62 \text{ tons},$$

the strain per square inch on the bottom section of the girder.

Applying this formula to the whole series of experiments, we obtain the following summary of results:—

Summary of Results.—1st Series of Experiments.

Beam 20 feet between the supports.

No of experiment.	Date.	Weight on middle of the beam, in tons.	Number of changes of load.	Strain per square inch on bottom.	Strain per square inch on top.	Deflection, in inches.	Remarks.
1	From March 21st to May 14th, 1860	2·96	596,790	4·62	2·58	·17	
2	From May 14th to June 26th, 1860	3·50	403,210	5·46	3·05	·23	
3	From July 25th to July 28th, 1860	4·68	5,175	7·31	4·08	·35	Broke by tension at a short distance from the centre of the beam.

The number of 1,005,175 changes was attained before fracture, with varying strains upon the bottom flange of 4·62 tons, 5·46 tons, and 7·31 tons per square inch.

Beam repaired.—2nd Series of Experiments.

4	August 9th, 1860 ..	4·68	158	7·31	4·08	...	The apparatus was accidentally set in motion.
5	August 11th and 12th	3·58	25,742	5·59	3·12	·22	
6	From August 13th, 1860 to October 16th, 1861	2·96	3,124,100	4·62	2·58	·18	
7	From October 18th, 1861 to January 9th, 1862	4·00	313,000	6·25	3·48	·20	Broke by tension as before, close to the plate riveted over the previous fracture.

Here the number of 3,463,000 changes was attained when fracture ensued.

From the above it is evident that wrought-iron girders, when loaded to the extent of a tensile strain of seven tons per square inch, are not safe, if that strain is subjected to alternate changes of taking off the load and laying it on again, provided a certain amount of vibration is produced by that process; and what is important to notice is, that from 300,000 to 400,000 changes of this description are sufficient to ensure fracture. It must, however, be borne in mind that the beam from which these conclusions are derived had sustained upwards of 3,000,000 changes with nearly five tons tensile strain on the square inch, and it must be admitted from the experiments thus recorded that five tons per square inch of tensile strain on the bottom of girders, as fixed by the Board of Trade, appears to be an ample standard of strength.

As regards compression, we have only to compare for practical purposes the difference between the resisting-powers of the material to tension and compression, and we shall require in a girder without a cellular top from one-third to three-fourths more material to resist compression than to resist tension; and as the strength of wrought iron in a state of compression is to its strength in a state of tension as about 3 to 4½, the area of the top and bottom will be nearly in that proportion, or, in other words, it will require that much more material in the top than the bottom to equalize the two forces.

In the experimental beam the area of the top was considerably in excess of that of the bottom, it having been constructed on data deduced from the experiments on tubes without cells, which required nearly double the area on the top to resist crushing. In the construction of large girders, where thicker plates are used, this proportion no longer exists, as much greater rigidity is obtained in the thicker plates, which causes a closer approximation to equal areas in the top and bottom of the girder; and from this we deduce that from one-third to three-fourths, and in some cases one-third additional area in the top has been found, according to the size of the girder, sufficient to balance the two forces under strain.

The foregoing experiments, however, were instituted to determine the safe measure of strength as respects tension, and it will be seen that in no case during the whole of the experiments was there any appearance of the top yielding to compression.

IX. The Bakerian Lecture.—*Contributions to Molecular Physics.—Being the Fifth Memoir of Researches on Radiant Heat.* By John Tyndall, *F.R.S., Member of the Academies and Societies of Holland, of Geneva, Göttingen, Zürich, Halle, Marburg, Breslau, Upsala, la Société Philomathique, Paris, Cam. Phil. Soc. &c.; Professor of Natural Philosophy in the Royal Institution.*

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Contents.

§ I.

The natural philosophy of the future must, I imagine, mainly consist in the investigation of the relations which subsist between the ordinary matter of the universe and the ether, in which this matter is immersed. Regarding the motions of the ether itself, as illustrated by the phenomena of reflexion, refraction, interference, and diffraction, the optical investigations of the last half century have left nothing to be desired; but as regards the atoms and molecules whence issue the undulations of light and heat, and their relations to the medium which they move, and by which they are set in motion, these investigations teach us nothing. To come closer to the origin of the ethereal waves—to get, if possible, some experimental hold of the oscillating atoms themselves —has been the main object of the researches in which I have been engaged for the

last five years. In these researches radiant heat has been used as an instrument for exploring molecular condition, and this is the object which I have kept constantly in view throughout the investigation which I have now the honour to submit to the Royal Society.

The first part of these researches is devoted to the more complete examination of a subject which was briefly touched upon at the conclusion of my Fourth Memoir—namely, the action of liquids, as compared with that of their vapours, upon radiant heat. The differences which exist between different gaseous molecules, as regards their power of emitting and absorbing radiant heat, have been already amply illustrated. When a gas is condensed to a liquid, the molecules approach and grapple with each other by forces which are insensible as long as the gaseous state is maintained. But though thus condensed and enthralled, the ether still surrounds the molecules. If, then, the powers of radiation and absorption depend upon them individually, we may expect that the deportment towards radiant heat which experiment establishes in the case of the free molecule, will maintain itself after the molecule has relinquished its freedom and formed part of a liquid. If, on the other hand, the state of aggregation be of paramount importance, we may expect to find on the part of liquids a deportment altogether different from that of their vapours.

MELLONI, it is well known, examined the diathermancy of various liquids, but he employed for this purpose the flame of an oil-lamp, covered by a glass chimney. His liquids, moreover, were contained in glass cells; hence the radiation from the source was profoundly modified before it entered the liquid at all, for the glass was impervious to a considerable part of the radiation. It was my wish to interfere as little as possible with the primitive emission, and also to compare the action of liquids with that of their vapours, when examined in a tube stopped with plates of rock-salt. I therefore devised an apparatus in which a layer of liquid of any thickness could be enclosed between two polished plates of rock-salt. It was skilfully constructed for me by Mr. BECKER, and the same two plates have already done service in more than six hundred experiments.

The apparatus consists of the following parts:—A B C (fig. 1) is a plate of brass, 3·4 inches long, ·2·1 inches wide, and 0·3 of an inch thick. Into it, at its corners, are rigidly fixed four upright pillars, furnished at the top with screws, for the reception of the nuts *q r s t*. D E F is a second plate of brass of the same size as the former, and pierced with holes at its four corners, so as to enable it to slip over the four columns of the plate A B C. Both these plates are perforated by circular apertures, *m n* and *o p*, 1·35 inch in diameter. G H I is a third plate of brass of the same area as D E F, and, like it, having its centre and its corners perforated. The plate G H I is intended to separate the two plates of rock-salt, which are to form the walls of the cell, and its thickness determines that of the liquid layer. Thus when the plates A B C and D E F are in position, a space of the form of a shallow cylinder is enclosed between them, and this space can be filled with any liquid through the orifice *k*. The separating plate

Fig. 1.

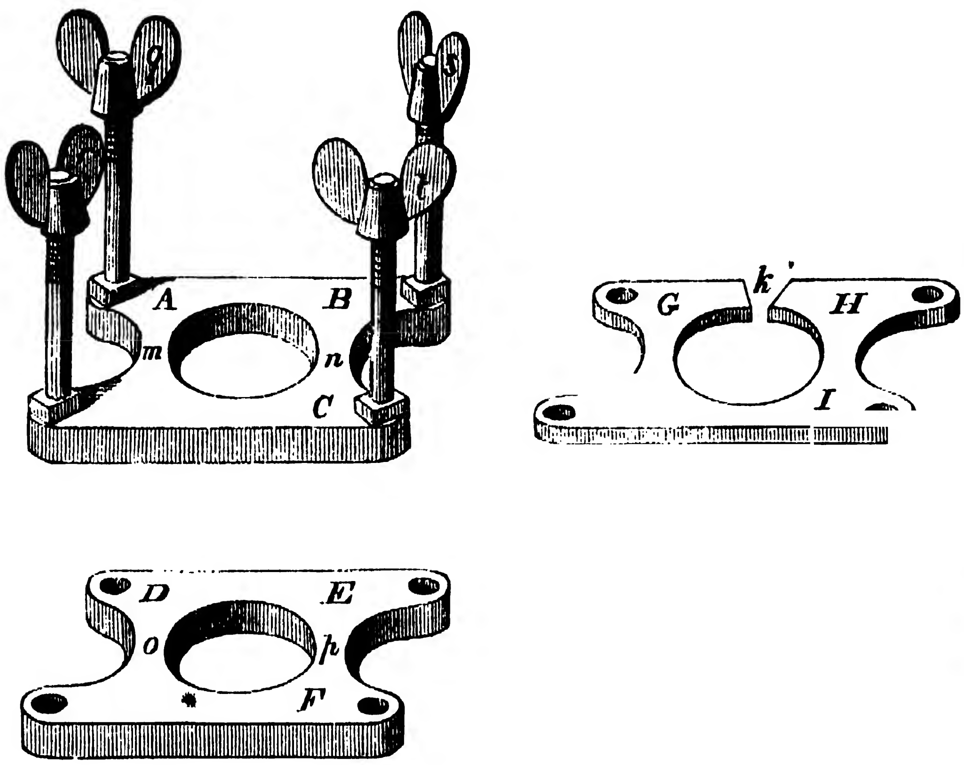
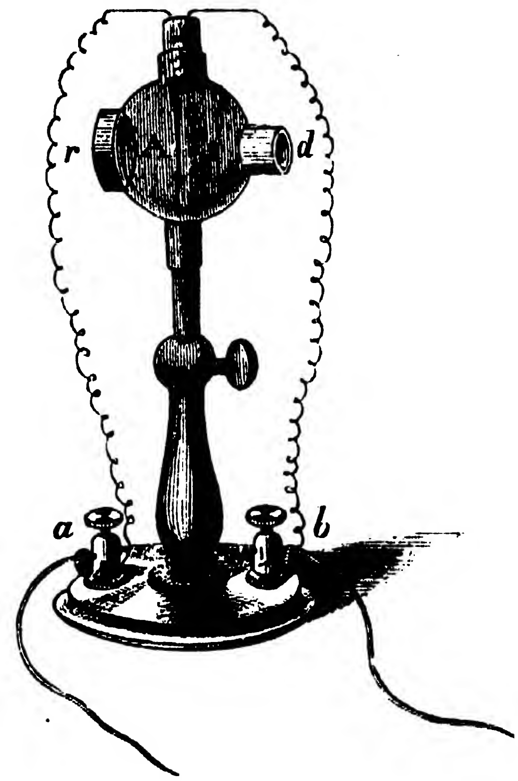


Fig. 2.



G H I was ground with the utmost accuracy, and the surfaces of the plates of salt were polished with extreme care, with a view to rendering the contact between the salt and the brass water-tight. In practice, however, it was found necessary to introduce washers of thin letter-paper between the plates of salt and the separating plate.

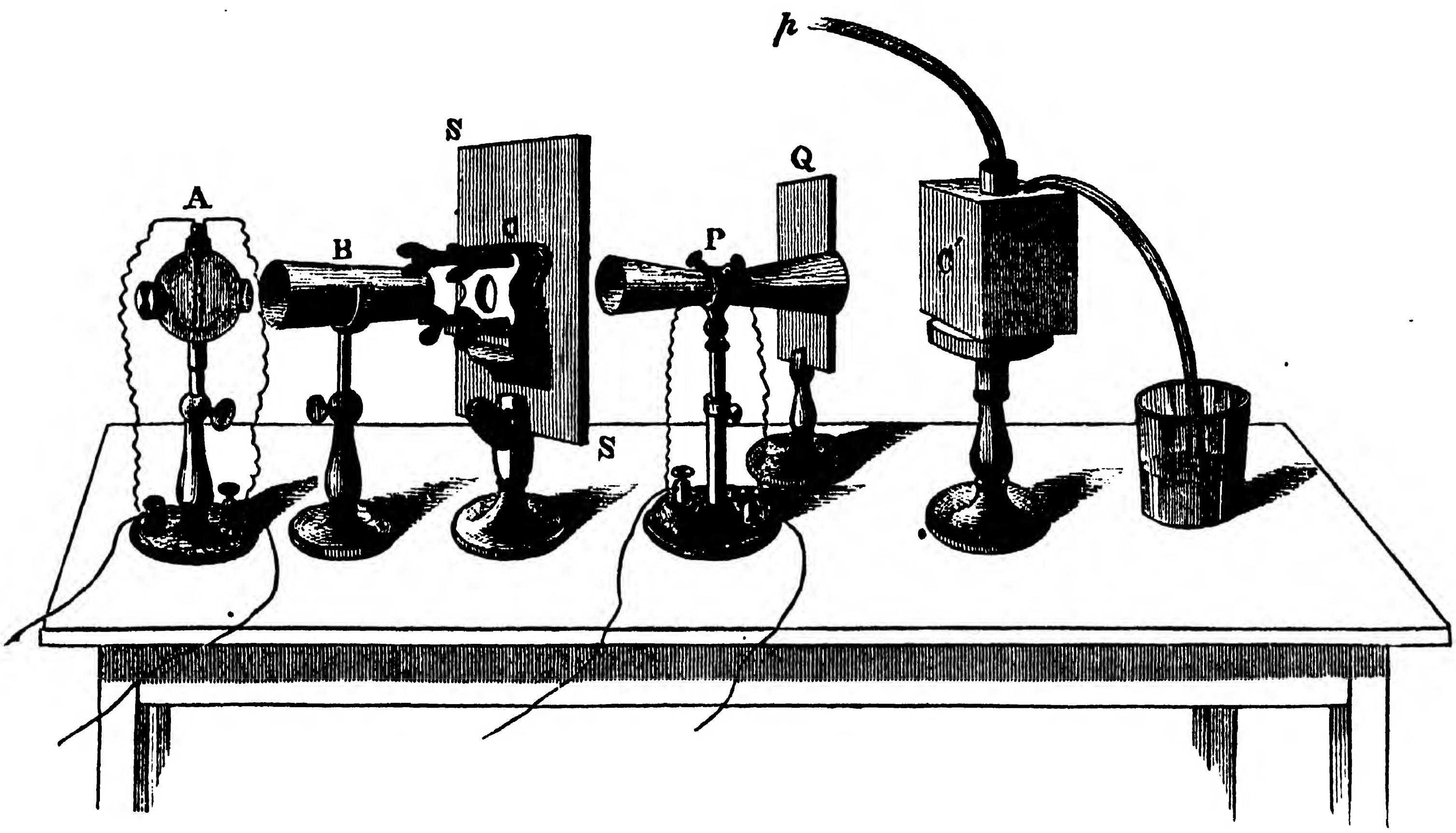
In arranging the cell for experiment, the nuts *q r s t* are unscrewed, and a washer of india-rubber is first placed on A B C. On this washer is placed one of the plates of rock-salt. On the plate of rock-salt is placed the washer of letter-paper, and on this again the separating plate G H I. A second washer of paper is placed on this plate, then comes the second plate of salt, on which another india-rubber washer is laid. The plate D E F is finally slipped over the columns, and the whole arrangement is tightly screwed together by the nuts *q r s t*. The use of the india-rubber washers is to relieve the crushing pressure which would be applied to the plates of salt if they were in actual contact with the brass plates; and the use of the paper washers is, as already explained, to render the cell liquid-tight. After each experiment, the apparatus is unscrewed, the plates of salt are removed and thoroughly cleansed; the cell is then remounted, and in two or three minutes all is ready for a new experiment.

My next necessity was a perfectly steady source of heat, of sufficient intensity to penetrate the most absorbent of the liquids to be subjected to examination. This was found in a spiral of platinum wire, rendered incandescent by an electric current. The frequent use of this source of heat led me to construct the lamp shown in fig. 2. A is a globe of glass 3 inches in diameter, fixed upon a stand, which can be raised and lowered. At the top of the globe is a tubulure, into which a cork is fitted, and through the cork pass two wires whose ends are united by the platinum spiral S. The wires are carried down to the binding screws *a b*, which are fixed in the foot of the stand

so that when the instrument is attached to the battery no strain is ever exerted on the wires which carry the spiral. The ends of the thick wire to which the spiral is attached are also of stout platinum; for when it was attached to copper wires, unsteadiness was introduced through oxidation. The heat issues from the incandescent spiral by the opening *d*, which is an inch and a half in diameter. Behind the spiral, finally, is a metallic reflector, *r*, which augments the flux of heat without sensibly changing its quality. In the open air the red-hot spiral is a capricious source of heat; but surrounded by its glass globe its steadiness is admirable.

The whole experimental arrangement will be immediately understood from the sketch

Fig. 3.



given in fig. 3. A is the platinum lamp just described, heated by a current from a Grove's battery of five cells. It is necessary that this lamp should remain perfectly constant throughout the day; and to keep it so, a tangent galvanometer and a rheocord are introduced into the circuit.

In front of the spiral, and surrounding the tubulure of its globe, is the tube B with an interior reflecting surface, through which the heat passes to the rock-salt cell C. This cell is placed on a little stage soldered to the back of the perforated screen S S, so that the heat, after having crossed the cell, passes through the hole in the screen, and afterwards impinges on the thermo-electric pile P. The pile is placed at some distance from the screen S S, so as to render the temperature of the cell C itself of no account. C′ is the compensating cube, containing water kept boiling by steam from the pipe *p*. Between the cube C′ and the pile P is the screen Q, which regulates the amount of heat falling on the posterior face of the pile. The whole arrangement is here exposed;

but in practice the pile P and the cube C′ are carefully protected from the capricious action of the surrounding air.

The experiments are thus performed. The empty rock-salt cell C being placed on its stage, a double silvered screen (not shown in the figure) is first introduced between the end of the tube B and the cell C; the radiation from the spiral being thus totally cut off, and the pile subjected to the action of the cube C′ alone. By means of the screen Q, the total heat to be adopted throughout the series of experiments is obtained: say that it is sufficient to produce a galvanometric deflection of 50 degrees. The double screen used to intercept the radiation from the spiral is then gradually withdrawn until this radiation completely neutralizes that from the cube C′, and the needle of the galvanometer points steadily to zero. The position of the double screen, once fixed, remains subsequently unchanged, the slight and slow alteration of the source being neutralized by the rheocord. Thus the rays in the first instance pass from the spiral through the empty rock-salt cell. A small funnel, supported by a suitable stand, dips into the aperture which leads into the cell, and through this the liquid is poured. The introduction of the liquid destroys the previous equilibrium, the galvanometer needle moves, and finally assumes a steady deflection; and from this deflection we can immediately calculate the quantity of heat absorbed by the liquid, and express it in hundredths of the entire radiation.

For example, the empty cell being placed upon its stand, and the needle being at 0°, the introduction of iodide of methyl into the cell produced a deflection of 30°·8. The total radiation on this occasion was 44°·2. Taking the force necessary to move the needle from 0° to 1° as our unit, the deflection 30°·8 corresponds to 32 such units, while the deflection 44°·2 corresponds to 58·3 such units. Hence the statement

$$58{\cdot}3 : 100 = 32 : 54{\cdot}9,$$

which gives an absorption of 54·9 per cent. for a layer of liquid iodide of methyl 0·07 of an inch in thickness.

§ 2.

The following Table contains the results obtained in this manner with the respective liquids there mentioned. It embraces both the deflection produced by the introduction of the liquid, and the quantity per cent. intercepted of the entire radiation. It has been intimated to me by some of my Continental friends that the publication of such details as would enable a reader to judge of the precision attainable by my apparatus would be desirable. In this paper I shall, to some extent, endeavour to satisfy this desire, making use, however, of my ordinary experiments.

TABLE I.—Radiation of heat through Liquids. Source of heat, red-hot platinum spiral. Thickness of liquid layer 0·07 of an inch.

Name of liquid.	Deflection.	Absorption per 100.
Iodide of Methyl	30°·8	54·9
Iodide of Ethyl	33·0	60·4

TABLE I. (continued).

Name of liquid.	Deflection.	Absorption per 100.
Benzol	35°·3	67·0
Amylene	37·7	74·8
Sulphuric Ether	39·0	79·4
Acetic Ether	39·6	81·6
Alcohol	41·0	86·6
Water*	43·3	91·4.
Total heat	44·2	100

In these experiments I employed a less delicate galvanometer than that used in my former researches. The experiments were made on the 29th of September, and on the following day I repeated them with the following results:—

TABLE II.—Radiation of heat through Liquids. Source of heat, red-hot platinum spiral. Thickness of liquid layer 0·07 of an inch.

Name of liquid.	Deflection.	Absorption per 100.
Iodide of Methyl	33°·5	53·7
Iodide of Ethyl	35·5	58·7
Benzol	37·5	64·4
Amylene	39·5	70·7
Sulphuric Ether	41·0	75·4
Acetic Ether	41·5	76·9
Formic Ether	42·4	80·0
Alcohol	43·5	84·2
Water	44·7	90·5
Total heat	46·7	100·0

On the 28th of October my most delicate galvanometer was at liberty, and with it I executed the experiments performed with the coarser one. The following are the results:—

TABLE III.—Radiation of heat through Liquids. Source, red-hot platinum spiral. Thickness of liquid layer 0·07 of an inch.

Name of liquid.	Deflection.	Absorption per 100.	
Bisulphide of Carbon	9°·0	12·5	
Chloroform	25·2	35·0	
Iodide of Methyl	36·0	53·2	54·3
Ditto, strongly coloured with iodine	36·0	53·2	
Iodide of Ethyl	38·2	59·0	59·6

* To prevent the water from attacking the cell, it was always first saturated with the substance of the cell itself, namely, transparent rock-salt.

TABLE III. (continued).

Name of liquid.	Deflection.	Absorption per 100.	
Benzol	39°·2	62·5	65·7
Amylene	42·0	73·6	72·3
Sulphuric Ether	42·6	76·1	77·4
Acetic Ether	43·4	78·0	79·3
Formic Ether	43·3	79·0	80·0
Alcohol	44·4	83·6	85·4
Water	45·6	88·8	90·9
Total heat	48·0	100·0	

I have here placed beside the results obtained with the delicate galvanometer, the means of those obtained with the coarser one. It is not my object to push these measurements to the last degree of nicety; otherwise the satisfactory agreement here exhibited might be made still better.

To render the experiments on liquid transmission more complete, I operated with layers of various thicknesses, employing throughout my most delicate galvanometer. The results of these measurements are recorded in the following series of Tables:—

TABLE IV.—Radiation of heat through Liquids. Source, red-hot platinum spiral. Thickness of liquid layer 0·02 of an inch.

Name of liquid.	Deflection.	Absorption per 100.
Bisulphide of Carbon	4°·0	5·5
Chloroform	12·0	16·6
Iodide of Methyl	26·0	36·1
Iodide of Ethyl	27·5	38·2
Benzol	31·3	43·4
Amylene	38·0	58·3
Boracic Ether	39·0	61·8
Sulphuric Ether	39·5	63·3
Formic Ether	40·0	65·2
Alcohol	40·5	67·3
Water	43·7	80·7
Total heat	48·0	100·0

TABLE V.—Thickness of liquid layer 0·04 of an inch.

Name of liquid.	Deflection.	Absorption per 100.
Bisulphide of Carbon	6°·1	8·4
Chloroform	18·0	25·0
Iodide of Methyl	33·0	46·5
Iodide of Ethyl	35·0	50·7

TABLE V. (continued).

Name of liquid.	Deflection.	Absorption per 100.
Benzol	37°·0	55·7
Amylene	40·0	65·2
Boracic Ether	41·0	69·4
Sulphuric Ether	42·0	73·5
Acetic Ether	42·1	74·0
Formic Ether	42·5	76·3
Alcohol	43·2	78·6
Water	45·0	86·1
Total heat	48·0	100·0

TABLE VI.—Thickness of liquid layer 0·14 of an inch.

Name of liquid.	Deflection.	Absorption per 100.
Bisulphide of Carbon	11°·0	15·2
Chloroform	28·6	40·0
Iodide of Methyl	40·0	65·2
Iodide of Ethyl	40·9	69·0
Benzol	41·5	71·5
Amylene	43·0	77·7
Sulphuric Ether	43·2	78·6
Acetic Ether	44·0	82·0
Formic Ether	44·5	84·0
Alcohol	44·8	85·3
Water	46·3	91·8
Total heat	48·0	100·0

TABLE VII.—Thickness of liquid layer 0·27 of an inch.

Name of liquid.	Deflection.	Absorption per 100.
Bisulphide of Carbon	12°·5	17·3
Chloroform	32·3	44·8
Iodide of Methyl	40·8	68·6
Iodide of Ethyl	41·5	71·5
Benzol	42·0	73·6
Amylene	44·1	82·3
Sulphuric Ether	44·8	85·2
Acetic Ether	45·0	86·1
Formic Ether	45·2	87·0
Alcohol	45·7	89·1
Water	46·1	91·0
Total heat	48·0	100·0

The foregoing results are collected together in the folloing Table:—

TABLE VIII.—Absorption of heat by Liquids. Source, platinum spiral heated to a bright redness by a voltaic current.

Liquid.	Thickness of liquid in parts of an inch.				
	0·02.	0·04.	0·07.	0·14.	0·27.
Bisulphide of Carbon	5·5	8·4	12·5	15·2	17·3
Chloroform	16·6	25·0	35·0	40·0	44·8
Iodide of Methyl	36·1	46·5	53·2	65·2	68·6
Iodide of Ethyl	38·2	50·7	59·0	69·0	71·5
Benzol	43·4	55·7	62·5	71·5	73·6
Amylene.......................	58·3	65·2	73·6	77·7	82·3
Sulphuric Ether..............	63·3	73·5	76·1	78·6	85·2
Acetic Ether	......	74·0	78·0	82·0	86·1
Formic Ether..................	65·2	76·3	79·0	84·0	87·0
Alcohol	67·3	78·6	83·6	85·3	89·1
Water..........................	80·7	86·1	88·8	91·8	91·0

Had it been desirable to push these measurements to the utmost limit of accuracy, I should have repeated each experiment, and taken the mean of the determinations. But considering the way in which the different thicknesses check each other, an inspection of the Table must produce the conviction that the results express, within small limits of error, the action of the bodies mentioned.

§ 3.

As liquids, then, those bodies are shown to possess very different capacities of intercepting the heat emitted by our radiating source; and we have next to inquire whether these differences continue after the molecules have been released from the bond of cohesion. We must, of course, test the vapours by waves of the same period as those applied to the liquids; and this our mode of experiment renders easy of accomplishment. The heat generated in a wire by a current of a given strength being invariable, it was only necessary, by means of the tangent compass and rheocord, to keep the current constant from day to day in order to obtain, both as regards quantity and quality, an invariable source of heat.

The liquids from which the vapours were derived were placed in a small long flask, a separate flask being devoted to each. The air above the liquid, and within it, being first carefully removed by an air-pump, the flask was attached to the experimental tube in which the vapours were to be examined. This tube was of brass, 49·6 inches long, and 2·4 inches in diameter, its two ends being stopped by plates of rock-salt. Its interior surface was polished. At the commencement of each experiment, the tube having been thoroughly cleansed and exhausted, the needle stood at zero*. The cock of the flask containing the volatile liquid was then carefully turned on, and the vapour allowed slowly to enter the experimental tube. The barometer attached to the tube was finely graduated, and the descent of the mercurial column was observed through a magnifying lens. When a pressure of 0·5 of an inch was obtained, the vapour was cut

* It is hardly necessary to remark that the principle of compensation described in my former memoirs was employed here also.

off and the permanent deflection of the needle noted. Knowing the total heat, the absorption in 100ths of the entire radiation could be at once deduced from the deflection. The following Table contains the results of the first series of experiments made with the platinum spiral as source.

TABLE IX.—Radiation of heat through Vapours. Source, red-hot platinum spiral. Tension, 0·5 of an inch.

Name of vapour.	Deflection.	Absorption per 100.	Mean.
Bisulphide of Carbon	17·0°	4·7	4·7
Bisulphide of Carbon	16·8	4·6	
Chloroform	22·5	6·2	6·3
Chloroform	22·8	6·3	
Iodide of Methyl	34·0	9·7	9·7
Iodide of Methyl	34·0	9·7	
Iodide of Ethyl	46·0	18·0	17·8
Iodide of Ethyl	45·5	17·6	
Benzol	48·5	20·4	20·4
Benzol	48·5	20·4	
Amylene	56·3	27·3	27·3
Amylene	56·2	27·2	
Total heat	78·3	100·0	

The absence of all caprice or uncertainty in the measurements is, I think, demonstrated by the foregoing Table, which is simply an average sample of the degree of coincidence obtained in separate measurements. Two determinations were made in each case; and it will be seen that while, in some instances, the second experiment yielded the same result as the first, in one instance only does the difference amount to half a degree of the galvanometer.

The foregoing measurements were executed on the 5th of October. On the 7th they were in part repeated, with the following results.

TABLE X.

Name of vapour.	Deflection.	Absorption.
Bisulphide of Carbon	16·5°	4·7
Chloroform	22·8	6·5
Iodide of Methyl	33·0	9·6
Iodide of Ethyl	45·0	17·7
Benzol	48·0	20·6
Amylene	55·3	27·5
Alcohol	55·7	28·1
Formic Ether	58·2	31·4
Sulphuric Ether	58·5	31·9
Acetic Ether	59·9	34·6
Total heat	78·0	100·0

Placing these results beside those recorded in Table IX., the manner in which they check each other will appear.

Comparison of Tables IX. and X.—Absorption.

	Table IX.	Table X.
Bisulphide of Carbon . . .	4·7	4·7
Chloroform	6·3	6·5
Iodide of Methyl	9·7	9·6
Iodide of Ethyl	17·8	17·7
Benzol	20·4	20·6
Amylene	27·3	27·5

The agreement, it will be seen, is as perfect as could be desired.

Augmenting the opening through which the heat passed from the source into the experimental tube, the total heat was increased, and the experiments with a few of the vapours were repeated. The total heat in the last case produced a deflection of 78°, which is equal to 350 units; the total heat now employed produced a deflection of 83°, which is equal to 605 units. It is easy to see that the experiments now to be recorded furnish a direct check on the calibration of the galvanometer. As long as the quality of the heat remains unchanged, the absorption ought to be the same with a high total heat as with a low one. But if experiment show that this is the case, it proves also that the calibration on which the calculation of the absorption depends, cannot be in error. The following results were obtained on the 8th of October:—

TABLE X *a*.

	Deflection.	Absorption.
Amylene	6°·0	28·7
Alcohol	66·4	29·2
Formic Ether	68·5	32·5
Formic Ether	68·5	32·5
Sulphuric Ether	69·2	33·6
Sulphuric Ether	69·1	33·4
Acetic Ether	69·7	34·5
Acetic Ether	69·7	34·5
Total heat	83·0	100·0

Placing the results beside those recorded in Table X., we have the following comparison:—

Comparison of Tables X. and X *a*.

Amylene	27·5	28·7
Alcohol	28·1	29·2
Formic Ether	31·4	32·5
Sulphuric Ether	31·9	33·5
Acetic Ether	34·6	34·5

The differences here are inconsiderable, and lean to neither side; within these limits, therefore, the calibration must be correct; it shall be tested more severely in another part of this paper.

§ 4.

We are now in a condition to compare the action of a series of volatile liquids with that of the vapours of those liquids upon radiant heat.

Commencing with the substance of the lowest absorptive energy, and proceeding to the highest, we have the following order of absorption:—

Liquids.	Vapours.
Bisulphide of Carbon.	Bisulphide of Carbon.
Chloroform.	Chloroform.
Iodide of Methyl.	Iodide of Methyl.
Iodide of Ethyl.	Iodide of Ethyl.
Benzol.	Benzol.
Amylene.	Amylene.
Sulphuric Ether.	Alcohol.
Acetic Ether.	Formic Ether.
Formic Ether.	Sulphuric Ether.
Alcohol.	Acetic Ether.
Water.	

Here, as far as amylene, the order of absorption is the same for both liquids and vapours. But from amylene downwards, though strong liquid absorption is in a general way paralleled by strong vapour absorption, the order of both is not the same. There is not the slightest doubt that next to water alcohol is the most powerful absorber in the list of liquids; but there is just as little doubt that the position which it occupies in the list of vapours is the correct one. This has been established by reiterated experiments. Acetic ether, on the other hand, though certainly the most energetic absorber in the state of vapour, falls behind both formic ether and alcohol in the liquid state. Still, on the whole, I think it is impossible to contemplate these results without arriving at the general conclusion that the act of absorption is in the main molecular, and that the molecule maintains its power as an absorber and radiator when it changes its state of aggregation. Should, however, any doubt linger as to the correctness of this conclusion, it will speedily disappear.

A moment's reflection will show that the comparison here instituted is not a strict one. We have taken the liquids at a common thickness, and the vapours at a common volume and pressure. But if the layers of liquid employed were turned bodily into vapour, the volumes obtained would *not* be the same. Hence the quantities of matter traversed by the radiant heat are neither equal nor proportional to each other in the two cases; and to render the comparison strict they ought to be proportional. It is easy, of course, to make them so; for the liquids being examined at a constant volume, their specific gravities give us the relative quantities of matter traversed by the radiant

heat, and from these, and the vapour-densities, we can immediately deduce the corresponding volumes of the vapour. Calling the quantity of matter q, the vapour-density d, and the volume V, we have

$$Vd=q,$$

or

$$V=\frac{q}{d}.$$

Dividing, therefore, the specific gravities of our liquids by the densities of their vapours, we obtain a series of volumes proportional to the masses of the liquids employed. The densities of both liquids and vapours are given in the following Table:—

Table of Densities.

	Vapour.	Liquid.
Bisulphide of Carbon	2·63	1·27
Chloroform	4·13	1·48
Iodide of Methyl	4·90	2·24
Iodide of Ethyl	5·39	1·95
Benzol	2·69	0·85
Amylene	2·42	0·64
Alcohol	1·59	0·79
Sulphuric Ether	2·56	0·71
Formic Ether	2·56	0·91
Acetic Ether	3·04	0·89
Water	0·63	1·00

Substituting for q the numbers of the second column, and for d those of the first, we obtain the following series of vapour volumes, whose weights are proportional to the masses of liquid employed.

Table of Proportional Volumes.

Bisulphide of Carbon	0·48
Chloroform	0·36
Iodide of Methyl	0·46
Iodide of Ethyl	0·36
Benzol	0·32
Amylene	0·26
Alcohol	0·50
Sulphuric Ether	0·28
Formic Ether	0·36
Acetic Ether	0·29
Water	1·60

Employing the vapours in the volumes here indicated, the following results were obtained:—

TABLE XI.—Radiation of heat through Vapours. Mass of vapour proportional to mass of liquid.

Name of vapour.	Tension in parts of an inch.	Deflection.	Absorption per 100.
Bisulphide of Carbon . . .	0·48	8·4, 8·5	4·3
Chloroform	0·36	13·0, 13·0	6·6
Iodide of Methyl	0·46	20·0, 20·4	10·2
Iodide of Ethyl	0·36	30·6, 30·6	15·4
Benzol	0·32	33·4, 33·1	16·8
Amylene	0·26	37·7	19·0
Sulphuric Ether	0·28	42·5, 42·6	21·5
Acetic Ether	0·29	44·0, 44·0	22·2
Formic Ether	0·36	44·5, 44·7	22·5
Alcohol	0·50	45·0, 44·9	22·7

Here the discrepancies revealed by our former series of experiments entirely disappear, and it is proved that for heat of the same quality the order of absorption for liquids and their vapours is the same. We may therefore safely infer that the position of a vapour as an absorber or radiator is determined by that of the liquid from which it is derived. Granting the validity of this inference, the position of *water* fixes that of *aqueous vapour*. From the first seven Tables of this memoir, or from the *résumé* of results in Table VIII., it will be seen that for all thicknesses water exceeds the other liquids in the energy of its absorption. Hence, if no single experiment on the vapour of water existed, we should be compelled to conclude, from the deportment of its liquid, that, weight for weight, aqueous vapour transcends all others in absorptive power. Add to this the direct and multiplied experiments by which the action of this substance on radiant heat has been established, and we have before us a body of evidence sufficient, I trust, to set this question for ever at rest, and to induce the meteorologist to apply without misgiving the radiant and absorbent property of aqueous vapour to the phenomena of his science.

§ 5.

The order and relative powers of absorption of our vapours, when equal volumes are compared, are given in Table X.: the chemical formulæ of the substances, and the number of atoms which their molecules embrace, are as follows:—

	Formula.	Number of atoms in molecules.
Bisulphide of Carbon	$C\,S_2$	3
Chloroform	$C\,H\,Cl_3$	5
Iodide of Methyl	$C\,H_3\,I$	5
Iodide of Ethyl	$C_2\,H_5\,I$	8
Benzol	$C_6\,H_6$	12
Amylene	$C_5\,H_{10}$	15
Alcohol	$C_2\,H_6\,O$	9
Formic Ether	$C_3\,H_6\,O_2$	11
Sulphuric Ether	$C_4\,H_{10}\,O$	15
Acetic Ether	$C_4\,H_8\,O_2$	14
Boracic Ether	$B\,C_6\,H_{15}\,O_3$	25

Here, for the first six vapours, the radiant and absorbent powers augment with the number of atoms contained in the molecules. Alcohol and amylene vapours, however, are nearly alike in absorptive power, the molecule of amylene containing 15 atoms, while that of alcohol embraces only 9. But in alcohol we have a third element introduced, which is absent in the amylene; the oxygen of the alcohol gives its molecule such a character as enables it to transcend that of the amylene, though the latter contains the greater number of atoms. Here the idea of *quality* superadds itself to that of number. Acetic ether also has a less number of atoms in its molecule than sulphuric ether; but whereas the latter has but one atom of oxygen, the former has two. Formic ether and sulphuric ether are almost identical in their absorptive powers for the heat here employed; still formic ether has but 11 atoms in its molecule, while sulphuric has 15. But formic ether possesses two atoms of oxygen, while sulphuric possesses only one. Two things here suggest themselves as influential on the absorbent and radiant power, which may be expressed by the terms *multitude* and *complexity*. As a molecule of multitude, amylene, for example, exceeds alcohol; as a molecule of complexity, alcohol exceeds amylene; and in this case, as regards radiant and absorbent power, the complexity is more than a match for the multitude. The same remarks may be made with reference to sulphuric and formic ether: the former excels in multitude, the latter in complexity, the excess in the one case almost exactly balancing that in the other. Adding two atoms of hydrogen and one of carbon to the formic ether, we obtain acetic ether, and by this addition the balance is turned; for though acetic ether falls short of sulphuric ether in multitude, it transcends it in absorbent and radiant power. Outstanding from all others, when equal volumes are compared, and signalizing itself by the enormous magnitude of its absorption, we have boracic ether, each molecule of which embraces no less than 25 atoms. The time now at my disposal enables me to do little more than glance at these singular facts; but I must direct the attention of chemists to the water molecule: its power as a radiant and an absorbent is perfectly unprecedented and anomalous, if the usually recognized formula be correct.

§ 6.

In Table III. a fact is revealed which is worth a little further attention. The measurements there recorded show that the absorption of a layer of iodide of methyl, strongly coloured with iodine (which, doubtless, had been liberated by the action of light), was precisely the same as that of a perfectly transparent layer of the liquid. The iodine, which produced so marked an effect on light, did not sensibly affect the radiant heat emitted by the platinum spiral. Here are the numbers:—

	Absorption.
Iodide of Methyl (transparent)	53·2
Iodide of Methyl (strongly coloured with iodine) . . .	53·2

In this case, the incandescent spiral, or a flame, was visible when looked at through the liquid; I therefore intentionally deepened the colour (a rich brown), by adding iodine, until the layer was of sufficient opacity to cut off wholly the light of a brilliant jet of gas. The transparency of the liquid to the radiant heat was not sensibly affected by the addition of the iodine. The luminous heat was of course cut off; but this, as compared with the whole radiation, was so small as to be insensible in the experiments.

It is known that iodine dissolves freely in the bisulphide of carbon, the colour of the solution in thin layers being a splendid purple; but in layers of moderate thickness it may be rendered perfectly opaque to light. I dissolved in the liquid a quantity of the iodine sufficient, when introduced into a cell 0·07 of an inch wide, to cut off wholly the light of the most brilliant gas-flame. Comparing the opaque solution with the transparent bisulphide, the following results were obtained:—

	Deflection.	Absorption.
Bisulphide of Carbon (opaque)	$9^{\circ}·0$	12·5
Bisulphide of Carbon (transparent) . . .	9·0	12·5

Here the presence of a quantity of iodine, perfectly opaque to a brilliant light, was without measureable effect upon the heat emanating from our platinum spiral. The liquid was sensibly thickened by the quantity of iodine dissolved in it.

The same liquid was placed in a cell 0·27 of an inch in width; that is to say, a solution which was perfectly opaque to light, at a thickness of 0·07 was employed in a layer of nearly four times this thickness. Here are the results:—

	Deflection.	Absorption.
Bisulphide of Carbon (transparent) . . .	$13^{\circ}·6$	18·8
Bisulphide of Carbon (opaque)	13·7	19·0

The difference between the two measurements lies within the limits of possible error.

Bisulphide of carbon is commonly used to fill hollow prisms, when considerable dispersion is sought for in the decomposition of white light. Such prisms, filled with the opaque solution, intercept entirely the luminous part of the spectrum, but allow the extra-red rays free passage. A heat-spectrum of the sun, or of the electric light, may be thus

obtained entirely separated from the luminous one. By means of a prism of the transparent bisulphide, I determined the position of the spectrum of the electric light upon a screen, and behind the screen placed a thermo-electric pile so that when the screen was removed the extra-red rays fell upon the pile. I then substituted an opaque prism for the transparent one: there was no visible spectrum on the screen; but the removal of the latter at once demonstrated the existence of an invisible spectrum by the thermo-electric current which is generated, and which was powerful enough to dash violently aside the needles of a large lecture-room galvanometer.

To what, then, are we to ascribe the deportment of iodine towards luminous and obscure heat? The difference between both qualities of heat is simply one of period: in the one case the waves which convey the energy are short and of rapid recurrence; in the other case they are long and of slow recurrence. The former are intercepted by the iodine, and the latter are allowed to pass. Why? There can, I think, be only one answer to this question—that the intercepted waves are those whose periods coincide with the periods of oscillation possible to the atoms of the dissolved iodine. Supposing waves of any period to impinge upon an assemblage of molecules of any other period, it is, I think, physically certain that a tremor of greater or less intensity will be set up among the molecules; but for the motion to *accumulate* so as to produce sensible absorption, coincidence of period is necessary. Briefly defined, therefore, transparency is synonymous with *discord*, while opacity is synonymous with *accord* between the periods of the waves of ether and those of the molecules of the body on which they impinge. The opacity, then, of our solution of iodine to light shows that its atoms are competent to vibrate in all periods which lie within the limits of the visible spectrum; while its transparency to the extra-red undulations demonstrates the incompetency of its atoms to vibrate in unison with the longer waves.

This simple conception will, I think, be found sufficient to conduct us with intellectual clearness through a multitude of otherwise perplexing phenomena. It may of course be applied immediately to that numerous class of bodies which are transparent to light, but opaque in a greater or less degree to radiant heat. Water, for example, is an eminent example of this class of bodies: while it allows the luminous rays to pass with freedom, it is highly opaque to all radiations emanating from obscure sources. A layer of this substance one-twentieth of an inch thick is competent, as MELLONI has shown, to intercept all rays issuing from bodies heated under incandescence. Hence we may infer that, throughout the range of the visible spectrum, the periods of the water-molecules are in discord with those of the ethereal waves, while beyond the red we have coincidence between both.

What is true of water is, of course, true in a less degree of glass, alum, calcareous spar, and of all the substances named in the first section of this paper. They are all in discord with the visible spectrum; they are all more or less in accord with the extra-red undulations of the spectrum.

Thus also as regards lampblack: the blackness of the substance is due to the accord which reigns between the oscillating periods of its atoms and those of the waves embraced within the limits of the visible spectrum. The substance which is thus impervious to the luminous rays is moreover the very one from which the whitest light of our lamps is derived. It can absorb all the rays of the visible spectrum, it can also emit them. But though in a far less degree than iodine, lampblack is also to some extent transparent to the longer undulations. MELLONI was the first to prove this; and in an experiment described in a former memoir, I myself found that 30 per cent. of the radiation from an obscure source found its way through a layer of lampblack, which cut off totally the light of the most brilliant jet of gas. I shall have occasion to show that, for certain sources of heat of long period, between 40 and 50 per cent. of the entire radiation is transmitted by a layer of lampblack which is perfectly opaque to our most brilliant artificial lights. Hence, in the case of lampblack, while accord exists between the periods of its atoms and those of the light-exciting waves, discord, to a considerable extent, exists between the periods of the same atoms and those of the extra-red undulations.

§ 7.

The power of varying at will the temperature of the platinum spiral, renders it peculiarly suitable for the examination of the influence of temperature on the transmission of radiant heat. To obtain sources of different temperatures, MELLONI resorted to lamps, to spirals heated to incandescence by the flame of alcohol, to copper laminæ heated by flame, and to the surfaces of vessels containing boiling water. No conclusions regarding temperature can, as will afterwards be shown, be drawn from such experiments; but by means of the platinum spiral we can go through all those changes of temperature, *retaining throughout the same vibrating atoms*, and we can therefore investigate how the alteration of the rate of vibration affects the rate of absorption. The following series of experiments were executed on the 9th of October, with a platinum spiral raised to barely visible redness, and vapours at a tension of 0·5 of an inch.

TABLE XII.—Radiation of heat through Vapours. Source of heat, platinum spiral barely visible in the dark.

Name of vapour.	Deflection.	Absorption per 100.
Bisulphide of Carbon . . .	7°·5	6·5
Bisulphide of Carbon . . .	7·45	6·4
Chloroform	10·5	9·1
Chloroform	10·5	9·1
Iodide of Methyl	14·5	12·5
Iodide of Methyl	14·5	12·5
Iodide of Ethyl	24·2	20·9
Iodide of Ethyl	24·5	21·1

TABLE XII. (continued).

Name of vapour.	Deflection.	Absorption per 100.
Benzol	31·0°	26·7
Benzol	30·0	25·9
Amylene	37·6	35·6
Amylene	37·8	35·9
Sulphuric Ether	41·1	43·4
Sulphuric Ether	41·0	43·4
Formic Ether	41·7	45·0
Formic Ether	41·8	45·3
Acetic Ether	43·6	49·8
Acetic Ether	43·4	49·3

On the 10th of October the following results were obtained with the same platinum spiral, raised to a white heat:—

TABLE XIII.—Radiation of heat through Vapours. Source, white-hot platinum spiral.

Name of vapour.	Deflection.	Absorption per 100.
Bisulphide of Carbon	3·5	2·9
Bisulphide of Carbon	3·4	2·8
Chloroform	6·7	5·6
Chloroform	6·7	5·6
Iodide of Methyl	9·2	7·7
Iodide of Methyl	9·4	7·9
Iodide of Ethyl	15·4	13·0
Iodide of Ethyl	15·0	12·6
Benzol	19·3	16·6
Benzol	19·0	16·4
Total heat	59·2	100·0
Amylene	27·6	22·6
Amylene	27·7	22·7
Formic Ether	30·5	25·0
Formic Ether	30·7	25·2
Sulphuric Ether	31·4	25·7
Sulphuric Ether	31·7	26·0
Acetic Ether	33·0	27·0
Acetic Ether	33·2	27·3
Total heat	60·0	100·0

With the same spiral, brought still nearer to its point of fusion, the following results were obtained with four of the vapours:—

TABLE XIV.—Radiation through Vapours. Source, platinum spiral at an intense white heat.

Name of vapour.	Deflection.	Absorption.
Bisulphide of Carbon . . .	14°·5	2·5
Bisulphide of Carbon . . .	14·5	2·5
Chloroform	23·0	3·9
Chloroform	23·0	3·9
Formic Ether	60·4	21·3
Formic Ether	60·5	21·3
Sulphuric Ether	62·3	23·6
Sulphuric Ether	62·5	23·8
Total heat	82·7	100·0

In the experiments recorded in the foregoing Table, a total heat of 82°·7, or 588 units, was employed; and to test whether the absorption calculated from this high total agreed with the absorptions calculated from a low total, a portion of the current was diverted, the branch passing through the galvanometer producing a deflection of 49°·4. This corresponds to 77 units. The source, it will be observed, is here quite unchanged; the rays are of the same quality, and pass through the tube in the same quantity as before; but in the one case the absorption is calculated from the deflection among the high degrees, and in the other case it is calculated from deflections among the low degrees of the galvanometer.

The experiments were limited to formic and sulphuric ether, with the following results:—

	Deflection.	Absorption.	Absorption from Table XIV.
Formic Ether . .	17°·7	23	21·3
Sulphuric Ether . .	19·1	24·8	23·7

The agreement is such as to prove that no material error can have crept into the calibration.

Placing the results obtained with the respective sources side by side, the influence of temperature on the transmission comes out in a very decided manner.

TABLE XV.—Absorption of heat by Vapours. Tension 0·5 of an inch.

Name of vapour.	Source, platinum spiral.			
	Barely visible.	Bright red.	White hot.	Near fusion.
Bisulphide of Carbon	6·5	4·7	2·9	2·5
Chloroform	9·1	6·3	5·6	3·9
Iodide of Methyl	12·5	9·6	7·8	
Iodide of Ethyl	21·0	17·7	12·8	
Benzol	26·3	20·6	16·5	
Amylene	35·8	27·5	22·7	
Sulphuric Ether	43·4	31·4	25·9	23·7
Formic Ether	45·2	31·9	25·1	21·3
Acetic Ether	49·6	34·6	27·2	

The gradual augmentation of penetrative power as the temperature is augmented is here very manifest. By raising the spiral from a barely visible heat to an intense white heat, we reduce the absorption, in the cases of bisulphide of carbon and chloroform, to less than one-half. At barely visible redness, moreover, 56·6 and 54·8 per 100 get through sulphuric and formic ether respectively; while, of the intensely white-hot spiral, 76·3 and 78·7 per 100 pass through the same vapours. By augmenting the temperature of solid platinum, we introduce into the radiation waves of shorter period, which, being in discord with the periods of the vapours, get more easily through them.

What becomes of the more slowly recurrent vibrations as the more rapid ones are introduced? Do the latter take the place of the former? This question is answered by experiments made with an opaque solution of iodine, and with lampblack. As the temperature of the platinum spiral increases from a dark heat to the most intense white heat, the absolute quantity transmitted through both these bodies steadily augments. But this heat is wholly obscure, for both the solution and the lampblack intercept all the luminous heat. Hence the conclusion that the augmentation of temperature which introduces the shorter waves augments at the same time the amplitude of the longer ones, and hence also the inference that a body like the sun must of necessity include in its radiation waves of the same period as those emitted by obscure bodies.

§ 8.

Running the eye along the numbers which express the absorptions of sulphuric and formic ether in Table XV., we find that, for the lowest heat, the absorption of the latter exceeds that of the former; for a bright red heat they are nearly equal, but the formic still retains a slight predominance; at a white heat, however, the sulphuric slips in advance, and at the heat near fusion its predominance is decided. I have tested this result in various ways, and by multiplied experiments, and placed it beyond doubt. We may at once infer from it that the capacity of the molecule of formic ether to enter into rapid vibration is less than that of sulphuric. By augmenting the temperature of the spiral we produce vibrations of quicker periods, and the more of these that are introduced, the

more transparent, in comparison with sulphuric ether, does formic ether become. Thus what I have called its complexity tells upon the vibrating periods of the formic ether; the atom of oxygen which it possesses in excess of sulphuric ether renders it more sluggish as a vibrator. Experiments made with a source of 212° Fahr. establish more decidedly the preponderance of the formic ether for vibrations of slow period.

Table XVI.—Radiation through Vapours. Source, Leslie's cube, coated with lamp-black. Temperature, 212° Fahr.

Name of vapour.	Absorption.
Bisulphide of Carbon	6·4
Iodide of Methyl	18·4
Chloroform	19·5
Sulphuric Ether	54·8
Formic Ether	60·9

For heat issuing from this source, the absorption by formic ether is 6·1 per cent. in excess of that by sulphuric.

Deeming the result worthy of rigid confirmation, I repeated the experiments, and obtained the following deflections:—

Table XVII.

Name of vapour.	Deflections.
Bisulphide of Carbon	9°·3
Iodide of Methyl	25·0
Chloroform	26·5
Iodide of Ethyl	34·0
Benzol	35·5
Amylene	46·8
Sulphuric Ether	47·3
Sulphuric Ether	47·7
Formic Ether	49·7
Formic Ether	49·9
Acetic Ether	51·4

When the absorptions were calculated from these deflections, the absorption of formic ether was found to be 6·3 per cent. in advance of that of sulphuric.

But in both Tables XVI. and XVII. we notice another case of reversal. In all the experiments with the platinum spiral recorded in Table XV., chloroform showed itself less energetic as an absorber than iodide of methyl; but in Tables XVI. and XVII. chloroform shows itself to be decidedly the more powerful of the two. Cases of this kind have, in my estimation, a peculiar significance, and I therefore take care to verify them. The experiments with all the vapours were therefore repeated, with the following results:—

TABLE XVIII.—Radiation through Vapours. Source, LESLIE's cube at 212° Fahr.

Name of vapour.	Deflection.	Absorption.
Bisulphide of Carbon . .	15°·0	6·6
Iodide of Methyl	38·3	18·8
Chloroform	40·7	21·6
Iodide of Ethyl	46·2	29·0
Benzol	50·0	34·5
Amylene	57·8	47·1
Sulphuric Ether	60·3	54·1
Formic Ether	62·1	60·4
Acetic Ether	64·3	69·9
Total heat	71·4	100·0

The absorption by formic ether is here also 6·3 per cent. in excess of that effected by sulphuric ether; while, as in the last two Tables, chloroform excels iodide of methyl.

Preserving the quality of the heat unchanged, but reducing its quantity from 71°·4 =227 units to 52°·3=86·5 units, the following results were obtained:—

TABLE XIX.

Name of vapour.	Deflection.	Absorption.
Iodide of Methyl	16°·5	18·3
Chloroform	18·5	20·6
Iodide of Ethyl	24·4	27·1
Benzol	30·0	33·3
Amylene	38·6	48·6
Sulphuric Ether	40·3	53·2
Formic Ether	42·8	60·0

Placing the figures of Tables XVI., XVIII. and XIX. side by side, we have an opportunity of seeing how results obtained on different days check each other.

TABLE XX.—Source, blackened cube of boiling water.

Name of vapour.	Absorptions from		
	Table XVI.	Table XVIII.	Table XIX.
Bisulphide of Carbon . . .	6·4	6·6	•
Iodide of Methyl	18·4	18·8	18·3
Chloroform	19·5	21·6	20·6
Iodide of Ethyl	——	29·0	27·1
Benzol	——	34·5	33·3
Amylene	——	47·1	48·6
Sulphuric Ether	54·8	54·1	53·2
Formic Ether	60·9	60·4	60·0
Acetic Ether	——	69·9	

Were it essential to my purpose, I should certainly be able to make even the small differences which here show themselves to disappear. But the agreement is such as to place the reliability of the experiments beyond doubt. *It will be seen that, contrary to the results obtained with a white-hot spiral, in all three cases, where a blackened cube of boiling water was the source, chloroform exceeds iodide of methyl, and formic ether exceeds sulphuric in absorbent power.* To confirm the demonstration, I once more resorted to the white-hot spiral, and obtained the following results:—

TABLE XXI.—Radiation through Vapours. Source, white-hot platinum spiral.

Name of vapour.	Deflection.	Absorption.
Chloroform	9°·8	4·5
Chloroform	9·5	4·5
Iodide of Methyl	16·0	7·3
Iodide of Methyl	15·8	7·3
Formic Ether	42·1	24·2
Formic Ether	42·3	24·5
Sulphuric Ether	43·6	26·3
Sulphuric Ether	43·5	26·2
Total heat	70·9	100·0

Here chloroform retreats once more behind iodide of methyl, and formic ether behind sulphuric.

The positions of sulphuric and formic ether are reversed within the range of the experiments made with the platinum spiral, but this is not the case with the chloroform and the iodide of methyl. Even when the spiral was at a barely visible heat, the iodide was decidedly the most opaque of the two; the same result was obtained with a spiral heated under redness, as proved by the following figures:—

Name of vapour.	Deflection.	Absorption.
Chloroform	8°·5	12·14
Chloroform	8·5	12·14
Iodide of Methyl	10·0	14·28
Iodide of Methyl	10·0	14·28
Total heat	47·3	100·0

Here the iodide is still predominant. Is it, then, a question of *temperature* merely? or is there a special flux emitted by the lampblack, to which chloroform is particularly opaque? In other words, is there a special accord between the rates of vibration of lampblack and chloroform? To answer this question I operated thus:—The platinum spiral was heated by only two cells, and the strength of this current was lowered by the introduction of resistance. When decidedly below a red heat, the spiral was plunged into boiling water. Bubbles of steam issued from it, proving that its temperature was

above 212° Fahr. By augmenting the resistance its heat was lowered, until it was no longer competent to produce the least ebullition. It was then withdrawn from the water, and employed as a source: the following are the results:—

TABLE XXII.—Source, platinum spiral at 100° C.

Name of vapour.	Deflection.	Absorption.
Bisulphide of Carbon . .	5°·7	7·03
Chloroform	14·0	16·8
Iodide of Methyl	15·3	18·0

No reversal was here obtained. The temperature was then reduced so that the total heat fell from 81 units to 59 units; but not even in this case (when the temperature was considerably below that of boiling water) could the reversal be obtained. The absorptions approach each other, but the iodide has still the advantage of the chloroform.

TABLE XXIII.—Source, platinum spiral, heated under 100° C.

Name of vapour.	Deflection.	Absorption.
Bisulphide of Carbon . . .	5°·2	9·2
Chloroform	10·0	17·3
Iodide of Methyl	10·8	18·2

It is not, therefore, temperature alone which determines the inversion: the experiments prove that there is a greater synchronism between the vibrating periods of chloroform and lampblack than between those of chloroform and platinum raised to the temperature of the lampblack. It will be seen, however, that as the temperature of the platinum falls, the opacity of the chloroform increases more quickly than that of the iodide: with an intensely white-hot spiral, as shown in Table XXI., the absorption of chloroform is to that of the iodide as 100 : 162, while, with the spiral heated to a temperature of 212° Fahr., the ratio of the absorptions is as 100 : 105.

§ 9.

We have hitherto occupied ourselves with the radiation from heated solids: I will now pass on to the examination of the radiation from flames. The first experiments were made with a steady jet of gas issuing from a small circular burner, the flame being long and tapering. The top and bottom of the flame were excluded, and its most brilliant portion was chosen as the source. The following results were obtained:—

TABLE XXIV.—Radiation of heat through Vapours. Source, a highly luminous jet of gas.

Name of vapour.	Deflection.	Absorption.	White-hot spiral.
Bisulphide of Carbon	8°·9	9·8	2·9
Chloroform	10·9	12·0	5·6
Iodide of Methyl	15·4	16·5	7·8
Iodide of Ethyl	17·7	19·5	12·8
Benzol	20·0	22·0	16·5
Amylene	27·5	30·2	22·7
Formic Ether	31·5	34·6	25·1
Sulphuric Ether	32·5	35·7	25·9
Acetic Ether	34·2	38·7	27·2
Total heat	53·8	100·0	

It is interesting to compare the heat emitted by the white-hot carbon with that emitted by the white-hot platinum; and to facilitate the comparison, I have placed beside the results in the last Table those recorded in Table XIII. The emission from the flame is thus proved to be far more powerfully absorbed than the emission from the spiral. Doubtless, however, the carbon, in reaching incandescence, passes through lower stages of temperature, and in those stages emits heat more in accord with the vapours. It is also mixed with the vapour of water and carbonic acid, both of which contribute their quota to the total radiation. It is therefore probable that the greater accord between the periods of the flame and those of the vapours is due to the slower periods of the substances which are unavoidably mixed with the body to which the flame mainly owes its light.

The next source of heat employed was the flame of a BUNSEN'S burner, the temperature of which is known to be very high. The flame was of a pale-blue colour, and emitted a very feeble light. The following results were obtained:—

TABLE XXV.—Radiation of heat through Vapours. Source, pale-blue flame of BUNSEN'S burner.

Name of vapour.	Deflection.	Absorption.	From Table XXIV. Luminous jet of gas.
Chloroform	5°·0	6·2	12·0
Bisulphide of Carbon	9·0	11·1	9·8
Iodide of Ethyl	11·3	14·0	19·5
Benzol	14·5	17·9	22·0
Amylene	19·6	24·2	30·2
Sulphuric Ether	25·8	31·9	35·7
Formic Ether	27·0	33·3	34·6
Acetic Ether	29·4	36·3	38·7
Total heat	50·6	100·0	100·0

The total heat radiated from the flame of BUNSEN's burner is greatly less than that radiated when the incandescent carbon is present in the flame. The moment the air is permitted to mix with the luminous flame, the radiation falls so considerably that the diminution is at once detected, even by the hand or face brought near the flame. Comparing Tables XXIV. and XXV., we see that the radiation from the BUNSEN's burner is, on the whole, less powerfully absorbed than that from the luminous gas jet. In some cases, as in that of formic ether, they come very close to each other; in the case of amylene, and a few other substances, they differ more markedly. But an extremely interesting case of reversal here shows itself. Bisulphide of carbon, instead of being first, stands decidedly below chloroform. With the luminous jet, the absorption of bisulphide of carbon is to that of chloroform as 100 : 122, while with the flame of BUNSEN's burner the ratio is 100 : 56; the removal of the carbon from the flame more than doubles the relative transparency of the chloroform. The case is of too much interest to be passed over without verification: here is the result obtained with a different total heat:—

	Deflection.	Absorption.
Chloroform	16°·5	8·4
Chloroform	16·0	8·2
Bisulphide of Carbon . . .	19·0	9·7
Bisulphide of Carbon . . .	19·4	9·9
Total heat	68·4	100·0

And again, with an intermediate total heat,—

	Deflection.	Absorption.
Chloroform	10°·2	8·4
Chloroform	10·0	8·4
Bisulphide of Carbon . . .	12·0	9·8
Bisulphide of Carbon . . .	11·8	9·7
Total heat	60·0	100·0

There is therefore no doubt that, while in the case of a platinum spiral at all temperatures, of a luminous gas flame, and, more especially, in the case of lampblack heated to 212° Fahr. the absorption of chloroform exceeds that of bisulphide of carbon, for the flame of BUNSEN's burner the bisulphide is the more powerful absorber of the two. The absorptive energy of the chloroform, as shown in Table XX., is more than three times that of the bisulphide, while in Table XXV. the action of the bisulphide is nearly twice that of the chloroform. We have here, moreover, another instance of the reversal of formic and sulphuric ether. For the luminous jet the sulphuric ether is decidedly the more opaque; for the flame of BUNSEN's burner it is excelled in opacity by the formic.

§ 10.

The main radiating bodies in the flame of a BUNSEN's burner are, no doubt, aqueous

vapour and carbonic acid. Highly heated nitrogen is also present, which may produce a sensible effect: the unburnt gas, moreover, in proximity with the flame, and warmed by it, may contribute to the radiation, even before it unites with the atmospheric oxygen. But the main source of the radiation is, no doubt, the aqueous vapour and the carbonic acid. I wished to separate these two constituents, and to study them separately. The radiation of aqueous vapour could be obtained from a flame of pure hydrogen, while that of carbonic acid could be obtained from an ignited jet of carbonic oxide. To me the radiation from the hydrogen-flame possessed a peculiar interest; for notwithstanding the high temperature of such a flame, I thought it likely that the accord between its periods of vibration and those of the cool aqueous vapour of the atmosphere would still be such as to cause the atmospheric vapour to exert a special absorbent power upon the radiation. The following experiments test this surmise:—

TABLE XXVI.—Radiation through Atmospheric Air. Source, a hydrogen-flame.

	Deflection.	Absorption.
Dry air	0°	0
Undried air	21·5	17·20
Total heat	60·4	100·0

Thus, in a polished tube 4 feet long, the aqueous vapour of our laboratory air absorbed 17 per cent. of the radiation from the hydrogen-flame. A platinum spiral, raised by electricity to a degree of incandescence not greater than that obtainable by plunging a wire into the hydrogen-flame, was used as a source of heat; of its radiation, the undried air of the laboratory absorbed

5·8 per cent.,

or one-third of the quantity absorbed when the flame of hydrogen was employed.

The plunging of a spiral of platinum wire into the flame reduces its temperature; but it at the same time introduces vibrations which are not in accord with those of aqueous vapour: the absorption by ordinary undried air of heat emitted by this composite source amounted to

8·6 per cent.

On humid days the absorption of the rays emitted by a hydrogen-flame exceeds even the above large figure. Employing the same experimental tube and a new burner, the experiments were repeated some days subsequently, with the following result:—

TABLE XXVII.—Radiation through Air. Source, hydrogen-flame.

	Absorption.
Dry air	0
Undried air	20·3

The undried air here made use of embraced the carbonic acid of the atmosphere; after the foregoing experiments, the air was conducted through a tube containing a solution

of caustic potash, in which the carbonic acid was intercepted, while the air charged itself with a little additional moisture. The absorption then observed amounted to

20·3 per cent.

of the entire radiation. The exact agreement of this with the last result is, of course, an accident; the additional humidity of the air derived from the solution of potash happened to compensate for the action of the carbonic acid withdrawn.

The other component of the flame of BUNSEN'S burner is carbonic acid; and the radiation of this substance is immediately obtained from a flame of carbonic oxide. With the air of the laboratory the following results were obtained:—

TABLE XXVIII.—Radiation through Atmospheric Air. Source, carbonic-oxide flame (very small).

	Deflection.	Absorption.
Dry air	0°	0
Undried air	10·0	16·1

Of the heat emitted by carbonic acid, 16 per cent. was absorbed by the common air of the laboratory. After the air had been passed through sulphuric acid, the aqueous vapour being thus removed while the carbonic acid remained, the absorption was 13·8 per cent.

An india-rubber bag was filled from the lungs; it contained therefore both the aqueous vapour and the carbonic acid of the breath. The air from the bag was conducted through a drying apparatus, the mixed air and carbonic acid being permitted to enter the experimental tube. The following results were obtained:—

TABLE XXIX.—Air from the lungs containing CO_2. Source, carbonic-oxide flame.

Tension in inches.	Deflection.	Absorption.
1	$7^{\circ}\cdot2$	12·0
3	15·0	25·0
5	20·0	33·3
30	30·8	50·0

Thus the tube filled with dry air from the lungs intercepted 50 per cent. of the entire radiation from a carbonic-oxide flame. It is quite manifest that we have here a means of testing with surpassing delicacy the amount of carbonic acid emitted under various circumstances by the act of expiration.

That pure carbonic acid is highly opaque to the radiation from the carbonic-oxide flame, is forcibly evidenced by the results recorded in the following Table.

TABLE XXX.

Radiation through dry Carbonic Acid. Source, carbonic-oxide flame.

Tension in inches.	Deflection.	Absorption.
1·0	33°·7	53·0
2·0	37·0	61·7
3·0	38·6	66·9
4·0	39·4	70·0
5·0	40·0	72·3
10·0	41·4	78·7

About four months subsequent to the performance of these experiments they were repeated, using as a source a much smaller flame of carbonic oxide. The absorptions were found somewhat less, but still very high. They follow in the next Table.

TABLE XXXI.

Radiation through dry Carbonic Acid. Source, small carbonic-oxide flame.

Tension in inches.	Deflection.	Absorption.
1·0	17°·3	48·0
2·0	20·0	55·5
3·0	21·7	60·3
4·0	22·8	65·1
5·0	24·0	68·6
10·0	26·0	74·3

For the rays emanating from the heated solids employed in all my former researches, carbonic acid proved to be one of the most feeble absorbers; but here, when the waves sent into it emanate from molecules of its own substance, its absorbent energy is enormous. The thirtieth of an atmosphere of the gas cuts off half the entire radiation; while at a tension of 4 inches, nearly 70 per cent. of the whole radiation is intercepted.

The energy of olefiant gas, both as an absorbent and a radiant, is well known; for the solid sources of heat just referred to, its power is incomparably greater than that of carbonic acid; but, for the radiation from the carbonic-oxide flame, the power of olefiant gas is feeble when compared with that of carbonic acid. This is proved by the experiments recorded in the following Table:—

TABLE XXXII.

Radiation through dry Olefiant Gas. Source, carbonic-oxide flame.

Tension in inches.	Deflection.	Absorption.
1	17°·0	24·2
2	26·0	37·1
4	33·0	49·1
Total heat . .	47·3	100·0

Four months subsequent to the performance of the above experiments, a second series were made with olefiant gas, and the following results obtained:—

TABLE XXXIII.

Radiation through dry Olefiant Gas. Source, small carbonic-oxide flame.

Tension in inches.	Deflection.	Absorption.	From Table XXXI.
1·0	11°·4	23·2	48·0
2·0	17·0	34·7	55·5
3·0	21·6	44·0	60·3
4·0	24·8	50·6	65·1
5·0	27·0	55·1	68·6
10·0	32·1	65·5	74·3

Beside the absorption by olefiant gas, I have placed that by carbonic acid derived from Table XXXI. The superior power of the acid is most decided in the smaller tensions; at a tension of an inch it is twice that of the olefiant gas. The substances approach each other more closely as the quantity of gas augments. Here, in fact, both of them approach perfect opacity; and as they draw near to this common limit, their absorptions, as a matter of course, approximate.

The temperature of a hydrogen-flame, as calculated by BUNSEN, is 3259° C., while that of a carbonic-oxide flame is 3042° C. The foregoing experiments demonstrate that accord subsists between the oscillating periods of these sources and the periods of aqueous vapour and carbonic acid at a temperature of 15° C. The heat of the flame goes to augment the amplitude, and not to quicken the vibration.

Sent through carbonic oxide, the radiation from the carbonic-oxide flame gave the following absorptions:—

TABLE XXXIV.

Radiation through Carbonic Oxide. Source, carbonic-oxide flame.

Tension in inches.	Deflection.	Absorption.
1	18°·0	29·0
2	27·0	43·5
4	34·0	56·4
10	37·3	65·5

The absorptive energy is here high—greater, indeed, than that of olefiant gas; it falls considerably short, however, of that exhibited by carbonic acid. This result shows us that the main radiant in the flame is its *product* of combustion, and not the carbonic oxide heated prior to combustion.

Wishing to examine the radiation from a flame whose product of combustion is sulphurous acid, through sulphurous acid, I resorted to the flame of bisulphide of carbon.

Here, however, we had carbonic acid mixed with the sulphurous acid of the flame. Of the heat radiated by this composite source, the absorption by an atmosphere of sulphurous acid amounted to

60 per cent.

The gas was sent from its generating retort through drying-tubes of sulphuric acid into a glass experimental tube 2·8 feet long. The comparative shortness of the tube, and the mixed character of the radiation, rendered the absorption less than it would have been had a source of pure sulphurous acid and a tube as long as that used in the other experiments been employed.

I subsequently caused the radiation from the carbonic-oxide flame to pass through a few of our vapours, with the following results:—

TABLE XXXV.

Radiation through Vapours (tension 0·5 inch). Source, carbonic-oxide flame.

Name of vapour.	Deflection.	Absorption.
Bisulphide of Carbon . . .	5°·5	9·8
Chloroform	6·0	10·7
Formic Ether	14·5	25·8
Sulphuric Ether	18·0	32·1
Total heat	43·0	100·0

The same vapours were employed to test the radiation from the hydrogen-flame, with the following results:—

TABLE XXXVI.

Radiation through Vapours (tension 0·5 inch). Source, hydrogen-flame.

Name of vapour.	Deflection.	Absorption.
Bisulphide of Carbon . . .	8°·8	11·9
Chloroform	9·9	13·4
Sulphuric Ether	32·0	42·2
Formic Ether	35·0	49·3
Total heat	48·5	100·0

We here find that, in the case of every one of the four vapours, the synchronism with hot aqueous vapour is greater than with hot carbonic acid. The temperature of the hydrogen-flame is higher than that of the carbonic oxide; but the radiation from the more intense source is most copiously absorbed. It has been already proved that, for waves of slow period, formic ether is more absorbent than sulphuric ether; while for waves of rapid period, the sulphuric ether is the more powerful absorber. For the radiation from hot carbonic acid, the absorption of sulphuric ether, as shown in Table XXXV., is between 6 and 7 per cent. in excess of that of formic ether; while for the radiation from hot aqueous vapour, the absorption by formic ether, as shown in Table XXXVI., is 7 per cent. in excess of that by sulphuric. That the periods of aqueous

vapour, as compared with those of carbonic acid, are slow, may therefore be inferred from these experiments.

The two following Tables illustrate the action of carbonic acid gas and olefiant gas respectively, on the radiation from a flame of hydrogen:—

TABLE XXXVII.—Radiation through Carbonic Acid Gas. Source, hydrogen-flame.

Tension in inches.	Deflection.	Absorption.
1	5°·5	7·4
2	9·5	12·8
4	11·0	14·9
30	19·0	25·7
Total heat	48·5	100·0

TABLE XXXVIII.—Radiation through Olefiant Gas. Source, hydrogen-flame.

Tension in inches.	Deflection.	Absorption.	From Table XXXVII.
1	12°·0	16·2	7·4
2	18·0	24·3	12·8
4	24·0	32·4	14·9
30	38·5	58·8	25·7
Total heat . .	48·5	100·0	100·0

A comparison of the last two columns, one of which is transferred from Table XXXVII., proves the absorption of the rays from a hydrogen-flame by olefiant gas to be about twice that of carbonic acid; while, when the source was a carbonic oxide flame, the absorption by carbonic acid at small tensions was more than twice that effected by olefiant gas.

§ 11.

Water at moderate thickness is a very transparent substance; that is to say, the periods of its molecules are in discord with those of the visible spectrum. It is also highly transparent to the extra-violet rays; so that we may safely infer from the deportment of this substance its incompetence to enter into rapid molecular vibration. When, however, we once quit the visible spectrum for the rays beyond the red, the opacity of the substance begins to show itself: for such rays, indeed, its absorbent power is unequalled. The synchronism of the periods of the water-molecules with those of the extra-red waves is thus demonstrated. I have already proved that undried atmospheric air manifests an extraordinary opacity for the radiation from a hydrogen-flame, and from this deportment I inferred the synchronism of the cold vapour of the air and the hot vapour of the flame. The vibrating-period of a molecule is, no doubt, determined by the elastic forces which separate it from other molecules, and it is worth inquiring how these forces are affected when a change so great as that of the passage of a vapour to a liquid occurs. The fact established in the earlier sections of this paper, that the order of absorption for liquids and their vapours is the same, renders it extremely probable

that the period of vibration is not materially affected by the change from vapour to liquid; for, if changed, it would probably be changed in different degrees for the different liquids, and the order of absorption would be thereby disturbed*. The following Table, in which the deportment of our series of liquids towards the radiation from a hydrogen-flame is recorded, will throw additional light upon this question:—

TABLE XXXIX.—Radiation though Liquids. Source, hydrogen-flame. Thickness of liquid layer 0·07 of an inch.

Name of liquid.	Absorption.	Transmission.
Bisulphide of Carbon . . .	27·7	72·3
Chloroform	49·3	50·7
Iodide of Ethyl	75·6	24·4
Benzol	82·3	17·7
Amylene	87·9	12·1
Sulphuric Ether	92·6	7·4
Formic Ether	93·5	6·5
Acetic Ether	93·9	6·1
Water	100·0	0·0

Through a layer of water 9·21 millimetres thick, MELLONI found a transmission of 11 per cent. of the heat of a Locatelli lamp. Here we employ a source of higher temperature, and a layer of water only one-fifth of the thickness used by MELLONI, and still we find the whole of the heat intercepted†. A layer of water, 0·07 of an inch in thickness, is sensibly opaque to the radiation from a hydrogen-flame. Hence we may infer the coincidence in period between cold water and aqueous vapour heated to a temperature of 3259° C.; and inasmuch as the period of the water-molecules has been proved to be extra-red, the period of the vapour-molecules in the hydrogen-flame must be extra-red also.

Another point of considerable interest may here be adverted to. Professor STOKES has demonstrated that a change of period is possible to those rays which belong to the violet and extra-violet end of the spectrum, the change showing itself by a degradation of the refrangibility. That is to say, vibrations of a rapid period are absorbed, and the absorbing substance has become the source of vibrations of a longer period. Efforts, I believe, have been made to obtain an analogous result at the red end of the spectrum, but hitherto without result; and it has been considered improbable that a change of period can occur which should *raise* the refrangibility of the light or heat. Such a change, I

* The general agreement, in point of colour between a liquid and its vapour, favours the idea that the period, at all events in the great majority of cases, remains constant when the state of aggregation is changed.

† From the opacity of water to the radiation from aqueous vapour, we may infer the opacity of aqueous vapour to the radiation from water, and hence conclude that the very act of nocturnal refrigeration which causes the condensation of water on the earth's surface gives to terrestrial radiation that particular character which renders it most liable to be intercepted by the aqueous vapour of the air.

believe, occurs when we plunge a platinum wire into a hydrogen-flame. The platinum is rendered white by the collision of molecules whose periods of oscillation are incompetent to excite vision. There is in this common experiment an actual breaking up of the long periods into short ones—a true rendering of unvisual periods visual. The change of refrangibility differs from that of Professor STOKES, firstly, in its being in the opposite direction—that is, from low to high; and secondly, in the circumstance that the platinum is heated by the collision of the molecules of aqueous vapour, and before their heat has assumed the *radiant form*. But it cannot be doubted that the same effect would be produced by radiant heat of the same period, provided the motion of the ether could be raised to a sufficient intensity. The effect in principle is the same, whether we consider the platinum wire to be struck by a particle of aqueous vapour oscillating at a certain rate, or by a particle of ether oscillating at the same rate. And thus, I imagine, by a chain of rigid reasoning, we arrive at the conclusion that a degree of incandescence, equal to that of the sun itself, might be produced by the impact of waves, of themselves incompetent to excite vision *.

The change of quality produced in the radiation by the introduction of a platinum spiral into a hydrogen-flame is illustrated by a series of experiments, executed for me by my assistant, Mr. BARRETT, and inserted subsequently to the presentation of this memoir.

TABLE XXXIX. *a.*—Radiation through Liquids. Sources: 1. hydrogen-flame; 2. hydrogen-flame and platinum spiral.

Name of liquid.	Transmission. Thickness of liquid 0·04 inch. Flame only.	Flame and spiral.	Thickness of liquid 0·07 inch. Flame only.	Flame and spiral.
Bisulphide of Carbon	77·7	87·2	70·4	86·0
Chloroform	54·0	72·8	50·7	69·0
Iodide of Methyl	31·6	42·4	26·2	36·2
Iodide of Ethyl	30·3	36·8	24·2	32·6
Benzol	24·1	32·6	17·9	28·8
Amylene	14·9	25·8	12·4	24·3
Sulphuric Ether	13·1	22·6	8·1	22·0
Acetic Ether	10·1	18·3	6·6	18·5
Alcohol	9·4	14·7	5·8	12·3
Water	3·2	7·5	2·0	6·4

Here the introduction of the platinum spiral changed the periods of the flame into others more in discord with the periods of the liquid-molecules, and hence the more

* Some time after this was written I learned that Dr. AKIN had previously inferred, from the paucity of luminous and extra-violet rays in the hydrogen-flame, that its periods must be extra-red. And he deduced from this that the heating of a platinumwire in a hydrogen-flame must consist of a change of period. A very interesting communication from Dr. AKIN on this and kindred subjects, will be found in the 'Reader' for the 26th of September 1863.—April 5th, 1864.

copious transmission when the spiral was employed. It will be seen that a transmission of 2 per cent. is here obtained through a layer of water 0·07 of an inch in thickness.

Another series of experiments, also executed by my assistant, gave the following results of the radiation of a hydrogen-flame through layers of water of five different thicknesses:—

Radiation through Water. Source, hydrogen-flame.

	Thickness of liquid.				
	0·02 inch.	0·04 inch.	0·07 inch.	0·14 inch.	0·27 inch.
Transmission per 100 . . .	5·8	2·8	1·1	0·5	0·0

Wishing to compare the radiation from a flame of ordinary coal-gas with that of our hydrogen-flame, I reduced the former to the dimensions of the latter. The flame thus diminished, had a blue base and bright top, and the whole of it was permitted to radiate through our series of liquids. The following results were obtained:—

TABLE XL.—Radiation through Liquids. Source, small gas-flame. Thickness of liquid layer 0·07 of an inch.

Name of liquid.	Deflection.	Absorption.	From Table XXXIX.
Chloroform	28°·7	39·8	49·3
Bisulphide of Carbon . . .	36·0	53·2	27·7
Iodide of Ethyl	41·7	72·3	75·6
Benzol	43·4	79·4	82·3
Amylene	45·0	86·1	87·9
Sulphuric Ether	46·6	93·3	92·6
Formic Ether	46·6	93·3	93·5
Alcohol	46·8	94·1	
Acetic Ether	46·9	94·4	93·9
Water	47·4	97·1	100·0
Total heat	48·0	100·0	

I have placed the results obtained with the hydrogen-flame in the third column of figures. For some of the liquids it will be observed that the absorption of the heat issuing from the small gas-flame is nearly the same as that of the heat issuing from the flame of hydrogen. A very remarkable difference, however, shows itself in the deportment of bisulphide of carbon, as compared with that of chloroform. For the small gas-flame chloroform is the most transparent body in the list; it is markedly more transparent than the bisulphide of carbon, while for the hydrogen-flame the bisulphide greatly excels the chloroform in transparency. The large luminous gas-flame previously experimented with differs also from the small one here employed. With the large flame, the absorption by the bisulphide is to that by the chloroform as

$$100:121,$$

while with the small flame the absorptions of the same two substances stand to each other in the ratio of

$$100 : 76.$$

Numerous experiments were subsequently made, with a view of testing this result, but in all cases the bisulphide was found more opaque than the chloroform to the radiation of the small gas-flame. The same result was obtained when a very small oil-flame was employed; and it came out in a very decided manner when the source of heat was a flame of bisulphide of carbon. *It was found moreover that, whenever two liquids underwent a change of position of this kind, the vapours of the liquids underwent a similar change*; in its finest gradations, the deportment of the liquid was imitated by that of its vapour.

§ 12.

And here we find ourselves in a position to offer solutions of various facts which have hitherto stood as enigmas in researches upon radiant heat. It was for a time generally supposed that the power of heat to penetrate diathermic substances augmented as the temperature of the source of heat became more elevated. KNOBLAUCH contended against this notion, showing that the heat emitted by a platinum wire plunged into an alcohol flame was less absorbed by certain diathermic screens than the heat of the flame itself, and justly arguing that the temperature of the spiral could not be higher than that of the body from which it derived its heat. A plate of glass being introduced between his source and his thermo-electric pile, the deflection of his needle fell from 35° to 19° when the source was the platinum spiral; while, when the source was the flame of alcohol, when the glass was introduced the deflection fell from 35° to 16°, proving that the radiation from the flame was intercepted more powerfully than that from the spiral—showing, in other words, that the heat emanating from the body of highest temperature possessed the least penetrative power. MELLONI afterwards corroborated this experiment.

Transparent glass allows the rays of the visible spectrum to pass freely through it; but it is well known to be highly opaque to the radiation from obscure sources—in other words, to waves of long period. A plate 2·6 millimetres thick intercepts all the rays from a source of 100° C., and allows only 6 per cent. of the heat emitted by copper raised to 400° C. to pass through it*. Now the products of the combustion of alcohol are aqueous vapour and carbonic acid, whose waves have been proved to be of slow period, and hence of that particular character which are most powerfully intercepted by glass; but by plunging a platinum wire into such a flame, we virtually convert its heat into heat of higher refrangibility; we break up the long periods into shorter ones, and thus establish the discord between the periods of the source and the periods of the diathermic glass, which, as before defined, is the physical cause of the transparency. On purely *à priori* grounds, therefore, we might infer that the introduction of the platinum

* MELLONI.

spiral would augment the penetrative power of the heat through the glass. MELLONI, with two plates of glass of different thicknesses, found the following transmissions for the flame and the spiral:—

For the flame.	For the platinum.
41·2	52·8
5·7	26·2

The same remarks apply to the transparent selenite examined by MELLONI. This substance is highly opaque to the extra-red undulations; but the radiation from an alcohol flame is almost wholly extra-red, and hence the opacity of the selenite to this radiation. The introduction of the platinum spiral shortens the periods and augments the transmission. Thus, with two specimens of selenite, of different thicknesses, MELLONI found the transmission to be as follows:—

Flame.	Platinum.
4·4	19·5
1·7	3·5

So far the results of MELLONI correspond with those of M. KNOBLAUCH; but the Italian philosopher pursues the matter further, and shows that M. KNOBLAUCH'S results, though true for the particular substances examined by him, are far from being applicable to diathermic media generally. MELLONI shows that in the case of *black* glass and *black* mica, a striking inversion of the effect is observed; that is to say, that through these substances the radiation from the flame is more copiously transmitted than the radiation from the platinum spiral. For two pieces of black glass of different thicknesses, he found the following transmissions:—

From the flame.	From the platinum.
52·6	42·8
29·9	27·1

And for two plates of black mica the following transmissions were found:—

From the flame.	From the platinum.
62·8	52·5
43·3	28·9

These results were left unexplained by MELLONI; but the solution is now easy. The black glass and the black mica owe their blackness to the carbon incorporated in them, and the blackness of this substance, as already remarked, proves the accord of its vibrating periods with those of the visible spectrum. But it has been proved that carbon is in a considerable degree pervious to the waves of long period—that is to say, to such waves as are emitted by a flame of alcohol. The case of the carbon is therefore precisely antithetical to that of the transparent glass—the former transmitting the heat of long period most freely, and the latter transmitting the heat of short period most freely. Hence it follows that the introduction of the platinum wire, by converting

the long periods of the flame into short ones, augments the transmission through the transparent glass and selenite, and diminishes it through the black glass and the black mica.

§ 13.

Lampblack, as already stated, is in accord with the undulations of the visible spectrum; it absorbs them all; but it is partially transparent to the waves of slow period. As, therefore, the waves issuing from a flame of hydrogen have been proved to be of slow period, we may with probability infer that its radiation will penetrate the lampblack. A plate of rock-salt was placed over an oil-lamp until the layer of soot deposited on it was sufficient to intercept the light of the brightest gas-flame. The smoked plate was introduced in the path of the rays from the hydrogen-flame, and its absorption was measured; the plate was then cleansed, and its absorption again determined. The difference of both gave the absorption of the layer of lampblack. The results were as follows:—

Table XLI.

	Deflection.	Absorption.
Smoked rock-salt . .	44°·2	82·7
Unsmoked plate . .	15·8	24·0

The difference between these gives us the absorption of the lampblack; it is 58·7 per cent.; and this corresponds to a transmission of

41·3

per cent. of the radiation from the hydrogen-flame.

Iodine, in a solution sufficiently opaque to cut off the light of our most brilliant lamps, transmitted of the heat of the hydrogen-flame

99 per cent.

In experimenting on liquids with heat of slow period, I noticed that the introduction of the empty rock-salt cell caused the needle to move through a much larger arc than when the source was a luminous one. This suggested to me that a greater proportion of the heat of slow period was absorbed by the rock-salt. I made a few experiments to test the diathermancy of the salt, with the following results:—

For the heat of a hydrogen-flame, the transmission through a perfectly transparent plate of rock-salt was

82·3 per cent.

For a spiral of platinum wire heated to whiteness by an electric current, the transmission was

87 per cent.

For the same spiral lowered to bright redness, the transmission was

84·4 per cent.

For the same spiral lowered to moderate redness, the transmission was

83·6 per cent.

Nothing was changed in these experiments but the heat of the spiral; the direction of the rays, and the size of the radiating body, remained throughout the same; still we find a gradually augmenting opacity on the part of the rock-salt as the temperature of the source is lowered. There cannot, I think, be a doubt that MM. De la Provostaye and Desains are right in their conclusion that rock-salt acts differently on different calorific rays, and is not, as Melloni supposed, equally transparent to all. For the heat of the hydrogen-flame it is more opaque than for that of the moderately red spiral.

§ 14.

This memoir ought perhaps to end here; I would, however, ask permission to make a few additional remarks on a subject which was briefly touched upon towards the conclusion of the first of this series of memoirs. I make these remarks with diffidence, for I have reason to know that authorities for whom I entertain the highest respect do not share my views regarding the connexion which subsists between the radiation and conduction of heat.

Let us suppose heat to be communicated to the superficial stratum of the molecules of any body; say, the molecules at the extremity of a metal bar. They vibrate, and the motion communicated by them to the external ether is dispatched in waves through space. The vibrating superficial molecules must also set in motion the ether *within* the body, and a portion of this motion will be transferred to the stratum of molecules next adjacent to the superficial ones, heat as a consequence appearing to penetrate the mass. But irrespective of the ether, the molecules of the body occupy positions which are determined by their attractive and repulsive forces; so that if any one molecule be forcibly moved from its position of equilibrium, it will of necessity disturb its neighbours. In a system of molecules so related, it is manifest that motion could be transmitted independently of the ether which surrounds them. If we could imagine the ether entirely away, the motion that we call heat would still be propagated from molecule to molecule through such a body. *Conduction* would manifest itself, while radiation would be absent through want of a medium.

In matter, however, as we have it, molecular motion is only in part transmitted *immediately* from molecule to molecule, being in part transmitted mediately by the ether. Now the quantity of motion transmitted by the ether to our second stratum of molecules cannot be the *whole* of that which the first or superficial stratum imparted to the ether. The ether must retard and indeed squander the internal molecular motion; and were the medium absent—were the cushion removed which interferes with the direct propagation of motion from molecule to molecule—conduction would be freer than at present; the heat would penetrate further into the mass than when the ether intervenes.

The reasoning just employed leads to the inference that those molecules which experience most resistance from the ether, must be the least competent to transfer the motion of heat from one to the other. The *direct* power of communication is enfeebled by the ether, and the motion obtained *indirectly* cannot make good the loss. We are thus led to the conclusion that the best radiators ought to prove themselves the worst conductors.

A broad consideration of the subject shows that the conclusion is in general harmony with observed facts. Organic substances are all exceedingly imperfect conductors of heat, and they are all excellent radiators. The moment we pass from the metals to their compounds we pass from a series of good conductors to bad ones, and from a series of bad radiators to good ones*.

In the earlier memoirs of MM. De le Provostaye and Desains†, and in that of MM. Wiedemann and Franz, I find the following facts:—The radiative power of platinum is five times that of silver; its conductive power is one-tenth that of silver. Platinum has more than twice the radiative power of gold; it has only one-seventh of the conducting-power. Zinc and tin are almost equal as conductors, and they are also nearly equal as radiators. Silver has about six times the conductive power of zinc and tin; it has only one-fourth of their radiative powers. Brass possesses but one-half the radiative energy of platinum; it possesses more than twice its conductivity. Other experiments of MM. De la Provostaye and Desains‡ confirm those hitherto referred to. Taking the absorbent power, as determined by these excellent experimenters, to express the radiating power, and multiplying their results by a common factor to facilitate comparison

* And we also pass, as a general rule, from a series of bodies which vibrate in accord with the visible spectrum to a series which vibrate in discord with the spectrum. The lowering of the rate of vibration is a consequence of chemical union. The comparative incompetence of *compound* bodies to oscillate in visual periods has incessantly declared itself in these researches. I would here refer to a most interesting illustration of the same kind, derived from the experiments of MM. De la Provostaye and Desains. These distinguished experimenters were the first to record the important fact that the qualities of heat emitted by bodies at the same temperature may be very unlike. Two experiments illustrate this fact. The first is recorded in the Comptes Rendus, vol. xxxiv. p. 951. One half of a cube was coated with lampblack, and the other half with cinnabar. The cube being filled with oil at a temperature of 173° C., it was found that the emission from the cinnabar was more copiously absorbed by a plate of glass than that from the lampblack. In the second experiment, they found that, while 39 per cent. of the radiation from a bright surface of platinum was transmitted by a plate of glass, only 29 per cent. of the radiation from the opposite surface of the same plate, which was coated with borate of lead, was transmitted. These results are quite in harmony with the views which I have ventured to enunciate. We may infer from them that the heat emitted by the respective *compounds*—the cinnabar and the borate of lead—is of slower period than that emitted by the *elements*; for experiment proves that as the periods are quickened the glass becomes more transparent. At a temperature of 100° C., moreover, the emission from borate of lead was found equal to that from lampblack (Comptes Rendus, vol. xxxviii. p. 442), while at a temperature of 550° C. it had only three-fourths of the emissive power of the lampblack. With reference to the theoretic views which these researches are intended to foreshadow, the results of MM. De la Provostaye and Desains are of the highest interest.

† Comptes Rendus, 1846, vol. xxii. p. 1139.

‡ Annales de Chimie, 1850, vol. xxx. p. 442.

with those of MM. Wiedemann and Franz on conduction, we obtain the following Table:—

Table XLII.—Comparison of Conduction and Radiation.

Name of metal.	Conduction.	Radiation.
Silver	100	11
Gold	53	27
Brass	24	42
Tin	15	90
Platinum . . .	8	100

We here find that, as the power of conduction diminishes, the power of radiation augments—a result, I think, completely in harmony with that to which a consideration of the molecular mechanism leads us. There is but one serious exception known to me to the law here indicated; this is copper, which MM. De la Provostaye and Desains place higher than gold as a radiator, though it is also higher as a conductor. When, however, the immense change in radiative power which the slightest film of oxide can produce, and the liability of heated copper to contract such a film, are taken into account, the apparent exception will not have too much weight ascribed to it. I have had a cube of brass coated electrolytically with copper, silver, and gold; and, of all its faces, that coated with copper has the least emissive power. This is probably due to some slight impurity contracted by the silver. What we know of the deportment of minerals also illustrates the law. Rock-salt I find to be a far better conductor than glass, while MM. De la Provostaye and Desains find the relative emissive powers of the two substances to be as 17 to 6: the radiant power of the salt is little more than one-third that of the glass. So also with regard to alum: as a conductor it is immensely behind rock-salt; as a radiator it is immensely in advance of it.

Royal Institution, March 1864.

X. *On Plane Water-Lines in two Dimensions.* * *By* WILLIAM JOHN MACQUORN RANKINE, *C.E., LL.D., F.R.SS. L. & E., Associate of the Institution of Naval Architects, &c.*

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SECTION I.—*Introduction, and Summary of known Principles.*

(1.) *Plane Water-Lines in two Dimensions defined.*—By the term "Plane Water-Line in two Dimensions" is meant a curve which a particle of liquid describes in flowing past a solid body, when such flow takes place in plane layers of uniform thickness. Such curves are suitable in practice for the water-lines of a ship, in those cases in which the vertical displacements of the particles of water are small compared with the dimensions of the ship; for in such cases the assumption that the flow takes place in plane layers of uniform thickness, though not absolutely true, is sufficiently near the truth for practical purposes, so far as the determination of good forms of water-line is concerned. As water-line curves have at present no single word to designate them in mathematical language, it is proposed, as a convenient and significant term, to call them *Neoïds* (from νηὸς, the Ionic genitive of ναῦς).

(2.) *General Principles of the Flow of a Liquid past a Solid.*—The most complete exposition yet published, so far as I know, of the principles of the flow of a liquid past a solid, is contained in Professor STOKES's paper "On the Steady Motion of an Incompressible Fluid," published in the Transactions of the Cambridge Philosophical Society for 1842. So far as those principles will be referred to in the present paper, they may be summed up as follows.

When a liquid mass of indefinite extent flows past a solid body in such a manner that as the distance from the solid body in any direction increases without limit, the motion of the liquid particles approaches continually to uniformity in velocity and direction, the *condition of perfect fluidity* requires that the three components u, v, w of the velocity of a liquid particle should be the three differential coefficients of one function of the coordinates (φ); viz.

$$u=\frac{d\varphi}{dx};\ v=\frac{d\varphi}{dy};\ w=\frac{d\varphi}{dz}; \qquad (1)$$

and the *condition of constant density* requires that the said function should fulfil the following condition,

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}=0. \qquad (2)$$

By giving to the function φ a series of different constant values, a series of surfaces are represented, to which each water-line curve is an orthogonal trajectory, so that if U=

constant be the equation of a series of surfaces each containing a continuous series of water-line curves (and one of which surfaces must be that of the solid body), the function U must satisfy the following condition,

$$\frac{dU}{dx}\cdot\frac{d\varphi}{dx}+\frac{dU}{dy}\cdot\frac{d\varphi}{dy}+\frac{dU}{dz}\cdot\frac{d\varphi}{dz}=0; \quad \ldots \ldots \ldots \quad (3)$$

or if ds' be an elementary arc of a water-line curve, and x', y', z' its coordinates, the following conditions must be satisfied,

$$\frac{dx'}{ds'}:\frac{dy'}{ds'}:\frac{dz'}{ds'}::\frac{d\varphi}{dx}:\frac{d\varphi}{dy}:\frac{d\varphi}{dz}; \quad \ldots \ldots \ldots \quad (4)$$

and these are the most general expressions of the geometrical properties of water-line curves in three dimensions.

When the inquiry is restricted to motion in two dimensions only, x and y, the terms containing dz and dz' disappear from the preceding equations; and it also becomes possible to express the same conditions by means of equations of a kind which are more convenient for the purposes of the present investigation, and which are as follows. Conceive the plane layer of liquid under consideration of thickness unity, to be divided into a series of elementary streams by a series of water-line curves, one of which must be the outline of the solid body; let U= constant be the equation of any one of those curves, U being a function of such a nature that dU is the volume of liquid which flows in a second along a given elementary stream; then the components of the velocity of a particle of liquid are

$$u=\frac{dU}{dy};\ v=-\frac{dU}{dx}; \quad \ldots \ldots \ldots \quad (5)$$

the condition of continuity is satisfied; and the condition of perfect fluidity requires that the function U should fulfil the following equation,

$$\frac{d^2U}{dx^2}+\frac{d^2U}{dy^2}=0. \quad \ldots \ldots \ldots \quad (6)$$

(When the motion of the liquid is not subject to the condition of being uniform in velocity and direction at an infinite distance in every direction from the solid, it is sufficient that

$$\frac{d^2U}{dx^2}+\frac{d^2U}{dy^2}=\text{function of U};$$

but cases of that kind do not occur in the present paper*.)

(3.) *Notation.*—It is purely a question of convenience whether the infinitely distant particles of the fluid are to be regarded as fixed and the solid as moving uniformly, or

* Professor WILLIAM THOMSON, in 1858, completed an investigation of the motion of a solid through a perfect liquid, so as to obtain expressions for the motion of the solid itself, involving twenty-one constants depending on the figure and mass of the solid and the density of the liquid; but as that investigation, though on the eve of publication, has not yet been published, I shall not here refer to it further.

the solid as fixed, and the infinitely distant particles of the fluid as moving uniformly with an equal speed in the contrary direction. Throughout the present paper, the solid will be supposed to move along the axis of x; so that v will represent the transverse component of the velocity of a particle of liquid on either supposition. The longitudinal component of the velocity of a liquid particle *relatively to the solid* will be denoted by u; and when that particle is at an infinite distance from the solid, by c; so that when the infinitely distant part of the liquid is regarded as fixed, the solid is to be conceived as moving with the velocity $-c$; and the longitudinal component of the velocity of a liquid particle relatively to the indefinitely distant part of the liquid will be denoted by $u-c$.

It is convenient to regard the function U as equivalent to an expression of the following kind,

$$\mathrm{U}=bc, \quad \ldots \ldots \ldots \ldots \quad (7)$$

c being the uniform velocity of flow at an infinite distance, and b what the value of y would be for the water-line under consideration if the solid were removed; in which case that line would become a straight line parallel to the axis of x. This enables us to substitute for equations (5) and (6) the following, in which *proportionate* velocities only are considered:—

$$\frac{u}{c}=\frac{db}{dy}, \quad \frac{v}{c}=-\frac{db}{dx} \quad \ldots \ldots \ldots \quad (8)$$

$$\frac{d^2b}{dy^2}+\frac{d^2b}{dx^2}=0. \quad \ldots \ldots \ldots \ldots \quad (9)$$

(4.) *General Characteristics of Water-Line Functions.*—Since at an infinite distance from the solid body we have $u=c$, $v=0$, it follows that, if the origin of coordinates be taken in or near the solid body, b must be a function of such a kind that, when either $x=\infty$, or $y=\infty$,

$$b=y.$$

Hence in a great number of cases that function is of the form

$$b=y+\mathrm{F}(x, y); \quad \ldots \ldots \ldots \ldots \quad (10)$$

where F is a function which either vanishes or becomes constant when x or y increases indefinitely.

It is plain that when the function b takes this form, the term F is the function for the motions of the liquid particles *relatively to still water*; that is to say,

$$\frac{u-c}{c}=\frac{db}{dy}-1=\frac{d\mathrm{F}}{dy}; \quad \frac{v}{c}=-\frac{db}{dx}=-\frac{d\mathrm{F}}{dx}, \quad \ldots \ldots \ldots \quad (11)$$

and also that the term F fulfils the equation

$$\frac{d^2\mathrm{F}}{dy^2}+\frac{d^2\mathrm{F}}{dx^2}=0. \quad \ldots \ldots \ldots \ldots \quad (12)$$

When the solid is symmetrical at either side of the axis of x (as it is in all the cases that will be considered in this paper), the axis of x itself, so far as it lies beyond the outline of the solid, is a water-line. Hence it is necessary that the equation of that axis, viz.

$$\left.\begin{array}{l} y=0, \\ \text{should be one of the solutions of the equation} \\ b=y+\mathrm{F}(x, y)=0, \end{array}\right\} \quad . \quad . \quad . \quad (13)$$

and consequently that F should vanish with y.

The vanishing of F when $y=\infty$, indicates that every straight line given by the equation $y=b$ either forms part of, or is an asymptote to, a water-line curve.

The vanishing of F when $x=\infty$, indicates that the further the water-lines are from the generating solid, the more nearly they approximate to parallel straight lines.

Every water-line curve is itself the outline of a solid capable of moving smoothly through a liquid.

(5.) *Water-Line Curves generated by a Circle, or Cyclogenous Neoïds.*—Conceive that a circular cylinder of indefinite height, and of the radius l, described about the axis of z, moves through the liquid along the axis of x. Then it is already known that the general equation of the water-line curves is the following,

$$b=y\left(1-\frac{l^2}{x^2+y^2}\right), \quad . \quad . \quad . \quad (14)$$

giving a series of curves of the third order. When $b=0$ this equation resolves itself into two, viz.

$$y=0; \quad x^2+y^2=l^2;$$

the first of which represents the axis of x, and the second the circular outline of the cylinder. For each other value of b, equation (14) represents a curve having two branches: one of them is an oval, contained within the circle, and not relevant to the problem in question; the other, being the real water-line, is convex in the middle and concave towards the ends, and has for an asymptote in both directions the straight line $y=b$.

For brevity's sake, let $x^2+y^2=r^2$. Then the component velocities of a particle of water *relatively to the solid* are given by the equations

$$\left.\begin{array}{l} \dfrac{u}{c}=\dfrac{db}{dy}=1-\dfrac{l^2}{r^2}+\dfrac{2l^2y^2}{r^4}=1+\dfrac{l^2(y^2-x^2)}{r^4}, \\ \dfrac{v}{c}=-\dfrac{db}{dx}=-\dfrac{2l^2xy}{r^4}, \end{array}\right\} \quad (15)$$

and the square of their resultant by the equation

$$\frac{u^2+v^2}{c^2}=\left(1-\frac{l^2}{r^2}\right)^2+\frac{4l^2y^2}{r^4}; \quad . \quad . \quad . \quad (16)$$

while the component and resultant velocities *relatively to still water* are given by the

following equations:—

$$\frac{u}{c}-1=\frac{l^2(y^2-x^2)}{r^4};\quad \frac{v}{c}=-\frac{2l^2xy}{r^4};\quad \frac{\sqrt{\{(u-c)^2+v^2\}}}{c}=\frac{l^2}{r^2}. \quad . \quad . \quad . \quad . \quad . \quad (17)$$

As a convenient name for water-line curves of this sort, it is proposed to call them *Cyclogenous Neoïds*, that is, *ship-shape curves generated from a circle.*

The water-line surfaces generated by a sphere are known; but no use will be made of them in this paper. (See paper by Dr. HOPPE, Quart. Journ. Math., March 1856.)

SECTION II.—*Properties of Water-line Curves generated from Ovals, or Oögenous Neoïds.*

(6.) *Derivation of other Water-Line Curves from Cyclogenous Neoïds.*—When a form of the function F has been found which satisfies equation (12) of art. 4 (that is to say, which fulfils the condition of liquidity), an endless variety of other forms of that function possessing the same property may be derived from the original form by differentiation and integration.

The original form, and also the derived forms, must possess the properties of vanishing for $x=\infty$ and for $y=\infty$, and of becoming $=0$ or a constant for $y=0$. The first of those properties excludes trigonometrical functions, and consequently exponential functions also, which are always accompanied by trigonometrical functions, and leaves available functions of the nature of potentials. The second property excludes derivation by means of differentiation and integration with respect to y, and leaves available differentiation and integration with respect to x.

The original form of the function F which will be used in this paper is that appropriate to cyclogenous neoïds, or water-line curves generated from a circle, as given in equation (14) of art. 5, viz.—

$$F=\frac{y}{r^2}\times \text{ constant.}$$

When one or more differentiations with respect to x are performed on this function, and the results substituted for F in equation (10), there are obtained curves which are real water-lines, but which are not suitable for the figures of ships, some of them being lemniscates, others shaped like an hour-glass, and others looped and foliated in various ways. It is otherwise as regards integration with respect to x; for that operation, being performed once, gives the expression for the ordinate in a class of curves all of which resemble possible forms of ships, and which are so various in their proportions, that every form of ships' water-lines which has been found to succeed in practice may be closely imitated by means of them. As that class of curves consists of certain ovals, and of other water-lines generated from those ovals, it is proposed to call them *Oögenous Neoïds* (from Ὠογενής).

(7.) *General Equation of Oögenous Neoïds.*—The integration with respect to x, already referred to, is performed as follows:—The coordinates of a particle of water being x and y, let x' denote the position of a moveable point in the axis of x: then the function to be integrated is

$$\overline{(x-x')^2+y^2}$$

for all values of x' between two arbitrary limits. Let $2a$ denote the distance between those limits: the most convenient position for the origin of coördinates is midway between them, so as to make the limits

$$x'=+a,\ x'=-a \text{ respectively.}$$

Then the following is the integral sought:

$$\int_{x'=-a}^{x'=+a} \frac{y\,dx'}{(x-x')^2+y^2} = \tan^{-1}\frac{a-x}{y} + \tan^{-1}\frac{a+x}{y}. \quad \dots \dots \quad (18)$$

This quantity evidently denotes *the angle contained between two lines drawn from the point* (x, y) *to the points* $(+a, 0)$ *and* $(-a, 0)$. For brevity's sake, in the sequel that angle will be occasionally denoted by θ; the points $(+a, 0)$ and $(-a, 0)$ will be called the *foci*; and their distance a from the centre will be called the *excentricity*.

Substituting this integral in the general equation (10), we find, for the water-line curves now under consideration, the following equation, which is the general equation of *oögenous neoïds*:—

$$b=y-f\theta=y-f\left(\tan^{-1}\frac{a-x}{y}+\tan^{-1}\frac{a+x}{y}\right). \quad \dots \dots \quad (19)$$

The coefficient f denotes an arbitrary length, which will be called the *parameter*.

(8.) *Geometrical Meaning of that Equation.*—The equation (19) represents a curve at each point of which the excess $(y-b)$ of the ordinate (y) above a certain minimum value (b) is proportional to the angle (θ) contained at that point between two straight lines drawn to the two foci. Except when $b=0$, the curve has an asymptote at the distance b from the axis of x, and parallel to that axis. Since the value of b is not altered by reversing the signs of x, and is only changed from positive to negative by reversing the sign of y, it follows that each curve consists of two halves, symmetrical about the axis of y; and that there are pairs of curves, symmetrical about the axis of x.

In Plate VIII. fig. 1, therefore, which represents a series of such curves, one quadrant only of the space round the origin or centre O is shown, the other three quadrants being symmetrical. A is one of the foci, at the distance $OA=a$ from the centre; the other focus, not shown in the figure, is at an equal distance from the centre in the opposite direction. BL is one quadrant of the primitive oval; and the wave-like curves outside of it are a series of water-lines generated from it, having for their respective asymptotes the series of straight lines parallel to OX, and whose distances from OX are a series of values of b.

The equation (19) embraces also a set of curves contained within the oval, and all traversing the two foci; but as these curves are not suited for the forms of ships' water-lines, no detailed description of them needs be given.

(9.) *Properties of Primitive Oval Neoïds.*—When in equation (19) b is made $=0$, so that the equation becomes

$$y-f\theta=0, \quad \dots \dots \dots \dots \quad (20)$$

there are two solutions; one of which, viz. $y=0$, represents the axis of x, agreeably to

the condition stated in article 4, equations (13). The other solution represents the oval LB.

The greater semiaxis of that oval, OL, will be called the *base* of the series of water-lines generated by the oval, and denoted by l; its value is found as follows:

$$\frac{db}{dy}=1+f\frac{d}{dy}\left(\tan^{-1}\frac{y}{a-x}+\tan^{-1}\frac{y}{a+x}\right)=1+f\left\{\frac{a-x}{(a-x)^2+y^2}+\frac{a+x}{(a+x)^2+y^2}\right\};$$

but at the point L we have

$$x=l;\ y=0;\ \frac{db}{dy}=0;$$

and therefore

$$0=1+f\left(\frac{1}{a-l}+\frac{1}{a+l}\right);$$

whence

$$l^2=a^2+2af. \quad \ldots \ldots \quad (21)$$

To find the *parameter* f when the base l and excentricity a are given, we have the formula

$$f=\frac{l^2-a^2}{2a}. \quad \ldots \ldots \quad (22)$$

The half-breadth, or minor semiaxis of the oval, $OB=y_0$, is the root of the following transcendental equation, found by making $x=0$ in equation (19),

$$y_0-2f\tan^{-1}\frac{a}{y_0}=0, \quad \ldots \ldots \quad (23)$$

which may be otherwise written as follows:—

$$\tan\frac{y_0}{2f}-\frac{a}{y_0}=0. \quad \ldots \ldots \quad (23\text{ A})$$

When the minor semiaxis y_0 and excentricity a are given, the parameter f is found by the equation

$$f=\frac{y_0}{2\tan^{-1}\frac{a}{y_0}}; \quad \ldots \ldots \quad (24)$$

and thence the base l can be computed by equation (21).

When the base l and half-breadth y_0 are given, the excentricity a is found by solving the following transcendental equation:—

$$ay_0-(l^2-a^2)\tan^{-1}\frac{a}{y_0}=0. \quad \ldots \ldots \quad (24\text{ A})$$

An oval neoïd differs from an ellipse in being fuller towards the ends and flatter at the sides; and that difference is greater the more elongated the oval is.

(10.) *Varieties of Oval Neoïds, and extreme cases.*—The excentricity a may have any value, from nothing to infinity; and the base l may bear to the half-breadth y_0 any proportion, from equality to infinity. When the excentricity $a=0$, the two foci coalesce with the centre O; the base l becomes equal to the half-breadth b; the oval becomes a

circle of the radius l; and the water-lines generated by it become cyclogenous neoïds, already described in article 5.

As the excentricity increases, the oval becomes more elongated. In Plate IX. fig. 3, PL is an oval whose length is to its breadth as $\sqrt{3}:1$, its focus being at A_0. The oval BL in Plate VIII. fig. 1 is more elongated, its length being to its breadth as 17 : 6 nearly. When the excentricity is infinite, the centre O and the further focus go off to infinity, leaving only one focus. The parameter f becomes equal to the focal distance LA. The oval is converted into a curve bearing the same sort of analogy to a parabola that an oval neoïd bears to an ellipse*; but instead of spreading to an infinite breadth like a parabola, it has a pair of asymptotes parallel to the axis of x, and at the distance $\pm\pi f$ to either side of it; and each generated water-line has two parallel asymptotes, at the respective distances b and $b+\pi f$ from the axis of x. The properties of these curves may be easily investigated by placing the origin of coordinates at the focus A, and substituting, in equation (19), $\tan^{-1}\frac{y}{x}$ for θ; but as their figure is not suitable for ships' water-lines, it is unnecessary here to discuss them in detail; and the same may be said of a class of curves analogous to hyperbolas, whose equation is formed by putting $-$ instead of $+$ between the two terms of the right-hand member of equation (18).

(11.) *Graphic Construction of Oval and Oögenous Neoïds.*—For the sake of distinctness, the processes of drawing these curves are represented in two figures,—fig. 2 showing the preliminary, and fig. 1 the final processes (see Plate VIII.).

The axis OY is to be divided into equal parts of any convenient length (which will be denoted by δy in what follows), and through the divisions are to be drawn a series of straight lines parallel to OX. (It is convenient to print those lines from a copper-plate divided and ruled by machinery.) They are shown in fig. 1 only, and not in fig. 2, to avoid confusion.

Suppose, now, that the problem is as follows:—*The base* OL *and excentricity* OA *being given, it is required to construct the oval neoïd and the water-lines generated by it.*

Through the focus A (Plate VIII. fig. 2) draw AD perpendicular to OX; about O, with the radius OL, describe the circular arc LD, cutting AD in D; from D draw DE perpendicular to OD, cutting OX in E; then (as equation (22) shows) AE will be $=2f$, the *double parameter*.

About A, with the radius $AE=2f$ thus found, describe a circle cutting AD in F. Then commencing at F, lay off on that circle a series of arcs, each equal to $2\delta y$ (the double of the length of the equal divisions of the axis OY). Through the points of division of the circle draw a series of radii, AG_1, AG_2, &c., cutting the axis OY in a series of points (some of which, from G_2 to G_{12}, are marked in fig. 2)†. (These radii make, with the line AD, a series of angles, $\frac{\delta y}{f}$, $\frac{2\delta y}{f}$, $\frac{3\delta y}{f}$, &c.)

* This curve is identical with the quadratrix of TSCHIRNHAUSEN.

† When the parameter is small, it is sometimes advisable to use a circle (such as a protractor) with a radius

Then about each of the points in the axis OY thus found, with the outer leg of the compasses starting from the focus A, describe a series of circles (shown in Plate VIII. fig. 1) AC_1, AC_2, AC_3, &c.

Each of those circles traverses the two foci; and the equation of any one of them is

$$-F=f\theta=n\delta y, \quad \ldots \ldots \ldots \ldots \quad (25)$$

where θ denotes the angle made at any point of the circle by straight lines drawn to the two foci, and n has the series of values 1, 2, 3, &c. Since F, as explained in article 4, is the characteristic function for the motion of the liquid particles relatively to still water, it is plain that each of the circles for which F = constant is a tangent to the directions of motion of all the particles that it traverses.

The paper is now covered, as in fig. 1, with a network made by a series of straight lines whose equations are of the form $y=n'\delta y$, crossed by a series of circles whose equations are of the form $f\theta=n\delta y$.

Consequently any curve drawn like those in Plate VIII. fig. 1, diagonally through the corners of the quadrangles of that network, will have for its equation

$$y-f\theta=(n'-n)\delta y=b,$$

and will accordingly be an oögenous neoïd, having for its asymptote the line $y=b$.

The primitive oval is drawn by starting from the point L, and traversing the network diagonally. As many curves as are required can be drawn by the eye with great precision, and the whole process is very rapid and easy (see Appendix).

When the problem is *with a given base and excentricity to draw an oögenous neoïd through a given point in the axis* OY, such as P, the process is modified as follows:—The axis OY must be so divided that P shall be at a point of division. Then, up to the describing of the circle about A with the radius AE, the process is the same as before. Then join AP (Plate VIII. fig. 2), and draw Ag making the angle PAg=APO, and cutting the axis OY in a point (such as G_{10}) which will be the centre of the circle traversing A and P. Then on the circumference of the circle about A, from g towards F, lay off a series of arcs each $=2\delta y$; through the points of division draw radii cutting the axis OY in the points G_9, G_8, &c., and complete the process as before.

(12.) *Graphic Construction of Cyclogenous and Parabologenous Neoïds.*—When the excentricity vanishes and the oval becomes a circle, all the circles composing the network become tangents to OX at the point O. They pass through the points where the primitive circular water-line is cut by the equidistant parallel lines. Their radii are in harmonic progression; the equation of any one of them is of the form

$$-F=\frac{l^2y}{x^2+y^2}=n\delta y, \quad \ldots \ldots \ldots \ldots \quad (26)$$

which is a larger multiple of the parameter than double, the length of the divisions being increased in the same proportion; or the points on the axis OY may be laid down by means of their distances from O, calculated by the formula $OG=a \cdot \cotan\theta$.

n having the series of values 1, 2, 3, &c.; and its radius is given by the formula

$$\frac{l^2}{n\delta y}. \quad \ldots \quad (26\text{ A})$$

When there is but one focus, as in the infinitely long curve described in article 10, the network of circles is changed into a set of straight lines radiating from the focus, and making with AX the series of angles given by the formula

$$f\theta = n\delta y. \quad \ldots \quad (27)$$

(13.) *Component and Resultant Velocities of Gliding.*—The component and resultant velocities with which the liquid particles glide along the water-lines are given by the following equations, in terms of the excentricity a, the parameter f, and the coordinates:—

$$\left.\begin{aligned}&\frac{u}{c}=\frac{db}{dy}=1+\frac{f(a-x)}{(a-x)^2+y^2}+\frac{f(a+x)}{(a+x)^2+y^2}; \quad \frac{v}{c}=-\frac{db}{dx}=-\frac{fy}{(a-x)^2+y^2}+\frac{fy}{(a+x)^2+y^2};\\&\frac{u^2+v^2}{c^2}=\frac{db^2}{dy^2}+\frac{db^2}{dx^2}=1+\frac{2f(a-x)}{(a-x)^2+y^2}+\frac{2f(a+x)}{(a+x)^2+y^2}+\frac{4f^2a^2}{\{(a-x)^2+y^2\}.\{(a+x)^2+y^2\}}.\end{aligned}\right\} \quad (28)$$

At the point of greatest (breadth that is, at the axis of y) these expressions take the following values:—

$$\frac{u_0}{c}=1+\frac{2fa}{a^2+y_0^2}=1+\frac{l^2-a^2}{a^2+y_0^2}=\frac{l^2+y_0^2}{a^2+y_0^2}; \quad \frac{v_0}{c}=0. \quad \ldots \quad (28\text{ A})$$

These equations are applicable to a *whole series of water-lines* (such as those shown in fig. 1), including the generating oval, and are the best suited for solving questions relating to such a series.

But when *one particular water-line* is in question, it is sometimes more convenient to use another set of equations, formed from the equations (28) by the aid of the following substitutions, in which θ, as before, denotes $\frac{y-b}{f}$:—

$$\left.\begin{aligned}&\{(a-x)^2+y^2\}.\{(a+x)^2+y^2\}=\frac{4a^2y^2}{\sin^2\theta};\\&\{(a-x)^2+y^2\}+\{(a+x)^2+y^2\}=2a^2+2x^2+2y^2=4a^2+4ay\cot\theta;\\\therefore\quad &x^2+y^2=a^2+2ay\cot\theta; \quad x=\sqrt{\{a^2-y^2+2ay\cot\theta\}}.\end{aligned}\right\} \quad \ldots \quad (29)$$

These substitutions being made in the equations (28), give the following results:—

$$\left.\begin{aligned}&\frac{u}{c}=1+\frac{f}{a}\sin^2\theta-\frac{f}{y}\cdot\cos\theta\sin\theta=1+\frac{f}{2a}-\frac{f\cos 2\theta}{2a}-\frac{f\sin 2\theta}{2y};\\&\frac{v}{c}=-\frac{fx}{ay}\sin^2\theta=-\frac{f}{ay}\sqrt{\{a^2-y^2+2ay\cot\theta\}}\sin^2\theta;\\&\frac{u^2+v^2}{c^2}=1+\frac{2f}{a}\sin^2\theta-\frac{2f}{y}\cos\theta\sin\theta+\frac{f^2}{y^2}\sin^2\theta\\&\qquad=1+\frac{f}{a}+\frac{f^2}{2y^2}-\left(\frac{f}{a}+\frac{f^2}{2y^2}\right)\cos 2\theta-\frac{f}{y}\sin 2\theta.\end{aligned}\right\} \quad \ldots \quad (30)$$

(14.) *Trajectories of Normal Displacement, and of Swiftest and Slowest Gliding.*—By the "trajectory of normal displacement" is meant a curve traversing all the points in a series of water-lines at which the directions of motion of the liquid particles relatively to still water are perpendicular to the water-lines; or, speaking geometrically, a curve traversing all the points at which the circles AC_1, AC_2, &c. of fig. 1, Plate VIII. cut the water-lines at right angles. To find the form of that trajectory, it is sufficient to make

$$\frac{u^2+v^2}{c^2}-\frac{u}{c}=0, \quad \ldots \ldots \ldots \ldots \quad (31)$$

employing the values of these ratios given by the equations (28). This having been done, it appears, after some simple reductions, that the equation of the *trajectory of normal displacement* is the following,

$$x^2-y^2=l^2, \quad \ldots \ldots \ldots \ldots \quad (32)$$

being that of a rectangular hyperbola LM, fig. 1, having its vertex at L, and its centre at O. Hence that curve is *similar for all oögenous and cyclogenous neoïds whatsoever*, being independent of the excentricity, and is identical for all oögenous and cyclogenous neoïds having the same base l.

By the "trajectory of swiftest and slowest gliding" is meant a curve traversing every point in a series of water-lines at which the velocity of gliding, $\sqrt{u^2+v^2}$, is a maximum or a minimum for the water-line on which that point is situated. To find the equation of that curve, it is necessary to solve the following equation,

$$\frac{d}{cdt}\left(\frac{u^2+v^2}{c^2}\right)=\left(\frac{u}{c}\cdot\frac{d}{dx}+\frac{v}{c}\cdot\frac{d}{dy}\right)\left(\frac{u^2+v^2}{c^2}\right)=0, \quad \ldots \ldots \quad (33)$$

the expression employed for $\frac{u^2+v^2}{c^2}$ being that given by the third of the equations (28). After a tedious but not difficult process of differentiation and reduction, which it is unnecessary to give in detail, an equation is found which resolves itself into three factors, viz.

$$x=0, \quad \ldots \ldots \ldots \ldots \ldots \quad (34)$$

being the equation of the axis OY, and

$$\sqrt{x^2+y^2}+y\pm\sqrt{l^2+y^2}=0, \quad \ldots \ldots \ldots \quad (35)$$

being the equations of the two branches LN and LP of a curve of the fourth order. This curve, too, is independent of the excentricity, and therefore *similar for all oögenous and cyclogenous neoïds whatsoever*, and identical for those having the same base l. It has also the following properties:—The straight line joining L with P makes an angle of 30° with the axis OX; there are a pair of straight asymptotes through O, making angles of 30° to either side of OX; and the two branches of the curve cut OX in the point L, at angles of 45°.

(15.) *Graphic Construction of those Trajectories.*—The curves described in the preceding article are easily and quickly constructed, with the aid of the series of equidistant lines parallel to OX, as follows:—In fig. 2, Plate VIII., let ST be any one of

those lines. With the distance SL in the compasses, lay off SH on that line; H will be a point in the hyperbola LM. Also from S lay off, on the axis of y, SI and SJ, each equal to the same distance SL. About the centre O, with the radius OI, draw a circular arc cutting ST in K; this will be a point in the branch LN. About the centre O, with the radius OJ, draw a circular arc cutting ST in k; this will be a point in the branch LP.

(16.) *Properties of the Trajectory of Swiftest and Slowest Gliding.*—The branch LN traverses a series of points of slowest gliding, where the water-lines are furthest apart; the branch LP traverses a set of points of swiftest gliding, where the water-lines are closest together; from O to P the axis of y traverses points of slowest gliding, and beyond P, points of swiftest gliding.

Hence every complete oögenous neoïd which cuts the axis of y between O and P, contains two points of swiftest and three of slowest gliding; and every complete oögenous or cyclogenous neoïd which cuts the axis of y at or beyond P, contains only one point of swiftest and two of slowest gliding.

(17.) *Water-Lines of Smoothest Gliding, or Lissoneoïds.*—At the point P itself, situated at the distance

$$\mathrm{OP}=\frac{l}{\sqrt{3}} \quad \ldots \ldots \ldots \ldots \quad (36)$$

from the centre, two maxima and a minimum of the velocity of gliding coalesce; and therefore not only the first, but the second and third differential coefficients of the velocity of gliding vanish; from which it follows that the velocity of gliding changes more gradually on those water-lines which pass through the point P, than on any other class of oögenous or cyclogenous neoïds.

It is proposed therefore to call this class of water-lines *Lissoneoïds* (from λισσός).

The oval neoïd whose length is to its breadth as $\sqrt{3}:1$ is itself a lissoneoïd; and every series of water-lines generated by an oval *more elongated* than this contains one lissoneoïd; for example, in the series of water-lines shown in fig. 1, the lissoneoïd is marked PQ.

The excentricity of the oval lissoneoïd is computed by solving equation (24 A) of article 9, when $y_0=\frac{l}{\sqrt{3}}$; and it is found to be

$$a=\cdot 732 l, \text{ or nearly } (\sqrt{3}-1)l. \quad \ldots \ldots \quad (36\ \text{A})$$

By giving the excentricity values ranging from $\cdot 732l$ to l, there are produced a series of lissoneoïds ranging from the oval PL in fig. 3, Plate IX., whose focus is at A_0, to the straight line PN, whose focus coalesces with L. PQ_1, PQ_2, and PQ_3 are specimens of the intermediate forms, having their foci respectively at A_1, A_2, and A_3. For a reason which will be explained in Section III., those curves are not shown beyond the trajectory of slowest gliding.

The *greatest speed of gliding*, for a lissoneoïd, is found by making $y_0^2=\frac{l^2}{3}$ in equation (28 A) of article 13; that is to say,

$$\frac{u_Q}{c}=\frac{4l^2}{3a^2+l^2}. \quad \ldots \ldots \ldots \ldots \quad (37)$$

(18.) *Orbits of the Particles of Water.*—The general expressions for the components of the velocity of a liquid particle relatively to still water have been given in equation (11) of article 4; and to apply those to the case of oögenous neoïds, it is only necessary to modify the equations (28) of article 13, by introducing the expression for $\frac{u-c}{c}$ instead of that for $\frac{u}{c}$, as follows:-

$$\left.\begin{aligned} \frac{u-c}{c}&=\frac{(l^2-a^2)\,.\,(a^2-x^2+y^2)}{\{(a-x)^2+y^2\}\,.\,\{(a+x)^2+y^2\}}\,; \quad \frac{v}{c}=\frac{-2(l^2-a^2)xy}{\{(a-x)^2+y^2\}\,.\,\{(a+x)^2+y^2\}}\,; \\ \frac{(u-c)^2+v^2}{c}&=\frac{(l^2-a^2)^2}{\{(a-x)^2+y^2\}\,.\,\{(a+x)^2+y^2\}}\,. \end{aligned}\right\} \quad (38)$$

From the last of these equations it appears that *the velocity of a particle relatively to still water is inversely as the product of its distances from the two foci.*

The only other investigation which will here be made respecting the orbit of a particle of water, is that of the relation between its direction and curvature at a given point, and its ordinate y.

It has already been explained, in article 11, that the direction of motion of a particle is a tangent to a circle traversing it and the two foci. The radius of that circle is

$$\frac{a}{\sin\frac{y-b}{f}}=\frac{a}{\sin\theta}\,;$$

and if ϕ be taken to denote the angle which the direction of the particle's motion relatively to still water makes with the axis of x, it is easily seen that

$$\cos\phi=\cos\theta-\frac{y}{a}\sin\theta. \quad (39)$$

While that angle undergoes the increment $d\phi$, the particle moves through an arc of its orbit whose length is $\frac{dy}{\sin\phi}$; consequently the curvature of that orbit at the arc in question is

$$\frac{1}{\varrho}=\frac{\sin\phi d\phi}{dy}=-\frac{d\,.\cos\phi}{dy}=\left(\frac{1}{f}+\frac{1}{a}\right)\sin\theta+\frac{y}{fa}\cos\theta=\frac{2}{l^2-a^2}\cdot\left\{\frac{l^2+a^2}{2a}\cdot\sin\theta+y\cos\theta\right\}. \quad (40)$$

For cyclogenous neoïds, we obtain the value of this expression by making

$$\sin\theta=\frac{y-b}{f},\ \cos\theta=1,$$

substituting l^2-a^2 for $2fa$, and then making $a=0$; the result being as follows,

$$\frac{1}{\varrho}=\frac{4}{l^2}\left(y-\frac{b}{2}\right)\,; \quad (40\text{A})$$

that is to say, the *curvature of the orbit varies as the distance of the particle from a line parallel to the axis of* **x**, *and midway between that axis and the undisturbed position of the particle.* This is the property of the looped or coiled elastic curve; therefore, *when*

the water-lines are cyclogenous, the orbit of each particle of water forms one loop of an elastic curve.

The general appearance of such an orbit is shown in fig. 6, Plate VIII. The arrow D shows the direction of motion of the solid body. The dotted line AC is supposed to be at the distance b from the axis of x. The particle starts from A, is at first pushed forwards, then deviates outwards and turns backwards, moving directly against the motion of the solid body as it passes the point of greatest breadth, as shown at B. The particle then turns inwards, and ends by following the body, and coming to rest at C, in advance of its original position.

When the water-lines are oögenous, the equations (39) and (40) show that the orbit is of the same general character with the looped elastic curve in fig. 6, but differs from it in detail to an extent which is greater the greater the excentricity a; and the difference consists mainly in a flattening of the loop, so as to make it less sharply curved at B.

When the excentricity increases without limit, the orbit approximates indefinitely to a "curve of pursuit," for which

$$\varphi=\theta,\ \frac{1}{\varrho}=\frac{\sin\theta}{f}. \qquad (40\text{B})$$

(19.) *Trajectory of Transverse Displacement.—Of Speed of Gliding equal to Speed of Ship.—Orthogonal Trajectories.*—The trajectories described in this article differ from those described in articles 14, 15, and 16 by being dependent upon the excentricity, and therefore not similar for all sets of oögenous neoïds.

By the "trajectory of transverse displacement" is meant the curve traversing all the points at which the liquid particles are moving at right angles to the axis OX, relatively to still water. It is determined from the first of the equations (28), by making

$$\frac{u}{c}-1=0;$$

from which is easily deduced the following equation,

$$x^2-y^2=a^2, \qquad (41)$$

being that of a rectangular hyperbola, with its centre at O and its vertex at the focus A.

The trajectory of the points where the speed of gliding is equal to the speed of the solid body, is found from the third of the equations (28), by making

$$\frac{u^2+v^2}{c^2}-1=0.$$

Its equation is

$$x^2-y^2=\frac{l^2+a^2}{2}, \qquad (42)$$

being that of a rectangular hyperbola, with its centre at O and its vertex between A and L, at a distance from O equal to half the hypothenuse of a right-angled triangle whose other sides are equal to the base and the excentricity respectively.

Let q= constant be the equation of one out of an indefinite number of orthogonal

trajectories to a set of oögenous neoïds. The function q, as is well known, must satisfy the equation

$$\frac{dq}{dx}\cdot\frac{db}{dx}+\frac{dq}{dy}\cdot\frac{db}{dy}=0.$$

Referring to equation (19) of article 7 for the value of b, it is easily seen that this condition is fulfilled by the following function,

$$q=x+\frac{f}{2}\text{ hyp. log. }\frac{(a+x)^2+y^2}{(a-x)^2+y^2}, \quad \ldots \quad (43)$$

which has also the following properties,

$$\frac{dq}{dx}=\frac{db}{dy}=\frac{u}{c}; \ \frac{dq}{dy}=-\frac{db}{dx}=\frac{v}{c}; \ \frac{d^2q}{dx^2}+\frac{d^2q}{dy^2}=0. \quad \ldots \quad (44)$$

Every orthogonal trajectory has a straight asymptote parallel to the axis of y, and expressed by the equation $x=q$.

The perpendicular distance between two consecutive orthogonal trajectories, like that between two consecutive water-lines, is inversely proportional to the velocity of gliding; hence, if a complete set of orthogonal trajectories were drawn on fig. 1, they would divide it into a network of small rectangles, the dimensions and area of any one of which would be expressed as follows:—

$$\frac{cdb}{\sqrt{u^2+v^2}}\times\frac{cdq}{\sqrt{u^2+v^2}}=\frac{c^2dbdq}{u^2+v^2}. \quad \ldots \quad (45)$$

For a series of cyclogenous neoïds, the equation of the orthogonal trajectories takes the following form,

$$q=x\left(1+\frac{l^2}{x^2+y^2}\right). \quad \ldots \quad (45\text{A})$$

(20.) *Disturbances of Pressure and Level.*—Let h denote the *head* at a given particle of liquid, being the sum of its elevation above a fixed level and of its pressure, expressed in units of height of the liquid itself. In a mass of liquid which is at rest, the head has a uniform value for every particle of the mass; let that value be denoted by h_0. Then when the mass of liquid is in the state of motion produced by the passage of a solid through it, the head at each particle, according to well-known principles, undergoes the change expressed by the following equation,

$$h-h_0=\frac{c^2-u^2-v^2}{2g}, \quad \ldots \quad (46)$$

being the height due to the difference between the squares of the speed of the solid body and of the speed of gliding; and in an open mass of water with a vessel floating in it, that change will take place by alterations in the level of surfaces of equal pressure. The trajectory of slowest gliding, LN (Plate VIII. fig. 1), will mark the summit of a swell thus produced, and so also will the axis of y between O and P; while the trajectory of swiftest gliding OP, and the axis of y beyond P, will mark the bottom of a hollow. These are the principal vertical disturbances, which, throughout this investigation, have been assumed

to be so small, compared with the dimensions of the body, as not to produce any appreciable error in the consequences of the supposition of motion in plane layers.

(21.) *Integral on which the Friction depends.*—Suppose a portion of an oögenous neoïd to be taken for the water-line of part of the side of a vessel, which part is of the depth δz, and that the resistance arising from friction between the water and the vessel is to be expressed—the law of that friction being, that it varies as the square of the velocity of gliding, and as the extent of rubbing surface.

That resistance is to be found (as already explained in a paper on Waves, published in the Philosophical Transactions for 1863) by determining the *work performed* in a second in overcoming friction, and dividing by the speed of the vessel; for thus is taken into account not only the direct resistance caused by the longitudinal component of the friction, but the resistance caused indirectly through the increase of pressure at the bow, and diminution of pressure at the stern, assuming the vertical disturbance to be unimportant.

Then for a part of the water-line which measures longitudinally dx, the extent of surface is

$$\delta z \cdot \frac{\sqrt{u^2+v^2}}{u} dx;$$

the friction on the unit of surface is

$$\frac{\mathrm{KW}(u^2+v^2)}{2g},$$

where W is the weight of a unit of volume of water, and K a coefficient of friction; and that friction has to be overcome through the distance $\sqrt{u^2+v^2}$, while the vessel advances through the distance c, giving as a factor

$$\frac{\sqrt{u^2+v^2}}{c}$$

Those three factors being multiplied together, and the result put under the sign of integration, give the following expression for the resistance,

$$\mathrm{R}=\frac{\mathrm{KW}c^2}{2g}\delta z \cdot \int \left(\frac{u^2+v^2}{c^2}\right)^2 \cdot \frac{c}{u} dx. \quad \ldots \ldots \ldots \quad (46\,\mathrm{A})$$

Another form of expression for the same integral is obtained by putting $\frac{c}{v} dy$ or $f\frac{c}{v} d\theta$ instead of $\frac{c}{u} dx$; and a third form by putting for the elementary area of the rubbing surface the following value,

$$dq;$$

where dq is the distance between the asymptotes of a pair of orthogonal trajectories, as explained in article 19. This gives for the resistance

$$\mathrm{R}=\frac{\mathrm{KW}c^2}{2g}\delta z \cdot \int \frac{u^2+v^2}{c^2} \cdot dq. \quad \ldots \ldots \ldots \quad (47)$$

In preparing these formulæ for integration, it is necessary to express the function to be integrated in terms of constants and of the independent variable only, x, y, θ, or q, as the case may be; for example, if y or θ is the independent variable, the expression of the function to be integrated is to be taken from the equations (30) of article 13.

Owing to the great complexity of that function, its exact integration presents difficulties which have not yet been overcome, although a probable approximate formula for the resistance has been arrived at by methods partly theoretical and partly empirical, as to which some further remarks will be made in the third section of this paper*.

There is one particular case only in which the exact integration of equation (46 A) is easy, that of a complete circular water-line of the radius l; and the result is as follows:—

$$R=\frac{KWc^2}{2g}\delta z\times 21\tfrac{1}{3}l. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

(22.) *Statement of the General Problem of the Water-Line of least Friction.*—It is evident that, by introducing under the sign of integration in equation (18) of article 7 an arbitrary function of x', the integral may be made capable of representing an arbitrary function of x and y, and will still satisfy the condition of perfect liquidity; and thus the equation

$$b=y+\int_{-a}^{+a}\frac{y\varphi(x')dx'}{(x-x')^2+y^2}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48\text{ A})$$

may be made to represent an arbitrary form of primitive water-line.

To find therefore, by the calculus of variations, the water-line enclosing a given area which shall have the least friction, will require the solution of the following problem:— To determine the function $\varphi(x')$ so that, with a fixed value of the integral $\int xdy$, the integral in equation (46 A) shall be a minimum.

(22 A.) *Another Class of Plane Water-Line Equations.*—A mode of expressing the conditions of the flow of water in plane layers past a solid, differing in form from that made use of in the preceding parts of this paper, consists in taking for independent variables, not the coordinates of the water-lines themselves, x and y, but the coordinates of their asymptotes (b), and of the asymptotes of their orthogonal trajectories (q). These new variables are connected with x and y, and with the velocity of gliding, by the following equations:—

$$\frac{u^2+v^2}{c^2}=\frac{dq}{dx}\cdot\frac{db}{dy}-\frac{dq}{dy}\cdot\frac{db}{dx}=\frac{1}{\frac{dx}{dq}\cdot\frac{dy}{db}-\frac{dy}{dq}\cdot\frac{dx}{db}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

It can be shown that in order to satisfy the condition of liquidity we must have

$$x=\frac{d\psi}{db},\quad y=\frac{d\psi}{dq}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

* See the Civil Engineer and Architect's Journal for October 1861, the Philosophical Transactions for 1863, the Transactions of the Institution of Naval Architects for 1864, and a Treatise on Shipbuilding, published in 1864.

where ψ denotes a function of b and q, such that

$$\frac{d^2\psi}{db^2}+\frac{d^2\psi}{dq^2}=0; \quad \ldots \ldots \ldots \ldots \quad (51)$$

and consequently that

$$\frac{c^2}{u^2+v^2}=\left(\frac{d^2\psi}{db\,dq}\right)^2+\left(\frac{d^2\psi}{db^2}\right)^2. \quad \ldots \ldots \ldots \quad (52)$$

The curves to which this method of investigation leads are inferior to oögenous neoïds as water-lines for ships, because they have comparatively sharp curvature amidships, which causes them to have small capacity for their length and breadth, and would give rise to comparatively sudden changes in the speed of gliding. They will therefore not be further discussed in the present paper, except to state that the simplest of them is the well-known cissoid.

SECTION III.—*Remarks on the Practical Use of Oögenous Water-Lines.*

(23.) *Previous Systems of Water-Lines.*—Owing principally to the great antiquity of the art of shipbuilding, and the immense number of practical experiments of which it has been the subject, that part of it which relates to the forms of water-lines has in many cases attained a high degree of excellence through purely empirical means. Excellence attained in that manner is of an uncertain and unstable kind; for as it does not spring from a knowledge of general principles, it can be perpetuated by mere imitation only.

The existing forms of water-lines, whose merits are known through their practical success, constitute one of the best tests of a mathematical theory of the subject; for if that theory is a sound one, it will reproduce known good forms of water-line; and if it is a comprehensive one, it will reproduce their numerous varieties, which differ very much from each other.

The geometrical system of CHAPMAN for constructing water-lines is wholly empirical; it consists in the use of parabolas of various orders, chosen so as to approximate to figures that have been found to answer in practice, and it has no connexion with any mechanical theory of the motion of the particles of water.

The first theory of ships' water-lines which was at once practically useful, and based on mechanical principles, was that of Mr. SCOTT RUSSEEL, explained in the first and second volumes of the Transactions of the Institution of Naval Architects. It consists of two parts; the first has reference to the dimensions of water-lines intended for a given maximum speed, and prescribes a certain relation between the length of those lines and the length of a natural wave which travels with that speed; the second part relates to the form of those lines, and prescribes for imitation the figures of certain natural waves, as being lines along which water is more easily displaced than along other lines. The figures thus obtained are known to be successful in practice; but it is also well known that there are other figures which answer well in practice, differing considerably from those wave-lines; and it is desirable that the mathematical theory of the subject should

embrace those figures also. It may further be observed that the figure of the solitary wave, as investigated experimentally by Mr. SCOTT RUSSELL (Reports of the British Association, 1845), and mathematically by Mr. EARNSHAW (Camb. Trans. 1845), is that of a wave propagated in a canal of small breadth and depth as compared with the dimensions of the wave, and in which particles of water originally in a plane at right angles to the direction of motion continue to be very nearly in a plane at right angles to the direction of motion, so as to have sensibly the same longitudinal velocity. This state of things is so different from the circumstances of the motions of the particles in the open sea, that it appears desirable to investigate the subject with special reference to a mass of water of unlimited breadth and depth, as has been done in the previous sections of this paper.

(24.) *Variety of Forms of Oögenous Neoïds, and their likeness to good known Forms of Water-Line.*—The water-lines generated from ovals which have been described in the second section of this paper, are remarkable for the great varieties of form and proportions which they present, and for the resemblance of their figures to those of the water-lines of the different varieties of existing vessels. There is an endless series of ovals, having all proportions of length to breadth, from equality to infinity; and each of those ovals generates an endless series of water-lines, with all degrees of fulness or fineness, from the absolute bluffness of the oval itself to the sharpness of the knife-edge. Further variations may be made by taking a greater or a less length of the curve chosen.

The ovals are figures suitable for vessels of low speed, it being only necessary, in order to make them good water-lines, that the vertical disturbance (as explained in art. 20) should be small compared with the vessel's draught of water. At higher speeds the sharper water-lines, more distant from the oval, become necessary. The water-lines generated by a circle, or "cyclogenous neoïds," are the "leanest" for a given proportion of length to breadth; and as the excentricity increases, the lines become "fuller." The lines generated from a very much elongated oval approximate to a straight middle body with more or less sharp ends. In short, there is no form of water-line that has been found to answer in practice which cannot be imitated by means of oögenous neoïds.

(25.) *Discontinuity at the Bow and Stern.—Best limits of Water-Lines.*—Amongst the endless variety of forms presented by oögenous water-lines, it may be well to consider whether there are any which there are reasons for preferring to the others. One of the questions which thus arise is the following:—Inasmuch as all the water-line curves of a series, except the primitive oval, are infinitely long and have asymptotes, there must necessarily be an abrupt change of motion at either end of the limited portion of a curve which is used as a water-line in practice, and the question of the effect of such abrupt change or discontinuity of motion is one which at present can be decided by observation and experiment only. Now it appears from observation and experiment, that the effect of the discontinuity of motion at the bow and stern of a vessel which has an entrance and run of ordinary sharpness and not convex, extends to a very thin layer of water only, and that beyond a short distance from the vessel's side the discontinuity

ceases, through some slight modification of the water-lines of which the mathematical theory is not yet adequate to give an exact account*.

Still, although the effect of the discontinuity in increasing resistance may not yet have been reduced to a mathematical expression, and although it may be so small that our present methods of experimenting have not yet detected it, it must have some value; and it is desirable so to select the limits of the water-line as to make that value as small as possible. In order that the abrupt change of motion may take place in as small a mass of water as possible, it would seem that the limits of the water-line employed in practice should be at or near the point of *slowest gliding*; that is, where the water-line curve is cut by the trajectory of slowest gliding LN, in Plate VIII. fig. 1, and Plate IX. fig. 3, as explained in articles 14, 15, and 16; and that conclusion is borne out by the figures of many vessels remarkable for economy of power.

(26.) *Preferable Figures of Water-Lines.*—In forming a probable opinion as to which, out of all the water-lines generated by a given oval, is to be preferred to the others, regard is to be had to the fact, that every point of maximum disturbance of the level of the water, whether upwards or downwards, that is to say, every point of maximum or minimum speed of gliding (see article 20), forms the origin of a wave, which spreads out obliquely from the vessel (as may easily be observed in smooth water), and so transfers mechanical energy to distant particles of water, which energy is lost. Hence such points should be as few as possible; and the changes of motion at them should be as gradual as possible; and these conditions are fulfilled by the curves described in article 17, by the name of "Lissoneoïds," being those which traverse the point P in the figures, and which may have any proportion of length to breadth, from $\sqrt{3}$ to infinity.

(27.) *Approximate Rules for Construction and Calculation.*—The description of those curves, already given in article 17, has been confined to those properties which are exactly true. The following rules are convenient approximations for practical purposes, *when the proportion of length to breadth is not less than* 4 : 1 (see Plate IX. figs. 3 & 4).

I. A tangent to the curve at Q, the point of slowest gliding, passes very nearly through the point P of greatest breadth.

II. The area PQR enclosed within the water-line is very nearly equal to the rectangle of the breadth PR and excentricity a. (When the length is not less than six times the breadth, this rule is almost perfectly exact.)

* In confirmation of this, experiments made on the steamers 'Admiral' and 'Lancefield,' by Mr. J. R. Napier and the author, may be specially referred to. The water-lines of the 'Admiral' are complete trochoids, and tangents to the longitudinal axis at the bow and stern. The engine-power required to drive her at her intended speed was computed from the frictional resistance, according to principles explained in publications already referred to in the note to article 21; and the result of the calculation was closely verified by experiment. The water-lines of the 'Lancefield' are only partly trochoidal, being straight from the point of contrary flexure to the bow, so that, instead of being tangents there to the longitudinal axis, they form with it angles of about $13\frac{1}{2}°$. Yet the same formula which gave the resistance of the 'Admiral' has been found to give also the resistance of the 'Lancefield' without any addition on account of the discontinuity of motion at the bow.

III. For the trajectory of slowest gliding, LN, there may be substituted, without practical error, a straight line cutting the axis OX in L at an angle of 45°; and when this has been done, the excentricity OA or a is almost exactly equal to the length

$$\times \cdot 634\left(=\frac{3}{3+\sqrt{3}}\right);$$

and this of course is also the ratio of the area to the circumscribed rectangle. The base OL or l also is very nearly equal to (the sum of the length and breadth) $\times \cdot 634$.

IV. Hence the following approximate construction. Given, the common length QR of a set of water-lines of smoothest gliding which are to have a common termination at Q, and their breadths RP_1, RP_2, RP_3, &c.: required, to find their areas, bases, and foci.

Through Q and R draw the straight lines QU and RU, making the angles RQU=45°, QRU=30°. Through their intersection U draw UV perpendicular to RQ. All the required foci will be in UV; and RV will be the length of the rectangles equivalent to each of the water-line areas; so that

$$\begin{aligned} \text{area } P_1\, QR_1 &= RV \times RP_1, \\ \text{area } P_2\, QR_2 &= RV \times RP_2, \\ \&c. \qquad & \qquad \&c. \end{aligned}$$

Through P_1, P_2, P_3, &c. draw lines parallel to RU, cutting QU in L_1, L_2, L_3, &c.: these points will be the ends of the bases required, through which draw the bases $L_1 O_1$, $L_2 O_2$, $L_3 O_3$, &c. parallel to QR, and cutting VU in A_1, A_2, A_3, &c.: these will be the required foci.

The bases and foci and the points P_1, P_2, P_3, &c. being given, the water-lines are to be constructed by the rules given in article 11.

(28.) *Lissoneoïds compared with Trochoids.*—In fig. 5, Plate IX., the full line PQ is a lissoneoïd, and the dotted line Pq a trochoid of the same breadth and area. The curves lie very near together throughout their whole course—the only differences being, that the trochoid is slightly less full and more hollow than the lissoneoïd, but at the same time the trochoid is the longer and has a greater frictional surface. Had the entrance of the trochoid consisted of a straight tangent from its point of contrary flexure (as in the bow of the 'Lancefield,' mentioned in the note to article 25), the two curves would have lain still closer together. The same likeness to a trochoid is found in all lissoneoïds whose length is more than about 3½ times the breadth.

(29.) *Combinations of Bow and Stern.*—Although there is reason to believe that water-lines of equal length and similar form at the bow and stern, such as are produced by using one neoïd curve throughout, are the best on the whole, still the naval architect, should he think fit, can combine two different öogenous neoïds for the bow and stern; or, according to a frequent practice, he may adapt the figure of the stern to motion of the particles in vertical layers instead of horizontal layers; provided he takes care in every case that the midship velocity of gliding (u_0, as given by equation (28 A) of article 13) is the same for each bow water-line and stern water-line at their point of junction.

(30.) *Provisional Formula for Resistance.*—Until the difficulty of integration, mentioned in article 30, shall have been overcome, or until more exact experimental data than we have at present shall have been obtained, the following provisional formula, analogous to that which has been found to agree with the results of experiment on trochoidal and nearly trochoidal lines, as well as some others, may be considered as a probable approximation for lissoneoïds,

$$R=\frac{KWc^2}{2g}\cdot\left(1+4\frac{(u_0-c)^2}{c^2}\right)LG; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

where G is the mean girth of the vessel under water; L her total length; u_0 the midship velocity of gliding, found, for a lissoneoïd, by equation (37) of article 17; c the speed of the ship; W the heaviness of water; and K a coefficient of friction (=about ·0036 for a clean surface of paint).

APPENDIX.

Note to Article 11.—The general process of constructing a series of curves whose equation is $\phi(x, y)+\psi(x, y)=$ constant, by drawing lines diagonally through a network consisting of two sets of curves whose equations are respectively $\phi(x, y)=$ constant and $\psi(x, y)=$ constant, is due to Professor CLERK MAXWELL.

SUMMARY OF THE CONTENTS.

Section I.—*Introduction, and Summary of known Principles.*

Section II.—*Properties of Water-Line Curves generated from Ovals, or Oögenous Neoïds.*

XI. *On the Joint-Systems of Ireland and Cornwall, and their Mechanical Origin.* *By the Rev.* Samuel Haughton, *M.D., Fellow of Trinity College, Dublin.*

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Contents.

INTRODUCTION.

In the Philosophical Transactions for 1858, I have given an account of the Joints of the Old Red Sandstone of the co. Waterford, and demonstrated in it the existence of the following Systems of Joints (page 348):—

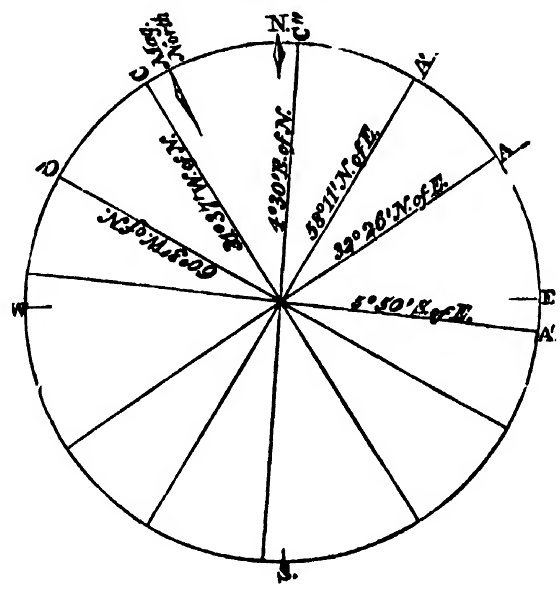
		First Conjugate System.	
A . .	81 observations	. 7° 46′ N. of E. (Mag.) . .	32° 26′ N. of E. (True.)
C . .	49 ,,	. 6° 57′ W. of N. ,, . .	31° 37′ W. of N. ,,
		Second Conjugate System.	
A′ . .	135 observations	. 33° 31′ N. of E. (Mag.) . .	58° 11′ N. of E. (True.)
C′ . .	47 ,,	. 35° 23′ W. of N. ,, . .	60° 3′ W. of N. ,,
		Third Conjugate System.	
A″ . .	14 observations	. 30° 30′ S. of E. (Mag.) . .	5° 50′ S. of E. (True.)
C″ . .	12 ,,	. 29° 10′ E. of N. ,, . .	4° 30′ E. of N. ,,
		Fourth Conjugate System.	
A‴ . .	1 observation	. 10° 0′ S. of E. (Mag.)	
C‴ . .	6 observations	. 9° 30′ E. of N. ,,	

Since the publication of the foregoing account, I have found the system (A, C) in Donegal, Mourne, Cornwall and elsewhere, and have reason to believe it to be the most important of all the joint-systems of Waterford. I shall therefore call it the Primary System of Conjugate Joints, and the systems (A′, C′) and (A″, C″) I shall call

correlated Secondary Systems of Conjugate Joints, as I believe that they can be shown to be simple mechanical consequences of the Primary System (A, C).

Neglecting for the present the system (A‴, C‴), the whole of the Joints of Waterford may be represented by the following diagram (fig. 1), to which I shall frequently have occasion to refer.

Fig. 1.



It will be observed that the Primary System (A) divides somewhat unequally the angle between (A′) and (A″).

Angle between A and A′	. .	25	45	Mean = 27 5.
,, C and C′	. .	28	26	
Angle between A and A″	. .	38	16	Mean = 37 11.
,, C and C′	. .	36	7	

Part I.—On the Joint-Systems of the Co. Donegal.

The following Tables give the bearings of all the joints observed, arranged according to their azimuths, together with the locality, and a statement of the rock in which the joint was observed. In each locality several joints of the same bearing were measured, but only one is recorded.

Table I.—East and West Joints in Donegal.

No.	Bearing.	Dip.	Locality.	Rock.
1	45° N. of E.	°......	Between Dunglow and Doocharry.	Granite.
2	45 ,,	......	Culdaff.	Quartzite.
3	35 ,,	......	The Mintiaghs.	,,
4	35 ,,	......	Urrismenagh.	Granite.
5	30 ,,	90	Gweebarra, South.	,,
6	30 ,,	......	The Mintiaghs.	Syenite.
7	30 ,,	......	Gap of Mamore.	Quartzite.
8	20 ,,	......	Barnesmore.	Granite.
9	20 ,,	......	Barnesbeg.	,,
10	15 ,,	80 N.	Doocharry Bridge.	,,

TABLE I. (continued).

No.	Bearing.	Dip.	Locality.	Rock.
11	10° N. of E.	°......	Glenleheen.	Granite.
12	10 "	......	Between Glenties and Gweebarra.	"
13	7 "	......	Barnesbeg.	"
14	7 "	......	Lackagh Bridge.	"
15	5 "	......	Barnesbeg.	"
16	5 "	......	Lackagh Bridge.	"
17	5 "	......	Milford.	Quartzite.
18	0 E.W.	90	Annagary.	Granite.
19	0 "	......	Fanad.	"
20	5 S. of E.	......	McSwyne's Gun.	Syenite.
21	10 "	......	Barnesmore.	Granite.
22	10 "	......	Malin.	Quartzite.
23	10 "	......	Moville.	Mica-slate.
24	10 "	......	Shalwy.	Carb. Limestone.
25	10 "	90	Between Doocharry and Fintown.	Granite.
26	35 "	90	Shalwy.	Old Red Conglomerate.
27	35 "	......	The Mintiaghs.	Quartzite.
28	40 "	......	The Mintiaghs.	Syenite.
29	40 "	......	Barnesbeg.	Granite.

TABLE II.—North and South Joints in Donegal.

No.	Bearing.	Dip.	Locality.	Rock.
1	40° E. of N.	°......	Culdaff.	Quartzite.
2	30 "	90	Shalwy.	Old Red Conglomerate.
3	15 "	......	Meen Banad.	Granite.
4	10 "	......	Shalwy.	Carb. Limestone
5	10 "	80 E.	Sheskina Roan.	Granite.
6	7 "	90	Annagary Hill.	"
7	5 "	......	McSwyne's Gun.	Syenite.
8	5 "	......	Glen (North).	Granite.
9	5 "	......	Fanad.	"
10	5 "	......	The Mintiaghs.	Syenite.
11	0 N.S.	......	Barnesbeg.	Granite.
12	0 "	......	Glen (North).	"
13	0 "	......	Sheskina Roan.	"
14	5 W. of N.	......	Barnesbeg.	"
15	5 "	......	Milford.	Syenite.
16	5 "	......	Glenleheen.	Granite.
17	5 "	90	Lough Anure.	"
18	7 "	......	Lackagh Bridge.	"
19	10 "	......	Between Glenties and Gweebarra.	"
20	10 "	......	Gweebarra (South).	"
21	10 "	80 E.	Doocharry Bridge.	"
22	10 "	90	Annagary Hill.	"
23	10 "	......	Between Dunglow and Doocharry.	"
24	10 "	90	Between Doocharry and Fintown.	"
25	10 "	......	Glen (North).	"
26	10 "	......	Lackagh Bridge.	"
27	10 "	......	Ards.	Quartzite.
28	15 "	......	Fintown Gap.	Gneiss.
29	20 "	......	Urrismenagh.	Granite.
30	20 "	......	Glen (North).	"
31	22 "	......	Urrismenagh.	"
32	25 "	......	Fanad.	"
33	25 "	......	Barnesbeg.	"
34	40 "	......	Gap of Mamore.	Quartzite.
35	42 "	......	Barnesbeg.	Granite.

If we arrange the 64 observations recorded in the preceding Tables according to their azimuths and the number of observations corresponding to each, we shall obtain the following:—

TABLE III.—Joints of Donegal, arranged according to azimuths and number of observations.

Azimuth.	No. of observations.
42° W. of N.	1
40 "	1
25 "	2
22 "	1
20 "	2
15 "	1
10 "	9
7 "	1
5 "	4
0 N.S.	3
5 E. of N.	4
7 "	1
10 "	2
15 "	1
30 "	1
40 "	1
45 N. of E.	2
35 "	2
30 "	3
20 "	2
15 "	1
10 "	2
7 "	2
5 "	3
0 E.W.	2
5 S. of E.	1
10 "	5
35 "	2
40 "	2
	Total 64

An inspection of this Table shows that, of the 64 observations, 24 are included in the angle between 10° E. of N. and 10° W. of N., and that 15 are included in the angle between 10° N. of E. and 10° S. of E.,—making a total of 39 in 64, or 61 per cent. of the whole number of observations contained between these two limits. Dividing the N.S. observations equally between those that lie to the E. and W. of North, we find the following mean azimuths:—

I. 7° 33′ W. of N. from $15\frac{1}{2}$ observations =24 per cent.
II. 5° 32′ E. of N. from $8\frac{1}{2}$ observations =13 per cent.

In like manner, dividing the E.W. observations equally between those that lie to the N. and S. of East, we find the following mean azimuths:—

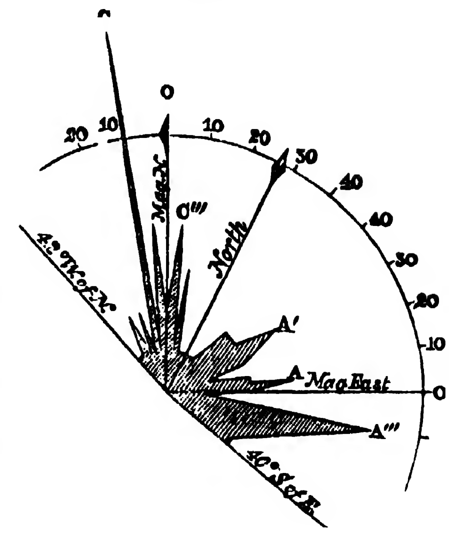
III. 6° 7′ N. of E. from 8 observations $=12\frac{1}{2}$ per cent.
IV. 7° 51′ S. of E. from 7 observations =11 per cent.

From Table III. it also appears that between 30° and 35° North of East, there are five observations, whose mean is

V. 32° 0′ N. of E. from 5 observations $=8$ per cent.

The accompanying diagram (fig. 2) is a graphical representation of Table III., and shows sufficiently well the five systems of joints that have been deduced from that Table.

Fig. 2.



Of the joints themselves, I. and III., II. and IV. are evidently conjugates, being nearly at right angles respectively, while V. belongs to a system different from the others.

Adopting the notation used by me already* in my paper on the Joints of the co. Waterford, and comparing the results there obtained with those here found, we may construct the following Table:—

TABLE IV.—Comparison of the Joints of Donegal and Waterford.

(C). West of North (Magnetic).		(A). North of East (Magnetic).		(C). West of North (true bearings†).		(A). North of East (true bearings).	
Donegal.	Waterford.	Donegal.	Waterford.	Donegal.	Waterford.	Donegal.	Waterford.
7° 33′	6° 57′	6° 7′	7° 46′	34° 13′	31° 37′	32° 47′	32° 26′

Angle of Axes $= \left\{ \begin{array}{l} 91° \ 26′ \text{ Donegal} \\ 89° \ 11′ \text{ Waterford} \end{array} \right\}$ from East to North.

(C‴). East of North (Magnetic).		(A‴). South of East (Magnetic).		(C‴). West of North (true bearings).		(A‴). North of East (true bearings).	
Donegal.	Waterford.	Donegal.	Waterford.	Donegal.	Waterford.	Donegal.	Waterford.
5° 32′	9° 30′	7° 51′	10° 0′	21° 8′	15° 10′	18° 49′	14° 40′

Angle of Axes $= \left\{ \begin{array}{l} 92° \ 19′ \text{ Donegal} \\ 90° \ 30′ \text{ Waterford} \end{array} \right\}$ from East to North.

(A′). North of East (Magnetic).		(A′). East of North (true bearings).	
Donegal.	Waterford.	Donegal.	Waterford.
32° 0′	33° 31′	31° 20′	31 49′

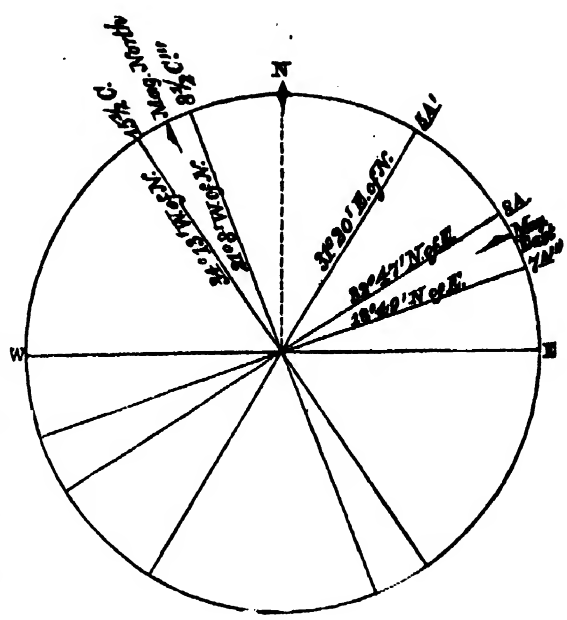
The agreement between the respective systems A, C and A′ is very remarkable; that between A‴ and C″, in the two localities, is perhaps as good as can be expected from the

* Philosophical Transactions, 1858, p. 333.

† The magnetic variation in Donegal was 26° 40′ W. and in Waterford 24° 40′ W.

limited number of observed planes, which was 15½ in Donegal, and only 7 in Waterford, where this conjugate system is very subordinate in importance.

Fig. 3.



The annexed diagram (fig. 3) shows the Donegal joints divided into two conjugate systems, (A, C) and (A''', C'''), together with one plane (A') of a third system; all of which are represented in the co. Waterford.

Of the whole 64 joints recorded, 44, or 69 per cent., are involved in these five planes; and the number of observations belonging to each is placed beside its designation in the diagram.

If we combine together the systems (A, A''') and (C, C'''), we obtain the following as the most simple combination of joints observable in Donegal.

	No. of observations.	Magnetic bearing.	True bearing.
I. Primary System (A—A''') . .	15 . .	0° 24′ S. of E.	26° 16′ N. of E.
II. Conjugate Primary (C—C''') .	24 . .	2° 55′ W. of N.	29° 35′ W. of N.
III. Secondary System (A′) . . .	5 . .	32° 0′ N. of E.	58° 48′ N. of E.

The angle between the Primary System and its Conjugate (A, C) is therefore found to be 93° 19′.

And the angle between the Primary and Secondary Systems (A, A′) is 32° 24′.

The angle between (A, C) and (A′, C′) in the co. Waterford has been already stated to be 27° 5′.

Part II.—ON THE JOINT-SYSTEMS OF THE MOURNE AND NEWRY MOUNTAINS.

The following Tables contain the observed joint-planes of the Mourne and Newry mountain district.

Table V.—North and South Joints of the Mourne and Newry Mountains.

No.	Bearing.	Dip.	Locality.	Rock.
1	45° W. of N.	......	Railway cutting *.	Granite.
2	45 "	70 N.	"	"
3	45 "	90	Back of Eagle Mountain.	"
4	45 "	65 N.	Campbell's Quarry, Newry.	"
5	45 "	90	" "	"
6	30 "	90	Summit of Eagle Mountain.	"
7	30 "	55 E.	Killowen.	Slate.
8	30 "	60 E.	"	"

* The railway cuttings are from the Main Line Station of the Dublin and Belfast Junction Railway to the open plain under Slieve Gullion.

TABLE V. (continued).

No.	Bearing.	Dip.	Locality.	Rock.
9	25° W. of N.	°	Railway cutting.	Granite.
10	25 "	90	Killowen.	Slate.
11	20 "	80 E.	Eagle Mountain (summit).	Granite.
12	15 "	......	Railway cutting.	"
13	15 "	75 E.	"	"
14	15 "	80 E.	"	"
15	15 "	......	"	"
16	15 "	90 to 65 E.	Wellington Inn *	"
17	15 "	90 to 65 E.	"	"
18	15 "	......	Rostrevor.	Slate.
19	15 "	......	Eagle Mountain (summit).	Granite.
20	15 "	90	Slieve Bingian.	"
21	10 "	70 E.	Railway cutting.	"
22	10 "	90	Rostrevor.	Slate.
23	5 "	90	Back of Eagle Mountain.	Granite.
24	5 "	90	Campbell's Quarry, Newry.	"
25	5 "	80 E.	" "	"
26	0 N.S.	90	Eagle Mountain.	"
27	0 "	90	"	"
28	0 "	90	"	"
29	5 E. of N.	80 E.	Campbell's Quarry, Newry.	"
30	10 "	......	Railway cutting.	"
31	15 "	90	Rostrevor.	Slate.
32	15 "	90	Killowen.	"
33	15 "	90	Eagle Mountain.	Granite.
34	15 "	90	Campbell's Quarry, Newry.	"
35	15 "	90	" "	"
36	15 "	80 E.	" "	"
37	20 "	90	" "	"
38	20 "	......	Railway cutting.	"
39	20 "	65 E.	Rostrevor.	Slate.
40	20 "	70 W.	"	"
41	22 "	......	Slieve Bingian.	Granite.
42	25 "	90	Railway cutting.	"
43	30 "	90	"	"
44	30 "	......	"	"
45	30 "	90	"	"
46	30 "	90	"	"
47	40 "	90	Campbell's Quarry, Newry.	"
48	40 "	......	Railway cutting.	"
49	45 "	......	"	"
50	45 "	80 E.	"	"
51	45 "	......	"	"
52	45 "	90	Summit of Eagle Mountain †.	"
53	45 "	90	" "	"
54	45 "	90	" "	"
55	45 "	80 W.	Campbell's Quarry, Newry.	"

* These joints are beautifully curved.

† These joints form the faces of the splendid granite columns, 150 feet in height, for which this mountain is celebrated.

TABLE VI.—East and West Joints of the Mourne and Newry Mountains.

No.	Bearing.	Dip.	Locality.	Rock.
1	40° N. of E.	90°	Columns at summit of Eagle Mountain.	Granite.
2	40 „	90	„ „ „	„
3	35 „	......	Railway cutting.	„
4	30 „	90	Summit of Eagle Mountain.	„
5	30 „	65 N.	Rostrevor.	Slate.
6	30 „	90	Railway cutting.	Granite.
7	20 „	65 N.	„	„
8	20 „	45 S.	Rostrevor.	Slate.
9	15 „	......	Railway cutting.	Granite.
10	15 „	......	„	„
11	15 „	.....	„	„
12	15 „	90	Campbell's Quarry.	„
13	10 „	......	„	„
14	10 „	90	Rostrevor.	Slate.
15	10 „	60 N.	„	„
16	10 „	90	Eagle Mountain.	Granite.
17	5 „	25 N.	Rostrevor.	Slate.
18	0 E.W.	80 S.	Railway cutting.	Granite.
19	0 „	......	„	„
20	0 „	90	„	„
21	0 „	60 N.	Eagle Mountain.	„
22	0 „	90	„	„
23	5 S. of E.	90	„	„
24	5 „	80 N.	Campbell's Quarry.	„
25	5 „	85 N.	Slieve Bingian.	„
26	20 „	......	„	„
27	22 „	......	„	„
28	25 „	......	„	„
29	25 „	70 N.	Campbell's Quarry.	„
30	25 „	......	Railway cutting.	„
31	30 „	90	Eagle Mountain.	„
32	30 „	70 N.	Railway cutting.	„
33	40 „	90	„	„
34	40 „	70 N.	Campbell's Quarry.	„

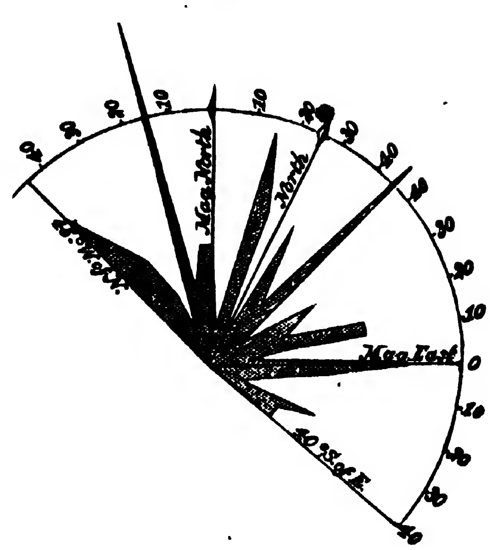
Collecting together into one Table the 89 observations, and arranging them by azimuths and the number of observations belonging to each, we find the following:—

TABLE VII.—Joints of the Mourne and Newry Mountains, arranged according to azimuths and number of observations.

Azimuth.	Number of observations.
45° W. of N.	5
30 ,,	3
25 ,,	2
20 ,,	1
15 ,,	9
10 ,,	2
5 ,,	3
0 N.S.	3
5 E. of N.	1
10 ,,	1
15 ,,	6
20 ,,	4
22 ,,	1
25 ,,	1
30 ,,	4
40 ,,	2
45 ,,	7
40 N. of E.	2
35 ,,	1
30 ,,	3
20 ,,	2
15 ,,	4
10 ,,	4
5 ,,	1
0 E.W.	5
5 S. of E.	3
20 ,,	1
22 ,,	1
25 ,,	3
30 ,,	2
40 ,,	2
Total	89

The accompanying diagram (fig. 4) represents graphically the preceding Table.

Fig. 4.



From an examination of Table VII., it appears that 71 per cent. of the total observations may be included in six systems of joints, each contained within limits of 10°.

1. Between 5° and 15° W. of N. . . . 14 observations.
2. ,, 15 and 25 E. of N. . . . 12 ,,
3. ,, 40 and 50 E. of N. . . . 7 ,,
4. ,, 10 and 20 N. of E. . . . 10 ,,
5. ,, 5 N. of E. and 5° S. of E. 9 ,,
6. ,, 45 and 20 W. of N. . . . 11 ,,

63 or 71 per cent. of 89 observations.

The precise bearings of these systems are—

1.	12° 8′ W. of N.	(C).	
2.	18 5 E. of N.		(C″).
3.	45 0 E. of N.	(A′).	
4.	14 0 N. of E.	(A).	
5.	1 6 S. of E.		(A″).
6.	46 26 W. of N.	(C′).	

Of these systems of joints, 1 and 4, 3 and 6 are evidently conjugates, and neither system closely corresponds with those of Waterford and Donegal, although that named (A, C) comes within a few degrees of (A, C) of those localities.

The Conjugate Joint-Systems of the Mourne and Newry Mountains are therefore as follows:—

Table VIII.—Conjugate Joints of the Mourne and Newry Mountains.

Designation.	Magnetic bearing.	True bearing*.	Angle from East to North.
A	14° 0′ N. of E.	39° 40′ N. of E.	88° 8′
C	12 8 W. of N.	37 48 W. of N.	
A′	45 0 E. of N.	19 20 E. of N.	88 34
C′	46 26 W. of N.	17 54 S. of E.	

In addition to the foregoing systems, which are evidently conjugate, we have the other systems, 2 and 5.

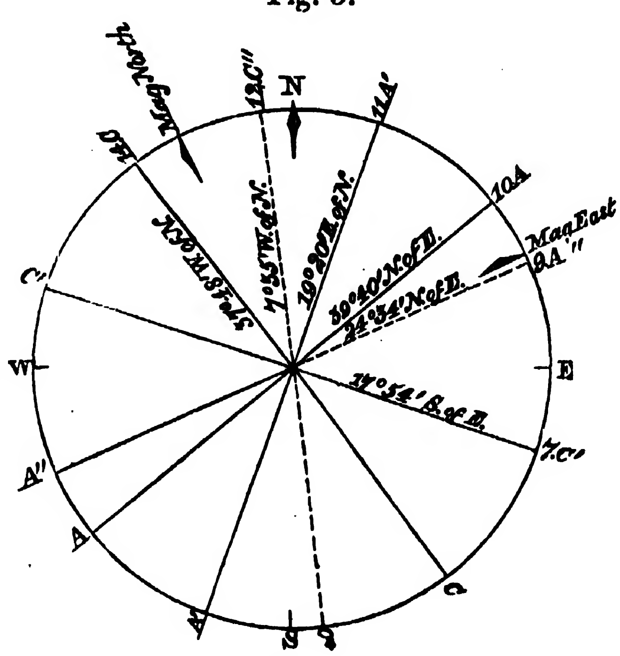
	Magnetic bearing.	True bearing.
System No. 2 (C″).	18° 5′ E. of N.	7° 35′ W. of N.
System No. 5 (A″).	1 6 S. of E.	24 34 N. of E.

The accompanying diagram (fig. 5) exhibits the six systems of joints just described, with the number of observations on which each is founded.

A very remarkable system of trap dykes penetrating the granite is exhibited in the railway cuttings from the Goragh Wood station to Slieve Gullion, in the Newry Mountains.

They are twenty-five in number, and are given in the following Table, numbered in order from Goragh Wood to Slieve Gullion. It appears evident from the Table that they are all reducible to four systems, two of which have directions closely corresponding with the joint-systems (C) and (C′) already found.

Fig. 5.



* The magnetic variation of the Mourne district was 25° 40′ W.

TABLE IX.—Trap Dykes in Granite between Goragh Wood and Slieve Gullion.

System (A″)?

No.	Bearing.	Dip.	
15	10° S. of E.	60° S.	Dolerite.
17	20 "	50 S.	Dolerite.

System C.

No.	Bearing.	Dip.	
1	10° W. of N.	°	
5	15 "	— E.	
7	5 "	90	
9	15 "	90	
13	15 "	70 E.	
16	15 "	70 E.	Basalt.
18	15 "	45 E.	
19	20 "	70 E.	
20	15 "	90	
21	15 "	70 E.	
22	15 "	65 E.	
23	15 "	70 E.	
24	10 "	90	
25	10 "	70 E.	
Mean	13° 34′ W. of N.		

System A′.

No.	Bearing.	Dip.
10	30° E. of N.	65° W.
12	55 "	90

System C′.

No.	Bearing.	Dip.
2	45° W. of N.	°
3	45 "	70 N.
4	35 "	70 N.
6	45 "	90
8	50 "	65 S.
11	50 "	75 N.
14	35 "	45 S.
Mean	43° 34′ W. of N.	

The trap dykes, with which other and distant parts of the Mourne district are intersected, correspond in general direction with the joint-systems already found, as is shown by the following, among other examples:—

Trap Dykes of Systems A and A″.

Bearing.	Dip.	Rock.	Locality.
5° N. of E.	50° S.	Felstone dyke in slate.	Rostrevor.
E.W.	90	Greenstone dyke (19 feet wide) in slate.	Rostrevor.
5 N. of E.	90	Greenstone dyke (12 feet wide) in slate.	Killowen.
10 N. of E.	......	Greenstone in slate.	Clogh More.
15 N. of E.	......	Dolerite in slate.	Slieve Bawn.
5 S. of E.	85 N.	Felstone porphyry in granite.	Slieve Bingian (little).
10 S. of E.	90	Felstone in slate.	Armer's Hole.
20 S. of E.	90	Elvan in granite.	Slieve Bingian.

Trap Dykes of System C.

Bearing.	Dip.	Rock.	Locality.
° N.S.	90°	Basalt dyke in granite.	Ballymacilreiny.
10 W. of N.	80 W.	Greenstone dyke in slate.	Carlingford Mountain.
5 W. of N.	30 W.	Syenite in slate.	Carlingford Pier.

We have therefore evidence of the existence of the following systems of Joints and Dykes in the Mourne and Newry Mountains:—

I. Primary System:—

(A). Joints . . . 14° 0′ N. of E. (Mag.)=39° 40′ N. of E.

II. Conjugate of Primary:—

(C). Joints . . . 12° 8′ W. of N. (Mag.)=37° 48′ W. of N.
Dykes . . . 13° 34′ ,, ,, =39° 14′ W. of N.

III. First Secondary System:—

(A′). Joints . . . 45° 0′ E. of N. (Mag.)=19° 20′ E. of N.

IV. Conjugate of First Secondary:—

(C′). Joints . . . 46° 26′ W. of N. (Mag.)=17° 54′ S. of E.
Dykes . . . 43° 34′ ,, ,, =20° 46′ S. of E.

V. Second Secondary System:—

(C″). Joints . . 18° 5′ E. of N. (Mag.)=7° 35′ W. of N.

The angles between the Primary and Secondary Systems are
Between Primary and First Secondary Systems—

$$\left.\begin{array}{lr} \text{A—A}' \ldots\ldots & 31^\circ\ 0' \\ \text{C—C}' \text{ (Joints)} \ldots & 34^\circ\ 18' \\ \text{C—C}' \text{ (Dykes)} \ldots & 30^\circ\ 0' \end{array}\right\} = 31^\circ\ 46'.$$

Between Conjugate to Primary and Second Secondary—

$$\left.\begin{array}{lr} \text{C—C}'' \text{ (Joints)} \ldots & 30^\circ\ 13' \\ \text{C—C}'' \text{ (Dykes and Joints)} \ldots & 31^\circ\ 39' \end{array}\right\} = 30^\circ\ 56'.$$

Part III.—ON THE JOINT-SYSTEMS OF CORNWALL.

The following notice of the Joint-Systems is founded on my own observations, with the exception of the two sets of observations made by Sir H. De la Beche and Mr. N. Whitley, which I was permitted to copy by tracing from the diagram deposited in the Museum of the Geological Society of Cornwall, in Penzance.

Sir H. De la Beche and other observers have noticed the relation between the directions of the lodes and of the joints in Cornwall. This relation is even more intimate than they supposed, and extends to the Secondary Joints, which are well represented in the mines by the Caunter Lodes, while the Conjugate Joints are represented by the Cross Courses.

Joint-Planes observed in Cornwall.

System A.

No.	Bearing.	Dip.	Locality.	Rock.
1	10° N. of E.	90°	Land's End.	Granite.
2	E.W.	90	Carn Bosavern.	"
3	E.W.	90	Carn Marth.	"
From the MS. observations of Sir H. De la Beche and Mr. N. Wihtley, in the Geological Society of Penzance.				
4	24 N. of E.	......	Land's End.	Granite.
5	21 "	......	"	"
6	3 "	......	"	"
7	6 "	......	Gunnis Lake	"
8	8 S. of E.	......	"	"
9	8 N. of E.	......	"	"
10	14 "	......	"	"
11	6 "	......	"	"
12	9 "	......	"	"
13	10 S. of E.	......	Trincomlee Hill.	"
Mean	6° 23′ N. of E.	......		

System C.

No.	Bearing.	Dip.	Locality.	Rock.
1	10° W. of N.	90°	Land's End.	Granite.
2	N.S.	90	Carn Bosavern, St. Just.	"
3	N.S.	90	Carn Brea.	"
4	N.S.	90	Carn Marth.	"
5	N.S.	90	Portreath.	Slate.
	From the MS. observations of Sir H. De la Beche and Mr. N. Whitley, in the Geological Society of Penzance.			
6	10 W. of N.	....	Pedn maen du.	Granite.
7	10 "	....	Carn Clog.	"
8	15 "	....	Carn Creis.	"
9	21 "	....	Mill Bay.	"
10	17 "	....	Carn Barra.	"
11	7 "	....	Carn Mellyn.	"
12	17 "	....	Tol pedn penwith.	"
13	7 "	....	Porth Chaple.	"
14	13 "	....	Pedn maen an mear.	"
Mean	9° 4′ W. of N.			

Other systems of Joint-Planes occur in the granite of Cornwall. Mr. WHITLEY has observed at Gunnis Lake the following joint-planes:—

21° E. of N. (Mag.)
16° „ „ } 19° 40′ E. of N.
22° „ „

and at Trincomlee Hill, 13° E. of N.

And I observed at Carn Brea well-marked joints, which seem to be conjugate to the foregoing, viz. 20° S. of E.

These joint-planes may possibly indicate the System (A″—C″) observed in the co. Waterford.

At Carn Marth I found joints bearing 40° W. of N., as in the Mourne Mountains.

The relation between the directions of the lodes and joints in Cornwall has been long known. I was desirous of testing it in the Carn Brea district, and therefore asked permission of the officers of Carn Brea, Dolcoath, and North Roskear Mines to make use of their maps for the purpose; the permission was readily granted, and I have here tabulated the results of my investigation.

Direction of Main Lodes (Mag.), 1840.

System A.

No.	Bearing.	Name of lode.	Mine.
1	15° N. of E.	Highbarrow.	Carn Brea.
2	9 „ „	Druid's.	„
3	5 „ „	Teague's.	„
4	16 „ „	Western part.	Dolcoath.
5	5 „ „	Eastern part.	„
6	11 „ „	South lode.	„
7	16 „ „	Entral, western.	„
8	5 „ „	Entral, eastern.	„
9	10 „ „	Main lode.	North Roskear.
10	E.W.	„ „	„
11	5 N. of E.	„ „	„
Mean	8° 45′ N. of E.		

Direction of Cross Courses conjugate to Main Lodes.

System C.

No.	Bearing.	Name of lode.
1	° N.S.	Carn Brea.
2	14 W. of N.	„ „
3	6 „ „	Great Cross Course, dividing Dolcoath from Cooks Kitchen.
Mean	6° 40′ W. of N.	

Direction of Caunter Lodes (A″ ?).

No.	Bearing.	Lode.	Mine.
1	16° S. of E.	Barncoose.	Carn Brea.
2	22 „ „	Vigors' Caunter.	„ „
3	39 „ „	Caunter.	Dolcoath.
4	40 „ „	Caunter.	N. Roskear.

Cross Course, conjugate to the Caunters.

(C″) 20° E. of N. in Carn Brea.

The following Systems may be regarded as fully established in Cornwall:—

I. *Primary System.*

(A) Joints . .	6° 23′ N. of E. (Mag.)*	=30° 20′ N. of E.	
Lodes . .	8° 45′ „ „	=34° 45′ „	

II. *Conjugate of Primary.*

(C) Joints . .	9° 4′ W. of N. (Mag.)	=33° 4′ W. of N.
Cross Courses	6° 40′ „ „	=32° 40′ „

The following System exists also, but is not so prominent.

III. *Secondary System.*

(A″) Joints and Cross Courses 20° 0′ S. of E. (Mag.) =4° N. of E.

IV. *Conjugate of Secondary.*

(C″) Joints . . .	18° 0′ E. of N. (Mag.)	6° W. of N.
Caunter Lodes	19° 0′ „ „	7° „

The angles between the Primary and Secondary Systems are as follows:—

Between Primary and Conjugate of Secondary,

(A—A″) . .	Joints	26° 23′	Mean . . 27° 28′
	Lodes	30° 45′	
(C—C″) . .	Joints	27° 4′	
	Cross Courses . .	25° 40′	

* I have assumed the mean variation in Cornwall to be 24° W. in 1860, and to have been 26° W. in the year 1840.

Part IV.—On the Joint-Systems of the Co. Fermanagh.

I have made the following observations on the Joints of the Carboniferous limestone of the co. Fermanagh.

System A.

No.	Bearing.	Dip.	Locality.
1	5° N. of E.	90°	Belmore Mountain.
2	E.W.	90	" "
3	15 S. of E.	90	Dunbar Quarry.
4	8 S. of E.	90	" "
Mean	4° $30'$ S. of E.		

System C.

No.	Bearing.	Dip.	Locality.
1	5° E. of N.	90°	Dunbar Quarry.
2	N.S.	86 E.	" "
3	5 W. of N.	80 W.	" "
4	7 E. of N.	90	" "
5	15 E. of N.	90	" "
6	10 W. of N.	90	Belmore Mountain.
7	N.S.	90	" "
8	10 W. of N.	90	" "
9	N.S.	90	" "
10	N.S.	80 E.	" "
Mean	0° $12'$ E. of N.		

System A′.

No.	Bearing.	Dip.	Locality.
1	20° N. of E.	90°	Carrickreagh Quarry.
2	35 " "	90	" "
3	27 " "	90	Belmore Mountain.
4	30 " "	90	" "
Mean	28° $0'$ N. of E.		

System C′.

No.	Bearing.	Dip.	Locality.
1	26° W. of N.	84° E.	Carrickreagh.
2	30 " "	78 W.	"
3	32 " "	80 W.	"
Mean	29° $20'$ W. of N.		

System (X).

No.	Bearing.	Dip.	Locality.
1	45° N. of E.	90	Carrickreagh.
2	45 „	90	„
3	45 „	90	„
4	45 „	76 S.E.	Dunbar.
5	45 „	90	Belmore.
Mean	45° N. of E.		

In these observations there is distinct evidence of a Primary and Secondary Conjugate System, and of a third series of planes (X), which is well represented in the Mourne Mountains.

I.	Primary System	4° 30′ S. of E.	(Mag.)	=21° 30′ N. of. E.
II.	Conjugate to Primary . .	0° 12′ E. of N.	„	=25° 48′ W. of N.
III.	Secondary System	28° 0′ N. of E.	„	=54° 0′ N. of E.
IV.	Conjugate to Secondary .	29° 20′ W. of N.	„	=55° 20′ W. of N.
V.	(X) . .	45° N. of E. .	„	=19° 0′ E. of N.

The angle between the Primary and Secondary Systems is thus found:

$$\left.\begin{array}{lll} A—A' & . . . & 32° \ 30' \\ C—C' & . . . & 29° \ 32' \end{array}\right\} = 31° \ 1'.$$

PART V.—GENERAL CONCLUSIONS FROM THE FOREGOING OBSERVATIONS.

Collecting together into one Table the results of the preceding observations, we find the following:—

TABLE X.—Primary and Secondary Joints (True Bearings).

	Waterford.	Donegal.	Mourne.	Cornwall.	Fermanagh.
Primary System (A)............	N. of E. 32° 26′	N. of E. 26° 16′	N. of E. 39° 40′	N. of E. 32° 34′	N. of E. 21° 30′
Conjugate to Primary (C) ...	W. of N. 31° 37′	W. of N. 29° 35′	W. of N. 38° 31′	W. of N. 32° 55′	W. of N. 25° 48′
First Secondary (A′)	N. of E. 58° 11′	N. of E. 58° 40′	N. of E. 70° 40′	....	N. of E. 54° 0′ [and 71° 0′]?
Conjugate to First Secondary (C′)	W. of N. 60° 3′	....	W. of N. 70° 40′	....	W. of N. 55° 20′
Second Secondary (A″)	S. of E. 5° 50′	....	....	N. of E. 4° 0′	....
Conjugate to Second Secondary (C″)	E. of N. 4° 30′	....	W. of N. 7° 35′	W. of N. 6° 30′	....

The only remarkable agreement as to direction of Joints disclosed by the preceding

Table, is that between Waterford and Cornwall. If we compare together the primary and secondary joints in each locality, we find the following Table:—

TABLE XI.—Angle between Primary and Secondary Joints.

	Waterford.	Donegal.	Mourne.	Cornwall.	Fermanagh.
Primary (A, C) and First Secondary (A′, C′)	+ 27° 5′	+ 32° 24′	+ 31° 46′	+	+ 31° 1′
Primary (A, C) and Second Secondary (A″, C″)	− 37° 11′	−	− 30° 56′	− 27° 28′	− ......

This Table discloses a very interesting and unexpected result, viz. that, in Waterford, Donegal, Mourne, and Fermanagh, the angle between the Primary and First Secondary Joint-Systems ranges between the narrow limits of 27° 5′ and 32° 24′, and that in Waterford, Mourne, and Cornwall the angle between the Primary and Second Secondary Joint-Systems ranges from 27° 28′ to 37° 11′.

I hope to be able to show that this important result of observation is an easy consequence of the mechanical theory of Joints; but before doing so, I shall prove by the following Table, that the theory of Conjugate Joints used in my paper on the co. Waterford applies equally well to the other districts examined by me.

TABLE XII.—Angles between Conjugate Joints, measured from East to North.

Systems.	Waterford.	Donegal.	Mourne.	Cornwall.	Fermanagh.
Primary (A, C)	89° 11′	93° 19′	88° 51′	90° 21′	94° 18′
Secondary (A′, C′)..............................	91° 52′	......	90° 0′	......	91° 20′
Secondary (A″, C″)	91° 20′	......	......	92° 30′	......

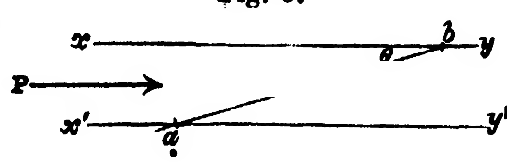
The tendency of Conjugate Joints to place themselves at right angles is plainly shown by the foregoing Table.

PART VI.—MECHANICAL THEORY OF ROCK JOINTS.

Rock masses are always arranged in sheets, whose dimensions in two rectangular directions are much greater than in the third direction at right angles to the first two. Hence, when a system of forces acts upon such a mass, its first effect will be to produce a system of fissures at right angles to the plane of the resultant force. When other forces are subsequently applied to the rock mass, already divided into bands by the parallel fissures, their effect upon these rock bands will be different according as they have a large vertical or large horizontal component. In the first case the bands of rock will simply bend and break across along new fissures at right angles to the original fissures. This cause produces the phenomena of conjugate joints, which have been described and accounted for by several writers, and have been fully discussed by myself in the paper, "On the Physical Structure of the Old Red Sandstone of Waterford," so often referred to by me.

I shall therefore direct my attention at present to the phenomena of Secondary Joints, which have been shown to take place along directions inclined at about 30°, on each side of the primary direction of the original joints. Let xy, $x'y'$ be the directions of two primary parallel joints, and let a force P, acting horizontally, tend to produce a fracture in some unknown direction, ab, making an angle θ with the directions xy, $x'y'$; if the band of rock were capable of bending, this direction would be at right angles to the direction xy; but if, from the pressure of superincumbent rock masses, or other cause, it be prevented from bending, it will fracture along the line ab, which is the plane of easiest fracture, and whose direction θ is thus found.

Fig. 6.



Let K denote the coefficient of cohesion of the rock, l the length of ab, and h the perpendicular distance between xy and $x'y'$, and let $\mu=\tan\lambda$ denote the coefficient of friction of the rock.

The mechanical condition of equilibrium is evidently as follows:—

$$\mathrm{P}\cos\theta=\mathrm{K}l+\mu\mathrm{P}\sin\theta,$$

or

$$\mathrm{P}\cos\theta=\frac{\mathrm{K}h}{\sin\theta}+\mu\mathrm{P}\sin\theta. \qquad (1)$$

Differentiating this equation, regarding P as a minimum and therefore $d\mathrm{P}=0$, we find

$$-\mathrm{P}\sin\theta=-\frac{\mathrm{K}h\cos\theta}{\sin^2\theta}+\mu\mathrm{P}\cos\theta. \qquad (2)$$

Equating the values of P deduced from (1) and (2), we find

$$\sin\theta(\sin\theta+\mu\cos\theta)=\cos\theta(\cos\theta-\mu\sin\theta);$$

from which follows

$$\cos 2\theta=\mu\sin 2\theta \qquad (3)$$

or

$$\cot 2\theta=\tan\lambda;$$

and, finally,

$$2\theta+\lambda=90°. \qquad (4)$$

The values of λ, determined for various kinds of rock, limestone, sandstone, and slate, vary from 30° to 38°, thus giving for θ corresponding values from 30° to 26°. These values agree very well with the angles between the systems of Primary and Secondary joints determined by the preceding observations in different localities; and I believe that the mechanical cause I have assigned is the true cause of these joints.

From this it follows that a single hypothesis as to the direction of a system of forces is sufficient to account for the existence of three conjugate systems of joints, involving six directions; and it also explains the rudely hexagonal jointing of many rocks, if the coefficient of friction be such as to render the angle θ nearly 30°.

Many other consequences flow from the preceding investigations, into which I have not time to enter, but which may be readily found and turned to practical use by the field geologist who prefers one hypothesis to many.

XII. *On the Spectra of some of the Fixed Stars.* *By* WILLIAM HUGGINS, *F.R.A.S.*, *and* W. A. MILLER, *M.D.*, *LL.D.*, *Treas. & V.P.R.S.*, *Professor of Chemistry, King's College, London.*

Received April 28,—Read May 26, 1864.

§ I. *Introduction.*

1. THE recent discovery by KIRCHHOFF of the connexion between the dark lines of the solar spectrum and the bright lines of terrestrial flames, so remarkable for the wide range of its application, has placed in the hands of the experimentalist a method of analysis which is not rendered less certain by the distance of the objects the light of which is to be subjected to examination. The great success of this method of analysis as applied by KIRCHHOFF to the determination of the nature of some of the constituents of the sun, rendered it obvious that it would be an investigation of the highest interest, in its relations to our knowledge of the general plan and structure of the visible universe, to endeavour to apply this new method of analysis to the light which reaches the earth from the fixed stars. Hitherto the knowledge possessed by man of these immensely distant bodies has been almost confined to the fact that some of them, which observation shows to be united in systems, are composed of matter subjected to the same laws of gravitation as those which rule the members of the solar system. To this may be added the high probability that they must be self-luminous bodies analogous to our sun, and probably in some cases even transcending it in brilliancy. Were they not self-luminous, it would be impossible for their light to reach us from the enormous distances at which, the absence of sensible parallax in the case of most of them shows, they must be placed from our system.

The investigation of the nature of the fixed stars by a prismatic analysis of the light which comes to us from them, however, is surrounded with no ordinary difficulties. The light of the bright stars, even when concentrated by an object-glass or speculum, is found to become feeble when subjected to the large amount of dispersion which is necessary to give certainty and value to the comparison of the dark lines of the stellar spectra with the bright lines of terrestrial matter. Another difficulty, greater because it is in its effect upon observation more injurious, and is altogether beyond the control of the experimentalist, presents itself in the ever-changing want of homogeneity of the earth's atmosphere, through which the stellar light has to pass. This source of difficulty presses very heavily upon observers who have to work in a climate so unfavourable in this respect as our own. On any but the finest nights the numerous and closely approximated fine lines of the stellar spectra are seen so fitfully that no observations of value

can be made. It is from this cause especially that we have found the inquiry, in which for more than two years and a quarter we have been engaged, more than usually toilsome; and indeed it has demanded a sacrifice of time very great when compared with the amount of information which we have been enabled to obtain.

2. Previously to January 1862, in which month we commenced these experiments, no results of any investigation undertaken with a similar purpose had been published. With other objects in view, two observers had described the spectra of a few of the brighter stars, viz. FRAUNHOFER in 1823*, and DONATI, whose memoir, "Intorno alle strie degli spettri stellari," was published in the Annali del Museo Fiorentino for 1862.

FRAUNHOFER recognized the solar lines D, E, *b*, and F in the spectra of the Moon, Venus, and Mars; he also found the line D in Capella, Betelgeux, Procyon, and Pollux; in the two former he also mentions the presence of *b*. Castor and Sirius exhibited other lines. DONATI'S elaborate paper contains observations upon fifteen stars; but in no case has he given the positions of more than three or four bars, and the positions which he ascribes to the lines of the different spectra relatively to the solar spectrum do not accord with the results obtained either by FRAUNHOFER or by ourselves. As might have been anticipated from his well-known accuracy, we have not found any error in the positions of the lines indicated by FRAUNHOFER.

3. Early in 1862 we had succeeded in arranging a form of apparatus in which a few of the stronger lines in some of the brighter stars could be seen. The remeasuring of those already described by FRAUNHOFER and DONATI, and even the determining the positions of a few similar lines in other stars, however, would have been of little value for our special object, which was to ascertain, if possible, the constituent elements of the different stars. We therefore devoted considerable time and attention to the perfecting of an apparatus which should possess sufficient dispersive and defining power to resolve such lines as D and *b* of the solar spectrum. Such an instrument would bring out the finer lines of the spectra of the stars, if in this respect they resembled the sun. It was necessary for our purpose that the apparatus should further be adapted to give accurate measures of the lines which should be observed, and that it should also be so constructed as to permit the spectra of the chemical elements to be observed in the instrument simultaneously with the spectra of the stars. In addition to this, it was needful that these two spectra should occupy such a position relatively to each other, as to enable the observer to determine with certainty the coincidence or non-coincidence of the bright lines of the elements with the dark lines in the light from the star.

Before the end of the year 1862 we had succeeded in constructing an apparatus which fulfilled part of these conditions. With this some of the lines of the spectra of Aldebaran, α Orionis, and Sirius were measured; and from these measures diagrams of these stars, in greater detail than had then been published, were laid before the Royal Society in February 1863. After the note was sent to the Society, we became acquainted with some similar observations on several other stars by RUTHERFURD, in SILLIMAN'S Journal

* GILBERT'S 'Annalen,' vol. lxxiv. p. 374.

for 1863*. About the same time figures of a few stellar spectra were also published by SECCHI†. In March 1863, the Astronomer Royal presented a diagram to the Royal Astronomical Society, in which are shown the positions of a few lines in sixteen stars‡.

Since the date at which our note was sent to the Royal Society our apparatus has been much improved, and in its present form of construction it fulfils satisfactorily several of the conditions required.

§ II. *Description of the Apparatus and Methods of Observation employed.*

4. This specially constructed spectrum apparatus is attached to the eye end of a refracting telescope of 8 inches aperture and 10 feet focal length, which is mounted equatorially in the observatory of Mr. HUGGINS at Upper Tulse Hill. The object-glass is a very fine one, by ALVAN CLARK of Cambridge, Massachusetts; the equatorial mounting is by COOKE of York; and the telescope is carried very smoothly by a clock motion.

As the linear spectrum of the point of light which a star forms at the focus of the object-glass is too narrow for the observation of the dark lines, it becomes necessary to spread out the image of the star; and to prevent loss of light, it is of importance that this enlargement should be in one direction only; so that the whole of the light received by the object-glass should be concentrated into a fine line of light as narrow as possible, and having a length not greater than will correspond to the breadth of the spectrum (when viewed in the apparatus) just sufficient to enable the eye to distinguish with ease the dark lines by which it may be crossed. No arrangement tried by us has been found more suitable to effect this enlargement in one direction than a cylindrical lens, which was first employed for this purpose by FRAUNHOFER. In the apparatus by which the spectra described in our "Note" of February 1863 were observed, the cylindrical lens employed was plano-convex, of 0·5 inch focal length. This was placed within the focus of the object-glass and immediately in front of the slit of the collimator.

The present form of the apparatus is represented in Plate X. figs. 1 & 2, where the cylindrical lens is marked *a*. This is plano-convex, an inch square, and of about 14 inches focal length. The lens is mounted in an inner tube, *b*, sliding within the tube *c*, by which the apparatus is adapted to the eye end of the telescope. The axial direction of the cylindrical surface is placed at *right angles* to the slit *d*, and the distance of the lens from the slit within the converging pencils from the object-glass is such as to give exactly the necessary breadth to the spectrum.

In consequence of the object-glass being over-corrected, the red and, especially, the violet pencils are less spread out than the pencils of intermediate refrangibility; so that the spectrum, instead of having a uniform breadth, becomes slightly narrower at the red end, and tapers off in a greater degree towards the more refrangible extremity§.

* Vol. xxxv. p. 71.

† Astronomische Nachrichten, No. 1405, March 3, 1863.

‡ Monthly Notices, Roy. Astron. Soc. vol. xxiii. p. 190.

§ The experiment was made of so placing the cylindrical lens that the axial direction of its convex cylin-

In front of the slit *d*, and over one half of it, is placed a right-angled prism *e*, for the purpose of reflecting the light which it receives from the mirror *f* through the slit. In the brass tube *c* are two holes: by one of these the light is allowed to pass from the mirror to the reflecting-prism *e*; and by means of the other, access to the milled head for regulating the width of the slit is permitted. Behind the slit, and at a distance equal to its focal length, is placed an achromatic collimating lens *g*, made by T. Ross; this has a diameter of 0·6 inch and a focal length of 4½ inches. These proportions are such that the lens receives the whole of the light which diverges from the linear image of the star when this is brought exactly within the jaws of the slit.

The dispersing portion of the apparatus consists of two prisms, *h*, each having a refracting angle of about 60°; they were made by T. Ross, and are of very dense and homogeneous flint glass. The prisms are supported upon a suitable mounting, which permits them to be duly levelled and adjusted. Since the feebleness of the light from the stars limits the observations for the most part to the central and more luminous portions of the spectrum, the prisms have been adjusted to the angle of minimum deviation for the ray D. A cover of brass, *k*, encloses this part of the apparatus; and by this means the prisms are protected from accidental displacement, and from dust.

The spectrum is viewed through a small achromatic telescope *l*, furnished with an object-glass of 0·8 inch diameter and 6·75 inches focal length. This telescope has an adjustment for level at *m*. The axis of the telescope can be lowered and raised, and the tube can be also rotated around the vertical axis of support at *n*. At the focus of the object-glass are fixed two wires, crossing at an angle of 90°. These are viewed, together with the spectrum, by a positive eyepiece *p*, giving a magnifying power of 5·7 diameters. As the eyes of the two observers do not possess the same focal distance, a spectacle-lens, corresponding to the focal difference between the two, was fitted into a brass tube, which slipped easily over the eyepiece of the telescope, and was used or withdrawn as was necessary.

This telescope, when properly adjusted and clamped, is carried by a micrometer-screw *q*, which was constructed and fitted to the instrument by Cooke and Sons. The centre of motion about which it is carried is placed approximatively at the point of intersection of the red and the violet pencils from the last prism; consequently it falls within

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drical surface should be *parallel* with the direction of the slit. The line of light is in this case formed by the lens; and the length of this line, corresponding to the visible breadth of the spectrum, is equal to the diameter of the cone of rays from the object-glass where they fall upon the slit. With this arrangement, the spectrum appears to be spread out, in place of being contracted at the two extremities. Owing to the large amount of dispersion to which the light is subjected, it was judged unadvisable to weaken still further the already feeble illumination of the extremities of the spectrum; and in the examination of the stellar spectra the position of the cylindrical lens with its axis at right angles to the slit, as mentioned in the text, was therefore adopted.

A *plano-concave* cylindrical lens of about 14 inches negative focal length was also tried. The slight advantage which this possesses over the convex form is more than balanced by the inconvenience of the increased length given to the whole apparatus.

the last face of the prism nearest the small telescope. All the pencils therefore which emerge from the prism are, by the motion of the telescope, caused to fall nearly centrically upon its object-glass. The micrometer screw has 50 threads to an inch; and each revolution is read to the hundredth part, by the divisions engraved upon the head. This gives a scale of about 1800 parts to the interval between the lines A and H of the solar spectrum. During the whole of the observations the same part of the screw has been used; and the measures being relative, the inequalities, if any, in the thread of this part of the screw do not affect the accuracy of the results. The eye lens for reading the divisions of the micrometer-screw is shown at *s*.

The mirror *f* receives the light to be compared with that of the star-spectrum, and reflects it upon the prism *e*, in front of the slit *d*. This light was usually obtained from the induction spark taken between electrodes of different metals, fragments or wires of which were held by a pair of small forceps attached to the insulating ebonite clamp *r*. Upon a moveable stand in the observatory was placed the induction coil, already described by one of us*, in the secondary circuit of which was inserted a Leyden jar, having 140 square inches of tinfoil upon each of its surfaces. The exciting battery, which, for the convenience of being always available, consisted of four cells of SMEE's construction, with plates 6 inches by 3, was placed without the observatory. Wires, in connexion with this and the coil, were so arranged that the observer could make and break contact at pleasure without removing his eye from the small telescope. This was the more important since, by tilting the mirror *f*, it is possible, within narrow limits, to alter the position of the spectrum of the metal relatively to that of the star. An arrangement is thus obtained which enables the observer to be assured of the perfect correspondence in relative position in the instrument of the stellar spectrum and the spectrum to be compared with it.

5. The satisfactory performance of this apparatus is proved by the very considerable dispersion and admirably sharp definition of the known lines in the spectra of the sun and metallic vapours. When it is directed to the sun, the line D is sufficiently divided to permit the line within it, marked in KIRCHHOFF's map as coincident with nickel, to be seen. The close groups of the metallic spectra are also well resolved.

When this improved apparatus was directed to the stars, a large number of fine lines was observed, in addition to those that had been previously seen. In the spectra of all the brighter stars which we have examined, the dark lines appear to be as fine and as numerous as they are in the solar spectrum. The great breadth of the lines in the green and more refrangible parts of Sirius and some other stars, as seen in the less perfect form of apparatus which was first employed, and which band-like appearance was so marked as specially to distinguish them, has, to a very great extent, disappeared; and though these lines are still strong, they now appear, as compared with the strongest of the solar lines, by no means so abnormally broad as to require these stars to be placed in a class apart. No stars sufficiently bright to give a spectrum have been observed to

* Philosophical Transactions, 1864, p. 141.

be without lines. The stars admit of no such broad distinctions of classification. Star differs from star alone in the grouping and arrangement of the numerous fine lines by which their spectra are crossed.

6. For the convenience of reference and comparison, a few of the more characteristic lines of twenty-nine of the elements were measured with the instrument. These were laid down to scale, in order to serve as a chart, for the purpose of suggesting, by a comparison with the lines measured in the star, those elements the coincidence of the lines of which with stellar lines was probable.

For the purpose of ensuring perfect accuracy in relative position in the instrument between the star-spectrum and the spectrum to be observed simultaneously with it, the following general method of observing was adopted:—The flame of a small lamp of alcohol, saturated with chloride of sodium, was placed centrally before the object-glass of the telescope, so as to furnish a sodium-spectrum. The sodium-spectrum was then obtained by the induction spark, and the mirror *f* was so adjusted that the components of the double line D, which is well divided in the instrument, should be severally coincident in the two spectra. The lamp was then removed, and the telescope directed to the sun, when FRAUNHOFER's line D was satisfactorily observed to coincide perfectly with that of sodium in the induction-spark. Having thus ascertained that the sodium lines coincided in the instrument with the solar lines D, it was of importance to have assurance from experiment that the other parts of the solar spectrum would also accurately agree in position with those corresponding to them in the spectrum of comparison. When electrodes of magnesium were employed, the components of the triple group characteristic of this metal severally coincided with the corresponding lines of the group *b*. C and F also agreed exactly in position with the lines of hydrogen; the coincidence of several of the principal lines of iron was also observed. The stronger of the FRAUNHOFER lines were measured in the spectra of the moon and of Venus, and these measures were found to be accordant with those of the same lines taken in the solar spectrum.

Before commencing the examination of the spectrum of a star, the alcohol-lamp was again placed before the object-glass of the telescope, and the correct adjustment of the apparatus obtained with certainty. The first observation was whether the star contained a double line coincident with the sodium line D. When the presence of such a line had been satisfactorily determined, we considered it sufficient in subsequent observations of the same star to commence by ascertaining the exact agreement in position of this known stellar line with the sodium line D.

Since from flexure of the parts of the spectrum apparatus the absolute reading of the micrometer might vary when the telescope was directed to stars differing greatly in altitude, the measure of the line in the star which was known to be coincident with that of sodium was always taken at the commencement and at the end of each set of measures. The distances of the other lines from this line, and not the readings of the micrometer, were then finally registered as the measures of their position; and these form the numbers given in the Tables, from which the diagrams of the star-spectra have been laid down.

The very close approximation*, not unfrequently the identity, of the measures obtained for the same line on different occasions, as well as the very exact agreement of the lines laid down from these measures with the stellar lines subsequently determined by a direct comparison with metallic lines the positions of which were known, have given the authors great confidence in the minute accuracy of the numbers and drawings which they have now the honour of laying before the Society.

§ III. *Observations on the Moon and Planets.*

7. It is well known that in the solar spectrum many additional remarkable lines make their appearance when light from the sun seen near the horizon reaches the observer, after having traversed a much greater length of our atmosphere than when the sun is viewed at greater altitudes. This circumstance suggested to us the importance of a careful examination of the solar light after reflexion from the moon and planets, in reference to the extent and analogous constitution of atmospheres possibly surrounding these bodies. As far as practicable, the spectra of the moon, Venus, Mars, Jupiter, and Saturn have been observed on several occasions with this special object in view.

8. *The Moon.*—All the astronomical phenomena in which we should expect to discover indications of an atmosphere about the moon, if such exist, agree in proving the non-existence of a lunar atmosphere of sensible amount. From the absence of appreciable refraction at the moon's limb, and from the *sudden* extinction during a total lunar eclipse of stars of even the tenth and eleventh magnitude at the limb of the moon, "we are," writes Sir JOHN HERSCHEL, "entitled to conclude that no amount of appreciable vapour is suspended near the surface of the moon, and .... the non-existence of an atmosphere at the moon's edge having the 1980th part of the density of the earth's atmosphere"†.

As by direct observation we know that the solar light is reflected from the *surface* of the moon, the light which reaches the earth after having undergone this reflexion must have passed through a length of lunar atmosphere, if such exist, at least equal to double the height of such atmosphere above that surface of the moon which is visible to us. From some parts of the moon, when the whole or a large part of its illuminated surface is turned towards the earth, the length of the column of lunar atmosphere which the solar light would have to traverse would be considerably greater.

The examination of lunar light by the spectroscope, and the comparison of the light reflected from different portions of the moon's illuminated surface with each other by this method, would take place under conditions favourable to the detection of an atmosphere of considerable extent, if such exist.

The moon was examined by us on April 12 and November 26, 1862, March 31 and

* These measures, on repeated observation, seldom varied more than a single division of the scale, or $\frac{1}{1800}$th of the distance between A and H.

† Outlines of Astronomy, 7th edition, par. 431, p. 284. See also a paper by Professor CHALLIS in the Monthly Notices of the Roy. Astron. Soc., vol. xxiii. p. 254, June 1863.

December 31, 1863, March 15 and 19, and April 12, 1864. The solar lines were perfectly well seen, appearing exceedingly sharp and fine. The line D was well divided, and its components were observed to coincide with those of sodium. Coincidence of the magnesium group with the three lines forming *b* was also observed. The lunar spectrum is indeed full of fine lines, and they were well seen from B to about halfway between G and H. On all these occasions no other strong lines were observed than those which are visible in the solar spectrum when the sun has a considerable altitude.

Previously to the observations of March 15 and 19 and April 12, 1864, the apparatus was directed to the sun when near the horizon, and the relative positions and characteristic appearances of the atmospheric lines in the orange and red were carefully observed. These portions of the spectrum were closely scrutinized when the moon's light was afterwards examined; but no indication of similar lines could be detected. On each of the three evenings just mentioned successive portions of the moon's illuminated surface from the centre to the circumference were brought before the slit of the spectrum apparatus. The quantity of light from different parts was observed to be very different, but not the smallest change in the lines of the spectrum could be perceived, either in respect of relative intensity or the addition or disappearance of any lines*.

The result of this spectrum analysis of the light reflected by the moon is wholly negative as to the existence of any considerable lunar atmosphere†.

9. *The Planets Venus, Mars, Jupiter, and Saturn.*—The very sensible and rapidly changing appearances of the disk of Jupiter, other than those due to the rotation of the planet, present very strong evidence of the existence of a very considerable atmosphere about Jupiter. The same, though in a much less marked degree, is probably true of Saturn and Mars. In addition, the diminished brightness of the disk of Jupiter near the periphery supports the inference that an atmosphere exists about that planet.

The planet Jupiter was observed on April 12, 1862, and April 14 and May 1, 1863.

* [With the spectrum apparatus described at page 421, the spectra of particular and very limited regions of the moon's surface can be examined. The opening of the slit of the apparatus corresponding to a spectrum that can be separately observed is about $\frac{1}{300}$ inch $\times$ $\frac{1}{100}$ inch. The image of the moon formed by the object-glass of the telescope has a diameter of 1·04 inch. Practically it is found that the light reflected from an area upon the surface of the moon of about one-third that of Tycho can be analyzed in the instrument.

The particular spot of the moon's surface under observation can be ascertained by means of the finder attached to the telescope. For this purpose, however, a special set of wires, accurately adjusted, and an eyepiece of considerable power are necessary. When the part of the moon's surface under observation presents marked inequalities of illumination, the spectra of these differently illuminated portions can be easily recognized by the differences in their comparative brightness. In these observations the cylindrical lens may be removed.—August 31, 1864.]

† [A remarkably favourable opportunity of observing the effect upon the solar spectrum of transmission through a very large extent of the earth's atmosphere presents itself on the occasion of an eclipse of the moon. We had made preparations to observe the copper-coloured light reflected from the moon during the eclipse of June 1, 1863. The small altitude of the moon on this occasion rendered the observation impossible, from the circumstance that the eye end of the telescope, increased in length by the spectrum apparatus, came too near the wall of the observatory.—August 31, 1864.]

The solar lines B, C, D, E, *b*, F, and G were seen, with numerous fine intermediate lines, and D, E, *b*, and F were measured; but no marked lines other than those usually present in the solar spectrum were detected.

[Since these observations were made, we have had a spectrum apparatus constructed by Mr. Browning, optician, of the Minories, which is similar in general arrangement to that already described, but possesses much less dispersive power. In this apparatus the cylindrical lens, the collimating lens, and the object-glass of the small telescope correspond exactly in diameter and in focal length with those of which a description has been given; but the eyepiece of the telescope is of less power, and has a magnifying power of about three diameters. A second eyepiece was occasionally used, magnifying nine diameters. Two prisms are employed; one has a refracting angle of 35°, the other a refracting angle of 45°.

With this apparatus, in the spectrum of Jupiter a strong line in the red is seen which is scarcely distinguishable with the more powerful instrument, and was from this cause overlooked in our earlier observations. The remarkable increase of visibility of this line is due to the much greater brilliancy of the spectrum in this apparatus; and this is much more than inversely proportional to the diminution of the dispersion, since, on account of the greatly reduced obliquity of incidence, the loss of light at the surfaces of the prisms by reflexion is much less. This saving of light in the spectrum apparatus is of very great importance in observations of the planetary spectra. The image of a planet in the telescope is not a point, but forms a disk of considerable magnitude relatively to the image of a star. Of this image, enlarged in one direction by the cylindrical lens, a very narrow section only, corresponding to the breadth of the slit, passes on through the collimating lens to the prisms; and this portion only of the total light collected by the object-glass becomes available to form the spectrum. On this account we have found the observations of the planets much more difficult than would be observations of stars possessing an equal apparent brilliancy.

This band of which we are now speaking in the spectrum of Jupiter occurs in a rather obscure part of the spectrum; moreover, by the instrument of greater dispersive power, it appears to be resolved into two or more lines, which are severally very faint, and are less visible than a single stronger line. The altitude of Jupiter being small (about 22° above the horizon) at the time of observation, it was of great importance to have satisfactory evidence that this band was not due to absorption by our atmosphere.

On June 16, 1864, the moon and Jupiter being near each other in the sky, the opportunity was seized to compare directly the moon's light with that of Jupiter under precisely similar conditions of atmosphere. The observations of this evening were decisive in showing that this band in the spectrum of Jupiter was due to a modification suffered by the solar light before reaching our atmosphere, and therefore due probably to absorption by the atmosphere of Jupiter.

On June 20, and on July 12 and 14, an observation still more crucial was obtained. The length of the opening of the slit is much greater than the diameter of the tele-

scopic image of Jupiter, even after elongation by the cylindrical lens. If, therefore, at the time of observation the light from the sky is sufficiently intense to form a visible spectrum, the spectrum of the sky is seen in the instrument together with the spectrum of Jupiter, and much exceeding it in breadth. When the period is so chosen that the degree of illumination of the sky is suitable in proportion to the intensity of the light of Jupiter, the solar lines and those due to our atmosphere are well seen in close contiguity with the lines in the spectrum of Jupiter, and occupying exactly similar relative positions. The sky-spectrum is seen under precisely similar conditions of altitude and of state of atmosphere. To the light of Jupiter under these circumstances of observation is added the light reflected from the small area of sky immediately between the observer and the planet. This light is, however, too faint in proportion to that of Jupiter to become a source of error. In the diagram, fig. 3, Plate X., the position of this band is shown relatively to the spectrum of the sky. The band at 914 of the scale appears to be coincident with, but *much stronger* than, a faint band in the sky-spectrum. This increase in the strength of the band is probably due to an absorptive action exerted by the atmosphere of the planet.

The bands at 882 and 1033 of the scale are less intense in the spectrum of Jupiter than in the spectrum of the light of the sky. This variation of intensity is probably due to the circumstance that the light from the southern sky, before it is reflected to the observer, on account of the position of the sun, which is then near the horizon, has had to traverse a very much larger amount, and a more dense portion, of our atmosphere than that traversed by the light received from Jupiter. It is in accordance with this explanation that these bands are also less intense in the spectrum of the moon when similarly compared with those of the sky.

Other lines less refrangible were perceived in the spectrum of Jupiter, but were not sufficiently distinct to be measured. The bands in the orange and the red to which we have referred, when examined in the spectrum apparatus of greater dispersive power, and with a much stronger illumination by directing the apparatus to the sun when near the horizon, are resolved into groups of lines. The stronger of these lines are represented in the upper spectrum of the diagram. The relative position of the band in the red due to lines of oxygen and nitrogen when the induction spark is taken in air, is shown below the spectrum of Jupiter. This band is in a small degree more refrangible than the strong band due to Jupiter.

If this band, at 914 of the scale, in Jupiter's spectrum consists of lines severally coincident with the lines composing the faint atmospheric band with which it appears to correspond in position, it would seem entitled to be regarded as an evidence of the similarity of Jupiter's atmosphere with our own, with respect at least to one of its constituents, or to one of the vapours diffused through it. The smaller intensity of the bands 882 and 1033 would appear to oppose the supposition that Jupiter's atmosphere is identical with our own. This negative evidence, however, cannot be regarded as of much weight, since telescopic observations show that the light which we receive from

Jupiter is for the most part reflected from clouds floating in its atmosphere at an elevation above the planetary surface. The solar light, therefore, would not traverse the lower and denser portions of Jupiter's atmosphere, corresponding to those of our own atmosphere in which the vapours, which probably produce these lines, appear to be chiefly present. The band about C, and that a little more refrangible at 838 of the scale, appear quite as strong in Jupiter as in the light from the sky. It may therefore be supposed that these bands are in part due to absorption at Jupiter, since the light from Jupiter suffers less absorption from our atmosphere than does the solar light reflected from the sky under the circumstances in which the observations were made.

With the exception of these bands in the orange and the red, the spectrum of Jupiter appeared to correspond exactly with that of the sky.—August 31, 1864.]

Saturn was observed on April 12, 1862, April 14, 1863, and April 12, 1864. Several solar lines were seen, but the spectrum was too faint to permit of any satisfactory determination as to the presence or absence of atmospheric lines.

[The spectrum of Saturn was observed with the apparatus and in the manner described when speaking of Jupiter, on June 13, 16, and 20. The spectrum was more difficult of observation, on account of the feebler brilliancy of Saturn, and its less favourable position. Bands in the red and orange were seen similar to those in the spectrum of Jupiter, and by measurement these bands were found to occupy positions in the spectrum corresponding to those of the bands of Jupiter.—August 31, 1864.]

The spectrum of Mars was observed on November 6, 1862, and April 17, 1863. The principal solar lines were seen, and no other strong lines were noticed.

[On August 10 and 29, 1864, we re-examined Mars, using the new spectrum apparatus. No lines in the red, similar to those of Jupiter and Saturn, were observed; but in the extreme red, probably about B and *a*, two or three strong lines were seen. With the exception of these, no lines were detected in the red, orange, yellow, and green portions of the spectrum, other than those of the solar spectrum. At about F the brilliancy of the spectrum diminishes in a remarkable manner, in consequence of a series of strong and nearly equidistant bands, which commences at F and continues towards the more refrangible end as far as the spectrum can be traced. The absorption of these bands is evidently the cause of the predominance of the red rays in the light of this planet.

The spectrum apparatus of greater power resolves these bands in the blue into groups of lines.—August 31, 1864.]

The light of Venus gives a spectrum of great beauty. The observations were chiefly made on April 17, 22, and 26, 1863. The line D was seen double. B, C, and numerous solar lines to a little distance beyond G, were distinctly visible; and the principal of these were measured and found to agree with corresponding lines in the solar spectrum. Lines other than these, and in the position in which the stronger atmospheric lines present themselves, were carefully looked for, but no satisfactory evidence of any such lines has been obtained. Venus was observed as early in the evening as possible, and while a considerable amount of daylight still remained.

The imperfect evidence which analysis by the prism affords of the existence of atmospheres around these planets, notwithstanding the high probability, amounting almost to certainty in the case of Jupiter, that such atmospheres do exist, may receive an explanation in the supposition that the light is chiefly reflected, not from the planetary surfaces, but from masses of cloud in the upper strata of their atmospheres. In this case the length of atmosphere which the light would have to traverse would be considerably lessened. With perhaps the exception of Mars, telescopic observations are in favour of such a supposition.

§ IV. *Observations on the Fixed Stars.*

10. The number of fixed stars which we have, to a greater or less extent, examined amounts to nearly 50. We have, however, concentrated our efforts upon three or four of the brighter stars, and two only of these have been mapped with any degree of completeness. These spectra are, indeed, as rich in lines as that of the sun, and even with these it may be advantageous to compare the spectra of additional metals when the season is again favourable. The few really fine nights that are available whilst the star is well situated for such observations, in respect of altitude and the time of sun-setting, necessarily make the *complete* investigation even of a single star the work of some years.

11. ALDEBARAN (α Tauri) (Plate XI.).—The light of this star is of a pale red. When viewed in the spectroscope, numerous strong lines are at once evident, particularly in the orange, the green, and the blue portions. The positions of about seventy of these lines have been measured, and their places have been given in the Table. Besides these, numerous other strong lines are visible, particularly in the blue, but they have not been measured, owing to the feebleness of the light; we have therefore not inserted them in the Table or in the diagram. A similar remark is applicable also to the results of our examination of α Orionis and β Pegasi.

We have compared the spectra of sixteen of the terrestrial elements by simultaneous observation with the spectrum of Aldebaran, of course selecting those in which we had reason, from the observations, to believe coincidence was most likely to occur. Nine of these spectra exhibited lines coincident with certain lines in the spectrum of the star. They are as follows:—sodium, magnesium, hydrogen, calcium, iron, bismuth, tellurium, antimony, and mercury.

(1) *Sodium.*—The double line at D was coincident with a double line in the stellar spectrum.

(2) *Magnesium.*—The three components of the group at *b*, from electrodes of the metal, were coincident with three lines in the star-spectrum.

(3) *Hydrogen.*—The line in the red corresponding to C, and the line in the green corresponding to F in the solar spectrum, were coincident with strong lines in the spectrum of Aldebaran.

(4) *Calcium.*—Electrodes of the metal were used; four lines in its spectrum were observed to coincide with four of the stellar lines.

(5) *Iron.*—The lines in the spectrum of this metal are very numerous, but not remarkable for intensity. There was a double line corresponding to E in the solar spectrum, and three other more refrangible well-marked lines coincident with lines in the star.

(6) *Bismuth.*—Four strong lines in the spectrum of this metal coincided with four in the star-spectrum.

(7) *Tellurium.*—In the spectrum of this metal also four of the strongest lines coincided with four in the spectrum of the star.

(8) *Antimony.*—Three of the lines in the spectrum of antimony were observed to coincide with stellar lines.

(9) *Mercury.*—Four of the brightest lines in the mercury-spectrum correspond in position with four lines of the star.

It must not be supposed that other lines in all the spectra of the elements above enumerated do not possess corresponding lines in the star-spectrum. Comparisons of this kind are extremely fatiguing to the eye, and are necessarily limited to the stronger lines of each spectrum. In no case, in the instances above enumerated, did we find any strong line in the metallic spectrum wanting in the star-spectrum, in those parts where the comparison could be satisfactorily instituted.

Seven other elements were compared with this star, viz. nitrogen, cobalt, tin, lead, cadmium, lithium, and barium. No coincidence was observed. With *nitrogen* three strong double lines were compared, with *cobalt* one strong single line and a double line, with *tin* five lines, with *lead* two strong lines, with *cadmium* three lines, with *barium* two of the strongest in the green, and with *lithium* the line in the orange, but were found to be without any strong lines in the star-spectrum corresponding with them. The positions of these several lines relatively to the star-spectrum are given in the diagram.

12. *α* ORIONIS (Betelgeux) (Plate XI.).—The light of this star has a decided orange tinge. None of the stars which we have examined exhibits a more complex or remarkable spectrum than this. Strong groups of lines are visible, especially in the red, the green, and the blue portions. In the blue comparatively few of these lines have been measured with accuracy; we have therefore not inserted them in the Table or the diagram. We have measured the position of about eighty lines in the brighter portions of this spectrum.

In the interval between the divisions 890 and 920 of the scale adopted in the diagram, is a shading as of fine lines. A fainter shading of the same character is observed between 990 and 1010, also from 1050 to 1069. A stronger similar shading occurs from 1145 to 1170, and from 1280 to 1300. A similar shaded band commences at 1420, and another at 1557.

The spectra obtained from sixteen elementary bodies were observed simultaneously with the spectrum of *α* Orionis. In five of these, viz. sodium, magnesium, calcium, iron, and bismuth, lines corresponding with certain stellar lines were found to exist.

(1) *Sodium.*—The lines coincident with D are fainter in this star than in Aldebaran.

(2) *Magnesium.*—Decided group of three stellar lines coincident with the group at *b*.

(3) *Calcium.*—Four lines of this metal were on two different occasions seen to be coincident with four lines in the spectrum of the star.

(4) *Iron.*—The double line of this metal at E, and three other more refrangible bright lines, coincide with lines in the star-spectrum.

(5) *Bismuth.*—In the spectrum of this metal also four lines were found to coincide with four in the stellar spectrum.

Thallium.—The bright green line so characteristic of this metal appears to coincide with one of the lines seen in the star-spectrum; but this line may be due to calcium, since the small difference between the position of the thallium line and that of one of the calcium lines very close to it would not be distinguishable with the dispersive power of the apparatus employed.

In the spectra of the other elements which we compared with that of the star, no coincidences occur.

Hydrogen.—There is no line coincident with the red line C of hydrogen; but in the star are two strong lines, one on either side of the position of C: there is also no line coincident with F. It is strikingly confirmatory of this method of analysis, that in all the stars hitherto examined by us in which a line corresponding to C exists, that corresponding to F is also found. When F is absent, C is also wanting.

In *nitrogen* three strong double lines were compared. In *tin* five lines, and in *lead* two bright lines were compared, but no coincidence was found.

Gold.—The strongest of the gold lines approximates closely in position to one in the spectrum of the star, but it is probably not coincident.

Three of the strong lines of *cadmium*, two of *silver*, four of *mercury*, two of *barium*, and one (the orange line) of *lithium* were observed to be not coincident with any of the lines visible in the star. In these comparisons, when barium was used, it was employed in the form of a nearly solid amalgam.

The opening of the slit was maintained at the same width (not more than the $\frac{1}{500}$th of an inch) for all the observations, both with Aldebaran and α Orionis. In the case of the fainter star which follows, it was very slightly widened.

13. β PEGASI.—The colour of this star is a fine yellow. In the general arrangement of the groups, in the gradation of the strength of the lines composing the groups, and in the absence of the hydrogen lines, this spectrum, though much fainter, is closely analogous with the spectrum of α Orionis, as figured in the Plate.

This star was carefully observed on many different occasions; but the faintness of the star, and the unfavourable state of the atmosphere on many of the nights of observation, did not permit the same number of lines to be measured, nor allow a comparison with an equal number of terrestrial elements. From November 10, 1862, when twelve lines were observed, to the present year, we have scrutinized the star carefully.

Nine of the elements were compared with the spectrum of β Pegasi. Two of these, viz. *sodium* and *magnesium*, and perhaps a third, viz. *barium*, furnish spectra in which there are lines which coincide with lines in the spectrum of the star.

The spectra of *iron* and *manganese* were also compared with that of the star, but the state of the atmosphere prevented any certain conclusion.

The lines in the spectra of *nitrogen*, *tin*, and *mercury* were not coincident with any definite lines in the star-spectrum. Neither of the *hydrogen* lines corresponding to C and F was present.

At the end of the paper we have given a Table of such measures of the lines in the spectrum of this star as we can depend upon. Although it appears to be as full of lines as either of the preceding stars, the observations are attended with great difficulty, owing to the insufficient amount of light.

The absence in the spectrum of α Orionis, and also in the spectrum of β Pegasi, which so closely resembles it in character, of lines corresponding to those of hydrogen, is an observation of considerable interest. It is of the more importance since the lines C and F are highly characteristic of the solar spectrum and of the spectra of by far the larger number of the fixed stars to which our observations have been extended.

These exceptions are further interesting as they seem to prove that the lines C and F are due to the luminous bodies themselves. Of this some doubt might be entertained, and it might be suspected that they are in some way due to our own atmosphere, if these lines were present in the spectra of *all* the stars without exception.

This absence of the lines corresponding to hydrogen is also the more entitled to consideration since it is so rare to find them wanting, amongst the considerable number of stellar spectra which we have observed.

14. Sirius.—The spectrum of this brilliant white star is very intense; but owing to its low altitude, even when most favourably situated, the observation of the finer lines is rendered very difficult by the motions of the earth's atmosphere. For the present we do not give any details of our measures. The lines in the green and blue appeared, in the less perfect form of spectroscope which we employed in the early part of 1863, of very great breadth, and were so figured in the diagram of the spectrum of this star given in our "Note" of February 1863. With our present instrument, possessing much greater dispersive power and a very narrow slit, these bands appear but little broader than F and G are at times seen in the solar spectrum. In February 1863, the breadth of the band corresponding to F measured $1\frac{1}{2}$ unit of the scale we then adopted; each unit corresponded to 15·5 units of our present scale. The micrometric measurement of this line in Sirius, in terms of our present scale, is only 3·7—that is, only about one-seventh of the breadth as seen with the wider slit and a dispersing arrangement having little more than one-third of the power of the present apparatus.

Three if not four elementary bodies have been found to furnish spectra in which lines coincide with those of Sirius, viz. sodium, magnesium, hydrogen, and probably iron.

(1) *Sodium.*—A double line in the star, though faint, coincides in position with the line of this metal.

(2) *Magnesium.*—Three lines in the star-spectrum coincide with the triple group of magnesium.

(3) *Hydrogen.*—Both the lines corresponding to F and C have intensely strong lines in the star-spectrum.

(4) *Iron.*—No direct comparison with this metal was made; but the cross wires having been set to a position corresponding with E of the solar spectrum, a faint line in the star was seen exactly to bisect the wires when the telescope was turned upon Sirius.

The whole spectrum of Sirius is crossed by a very large number of faint and fine lines.

It is worthy of notice that in the case of Sirius, and a large number of the white stars, at the same time that the hydrogen lines are abnormally strong as compared with the solar spectrum, all the metallic lines are remarkably faint.

On the 27th February, 1863, and on the 3rd of March of the same year, when the spectrum of this star was caused to fall upon a sensitive collodion surface, an intense spectrum of the more refrangible part was obtained. From want of accurate adjustment of the focus, or from the motion of the star not being exactly compensated by the clock movement, or from atmospheric tremors, the spectrum, though tolerably defined at the edges, presented no indications of lines. Our other investigations have hitherto prevented us from continuing these experiments further; but we have not abandoned our intention of pursuing them.

15. *α* Lyræ (Vega).—This is a white star having a spectrum of the same class as Sirius, and as full of fine lines as the solar spectrum. Many of these we have measured, but our investigation of this star is incomplete.

We have ascertained the existence, in the stellar spectrum, of a double line at D corresponding to the lines of *sodium*, of a triple line at *b* coinciding with the group of *magnesium*, and of two strong lines coincident with the lines of *hydrogen* C and F.

16. Capella.—This is a white star with a spectrum closely resembling that of our sun. The lines are very numerous; we have measured more than twenty of them, and ascertained the existence of the double *sodium* line at D, but we defer giving details until we have completed our comparison with the spectra of other metals.

From this star we obtained (on February 27, 1863) a photograph of the more refrangible end of the spectrum; but the apparatus was not sufficiently perfect to exhibit any stellar lines.

17. Arcturus (*α* Boötis).—This is a red star the spectrum of which somewhat resembles that of the sun. In this also we have measured upwards of thirty lines, and have ascertained the existence of a double *sodium* line at D; but our comparisons with other metallic spectra are not yet complete.

18. Pollux.—In the spectrum of this star, which is rich in lines, we have measured twelve or fourteen, and have observed coincidences with the lines of *sodium*, *magnesium*, and probably of *iron*. At any rate there is a line which we believe occupies the position of E in the solar spectrum.

α Cygni and Procyon are both full of fine lines. In each of these spectra we observed a double line coincident with the *sodium* D.

19. The following stars have also been observed: numerous lines are seen in the spectrum of each; and in some, several of the lines were measured; but we have not instituted any comparisons with the metallic spectra as yet.

Castor; ϵ, ζ, and η *Ursæ majoris*; α and ϵ *Pegasi*; α, β, and γ *Andromedæ*, the last an interesting spectrum; *Rigel*, a spectrum full of fine lines; η *Orionis*; α *Trianguli*; γ and ϵ *Cygni*; α, β, γ, ϵ, and η *Cassiopeiæ*; γ *Geminorum*; β *Canis majoris*; β *Canis minoris*; *Spica*; γ, δ, and ϵ *Virginis*; α *Aquilæ*; *Cor Caroli*; β *Aurigæ*; *Regulus*; β, γ, δ, ϵ, ζ, and η *Leonis*.

§ V. *General Observations.*

20. *On the Colours of the Stars.*—From the earliest ages it has been remarked that certain of the stars, instead of appearing to be white, shine with special tints; and in countries where the atmosphere is less humid and hazy than our own, this contrast in the colour of the light of the stars is said to be much more striking. Various explanations of the contrast of colours, by SESTINI and others, founded chiefly on the difference of the wave-lengths corresponding to the different colours, have been attempted, but as yet without success. Probably in the constitution of the stars as revealed by spectrum analysis, we shall find the origin of the differences in the colour of stellar light*.

Since spectrum analysis shows that certain of the laws of terrestrial physics prevail in the sun and stars, there can be little doubt that the immediate source of solar and stellar light must be solid or liquid matter maintained in an intensely incandescent state, the result of an exceedingly high temperature. For it is from such a source alone that we can produce light even in a feeble degree comparable with that of the sun.

The light from incandescent solid and liquid bodies affords an unbroken spectrum containing rays of light of every refrangibility within the portion of the spectrum which is visible. As this condition of the light is connected with the state of solidity or liquidity, and not with the *chemical* nature of the body, it is highly probable that the light when first emitted from the photosphere, or light-giving surface of the sun and of the stars, would be in all cases identical.

The source of the difference of colour, therefore, is to be sought in the difference of the constituents of the investing atmospheres†. The atmosphere of each star must

* In connexion with this subject we quote the following passage from SMYTH's 'Speculum Hartwellianum,' 4to, 1860, p. 315:—"Sir DAVID BREWSTER observes that there can be no doubt that in the spectrum of every coloured star certain rays are wanting which exist in the solar spectrum; but we have no reason to believe that these defective rays are absorbed by any atmosphere through which they pass. And in recording the only observation perhaps yet made to analyze the light of the coloured stars, he says, 'In the orange-coloured star of the double star ζ Herculis, I have observed that there are several defective bands. By applying a fine rock-salt prism, with the largest possible refracting angle, to this orange star, as seen in Sir JAMES SOUTH's great achromatic refractor, its spectrum had the annexed appearance [in the Campden Hill Journal], clearly showing that there was one defective band in the red space, and two or more in the blue space. Hence the colour of the star was orange, because there was a greater defect of blue than of red rays.'"

† The presence in the atmospheres of Aldebaran and α Orionis of metals, such as iron, which require an

vary in nature as the constituents of the star vary; and observation has shown that the stars do differ from the sun and from each other in respect of the elements of which they consist. The light of each star therefore will be diminished by the loss of those rays which correspond in refrangibility to the bright lines which the constituents of each atmosphere would, in the incandescent state, be capable of emitting. In proportion as these dark lines preponderate in particular parts of the spectrum, so will the colours in which they occur be weaker, and consequently the colours of other refrangibilities will predominate.

Of this the spectrum of α Orionis affords a good example. The green and blue parts of the spectrum are comparatively dark, from the numerous and close groups of dark lines. In the orange they are less strong. Hence it might be anticipated that the light of the star would be characterized by "an orange tinge," as noted by SMYTH. β Pegasi is described by SMYTH as "deep yellow;" and the appearance exhibited by its spectrum, which closely resembles that of α Orionis, though much fainter, supports the same view.

Aldebaran is recorded by SMYTH as of a "pale rose tint." In the spectrum of this star, with the exception of the hydrogen line C, there are but few strong lines in the red, whilst the orange portion is considerably subdued by dark lines, which are less numerous in the green and blue. Sirius, on the contrary, is "brilliant white" (SMYTH); and the continuous brightness of the spectrum, with the exception of five strong lines, is, as compared with Aldebaran and α Orionis, unaffected by the dark lines which cross it. The spectrum is indeed crowded with numerous fine lines; but the intensity of these lines is extremely feeble as contrasted with those of the stars just mentioned. It may be that the length of the stellar atmosphere through which the light passes is less, relatively to the intensity of radiation from the photosphere, and so is insufficient to produce lines of the same degree of blackness as would be produced if the atmosphere were denser. The great intensity, however, of the light of Sirius would rather lead to the conclusion that the atmosphere of vapours is itself highly incandescent. If so, might it not to some extent replace with its own light, the light which it has absorbed from the photosphere behind it? It matters little, however, for the present purpose, whether or not either of these suppositions be adopted. There is at all events a most striking difference between the effect on the colour of the star of the closely grouped and very dark lines in the green and blue portions of the spectrum of α Orionis and of the corresponding portion of the spectrum of Sirius, in which the dark lines are faint and wholly unequal to produce any noticeable subduing of the blue and green rays.

We have not yet had an opportunity of testing by experiment whether this hypo-

exceedingly high temperature to convert them into vapour, renders untenable the supposition, which might otherwise have been entertained, that the orange and red tints of the light of these stars might be due to an inferior degree of incandescence of the photosphere as compared with the temperature of the stars the light of which is white.

thesis of the origin of the colours of the light of the stars is also applicable to the remarkable exceptional class of stars the light of which is of a decided green, blue, or violet colour. Such stars are usually very small, and they are always so closely approximated to other more brilliant stars, that it is scarcely possible, with the apparatus which we employ, to obtain separate images of the two spectra: and even were such separation easily practicable, the light of the strongly coloured star is usually so feeble that its satisfactory prismatic analysis would be a matter of great difficulty.

[One of the objects proposed in the construction of the spectrum apparatus with which the additional observations on Jupiter, Saturn, and Mars were made, and which has been described (p. 421) in connexion with those observations, was to make it available for the prismatic observation of some double and multiple stars.

Before commencing the observation of the spectra of the components of a double star, it is necessary that the position-angle of the stars should be approximatively known. The spectrum apparatus has then to be rotated upon the end of the telescope until the direction of the slit becomes perpendicular to a line joining the stars. When the instrument is in this position, the images of the stars are elongated by the cylindrical lens into two short lines of light parallel with the slit, and separated from each other by a small interval. If the telescope be now moved in a direction at right angles to that of the slit, either of the elongated stellar images can, at pleasure, be made to fall upon the slit and form its spectrum in the instrument. By adopting this method of observation, the spectra of the components of β Cygni were separately examined. These spectra, especially that of B, are so faint that the lines are seen with difficulty, and scarcely admit of being measured. Since, however, on account of the strongly contrasted colours of these stars, considerable interest attaches to a comparative examination of their spectra, we have represented in fig. 4, Plate X., the appearances which these spectra present to the eye, though we have not yet measured the lines and bands in them. These figures must be regarded as eye-estimations only of the general features of the two spectra. The spectra contain, doubtless, many other lines; and the positions of the lines inserted in the drawings, with the exception of *b* and D, were not measured, but only roughly estimated. The distinctive characteristics of these spectra are in accordance with the theory of the origin of the colours of the stars proposed in the foregoing paragraphs. In the case of both stars, the portions of the spectrum which correspond to the colours which are deficient in the light of the star, are those which are most strongly shaded with bands of absorption. Thus in the spectrum of A, the light of which is yellow tinted with orange, the absorption is greatest in the violet and blue; for the strong lines in the orange and red, since they are narrow, would diminish in a much smaller degree the light of these refrangibilities. The yellow and part of the green are free from *strong* lines.

The light of the star B appears to us to be blue, though in some states of the atmosphere the star becomes greenish blue, green, and even greenish white. These changes are probably due to the comparatively greater absorptive action of the vapours in the

air upon the more refrangible portions of the spectrum; in proportion to which absorption the other parts of the spectrum become relatively exalted, and thus predominate more or less in the eye.

This inequality of the absorptive action of the vapours of the atmosphere upon different parts of the spectrum becomes very evident if the eyepiece of the telescope be put out of focus (without the focus) so as to bring the blue and red rays to a focus in the centre of an expanded image of the star. In the case of B of β Cygni, the centre appears purple, surrounded with a margin of green. In proportion to the changes in the atmosphere by the passage of masses of vapour or thin cloud, will be the variations of these colours. The green becomes greener; but the blue and the violet are affected in a much greater degree, at times fading almost completely; then the colours resume their former tints and brightness. Several such changes may sometimes occur during one observation.

The spectrum B observed under conditions of atmosphere in which the colour of the star was blue, was remarkable for the faintness of the orange and yellow portions compared with the rest of the spectrum. The diminished brightness of these parts appears to be produced by several groups of closely set fine lines, while towards the more refrangible limit of the spectrum a few strong lines separated by considerable intervals are seen.

The observation of this star, on account of the faintness of its spectrum, is so difficult and fatiguing to the eye that we have not been able to examine it more accurately or in greater detail.

We have by the same method of observation examined the spectra of the components of α Herculis. The spectrum of A is remarkable for the great strength of the groups of lines in the green, blue, and violet; fainter bands are visible in the yellow and orange, also two strong bands in the red. This arrangement of the bands of absorption agrees with the orange colour which strongly predominates in the light of this star.

B is bluish green in colour. The more refrangible portions of its spectrum are very bright in consequence of the absence of any strong bands. The yellow and the orange parts are crossed by several groups of lines.—August 31, 1864.]

The suggestive fact that stars of these more highly refrangible colours are always observed in close contiguity with much brighter stars, generally of an orange or red tint, would afford countenance to the supposition that these exceptional colours are due to some special physical conditions essentially connected with the stellar systems of which they seem to form a part.

Arago* remarks, "Among the sixty or eighty thousand *isolated* stars, the positions of which are to be found in the catalogues of astronomers, there are none, I think, inscribed with any other indications in regard to colour, than white, red, and yellow. The physical conditions which determine the emission of blue and green light appear, then, to exist only in *multiple* stars."

* Popular Astronomy, translated by Smyth and Grant, vol. i. p. 295.

These stars are without exception feeble in the intensity of their light. The explanation is not admissible, that the faint blue or violet light is due to a less intense incandescence of the radiating surface, since it is precisely these more refrangible rays which would be the first to fail as the temperature diminished, and upon this supposition the star should be dull red. It is of course to be supposed that in the process of gradual cooling some bodies which are less volatile than others would cease to exist in the atmosphere at an earlier period than others, or that they might enter into new combinations more readily than others, and so modify the tint of the light emitted.

The existence around these blue stars of an extended atmosphere of "fog" will not explain the absorption of the *less* refrangible portion of the luminous spectrum.

21. These spectrum observations are not without interest also when viewed in connexion with the *nebular hypothesis* of the cosmical origin of the solar system and fixed stars. For if it be supposed that all the countless suns which are distributed through space, or at least those of them which are bright to us, were once existing in the condition of nebulous matter, it is obvious that, though certain constituents may have been diffused throughout its mass, yet the composition of the nebulous material must have differed at different points; otherwise, during the act of agglomeration, each system must have collected and condensed equal proportions of similar materials from the mass around. It cannot be supposed that similarity in physical properties has caused the association of the different elements: we find, for example, some of the least volatile of the metals, such as iron, associated with highly volatile elements, such as mercury and tellurium, in the same star.

If we may so say, there seems to be some analogy between this irregular distribution of the elements in different centres in space, and the manner in which the components of the earth's crust are distributed. Upon the earth there are certain very generally diffused elements, such as oxygen, hydrogen, carbon, silicon, iron, aluminium, and calcium, which occur in all parts; whilst there are others which, like silver, tin, lead, and other metals, are accumulated at particular points only. Whatever may have been the physical causes which may have produced this separation, we see abundant evidence of the advantage of this distribution in their application to the purposes of man—smallness in relative amount being compensated for by the accumulation of the material in denser deposits, which allow of their comparatively easy extraction to supply the wants of mankind. If this arrangement be admitted as designed in the case of the earth, is it going beyond the limits of fair deduction to suppose that, were we acquainted with the economy of those distant globes, an equally obvious purpose might be assigned for the differences in composition which they exhibit?

22. The additional knowledge which these spectrum observations give us of the nature and of the structure of the fixed stars, seems to furnish a basis for some legitimate speculation in reference to the great plan of the visible universe, and to the special object and design of those numerous and immensely distant orbs of light.

The closely marked connexion, in similarity of plan and mode of operation, in those

parts of the universe which lie within the range of experiment, and so of our more immediate knowledge, renders it not presumptuous to attempt to apply the process of reasoning from analogy to those parts of the universe which are more distant from us.

Upon the earth we find that the innumerable individual requirements which are connected with the present state of terrestrial activity, are not met by a plan of operation distinct for each, but are effected in connexion with the special modifications of a general method embracing a wide range of analogous phenomena. If we examine living beings, the persistence of unity of plan observable amidst the multiform varieties of special adaptation of the vertebrate form of life may be cited as an example of the unity of operation referred to. In like manner the remarkably wide range of phenomena which are shown to be reciprocally interdependent and correlative of each other, by the recent great extension of our knowledge in reference to the relation of the different varieties of force and their connexion with molecular motion, exhibits a similar unity of operation amidst the changes of the bodies which have not life.

The observations recorded in this paper seem to afford some proof that a similar unity of operation extends through the universe as far as light enables us to have cognizance of material objects. For we may infer that the stars, while differing the one from the other in the kinds of matter of which they consist, are all constructed upon the same plan as our sun, and are composed of matter identical, at least in part, with the materials of our system.

The differences which exist between the stars are of the *lower order*, of differences of *particular adaptation*, or special modification, and not differences of the *higher order* of distinct *plans of structure.*

There is therefore a probability that these stars, which are analogous to our sun in structure, fulfil an analogous purpose, and are, like our sun, surrounded by planets, which they by their attraction uphold, and by their radiation illuminate and energize. And if matter identical with that upon the earth exists in the stars, the same matter would also probably be present in the planets genetically connected with them, as is the case in our solar system.

It is remarkable that the elements most widely diffused through the host of stars are some of those most closely connected with the constitution of the living organisms of our globe, including hydrogen, sodium, magnesium, and iron. Of oxygen and nitrogen we could scarcely hope to have any decisive indications, since these bodies have spectra of different orders. These forms of elementary matter, when influenced by heat, light, and chemical force, all of which we have certain knowledge are radiated from the stars, afford some of the most important conditions which we know to be indispensable to the existence of living organisms such as those with which we are acquainted. On the whole we believe that the foregoing spectrum observations on the stars contribute something towards an experimental basis on which a conclusion, hitherto but a pure speculation, may rest, viz. that at least the brighter stars are, like our sun, upholding and energizing centres of systems of worlds adapted to be the abode of living beings.

Table of Stellar Spectra.

Aldebaran.

822·5	H	1107	
855·5		1112	Te
872·5		1117·5	Sb
880		1143 *d*	
893·5		1158	
900		1164	Hg
903·5		1171·5	
907·5		1178	
915		1187	
918	Ca	1192	
923	Hg	1202	
933	Ca	1210	
945·5	Sb	1224·5	
951·5		1240	
954·5		1241·5	
956		1250	
966·5	Sb	1252	Fe
972·5		1269·5	Fe
976	Te	1272	
982		1277	Bi
986·5		1282	
993		1291·5	
1000	Na	1297·5	Mg
1002		1300·5	
1013	Ca	1314	Bi
1023		1323	
1028		1328	
1031		1351	
1036·5	Hg	1420	Fe
1040		1442·5	Fe
1044	Hg	1483	H
1058			
1062	Bi		
1067	Te		
1076			
1086·5	Te		
1095			
1100			
1105	Ca		

α Orionis.

840		1139·5	
860		1144	
870		1145.5	
881		1148	
887		1151	
890		1158·5	
899		1167	
911		1169·5	
918	Ca	1176·5	
920		1183·5	
929		1187	
933	Ca	1191·5	
936		1198	
946		1201·5	
966		1210	Te
968·5		1214	
976		1220·5	
983		1225	
992		1237	
1000	Na	1243	
1002		1252	Fe
1010·5		1262	
1013	Ca	1269·5	Fe
1030		1277	Bi
1040		1280·5	
1050·5		1285·5	
1062	Bi	1291·5	
1069·5		1297·5	Mg
1079·5		1300·5	
1085·5		1303	
1090		1314	Bi
1091·5		1334	
1099		1350	
1105	Ca	1356	
1109·5		1361	
1116·5		1416	
1123·5		1420	Fe
1132		1442·5	Fe
1135·5		1557	

β Pegasi.

896	
923	
1000	Na
1002	
1014	
1165	
1220	
1276·5	
1291·5	
1297·5	Mg
1300·5	
1350·5	
1392·5	
1425	
1515	
1732	
1835	

XIII. *On the Spectra of some of the Nebulæ.* *By* WILLIAM HUGGINS, *F.R.A.S.* *A Supplement to the Paper " On the Spectra of some of the Fixed Stars.* *By* WILLIAM HUGGINS, *F.R.A.S., and* W. A. MILLER, *M.D., LL.D., Treas. and V.P.R.S."* *Communicated by Professor* W. A. MILLER, *M.D., LL.D.*

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THE concluding paragraphs of the preceding paper refer to the similarity of essential constitution which our examination of the spectra of the fixed stars has shown in all cases to exist among the stars, and between them and our sun.

It became therefore an object of great importance, in reference to our knowledge of the visible universe, to ascertain whether this similarity of plan observable among the stars, and uniting them with our sun into one great group, extended to the distinct and remarkable class of bodies known as nebulæ. Prismatic analysis, if it could be successfully applied to objects so faint, seemed to be a method of observation specially suitable for determining whether any essential physical distinction separates the nebulæ from the stars, either in the nature of the matter of which they are composed, or in the conditions under which they exist as sources of light. The importance of bringing analysis by the prism to bear upon the nebulæ is seen to be greater by the consideration that increase of optical power alone would probably fail to give the desired information; for, as the important researches of Lord ROSSE have shown, at the same time that the number of the clusters may be increased by the resolution of supposed nebulæ, other nebulous objects are revealed, and fantastic wisps and diffuse patches of light are seen, which it would be assumption to regard as due in all cases to the united glare of suns still more remote.

Some of the most enigmatical of these wondrous objects are those which present in the telescope small round or slightly oval disks. For this reason they were placed by Sir WILLIAM HERSCHEL in a class by themselves under the name of Planetary Nebulæ. They present but little indication of resolvability. The colour of their light, which in the case of several is blue tinted with green, is remarkable, since this is a colour extremely rare amongst single stars. These nebulæ, too, agree in showing no indication of central condensation. By these appearances the planetary nebulæ are specially marked as objects which probably present phenomena of an order altogether different from those which characterize the sun and the fixed stars. On this account, as well as because of their brightness, I selected these nebulæ as the most suitable for examination with the prism.

The apparatus employed was that of which a description was given at page 421. A second eyepiece was used in these observations, having a magnifying power of nine

diameters. For the greater part of the following observations on the nebulæ, the cylindrical lens is not necessary, and was removed from the instrument. The numbers and descriptions of the nebulæ, and their places for the epoch 1860, January 0, included within brackets, are taken from the last Catalogue of Sir JOHN HERSCHEL*.

[No. 4373. 37 H. IV. R.A. $17^{h}\ 58^{m}\ 20^{s}$. N.P.D. $23^{\circ}\ 22'\ 9''\cdot5$. A planetary nebula; very bright; pretty small; suddenly brighter in the middle, very small nucleus.] In Draco.

On August 29, 1864, I directed the telescope armed with the spectrum apparatus to this nebula. At first I suspected some derangement of the instrument had taken place; for no spectrum was seen, but only a short line of light perpendicular to the direction of dispersion. I then found that the light of this nebula, unlike any other ex-terrestrial light which had yet been subjected by me to prismatic analysis, was not composed of light of different refrangibilities, and therefore could not form a spectrum. A great part of the light from this nebula is monochromatic, and after passing through the prisms remains concentrated in a bright line occupying in the instrument the position of that part of the spectrum to which its light corresponds in refrangibility. A more careful examination with a narrower slit, however, showed that, a little more refrangible than the bright line, and separated from it by a dark interval, a narrower and much fainter line occurs. Beyond this, again, at about three times the distance of the second line, a third, exceedingly faint line was seen. The positions of these lines in the spectrum were determined by a simultaneous comparison of them in the instrument with the spectrum of the induction spark taken between electrodes of magnesium. The strongest line coincides in position with the brightest of the air lines. This line is due to nitrogen, and occurs in the spectrum about midway between *b* and F of the solar spectrum. Its position is seen in Plate XI.†

The faintest of the lines of the nebula agrees in position with the line of hydrogen corresponding to FRAUNHOFER'S F. The other bright line was compared with the strong line of barium 2075‡: this line is a little more refrangible than that belonging to the nebula.

Besides these lines, an exceedingly faint spectrum was just perceived for a short distance on both sides of the group of bright lines. I suspect this is not uniform, but is crossed with dark spaces. Subsequent observations on other nebulæ induce me to regard this faint spectrum as due to the solid or liquid matter of the nucleus, and as quite distinct from the bright lines into which nearly the whole of the light from the nebula is concentrated.

In the diagram (fig. 5, Plate X.) the three principal lines only are inserted, for it would be scarcely possible to represent the faint spectrum without greatly exaggerating its intensity.

The colour of this nebula is greenish blue.

* Philosophical Transactions, Part. I. 1864, pp. 1–138.

† See also Philosophical Transactions, 1864, p. 156, and Plate I.

‡ Ibid. p. 156.

[No. 4390. 2000 h. Σ 6. R.A. $18^h\ 5^m\ 17^s{\cdot}8$. N.P.D. 83° 10′ 53″·5. A planetary nebula; very bright; very small; round; little hazy.] In Taurus Poniatowskii.

The spectrum is essentially the same as that of No. 4373.

The three bright lines occupy the same positions in the spectrum, which was determined by direct comparison with the spectrum of the induction spark. These lines have also the same relative intensity. They are exceedingly sharp and well defined. The presence of an extremely faint spectrum was suspected. In connexion with this it is important to remark that this nebula does not possess a distinct nucleus.

The colour of this nebula is greenish blue.

[No. 4514. 2050 h. 73 H. IV. R.A. $19^h\ 41^m\ 7^s{\cdot}5$. N.P.D. 39° 49′ 41″·7. A planetary nebula with a central star. Bright; pretty large; round; star of the 11th magnitude in the middle.] In Cygnus.

The same three bright lines were seen. Their positions in the spectrum were verified by direct comparison with the induction spark. In addition to these a spectrum could be traced from about D to about G of the solar spectrum. This spectrum is much stronger than the corresponding spectrum of 4373. This agrees with the greater brightness of the central star, or nucleus. The opinion that the faint continuous spectrum is formed alone by the light from the bright central point was confirmed by the following observation. When the cylindrical lens was removed, the three bright lines remained of considerable length, corresponding to the diameter of the telescopic image of the nebula; but the faint spectrum became as narrow as a line, showing that this spectrum is formed by light which comes from an object of which the image in the telescope is a point.

Lord Rosse remarks of this nebula, "A very remarkable object, perhaps analogous to H. 450"*.

The colour of this nebula is greenish blue.

[No. 4510. 2047 h. 51 H. IV. R.A. $19^h\ 36^m\ 3^s{\cdot}0$. N.P.D. 104° 28′ 52″·5. A planetary nebula. Bright; very small; round.] In Sagittarius.

This nebula is less bright than those which have been described. The two brighter of the lines were well defined, and were directly compared with the induction spark. The third line was seen only by glimpses. I had a suspicion of an exceedingly faint spectrum.

The colour of this nebula is greenish blue.

Lord Rosse remarks, "Centre rather dark. The dark part is a little north preceding the middle"†.

[No. 4628. 2098 h. 1 H. IV. R.A. $20^h\ 56^m\ 31^s{\cdot}2$. N.P.D. 101° 55′ 4″·8. An exceedingly interesting object. Planetary; very bright; small; elliptic.] In Aquarius.

The three bright lines very sharp and distinct. They were compared for position with the induction spark. Though this object is bright, an indication only of the faint

* Philosophical Transactions, Part III. 1861, p. 733. For a figure of H. 450 see Philosophical Transactions, 1850, Plate XXXVIII. fig. 15.

† Ibid. 1861, Part III. p. 732.

spectrum was suspected. This nebula contains probably a very small quantity of matter condensed into the liquid or solid state.

The colour of the light of this nebula is greenish blue.

Lord ROSSE has not detected any central star, nor any perforation, as seen in some of the other planetary nebulæ. He represents it with ansæ, which probably indicate a nebulous ring seen edgeways*.

[No. 4447. 2023 h. 57 M. R.A. $18^h\ 48^m\ 20^s$. N.P.D. 57° 8′ 57″·2. An annular nebula; bright; pretty large; considerably elongated.] In Lyra†.

The apparent brightness of this nebula, as seen in the telescope, is probably due to its large extent, for the faintness of its spectrum indicates that it has a smaller intrinsic brightness than the nebulæ already examined. The brightest of the three lines was well seen. I suspected also the presence of the next in brightness. No indication whatever of a faint spectrum. The bright line looks remarkable, since it consists of two bright dots corresponding to sections of the ring, and between these there was not darkness, but an excessively faint line joining them. This observation makes it probable that the faint nebulous matter occupying the central portion is similar in constitution to that of the ring. The bright line was compared with the induction-spark ‡.

[No. 4964. 2241 h. 18 H. IV. R.A. $23^h\ 19^m\ 9^s{\cdot}9$. N.P.D. 48° 13′ 57″·5. Planetary; very bright; pretty small, round, blue.]

With a power of 600 this nebula appears distinctly annular. The colour of its light is greenish blue §. The spectrum formed by the light from this nebula corresponds with that of 37 H. IV. represented in fig. 5, Plate X.

* Philosophical Transactions, 1850, p. 507 and Plate XXXVIII. fig. 14.

† Lord ROSSE, in his description of this nebula, remarks, "The filaments proceeding from the edge become more conspicuous under increasing magnifying power within certain limits, which is strikingly characteristic of a cluster; still I do not feel confident that it is resolvable."—Philosophical Transactions, 1844, p. 322 and Plate XIX. fig. 29.

In 1850 Lord ROSSE further remarks, "I have not yet sketched it with the 6-feet instrument, because I have never seen it under favourable circumstances: the opportunities of observing it well on the meridian are comparatively rare, owing to twilight. It was observed seven times in 1848. and once in 1849. The only additional particulars I collect from the observations are that the central opening has considerably more nebulosity, and there is one pretty bright star in it, s. f. the centre, and a few other very minute stars. In the sky round the nebula and near it there are several very small stars which were not before seen; and therefore the stars in the dark opening may possibly be merely accidental. In the annulus, especially at the extremities of the minor axis, there are several minute stars, but there was still much nebulosity not seen as distinct stars."—Philosophical Transactions, 1850, p. 506.

"Nothing additional since 1844, except a star s. f. the middle."—Philosophical Transactions, 1861, p. 732.

‡ Already in 1850 Lord ROSSE had discovered a connexion in general plan of structure between some of the nebulæ which present small planetary disks in ordinary telescopes, and the annular nebula in Lyra. His words are, "There were but two annular nebulæ known in the northern hemisphere when Sir JOHN HERSCHEL's Catalogue was published; now there are seven, as we have found that five of the planetary nebulæ are really annular. Of these objects, the annular nebula in Lyra is the one in which the form is the most easily recognized."—Philosophical Transactions, 1850, p. 506.

§ For Lord ROSSE's observations of this nebula, see Philosophical Transactions, 1844, p. 323; ibid. 1850, p. 507 and Plate XXXVIII. fig. 13; ibid. 1861, p. 736 and Plate XXX. fig. 40.

In the spectrum of this nebula, however, in addition to the three bright lines, a fourth bright line, excessively faint, was seen. This line is about as much more refrangible than the line agreeing in position with F as this line is more refrangible than the brightest of the lines, which coincides with a line of nitrogen.

[No. 4532. 2060 h. 27 M. R.A. $19^h\ 53^m\ 29^s\cdot3$. N.P.D. 67° 39′ 43.″ Very bright; very large; irregularly extended. Dumb-bell.] In Vulpecula.

The light of this nebula, after passing through the prisms, remained concentrated in a bright line corresponding to the brightest of the three lines represented in fig. 5, Plate X. This line appeared nebulous at the edges. No trace of the other lines was perceived, nor was a faint continuous spectrum detected.

The bright line was ascertained, by a simultaneous comparison with the spectrum of the induction spark, to agree in position with the brightest of the lines of nitrogen.

Minute points of light have been observed in this nebula by Lord ROSSE, OTTO STRUVE, and others;* the spectra of these bright points, especially if continuous like those of stars, are doubtless invisible from excessive faintness.

By suitable movements given to the telescope, different portions of the image of the nebula formed in the telescope were caused successively to fall upon the opening of the slit, which was about $\frac{1}{10}$ inch by $\frac{1}{300}$ inch. This method of observation showed that the light from different parts of the nebula is identical in refrangibility, and varies alone in degree of intensity.

In addition to these objects the following were also observed:—

[No. 4294. 92 M. R.A. $17^h\ 12^m\ 56^s\cdot9$. N.P.D. 46° 43′ 31″·2.] In Hercules. Very bright globular cluster of stars. The bright central portion was brought upon the slit. A faint spectrum similar to that of a star. The light could be traced from between C and D to about G.

Too faint for the observation of lines of absorption.

[No. 4244. 50 H. IV. R.A. $16^h\ 43^m\ 6^s\cdot4$. N.P.D. 42° 8′ 38″·8. Very bright; large; round.] In Hercules. The spectrum similar to that of a faint star. No indication of bright lines.

[No. 116. 50 h. 31 M. R.A. $0^h\ 35^m\ 3^s\cdot9$. N.P.D. 49° 29′ 45″·7.] The brightest part of the great nebula in Andromeda was brought upon the slit.

The spectrum could be traced from about D to F. The light appeared to cease very abruptly in the orange; this may be due to the smaller luminosity of this part of the spectrum. No indication of the bright lines.

[No. 117. 51 h. 32 M. R.A. $0^h\ 35^m\ 5^s\cdot3$. N.P.D. 49° 54′ 12″·7. Very very bright; large; round; pretty suddenly much brighter in the middle.]

This small but very bright companion of the great nebula in Andromeda presents a spectrum apparently exactly similar to that of 31 M.

The spectrum appears to end abruptly in the orange; and throughout its length

is not uniform, but is evidently crossed either by lines of absorption or by bright lines.

[No. 428. 55 Androm. R.A. $1^h\ 44^m\ 55^s{\cdot}9$. N.P.D. $49°\ 57'\ 41''{\cdot}5$. Fine nebulous star with strong atmosphere.] The spectrum apparently similar to that of an ordinary star *.

[No. 826. 2618 h. 26 IV. R.A. $4^h\ 7^m\ 50^s{\cdot}8$. N.P.D. $103°\ 5'\ 32''{\cdot}2$. Very bright cluster.] In Eridanus. The spectrum could be traced from the orange to about the blue. No indication of the bright lines.

Several other nebulæ were observed, but of these the light was found to be too faint to admit of satisfactory examination with the spectrum apparatus.

It is obvious that the nebulæ 37 H. IV., 6 Σ., 73 H. IV., 51 H. IV., 1 H. IV., 57 M., 18 H. IV. and 27 M. can no longer be regarded as aggregations of suns after the order to which our own sun and the fixed stars belong. We have in these objects to do no longer with a special modification only of our own type of suns, but find ourselves in the presence of objects possessing a distinct and peculiar plan of structure.

In place of an incandescent solid or liquid body transmitting light of all refrangibilities through an atmosphere which intercepts by absorption a certain number of them, such as our sun appears to be, we must probably regard these objects, or at least their photo-surfaces, as enormous masses of luminous gas or vapour. For it is alone from matter in the gaseous state that light consisting of certain definite refrangibilities only, as is the case with the light of these nebulæ, is known to be emitted.

It is indeed *possible* that suns endowed with these peculiar conditions of luminosity may exist, and that these bodies are clusters of such suns. There are, however, some considerations, especially in the case of the planetary nebulæ, which are scarcely in accordance with the opinion that they are clusters of stars.

Sir JOHN HERSCHEL remarks of one of this class, in reference to the absence of central condensation, "Such an appearance would not be presented by a globular space uniformly filled with stars or luminous matter, which structure would necessarily give rise to an apparent increase of brightness towards the centre in proportion to the thickness traversed by the visual ray. We might therefore be inclined to conclude its real constitution to be either that of a hollow spherical shell or of a flat disk presented to us (by a highly improbable coincidence) in a plane precisely perpendicular to the visual ray"†. This absence of condensation admits of explanation, without recourse to the supposition of a shell or of a flat disk, if we consider them to be masses of glowing gas. For supposing, as we probably must do, that the whole mass of the gas is luminous, yet it would follow, by the law which results from the investigations of KIRCHHOFF, that the light emitted by the portions of gas beyond the surface visible to us, would

* "Looked at eight times, but saw no nebulous atmosphere."—Lord ROSSE, Philosophical Transactions, 1861, p. 712.

† Outlines of Astronomy, 7th edit. p. 646.

be in great measure, if not wholly, absorbed by the portion of gas through which it would have to pass, and for this reason there would be presented to us a *luminous surface* only*.

Sir JOHN HERSCHEL further remarks†, "Whatever idea we may form of the real nature of the planetary nebulæ, which all agree in the absence of central condensation, it is evident that the intrinsic splendour of their surfaces, *if continuous*, must be almost infinitely less than that of the sun. A circular portion of the sun's disk, subtending an angle of 1′, would give a light equal to that of 780 full moons, while among all the objects in question there is not one which can be seen with the naked eye." The small brilliancy of these nebulæ is in accordance with the conclusions suggested by the observations of this paper; for, reasoning by analogy from terrestrial physics, glowing or luminous gas would be very inferior in splendour to incandescent solid or liquid matter.

Such gaseous masses would be doubtless, from many causes, unequally dense in different portions; and if matter condensed into the liquid or solid state were also present, it would, from its superior splendour, be visible as a bright point or points within the disk of the nebula. These suggestions are in close accordance with the observations of Lord ROSSE.

Another consideration which opposes the notion that these nebulæ are clusters of stars is found in the extreme simplicity of constitution which the three bright lines suggest, whether or not we regard these lines as indicating the presence of nitrogen, hydrogen, and a substance unknown.

It is perhaps of importance to state that, except nitrogen, no one of thirty of the chemical elements the spectra of which I have measured has a strong line very near the bright line of the nebulæ. If, however, this line were due to nitrogen, we ought to see other lines as well; for there are specially two strong double lines in the spectrum of nitrogen, one at least of which, if they existed in the light of the nebulæ, would be easily visible‡. In my experiments on the spectrum of nitrogen, I found that the

* Sir WILLIAM HERSCHEL in 1811 pointed out the necessity of supposing the matter of the planetary nebulæ to have the power of intercepting light. He wrote:—"Admitting that these nebulæ are globular collections of nebulous matter, they could not appear equally bright if the nebulosity of which they are composed consisted only of a luminous substance perfectly penetrable to light. .... Is it not rather to be supposed that a certain high degree of condensation has already brought on a sufficient consolidation to prevent the penetration of light, which by this means is reduced to a superficial planetary appearance?"

"Their planetary appearance shows that we only see a superficial lustre such as opaque bodies exhibit, and which could not happen if the nebulous matter had no other quality than that of shining, or had so little solidity as to be perfectly transparent."—Philosophical Transactions, 1811, pp. 314, 315.

† Outlines of Astronomy, 7th edit. p. 646.

‡ Philosophical Transactions, 1864, p. 154 and Plate I.

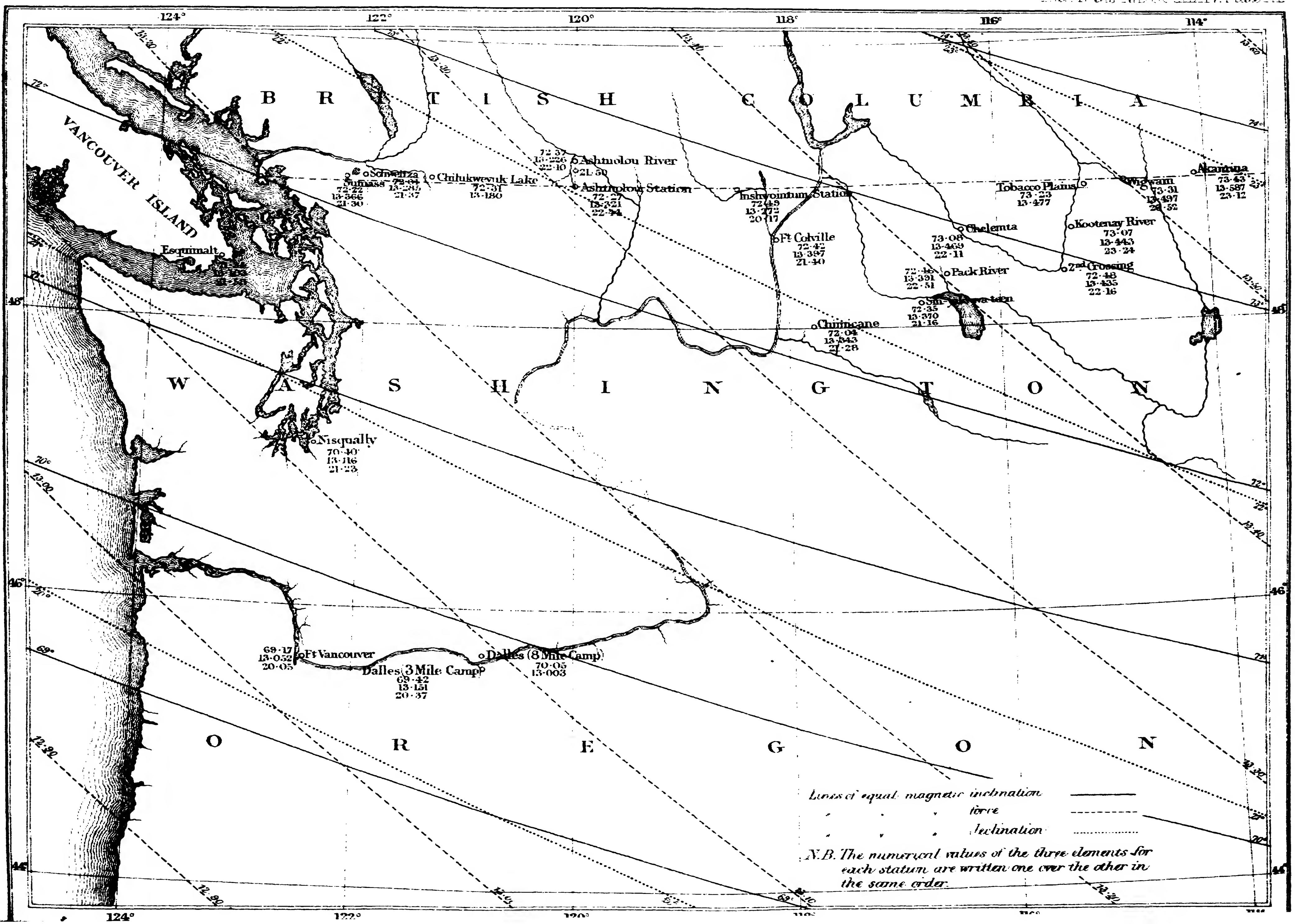
For the purpose of ascertaining whether the absence of the other bright lines of nitrogen might be connected with the presence of hydrogen, I arranged an apparatus in which, while the spectrum of the induction-spark in a current of nitrogen was being observed, a current of hydrogen could be introduced, and the propor-

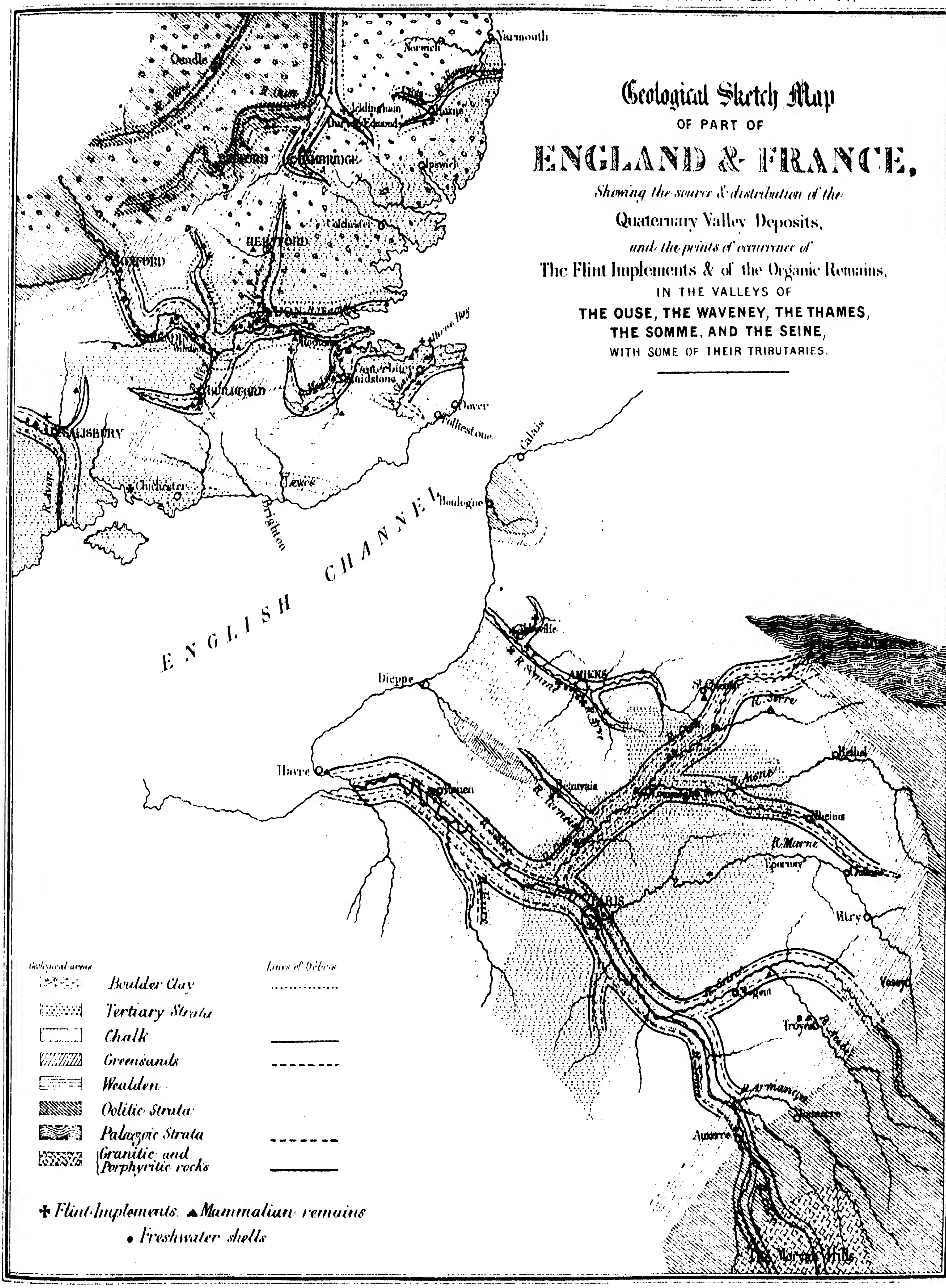
character of the brightest of the lines of nitrogen, that with which the line in the nebulæ coincides, differs from that of the two double lines next in brilliancy. This line is more nebulous at the edges, even when the slit is narrow and the other lines are thin and sharp. The same phenomenon was observed with some of the other elements*. We do not yet know the origin of this difference of character observable among lines of the same element. May it not indicate a physical difference in the atoms, in connexion with the vibrations of which the lines are probably produced? The speculation presents itself, whether the occurrence of this one line only in the nebulæ may not indicate a form of matter more elementary than nitrogen, and which our analysis has not yet enabled us to detect.

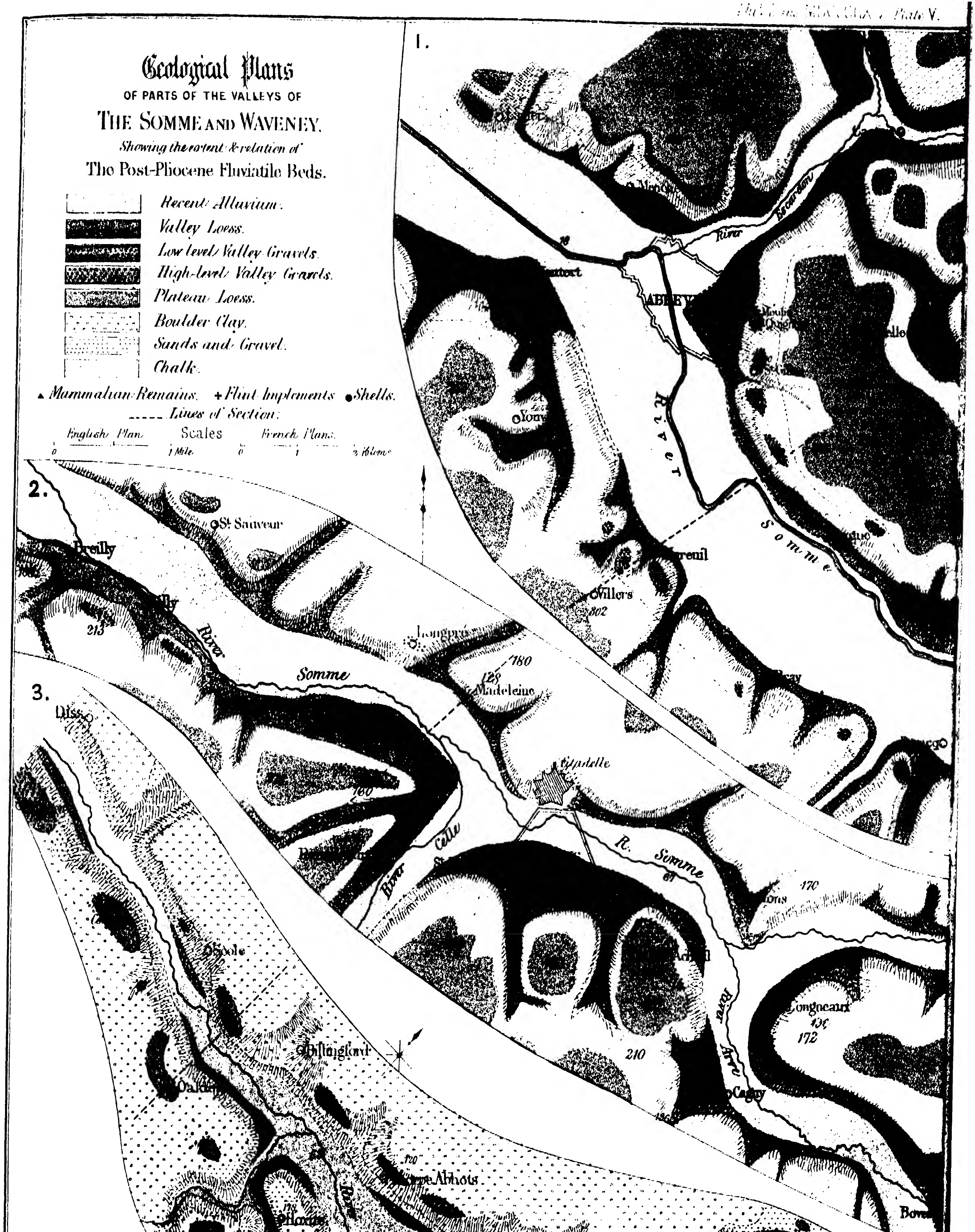
Observations on other nebulæ which I hope to make, may throw light upon these and other considerations connected with these wonderful objects.

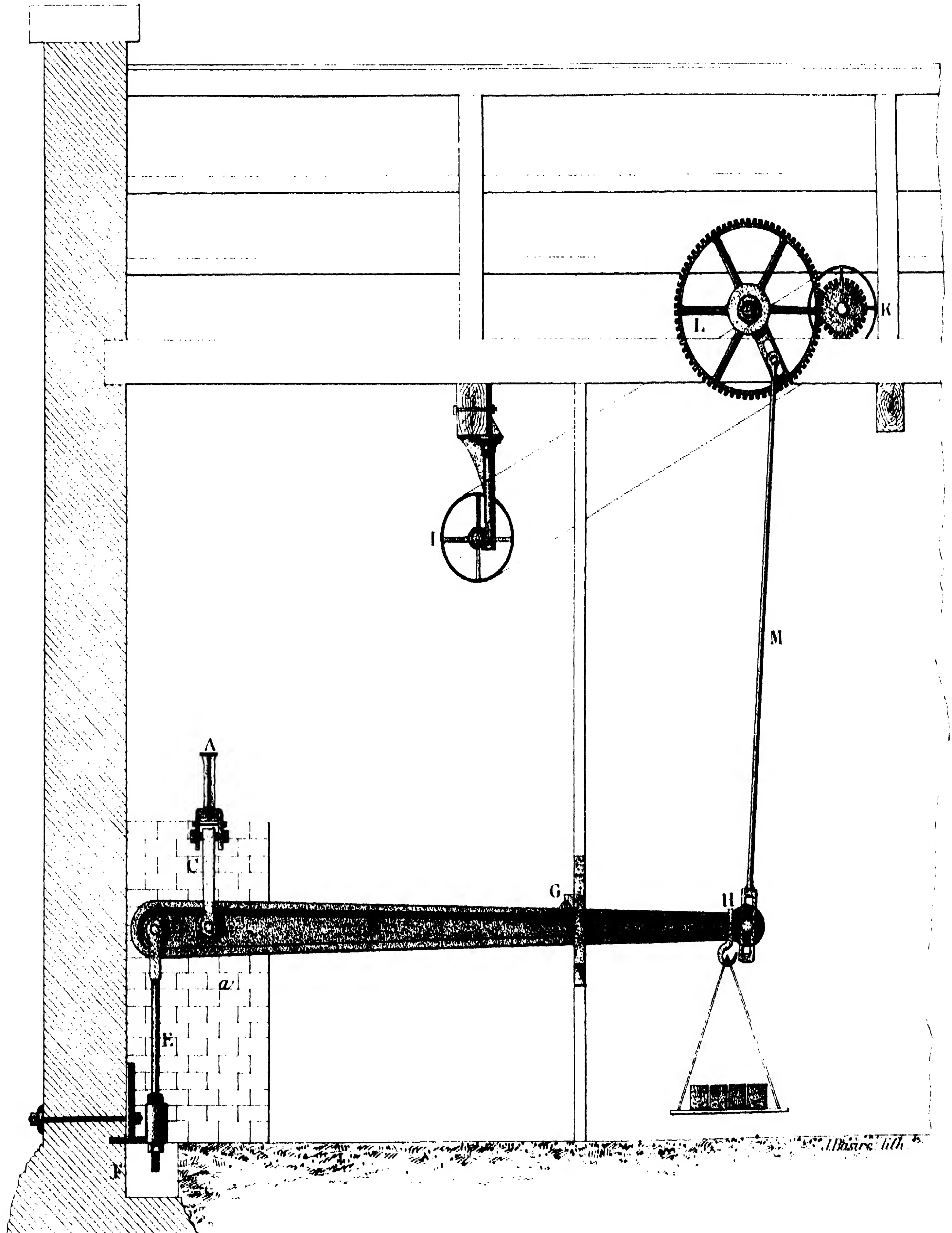
tion of the two gases to each other easily regulated. With this apparatus the fading out of the bright lines of nitrogen, as the proportion of this gas to hydrogen was diminished, and again their increase in brilliancy when the current of nitrogen was made stronger, were carefully observed, but without detecting any marked variation in the relative brightness of the lines.

* Philosophical Transactions, 1864, pp. 143, 150.


BRITISH COLUMBIA
WASHINGTON
OREGON
VANCOUVER ISLAND
Esquimalt
Sumass 72·22 13·366 21·30
Schweltza 72·34 13·285 21·37
Chilukweyuk Lake 72·31 13·180
Ashtnolou River 72·37 13·226 22·10
Ashtnolou Station 72·27 13·321 22·44
Inshwointum Station 72·49 13·272 20·17
Ft Colville 72·42 13·397 21·40
Chelemta 73·08 13·469 22·11
Pack River 72·46 13·391 22·51
Chiluicane 72·04 13·343 21·28
Tobacco Plains 73·23 13·477
Kootenay River 73·07 13·443 23·24
2nd Crossing 72·48 13·435 22·16
Wigwam 73·31 13·497 23·52
Akamina 73·43 13·587 23·12
Nisqually 70·40 13·116 21·23
Ft Vancouver 69·17 13·052 20·05
Dalles (3 Mile Camp) 69·42 13·151 20·37
Dalles (8 Mile Camp) 70·05 13·003
Lines of equal magnetic inclination
" " " force
" " " declination
N.B. The numerical values of the three elements for each station are written one over the other in the same order.


Geological Sketch Map
OF PART OF
ENGLAND & FRANCE,
Showing the source & distribution of the
Quaternary Valley Deposits,
and the points of occurrence of
The Flint Implements & of the Organic Remains,
IN THE VALLEYS OF
THE OUSE, THE WAVENEY, THE THAMES,
THE SOMME, AND THE SEINE,
WITH SOME OF THEIR TRIBUTARIES.
Geological areas
Lines of Débris
Boulder Clay
Tertiary Strata
Chalk
Greensands
Wealden
Oolitic Strata
Palæozoic Strata
Granitic and Porphyritic rocks
✚ Flint Implements. ▲ Mammalian remains
• Freshwater shells
ENGLISH CHANNEL
Yarmouth
Norwich
Ipswich
Colchester
HERTFORD
OXFORD
SALISBURY
Chichester
Brighton
Dover
Folkestone
Calais
Boulogne
Abbeville
AMIENS
Dieppe
Havre
Beauvais
PARIS
Epernay
Vitry
Troyes
Auxerre


Geological Plans
OF PARTS OF THE VALLEYS OF
THE SOMME AND WAVENEY.
Showing the extent & relation of
The Post-Pliocene Fluviatile Beds.
Recent Alluvium.
Valley Loess.
Low level Valley Gravels.
High-level Valley Gravels.
Plateau Loess.
Boulder Clay.
Sands and Gravel.
Chalk.
▲ Mammalian Remains. + Flint Implements ● Shells.
Lines of Section.
English Plan
Scales
French Plans
1 Mile
1.
River
Somme
Willers
2.
St. Sauveur
Breilly
213
River
Somme
Longpré
180
Madeleine
Citadelle
R. Somme
Longueau
172
210
Cagny
3.
Diss
Scole
Billingford
Hoxne
113
Waveney
J Basire lith.


A
C
a
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G
H
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L
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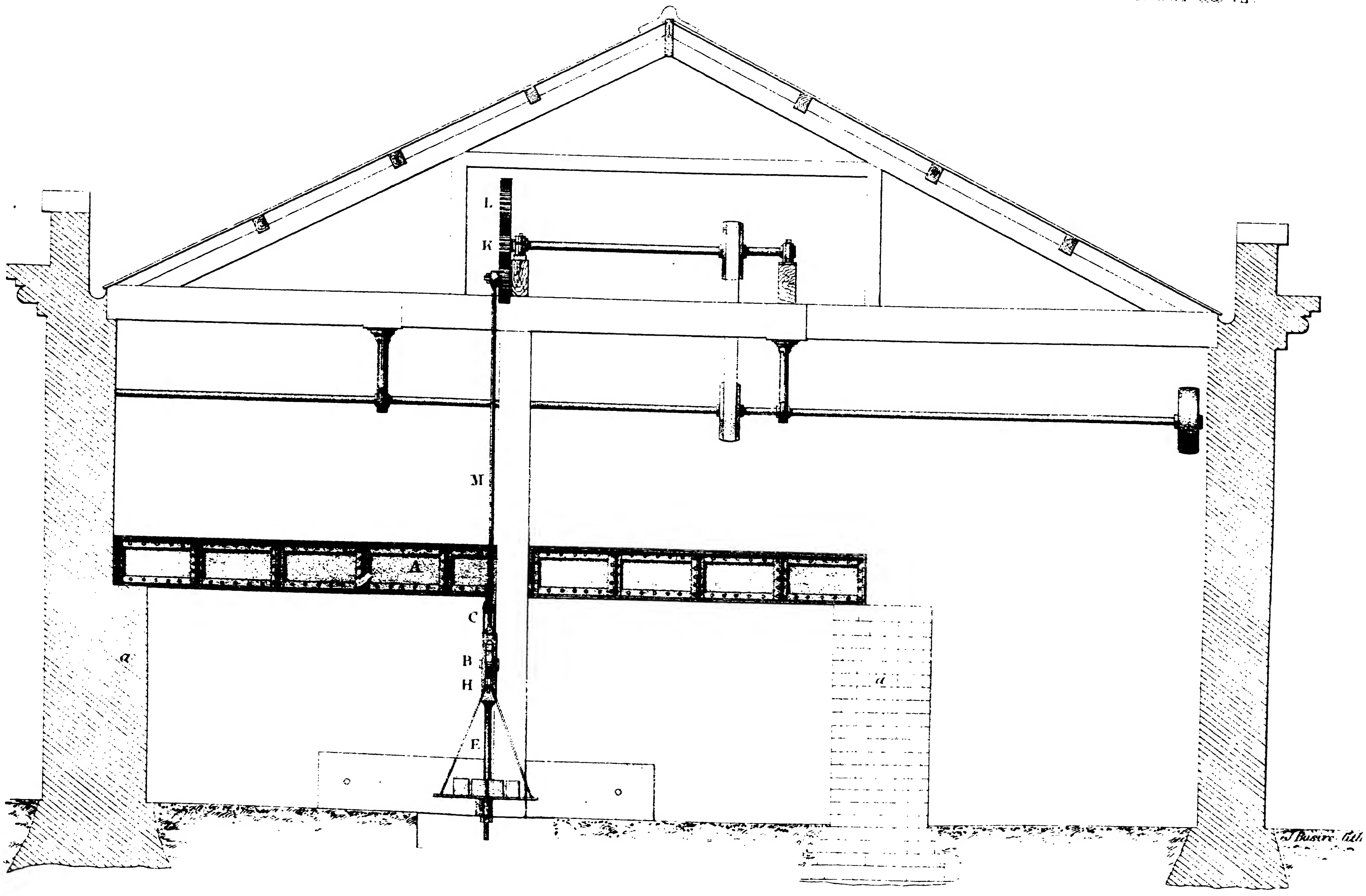
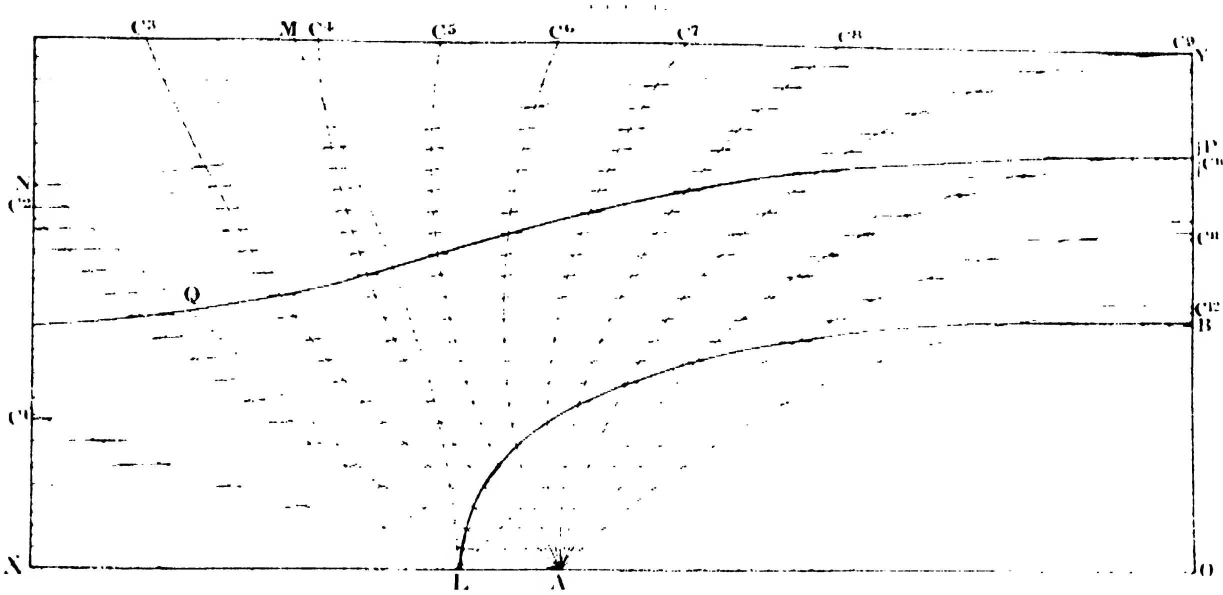
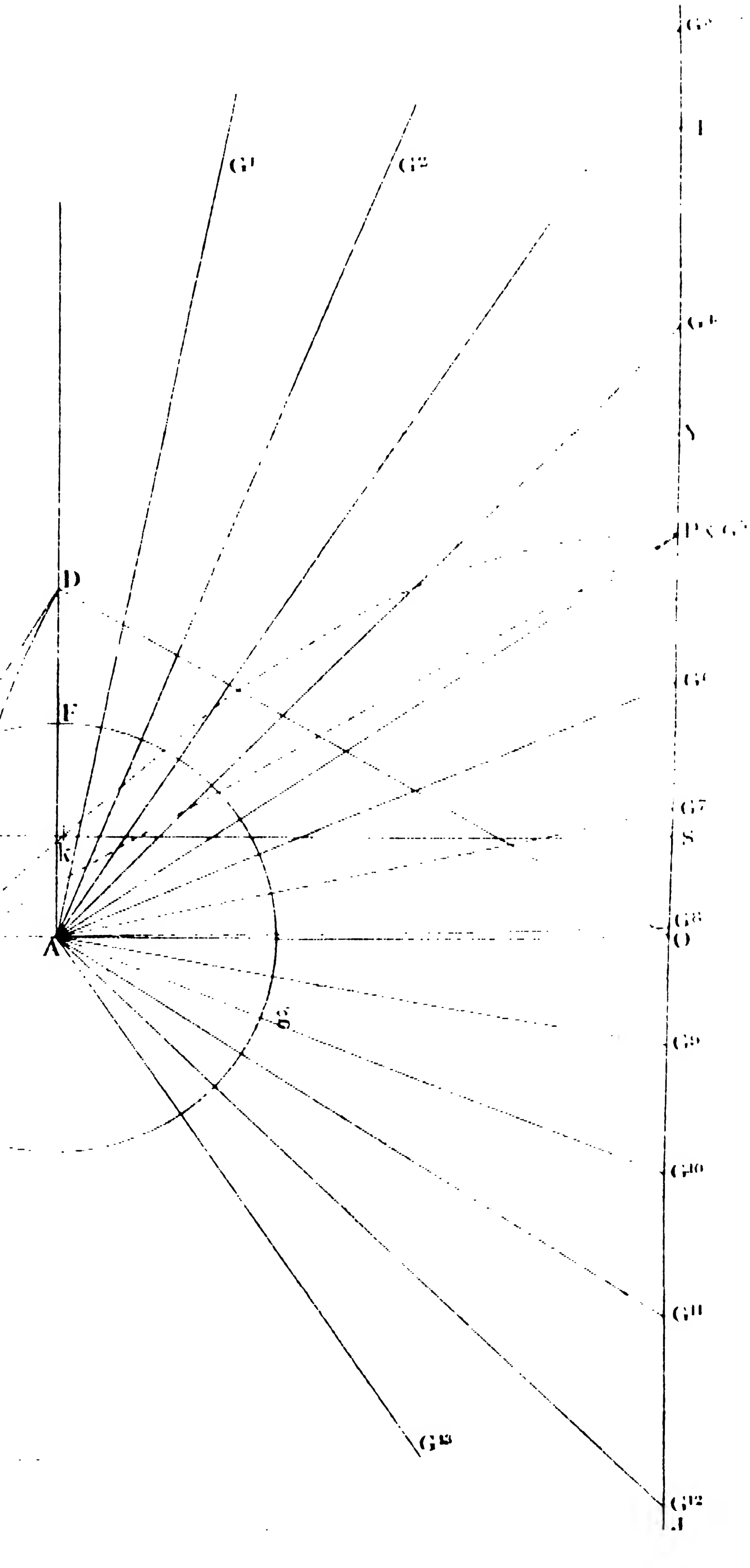
M
J. Basire lith.

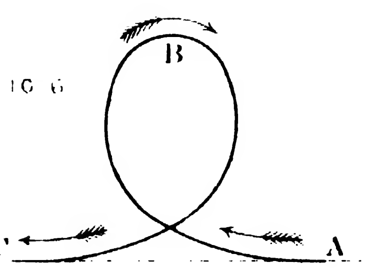

L
K
M
A
C
B
H
E
a
a
J. Basire lith.





Drawn by W. J. M. R.

FIG. 6



RANKINE ON PLANE WATER-LINES.

FIG.

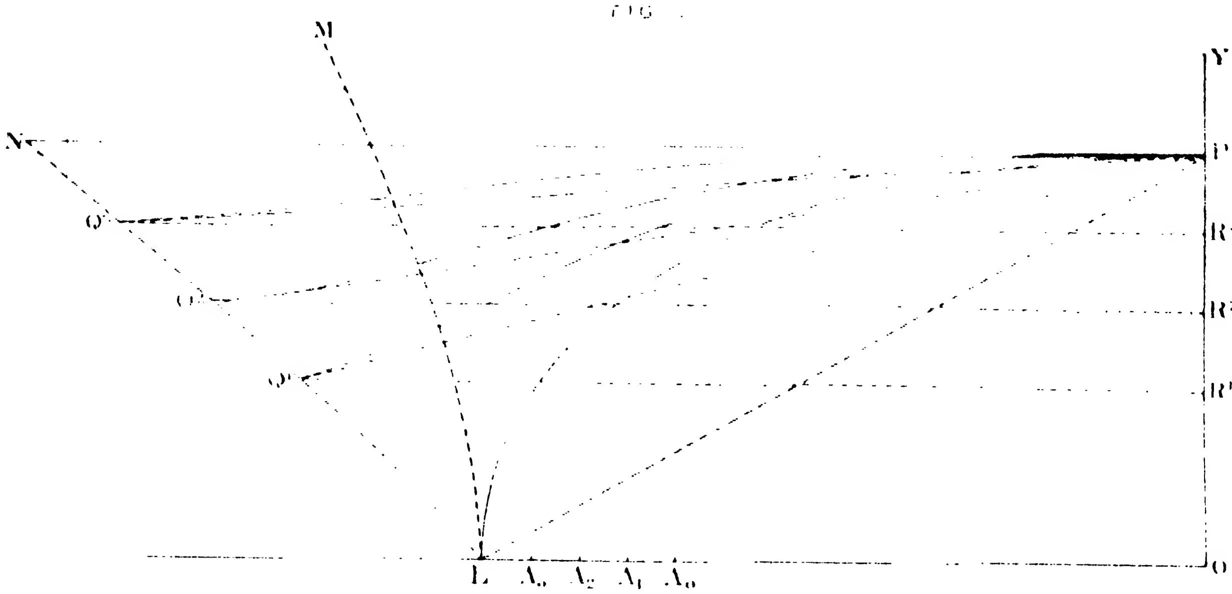


FIG. 4

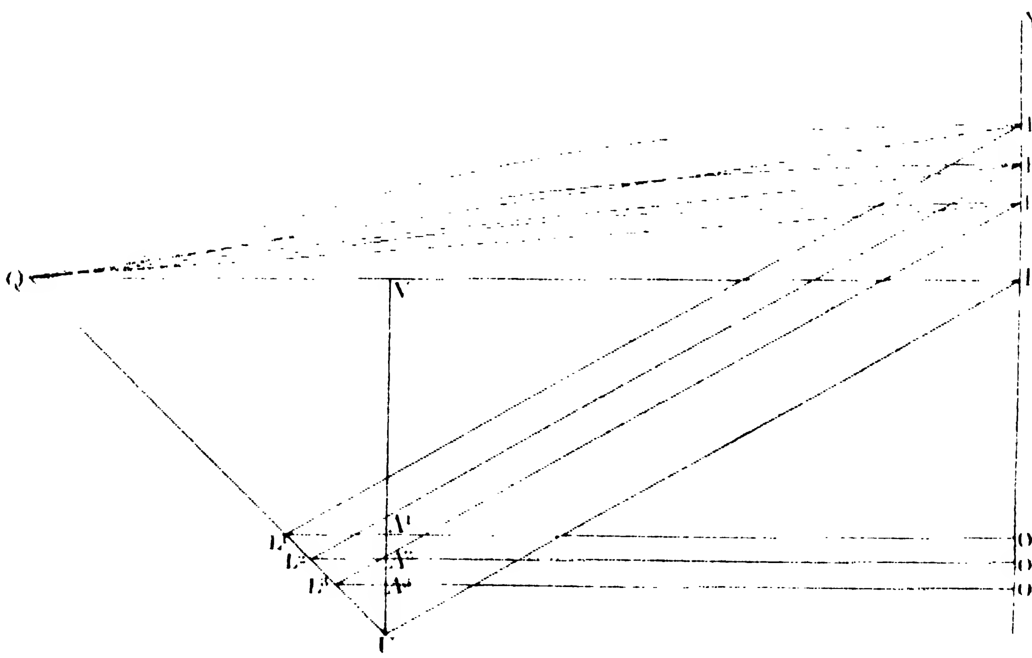
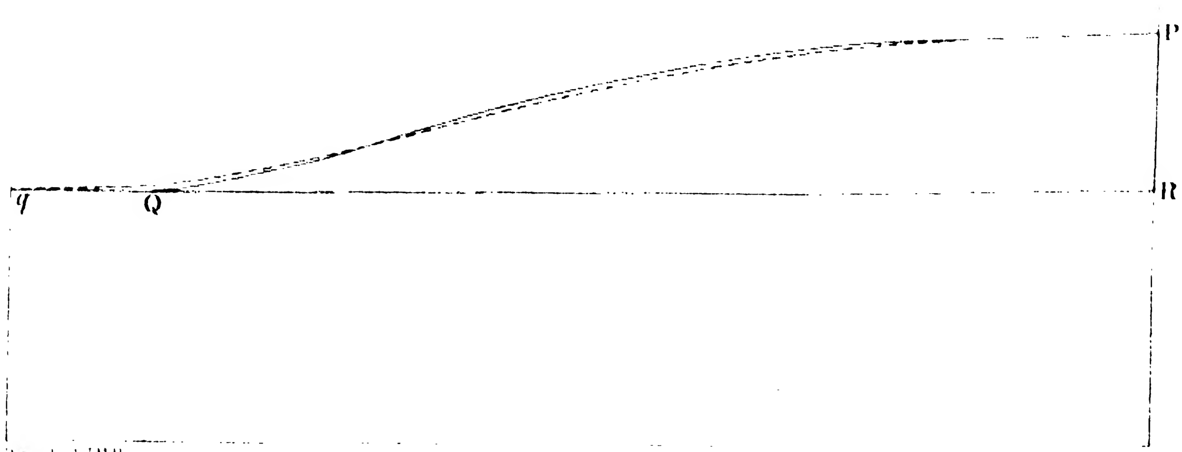


FIG. 5



Drawn by W.J.M.R.

J. Basire lith.

Fig. 1.

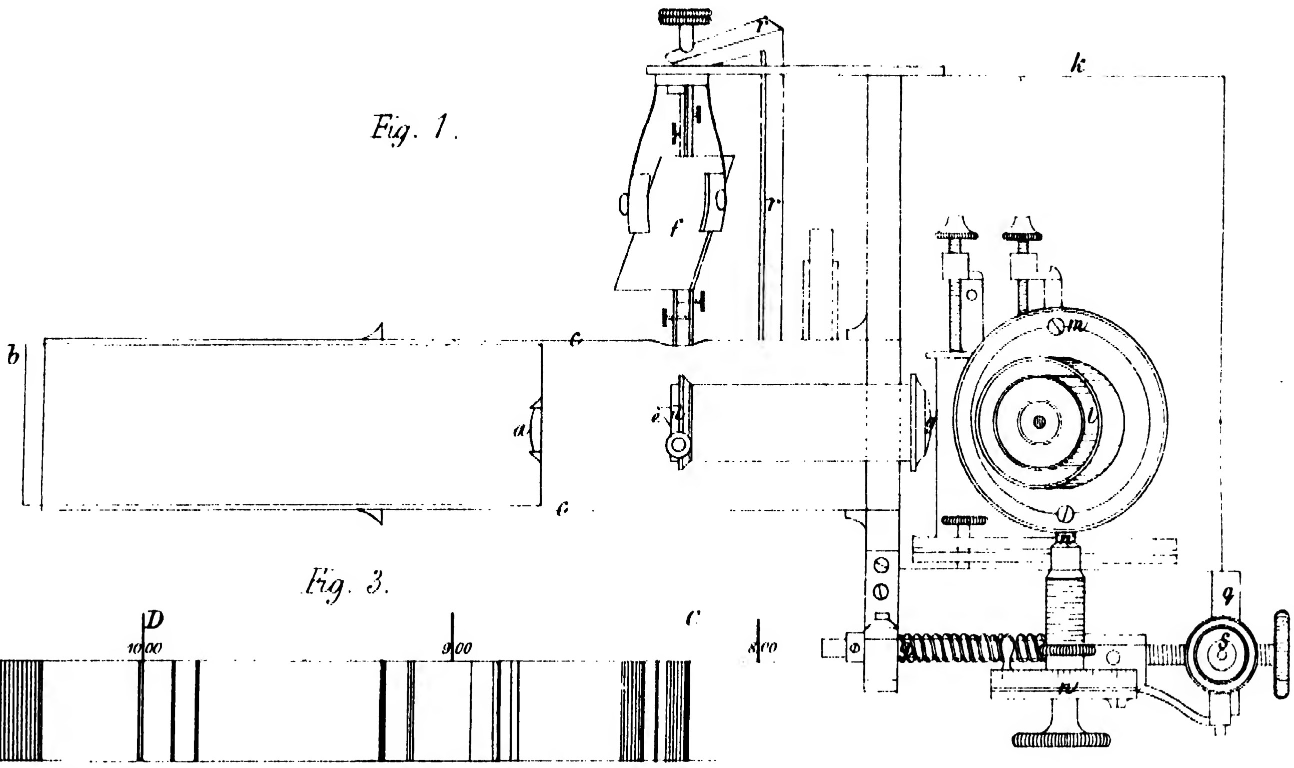


Fig. 3.

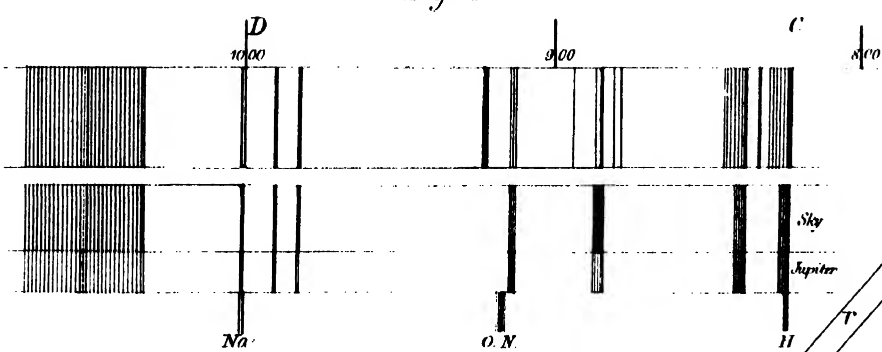


Fig. 2.

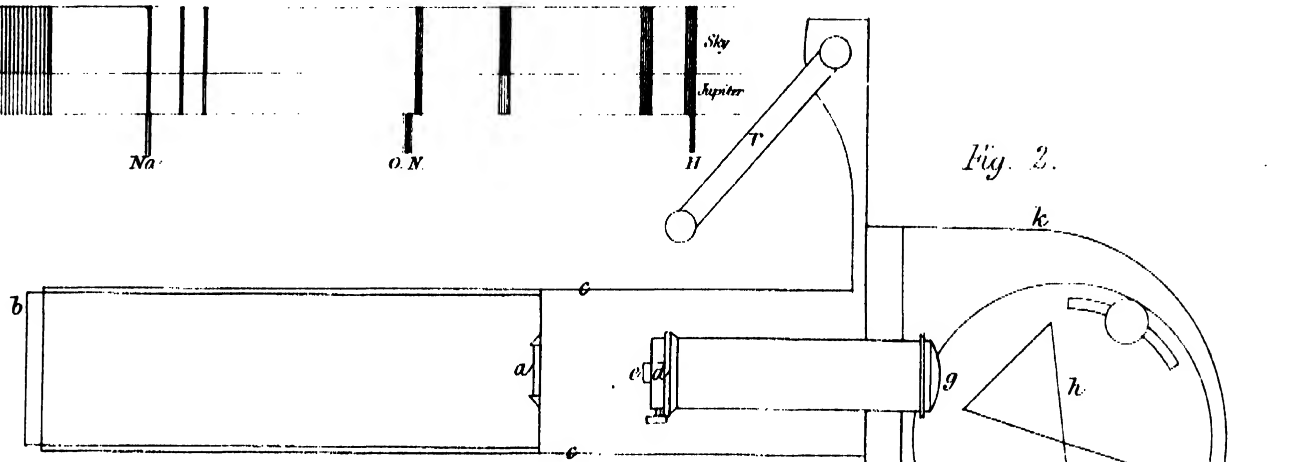
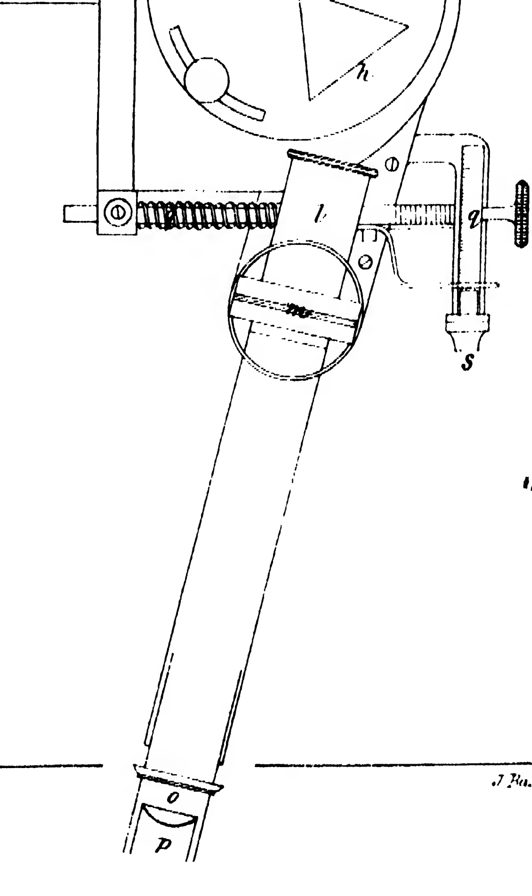
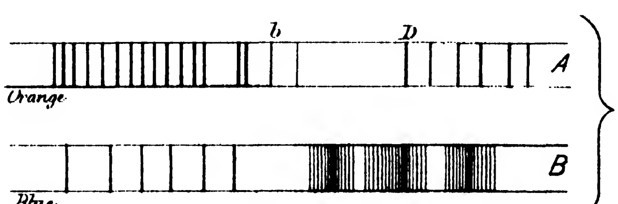
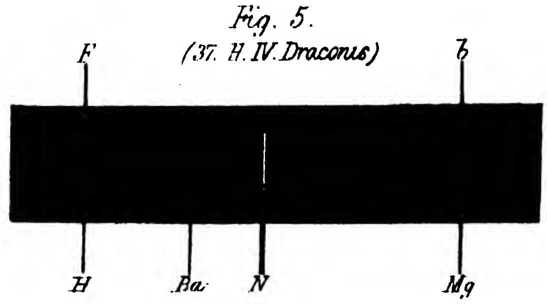



Fig. 4.



β Cygni.

Fig. 5.
(37. H. IV. Draconis)



J. Basire, lith.

XIV. *On the Arrangement of the Muscular Fibres in the Ventricles of the Vertebrate Heart, with Physiological Remarks.* *By* JAMES BELL PETTIGREW, *M.D. Edin.; Assistant in the Museum of the Royal College of Surgeons of England; Extraordinary Member and late President of the Royal Medical Society of Edinburgh, &c. &c. Communicated by* JOHN GOODSIR, *Esq., F.R.SS. L. and E., Professor of Anatomy in the University of Edinburgh.*

Received March 26,—Read April 23, 1863.

THE principal part of the following communication was presented to the Royal Society of London, in November 1859, and formed the subject of the Croonian Lecture for 1860. An abstract of it was published in the 'Proceedings' of the Society for April of that year. It was subsequently withdrawn for extension and revision, and I have to express my regret that the time occupied in this work has, from various unforeseen causes, been much longer than I anticipated.

The paper, as now presented, consists of four parts or sections,—the first section being devoted to the anatomy of the ventricle of the fish; the second to the anatomy of the ventricle of the reptile; the third and fourth treating of the ventricles of the bird and mammal. I have adopted this arrangement, because the structure of the ventricle in the fish and reptile is to a certain extent rudimentary, and a knowledge of it forms an appropriate introduction to the more intricate structure met with in the ventricles of the bird and mammal.

Of the ventricles more particularly examined in the fish, may be enumerated those of the salmon, shark, sunfish, fishing frog, turbot, and cod; in the reptile, those of the frog, turtle, tortoise, snake, and alligator; in the bird, those of the duck, goose, swan, turkey, capercailzie, eagle, and emu; and in the mammal, those of man, the mysticetus, dugong, porpoise, seal, armadillo, lion, giraffe, camel, horse, ox, ass, sheep, hog, hedgehog, dog, and deer.

VENTRICLE OF THE FISH.

In the fish, as is well known, the heart consists of two portions, one auricle and one ventricle. The shape of the ventricle in the salmon, which has been selected as typical of this division of vertebrate animals, is that of a three-sided pyramid, the base of which is perforated by two openings. These openings conduct to a conical-shaped ventricular cavity of comparatively small dimensions, the capacity of which is increased by its giving off numerous canals, which tunnel the ventricular wall in all directions, particularly towards the apex. The walls of the ventricle are accordingly of great thickness, and destitute of that solidity which characterizes the walls of the ventricles of the bird and mammal.

The fibres composing the ventricle of the fish consist of three layers—an external layer, a central transverse layer, and an internal one. The fibres of the external layer issue from the auriculo-ventricular opening and the opening for the bulbus arteriosus—some arising from the tendinous rings surrounding these apertures, others being continuous with corresponding fibres in the interior. Their course on the base is from before backwards, and is more or less circular; *i. e.* they flow in curves from either side of the auriculo-ventricular and arterial openings, towards the basal margin, over which they bend in graceful folds, to appear on the anterior and posterior borders and surfaces. On the borders, especially the posterior ones, they arrange themselves in parallel lines, and are continuous with each other at the angles and at the apex, where they are also continuous with the fibres of the internal layer. On the surfaces, the fibres of the superficial layer pursue a somewhat vertical direction, a certain number of them curving slightly upon themselves, and dipping beneath others having a more superficial position. The object of this arrangement is, to permit the superficial layer to furnish fibrous filaments which traverse the wall of the ventricle in a direction from without inwards, and which from this circumstance may be designated the perforating fibres. These perforating fibres connect the external and internal layers with each other, and with the fibres of the transverse or circular layer. Their function is obviously to approximate the various layers during the contraction of the ventricle, this approximation being rendered necessary by the ventricular walls of the fish, as has been explained, being tunnelled in various directions by canals proceeding from the ventricular cavity—these requiring to be emptied of blood during the systole.

When the external layer, which in the fish is comparatively thin, is removed, the transverse layer, with here and there the cut ends of the perforating fibres, is exposed. The fibres of the transverse layer are more or less circular, and differ from the fibres of the superficial layer in running at right angles to them; they moreover occur in fasciculi arranged in parallel lines, and may be readily separated from each other. The transverse layer is of considerable thickness, and is connected with the external and internal ones by fibres which it gives off to, and receives from both. The appearance presented by the ventricle of the fish, when the transverse layer is taken away, is somewhat porous, owing to the ends of the perforating fibres, and the orifices of the canals which permeate the substance of the ventricle from within, being at this stage of the dissection exposed. On tracing the perforating fibres whose ruptured extremities are thus brought into view, their connexion with the fibres constituting the internal layer may be clearly made out.

The fibres of the internal layer are continuous, in many instances, with the external fibres at the base, apex, and other portions of the ventricular wall, and are best exposed by cutting into the ventricular cavity, and dissecting from within outwards. They proceed in well-marked fascicular bundles from apex to base, and resemble in their general character the carneæ columnæ of the ventricles of the bird and mammal. They differ, however, from the carneæ columnæ in question in having a more highly reticulated structure. Situated within the reticulations, are a vast number of minute orifices commu-

nicating with the ventricular cavity, and conducting to canals of various sizes. These canals freely unite with each other, and, as they penetrate the ventricular wall only to certain depths, may on this account be denominated terminal canals. They serve to increase the size of the ventricular cavity, and render the ventricle lighter than it would otherwise be.

The ventricle of the fish may be regarded as a conical-shaped muscular bag, the fibres of which are curiously interwoven to secure the greatest amount of strength with the least possible material, and, what is not less desirable in a physiological point of view, to ensure that the organ shall contract in all directions, the more thoroughly to eject the blood from its interior. There is, however, in the ventricle under consideration, as far as I have been able to discover, no principle in the arrangement similar to that which, as I shall endeavour to explain, occurs in the ventricles of the higher vertebrata.

VENTRICLE OF THE REPTILE.

The form of the ventricle of the reptile's heart is intermediate between the well-defined pyramidal shape in the fish, and the finely rounded conical form of the ventricles in the bird and mammal. Thus it has the dorsal surface flattened, as in the fish, while the two anterior surfaces present a somewhat convex outline.

The arrangement of the fibres in the ventricle of the reptile is so similar in many respects to that met with in the ventricle of the fish, that a separate description appears unnecessary. There are, however, points of difference deserving of notice. The fibres composing the external and internal layers in the ventricle of the reptile are more decidedly vertical than in that of the fish, and run with fewer interruptions from the base to the apex, and from the apex to the base. This difference is well seen in the ventricle of the python and alligator, and is an approach to what is found in similar layers in the ventricles of the bird and mammal. The external layer in the ventricle of the reptile is thinner than in the ventricle of the fish, the internal layer being comparatively much thicker. The transverse layer, in the ventricle of the reptile, is also thinner than in the ventricle of the fish, the perforating fibres which run between the external and internal layers being on the contrary increased both in number and size. The perforating fibres have further a tendency to split up and give offsets to run athwart the ventricle in the direction of the transverse layer, and in this manner supply the deficiency in its thickness. The ventricular cavity of the reptile is smaller than in the fish, while the number of terminal canals which proceed from it to ramify in the ventricular wall is greater. This circumstance renders the wall of the ventricle of the reptile at once thicker and less dense than that of the fish. Lastly, the ventricular cavity of the reptile is variously shaped, according as the septum is absent or present and partially or fully developed.

VENTRICLES OF THE BIRD.

The ventricles of the bird so closely resemble those of the mammal in appearance and structure, that one description will suffice for both. Care however will be taken,

when explaining the construction of the right ventricle of the mammal, to discuss at length the peculiar fleshy valve which in the bird occupies the right auriculo-ventricular orifice. This valve constitutes, I may remark, the distinguishing feature between the ventricles of the bird and mammal.

VENTRICLES OF THE MAMMAL.

The ventricles in the mammal are subject to considerable variation as regards shape. In the porpoise they have their dorsal or posterior surface flattened, and their anterior surface very slightly rounded, as in the higher reptiles.

In the mysticetus the ventricles are compressed laterally, and so resemble those of the higher fishes. In the dugong and rhytina, they are characterized by having two very distinct and widely separated apices, as shown in Plate XIV. fig. 42.

The general appearance presented by the ventricles of the mammal is familiar to all, it being that of an irregular cone slightly twisted upon itself. The posterior surface of the cone is flattened, as in the ventricle of the fish and reptile, and on account of the obliquity of the base is shorter than the anterior surface. The anterior surface, which is divided into two by an oblique sulcus or furrow, is, on the contrary, rounded and prominent, and presents a characteristic convex outline. Two margins or borders are usually described—a right inferior or acute margin (margo acutus), and a left superior or obtuse one (margo obtusus). As, however, these margins vary somewhat in different hearts, no general description concerning them can be strictly applicable. In the hearts of the ass, American elk, and deer tribe generally, the ventricles are rounded and taper towards the apex, so that the right margin appears almost as obtuse as the left; while in those of the armadillo and carnivora, the ventricles, which are not taper but purse-shaped, have likewise the right side very obtuse*. The idea therefore of an acute margin ought perhaps to be confined to the right side of the human heart and a few others, such as the heart of the seal and hog, both of which bear a considerable resemblance to that of man.

The base of the cone formed by the ventricles of the mammal, as is well known, is perforated by four openings. These openings are surrounded by fibrous rings, and conduct to conical-shaped *spiral* cavities†, which vary somewhat in size according as the carneæ columnæ are absent‡ or present, and the musculi papillares feebly or fully developed.

Considered as a muscle, the heart, and especially the ventricular portion of it, is peculiar. Being in the strictest acceptation of the term an involuntary muscle, its fibres nevertheless possess the dark colour, and transverse markings, which are charac-

* Between these extremes in shape may be ranked the ventricles of the camel, horse, ox, giraffe, calf, hare, rabbit, &c.

† See photograph of a wax cast of the interior of the ventricles of a deer's heart, Plate XII. figs. 16 & 17, and transverse sections, Plate XV. figs. 49 to 53 inclusive.

‡ The ventricles of the American elk are devoid of carneæ columnæ, as are likewise those of the red deer (Plate XV. fig. 48).

teristic of the voluntary muscles. Unlike the generality of voluntary muscles, on the other hand, the fibres of the ventricles, as a rule, have neither origin nor insertion; *i. e.* they are continuous alike at the apex of the ventricles and at the base. They are further distinguished by the almost total absence of cellular tissue as a connecting medium *—the fibres being held together partly by splitting up and running into each other, and partly by the minute ramifications of the cardiac vessels and nerves †.

The manner in which the fibres are attached to each other, while it necessarily secures to the ventricles considerable latitude of motion, also furnishes the means whereby the fibres composing them may be successfully unravelled; for it is found that by the action of certain reagents, and the application of various kinds of heat, as in roasting and boiling ‡, the fibres may be prepared so as readily to separate from each other, in layers of greater or lesser thickness.

The crowning difference, however, and that which it is the especial object of the present paper to treat, is the arrangement of the fibres themselves—an arrangement so unusual and perplexing, that it has long been considered as forming a kind of Gordian knot in anatomy. Of the complexity of the arrangement I need not speak, further than to say that VESALIUS, ALBINUS, HALLER §, and DE BLAINVILLE ‖ all confessed their inability to unravel it.

Of those who have written more particularly on the structure of the mammalian heart, may be mentioned LOWER ¶ (1669), BARTHOLIN ** (1678), WINSLOW †† (1711),

* The little cellular tissue there is, is found more particularly at the base and apex of the ventricles, and is so trivial as to be altogether, though wrongly, denied by some. See article "On the Fibres of the Heart," by Mr. SEARLE, in the 'Cyclopædia of Anatomy and Physiology,' p. 652.

† When the vessels of the ventricles are injected in the cold state with some material which will stand heat (as, for example, a mixture of starch and water), and the heart boiled, the larger trunks from either coronary artery are found to give off a series of minute branches which penetrate the ventricular wall in a direction from without inwards—these branches, when the dissection is conducted to a certain depth, appearing like so many bristles transfixing the ventricular wall. As, moreover, the cardiac nerve-trunks accompany the trunks of the coronary vessels, while the nerve-filaments cross the smaller branches of the vessels, and the muscular fibres, (to both of which they afford a plentiful supply of nerve-twigs,) the influence exerted by the vessels and nerves. in uniting or binding the muscular fibres to each other, is very considerable. Vide Inaugural Prize Dissertation by the author, "On the Arrangement of the Cardiac Nerves, and their connexion with the Cerebro-spinal and Sympathetic Systems in Mammalia," deposited in the University of Edinburgh Library, March 1861.

‡ Of the various modes recommended for preparing the ventricles prior to dissection, I prefer that of continued boiling. The time required for the human heart, and those of the small quadrupeds, as the sheep, hog, calf, and deer, may vary from four to six hours; while for the hearts of the larger quadrupeds, as the horse, ox, ass, &c., the boiling should be continued from eight to ten hours; more than this is unnecessary. A good plan is to stuff the ventricular cavities loosely with bread crumbs, bran, or some pliant material before boiling, in order, if possible, to distend without overstretching the muscular fibres. If this plan be adopted, and the ventricles soaked for a fortnight or so in alcohol before being dissected, the fibres will be found to separate with great facility. VAUST recommended that the heart should be boiled in a solution of nitre; but nothing is gained by this procedure.

§ El. Phys. tom. i. p. 351.

‖ Cours de Physiologie, &c. tom. ii. p. 359.

¶ Tractatus de Corde, &c. London, 1669.

** Dissert. de Cordis structura et usu. Hafniæ, 1678.

†† "Sur les Fibres du Cœur et sur ses Valves," Mém. de l'Acad. Roy. de Paris, 1711.

SENAC * (1749), HALLER † (1757), WOLFF ‡ (1780–1792), GERDY § (1823), and REID ‖ (1839). The writings of these investigators, although differing in minor matters, agree on the whole, as may be seen by a hasty reference to the more prominent views entertained by them. As early as 1669, Dr. RICHARD LOWER promulgated the idea that the external fibres of the ventricles of the mammal proceed in a spiral direction from left to right downwards; the internal fibres proceeding from left to right upwards. The fibres, according to this author, are continuous at the apex, and form an imperfect figure of 8. In this opinion LOWER was followed by GERDY, who, however, maintained that the external and internal fibres make a more perfect figure of 8 than that given by LOWER, and added that all the fibres of the heart form loops, the apices of which look towards the apex of the heart. This idea of GERDY'S with reference to the looped arrangement of the fibres at the apex was combated in recent times by Dr. DUNCAN, jun., of Edinburgh ¶, who says, GERDY commits a grave error when he asserts that all the fibres of the heart form loops *the apices of which look towards the apex of the heart*, since the number of tops (and by this Dr. DUNCAN means loops) *which look in the opposite direction, or towards the base*, is not less. Adopting the suggestion of Dr. DUNCAN, it follows that the fibres composing the ventricles form twisted loops, *which look alike towards the apex and the base*. FREDERICK CASPAR WOLFF furthered the investigation, by showing the possibility of dividing the muscular substance composing the ventricles into bands; whilst SENAC in the last century, and Dr. JOHN REID in this, gave a new interest to the subject, by showing that the fibres of the external and internal surfaces of the ventricles are more vertical in direction than the deeper or more central fibres, which more approach to the circular. Such are a few of the more important facts ascertained by successive investigators.

Having myself in the summer of 1858 made numerous dissections, upwards of 100 of which are preserved in the Anatomical Museum of the University of Edinburgh **, I have arrived at results which appear to me to throw additional light on this complex question, and which seem to point to a law in the arrangement, simple in itself, and apparently comprehensive as to detail. This law will be adverted to subsequently.

Summary of Facts established in the present Memoir.

The following are a few of the more salient points demonstrated, which the reader

* Traité de la structure du Cœur, de son action, &c. Paris, 1749.

† Elementa Physiologiæ, tom. i. 1757.

‡ "Dissertationes de ordine fibrarum musculariun Cordis," in Acta Acad. Petropolit. 1780–1792.

§ Recherches, Discussions et Propositions d'Anatomie, Physiologie, &c. 1823.

‖ Cycl. of Anat. and Phys., article "Heart." London, 1839.

¶ See extract from Dr. DUNCAN'S unpublished manuscript, given by Dr. JOHN REID in Cycl. of Anat. and Phys., article "Heart," p. 592.

** These dissections obtained the Senior Anatomy Gold Medal of the University, in the winter of 1859.

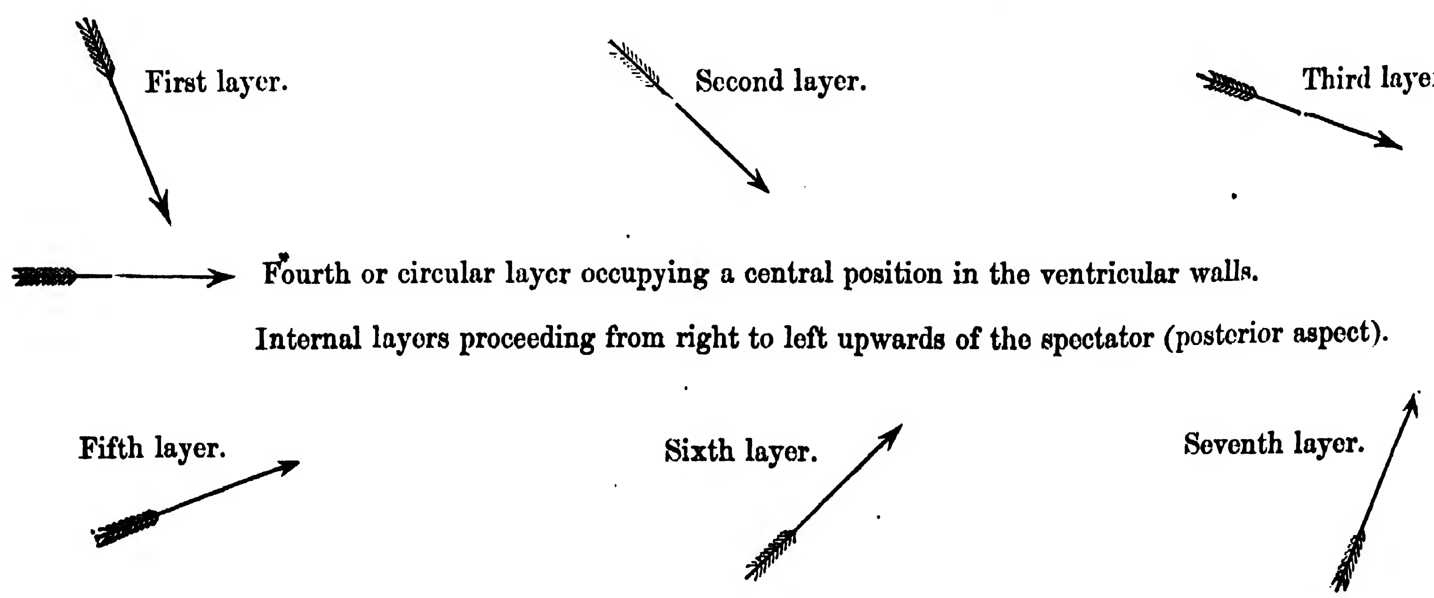
may corroborate by a reference to the accompanying Plates, engraved from photographs taken by myself from the dissections.

I. By exercising due care, I have ascertained that the fibres constituting the ventricles are rolled upon each other in such a manner as readily admits of their being separated by dissection into layers or strata, the fibres of each layer being characterized by having a different direction.

II. These layers, owing to the difference in the direction of their fibres, are well marked, and according to my finding, are seven in number—viz. three external, a fourth or central, and three internal.

III. There is a gradational sequence in the direction of the fibres constituting the layers, whereby they are made gradually to change their course from a nearly vertical direction to a horizontal or transverse one, and from the transverse direction, back again to a nearly vertical one. Thus, in dissecting the ventricles from without inwards, the fibres of the first layer, which run in a spiral direction from left to right downwards, are more vertical than those of the second layer, the second than those of the third, the third than those of the fourth—the fibres of the fourth layer having a transverse direction, and running at nearly right angles to those of the first layer. Passing the fourth layer, which occupies a central position in the ventricular walls and forms the boundary between the external and internal layers, the order of arrangement is reversed, and the fibres of the remaining layers, viz. five, six, and seven, gradually return in an opposite direction, and in an inverse order, to the same relation to the vertical as that maintained by the fibres of the first external layer. This remarkable change in the direction of the fibres constituting the several external and internal layers, which is observed to occur in all parts of the ventricular walls, whether they be viewed anteriorly, posteriorly, or septally, has in part been figured by SENAC *, and imperfectly described by REID †, but has not, so far as I am aware, been prominently brought forward by any one. A few arrows will illustrate the gradation in direction referred to.

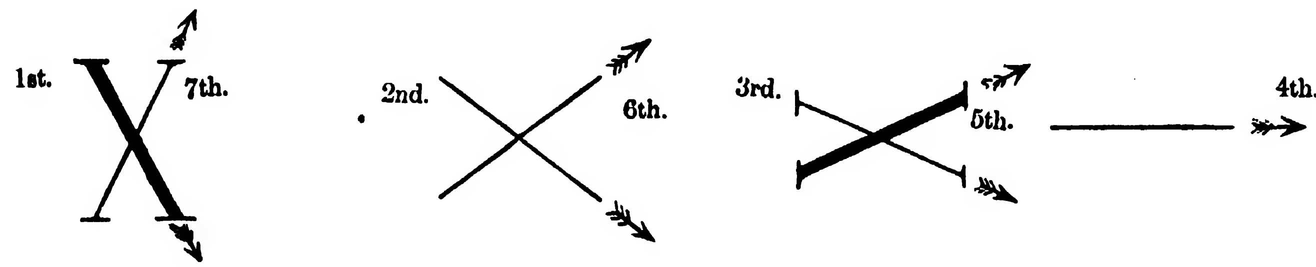
External layers proceeding from left to right downwards of the spectator (posterior aspect).



* *Op. cit.* tom. i. and Appendix to tom. ii.

† *Op. cit.* p. 591.

IV. The fibres composing the external and the internal layers are found at different depths from the surface, and from the fact of their pursuing opposite courses cross each other,—the fibres of the first external and last internal layers crossing with a slight deviation from the vertical, as in the letter X; the succeeding external and internal layers, until the fourth or central layer, which is transverse, is reached, crossing at successively wider angles, as may be represented by an ⋈ placed horizontally:—



V. The fibres composing corresponding external and internal layers, such as layers one and seven, two and six, &c., are continuous in the left ventricle at the left apex, and in the right ventricle in the track for the anterior coronary artery, the fibres of both ventricles being for the most part continuous likewise at the base *.

VI. From this distribution of the fibres, it follows that the first and seventh layers embrace in their convolutions those immediately beneath them, while these in turn embrace those next in succession, and so on until the central layer is reached—an arrangement which may in part explain, alike, the rolling movements and powerful action of the ventricles.

VII. The fibres of the right and left ventricles anteriorly and septally are to a certain extent independent of each other; whereas posteriorly many of them are common to both ventricles; *i. e.* the fibres pass from the one ventricle to the other,—an arrangement which induced WINSLOW † to regard the heart as composed of two muscles enveloped in a third. It will be evident from this distribution of the fibres, that while the ventricles are for obvious reasons intimately united, they nevertheless admit of being readily separated.

VIII. If the hinge-like mass of fibres (common fibres) which unite the right ventricle to the left posteriorly be cut through, and the right ventricle with its portion of the septum detached, the left ventricle will be found to be nearly as complete as it was before the separation took place, and to consist of four sets of conical spiral fibres—two external and two internal sets.

On the other hand the right ventricle, and its share of the septum, consists only of conical-shaped spiral fragments of fibres, or at most of flattened rings—a circumstance which, when taken in connexion with others to be mentioned presently, has induced me

* The late Dr. DUNCAN, jun., of Edinburgh, was aware of the fibres forming loops at the base, but seems to have had no knowledge of the continuity being occasioned by the union of the fibres of corresponding external and internal layers, or that these basal loops were prolongations of like loops formed by similar corresponding external and internal layers at the apex—a view which the author believes is here set forth for the first time.

† Mémoires de l'Académie Royale des Sciences, 1711, p. 197.

to regard the left ventricle as the typical or complete one, the right ventricle being a mere segment or portion nipped off at some period or other from the left.

IX. If the right ventricular walls be cut through immediately to the right of the track for the anterior and posterior coronary arteries, so as to detach the right ventricle without disturbing the septum, and the septum be regarded as forming part of the left ventricular wall, it will be found that the fibres from the right side of the septum, at no great depth from the surface, together with the external fibres from the left ventricular wall generally, enter the left apex in two sets; and if their course in the interior be traced, they are observed to issue from the left auriculo-ventricular opening, also in two sets; in other words, the left ventricle is bilateral. I would particularly direct the attention of investigators to this bilateral distribution of the fibres, as it has been hitherto overlooked, and furnishes the clue to the arrangement of the fibres of the left ventricle.

X. The double entrance of the fibres at the left apex, and their exit in two portions from the auriculo-ventricular opening at the base, are regulated with almost mathematical precision; so that while the one set of fibres invariably enters the apex posteriorly, and issues from the auriculo-ventricular opening anteriorly, the other set as invariably enters the apex anteriorly, and escapes from the auriculo-ventricular opening posteriorly. But for this disposition of the fibres, the apex and the base would have been like the barrel of a pen cut slantingly or lopsided, instead of bilaterally symmetrical as they are.

XI. The two sets of fibres which constitute the superficial or first external layer of the left ventricle, and which enter the left apex in two separate portions or bundles, are for the most part continuous in the interior with the musculi papillares, to the free ends of which the chordæ tendineæ are attached. These columns occupy different portions of the left ventricular cavity, and give a very good idea of the symmetry which prevails throughout the left ventricular walls.

Lastly. The apex is opened into and enlarged, and the auriculo-ventricular orifice widened, by the removal of consecutive external and internal layers, from the fact of the left ventricular cavity tapering in two directions and forming a double cone.

There are other points worthy of mention, such as the construction of the septum, fleshy pons, and conus arteriosus, the varying thickness of the right and left ventricular walls, the shape of the right and left ventricular cavities, &c. To these, however, allusion will be more conveniently made subsequently.

As the structure of the ventricles, with one or two exceptions, is the same in all mammals, *man included*, I have chosen to describe the arrangement of the fibres in the ventricles of the sheep and calf, from the readiness with which the hearts of these animals may be obtained. My descriptions, however, will by no means be confined to them.

Points to be attended to in the dissection of the left or typical ventricle. The points to be kept more particularly in view when dissecting the left ventricle are these:—

1st. The different angles made by the fibres of the several layers, with an imaginary vertical line drawn from base to apex, as they issue from the auriculo-ventricular opening and enter the apex.

2nd. The varying direction of the fibres on the body of the ventricle, produced by the different angles which the fibres of the several layers make with the imaginary vertical referred to.

3rd. The double entrance of the fibres at the apex, and their exit in two sets from the left auriculo-ventricular opening.

4th. The manner in which the apex is opened into, and enlarged, by the removal of successive layers.

5th. The gradual increase in the thickness of the layers when the dissection is conducted from without inwards. This is in all probability owing to the more internal or deeper layers being the first formed. Thus the seventh or deepest internal layer, which I am inclined to think is developed before the sixth, goes on increasing *pari passu* with it; while the seventh and sixth increase equally with the fifth, which is a later formation; and so on until the first, which is the thinnest layer, is reached. That this explanation has its foundation in truth, is probable from the fact that, in imitating the process by which I believe the left ventricle is formed, the seventh or most internal layer supplies a basis of support for the more superficial layers—just in the same way that the smaller and more central turns of a shell form the basis of support for the peripheral or more superficial turns*.

External layers of the left ventricle of the Mammal.

Superficial or first external layer. On looking at the left auriculo-ventricular opening of the sheep and calf posteriorly (Plate XII. fig. 1, *b*, and Plate XV. fig. 46, *b*), when the serous membrane, fat, vessels, and nerves have been removed, the fibres are seen to issue from it in fascicular bundles, and to curve over its margin all round (*d*, *f*).

The fibres on leaving the opening lose to a considerable extent their fascicular character, and naturally arrange themselves in two sets,—the one set proceeding from the anterior portion of the opening (*d*), and the anterior half of the septum (*e*); the other from the posterior portion of the opening (*f*), and the posterior half of the septum (*g*). On the body of the ventricle the fibres spread out to form a smooth muscular sheet, both sets pursuing a spiral nearly vertical direction from left to right of the spectator. In their course they gradually change their position on the ventricular wall, the fibres from the anterior portion of the opening and the anterior half of the septum winding round and appearing on the *posterior surface*, the fibres from the posterior portion of the opening and the posterior half of the septum, on the contrary, winding round and appearing on the *anterior surface*. On nearing the apex, which they do in graceful curves (Plate XII. fig. 9), the two sets of fibres become more strongly defined, the fibres of either set converging and overlapping to a greater or less extent. At the apex both sets (*f g*, *e d*) curve rapidly round and form a whorl or vortex of great beauty,—the fibres from the anterior portion of the opening and septum curving into those from

* For a detailed account of this view, see p. 484.

the posterior portion of the opening and septum, and entering the apex *posteriorly* (Plate XII. fig. 10, *d*), to become continuous with the fibres of the carneæ columnæ and anterior musculus papillaris* (Plate XII. fig. 13, *y*); the fibres from the posterior portion of the opening and septum curving into those from the anterior portion of the opening and septum, and entering the apex *anteriorly* (Plate XII. fig. 10, *g*), to become continuous with the fibres of the carneæ columnæ and posterior musculus papillaris* (Plate XII. fig. 13, *x*).

The fibres, therefore, which issue from the auriculo-ventricular orifice anteriorly, enter the apex posteriorly, and *vice versâ*—an arrangement which is accounted for by the fibres of the superficial or first external layer, from the time they leave the base until they reach the apex, making one turn and a half of a spiral. As the fibres of the carneæ columnæ and musculi papillares, which constitute the seventh or last internal layer, also make a turn and a half, from the time they leave the apex until they reach the base, the external and internal fibres always return to points not wide of those from which they started. It was, no doubt, this circumstance which induced LOWER and GERDY to describe the external and internal fibres as forming a more or less perfect figure of 8†, these investigators differing as to the completeness of the figure from having, in all probability, described different and deeper layers‡. The bilateral distribution of the fibres, which extends to all the layers, has hitherto escaped observation, but is clearly established by my dissections. Viewed in connexion with the musculi papillares and the segments of the bicuspid valve, it is, as I shall endeavour to show, of considerable physiological importance. The object of the two sets of fibres curving into each other at the apex §, is evidently threefold: first, to secure symmetry, structural and

* The fibres of the carneæ columnæ and musculi papillares pursue a spiral nearly vertical direction, from right to left upwards, so that they cross the fibres of the superficial or first external layer; for an explanation of the course and direction of the fibres of the first and seventh layers, see A, B, C, D, E of diagrams 3 & 4, Plate XVI.

† WINSLOW altogether, though wrongly, denied the crossing of the external and internal fibres (Mémoires de l'Acad. Roy. 1710, p. 197).

‡ Great assistance may be obtained in comprehending the scheme of the arrangement of the fibres, by occasionally referring to the diagrams contained in Plate XVI. In these diagrams the fibres are represented by lines drawn at intervals, the object being to furnish the reader with a transparent ventricle, which will enable him to analyze its structure by tracing the fibres composing the several layers throughout their entire extent. Thus at A A′ of diagram 4, the fibres of the superficial or first external layer are indicated, the fibres of the seventh or last internal layer being seen at E E′ of diagrams 4 & 6. In diagram 5 the fibres of the second layer are represented by the lines marked B B′, the fibres of the sixth or corresponding layer being marked D D′. In diagram 4 the fibres of the third and fifth layers are marked B and D, and the fibres of the fourth or central layer are seen at C of diagrams 4 & 6, and at C C′ C″ of diagram 5. In diagram 3 the lines are drawn at still wider intervals, and show how the external fibres A, B, become internal (D, E) by turning upon themselves at the apex C, where they also enter the interior. Diagrams 7 & 10 show how the external fibres A B, C D enter the apex in two sets at opposite points, viz. at E and F, while diagram 11 shows how the internal fibres are arranged in two sets (X and Y) in the interior.

§ The auriculo-ventricular orifice at the base is also closed by two symmetrical structures, viz. the anterior and posterior segments of the bicuspid valve.

funçtional; second, to obtain for the apex, which is the weak part of the ventricular wall, great strength with comparatively little material *; and third, to procure for the fibres constituting the external and internal layers, which envelope all the others, a latitude and universality of motion which go far to account for the freedom and force with which the left ventricle contracts.

The fibres of the superficial or first external layer arise, as a rule, from the fibrous ring surrounding the aorta (Plate XII. fig. 1, *a*, and Plate XV. fig. 46, *a*), and from the auriculo-ventricular tendinous ring (Plate XV. fig. 46, *n*). A few, however, are continuous, beneath the auriculo-ventricular tendinous ring, with the fibres of the carneæ columnæ † (Plate XV. fig. 47, *d*).

The fibres of the superficial or first external layer, with their internal continuations the fibres of the seventh or last internal layer, overlap and embrace the fibres of all the other layers, viz. those of two, three, four, five, and six,—an arrangement due, not, as was supposed, to the greater length of the fibres composing the first external and the last internal layers, but to the direction and position of the fibres of these layers on the ventricle—the fibres of the superficial or first external layer and those of the seventh or last internal one pursuing an almost vertical spiral direction on the body and wider portion of the ventricle, and appearing longer by twisting rapidly round the apex or narrow part, the fibres of the deeper layers pursuing a more oblique spiral direction on the body or wider portion of the ventricle, and appearing shorter from not reaching to the apex or narrow portion.

Second external layer of the left ventricle (*Mammal*). The fibres of the second external layer (Plate XII. fig. 2), like those of the first, advance spirally from left to right downwards in two separate sets (*f g*, *d d' e*),—the one set proceeding from rather more than the anterior half of the septum, and the anterior and inner portion of the auriculo-ventricular opening (*d*); the other from rather less than the posterior half of the septum, and the posterior and outer part of the opening (*f*). At the apex the fibres from the anterior portions of the opening and septum go to form the posterior half of the apicial orifice (Plate XII. figs. 2 & 11, *k*), where they become continuous with the anterior fibres of the sixth layer (Plate XII. fig. 6, *n n''*); while those from the posterior portions of the opening and septum enter into the formation of the anterior half of the apicial orifice (Plate XII. fig. 11, *l*), and become continuous with the posterior fibres of the sixth layer (Plate XII. fig. 6, *o o'*). I use the term apicial orifice, because the apex is opened into when the two sets of fibres which constitute the first layer

* The apex in the ventricle of even the horse does not exceed the eighth of an inch in thickness. This deficiency in the thickness of the ventricular wall, which, during the dilatation of the organ and the first stage of contraction, ensures great freedom of motion, is prevented from operating injuriously during the latter stages of contraction, by such portions of the musculi papillares as are situated at the apex plaiting into each other so as entirely to obliterate the apicial cavity.

† It is a great mistake to imagine that all the fibres of the ventricles arise from the auriculo-ventricular tendinous rings, the fact being that, with the exception of the fibres of the first and seventh layers, they are continuous beneath them. These rings are more fully developed anteriorly than posteriorly, and the ring which belongs to the left ventricle is stronger than that which belongs to the right.

(Plate XII. fig. 11, *f g*), are removed. The fibres of the second layer are similarly arranged at the apex to those of the first; *i. e.* they converge slightly and curve upon each other prior to doubling upon themselves to alter their direction and enter the interior. The fibres of the second layer differ from those of the first in being more fascicular, and in issuing from the auriculo-ventricular orifice and entering the apex more obliquely, the effect of which is to render their direction on the body of the ventricle slightly more transverse. They vary also somewhat from the fibres of the first layer in not quite extending either to the apex (Plate XII. compare *k* and *l* with *f g* of fig. 11) or the base (Plate XII. compare *n′ n″* with *q* of fig. 6)—an arrangement which, as it also prevails in the deeper layers, satisfactorily accounts for the ventricular wall tapering towards the apex and the base respectively, as shown in a vertical section (Plate XII. figs. 13 & 14, *s*). The varying thickness of the ventricular wall towards the apex was well known to GERDY*, DUNCAN†, and REID‡, and wrongly, as I think, attributed by them to a supposed difference in the length of the fibres composing the different portions of it, rather than to the position and direction of the fibres themselves, which appear to me to afford the true explanation. The fibres of the second layer further differ from those of the first in their arrangement at the base, most of them being continuous in this direction with the fibres of the sixth layer (Plate XII. fig. 6, *o n*). The continuity of the fibres of the second external layer at the base with corresponding internal fibres, is strictly analogous to the continuity of the external fibres with the internal ones at the apex—the only difference being that at the apex the external fibres, in order to become internal, *involute* or turn in, whereas at the base the internal fibres, in order to become external, *evolute* or turn out. That the same principle which turns in the external fibres and secures their continuity with corresponding internal ones at the apex, also turns out the internal fibres and renders them continuous with corresponding external fibres at the base, is probable from the fact, that the borders formed by the union of the external with the internal fibres at the base and at the apex are convex, and identical as regards structure; in other words, the fibres composing both borders advance spirally, the *external* fibres winding from above downwards and bending over the circular edge, forming the apicial orifice in a direction *from without inwards* (Plate XII. fig. 11, *k l*, and Plate XVI. fig. 55, *e f*), the *internal* winding from below upwards and bending over the convex border surrounding the auriculo-ventricular or basal orifice in a direction *from within outwards* (Plate XII. fig. 7, *n′*, and fig. 8, *o″*). The borders which limit the ventricle towards the apex and the base when the first layer is removed are consequently composed of loops or doublets of fibres (Plate XVI. fig. 55, *e f*, and Plate XV. fig. 47 *d*). Dr. JOHN REID§, in speaking of the left apex, says, "when the point is removed the *circular edge* is left entire, and is formed of another series of fibres, which, like those taken away, advance spirally from the base to the apex, and turning over the edge ascend in the opposite direction, continuing their course after being reflected." The converse of these remarks holds true of the fibres at the base.

* *Op. cit.* † *Loc. cit.* p. 591. ‡ *Loc. cit.* p. 591. § *Loc. cit.* p. 592.

Third external layer of the left ventricle (Mammal). The fibres of the third external layer (Plate XII. fig. 3) resemble in their more important features the fibres of the second layer just described, and advance spirally from the base to the apex and from left to right in two distinct sets (*f g*, *d d' e*). They differ, however, from those of the second layer in their position on the ventricle. The set which enters more particularly into the formation of the anterior half of the apicial orifice proceeds from the posterior third of the septum, and the posterior half and anterior third of the auriculo-ventricular opening; while that which enters into the formation of the posterior half of the apicial orifice (*k*) proceeds from the remaining anterior part of the auriculo-ventricular opening, and the anterior two-thirds of the septum. Arrived at the apex, which is now greatly widened, they bend or double upon themselves in a direction from without inwards, and reverse their course to enter the interior, where they become continuous with the two sets of fibres forming the fifth layer (Plate XII. fig. 5, *o o'*, *n n'*). The fibres of the third layer differ from those of the second in being arranged in smooth fascicular bands, and in issuing from the auriculo-ventricular opening and entering the apex very obliquely —an arrangement which causes the fibres on the body of the ventricle (*f d'*) to pursue an almost transverse direction. They differ also from the fibres of the second layer, in not extending quite so far either towards the apex (Plate XII. compare *k* of fig. 3 with *k* of fig. 2) or the base (compare *n' n''* of fig. 5 with *n' n''* of fig. 6), and in being confined to more central portions of the ventricle—a circumstance which, as has been already explained, accounts for a vertical section of the ventricular wall tapering towards the apex and the base. The fibres of the third layer further differ from those of the second in exhibiting a less degree of crowding at the apex—for the very obvious reason that the apicial orifice, on account of the conical shape of the ventricle, becomes larger and larger with the removal of each successive layer.

Fourth or central layer of the left ventricle (Mammal). When the three external layers have been removed, the fourth or central layer (Plate XII. fig. 4) is exposed. This layer may be denominated the circular layer, or layer of transition, from the fact that the fibres entering into its formation are circular, and form the boundary between the external and internal layers. It differs from the other layers as regards the quality and the direction of its fibres, and as regards its position in the ventricular wall—the fibres composing it being aggregated into strong fascicular bands, whose course is neither from left to right downwards as in the external layers, nor from right to left upwards as in the internal ones, but horizontal or transverse. This peculiarity in direction, which causes the fibres of the fourth layer (*f g*, *d d' e*) to run at nearly right angles to those of the first (*j*) and seventh layers (Plate XII. fig. 7, *o n*), is accounted for by the fact that in the fourth layer, the fibres of the third layer terminate or double upon themselves, while the fibres of the fifth layer (Plate XII fig. 4, *k*) begin. In other words, the fourth layer, while it belongs neither to the third layer nor the fifth, forms the connecting or transition link to both, as may be seen by a reference to Plate XII. fig. 4, where the fibres (*d d' e*) are seen to turn directly upon themselves (*k*).

The fibres of the fourth layer, like those of the other layers, are composed of two sets ($f\,g$, $d\,d'\,e$), the one proceeding from the entire septum and a limited portion of the auriculo-ventricular opening on either side of the septum anteriorly and posteriorly, the other from the outer aspect of the auriculo-ventricular opening and its remaining anterior and posterior portions. Although the fibres of the fourth or circular layer form the boundary between the external and internal layers, they do not on this account occupy the centre of the ventricular wall, as may be seen on transverse section (Plate XV. fig. 50). On the contrary, the circular layer (e) is found considerably nearer the exterior (c) than the interior (d) of the ventricular wall—an arrangement which was to be expected, since the layers, as was stated, increase in thickness from without inwards. The aggregate of the external layers, or those to the outside of the circular layer, is consequently much less than the aggregate of the internal ones. The fibres of the fourth layer are from their position embraced by the fibres of all the other layers—viz. those of one and seven, two and six, and three and five. The amount of spiral made by them is rather less than one full turn (Plate XII. fig. 4, $f\,g$, $d\,d'\,e$).

Internal layers of the left ventricle of the Mammal.

Fifth layer. When the three external and the fourth or central layer are removed, one is immediately struck with the change, or rather complete reversal, in the direction of the fibres. Thus the fibres of the fifth layer (Plate XII. fig. 5), which are the internal continuations or counterparts of those forming layer three, instead of proceeding in a spiral direction from base to apex and from left to right, proceed in a spiral direction from apex to base and from right to left; the reason of which is obvious when it is recollected that the fibres of the fifth layer, when the layers are numbered from without inwards, form the first of the internal layers—*i. e.* of those layers found to the inside of the central layer, where, as has been explained, the order in the arrangement of the fibres is reversed.

The fibres of the fifth layer, like the other layers described, consist of two sets (o, n); they, however, pursue a very oblique course, and in this respect resemble the fibres of the third layer, which they cross at a very obtuse angle. They moreover fold or double upon themselves in an outward direction at the base (Plate XII. compare n' with n'' of fig. 5 *), and in so doing alter their direction (n''), and become continuous with the two sets of fibres forming the third external layer (Plate XII. fig. 3, $f\,g$, $d\,e$). The fibres of the fifth layer are also continuous with the two sets of fibres forming the third layer at the apex (Plate XII. fig. 3, k). The amount of spiral made by them from the time they leave the apex until they reach the base, rather exceeds one full turn (Plate XII. fig. 5, $o\,o'$, $n\,n'\,n''$). The fibres of the third and fifth layers embrace or overlap the fibres of the fourth or central layer, they themselves being embraced by the fibres of the first and seventh and the second and sixth layers.

Sixth layer of the left ventricle (Mammal). The fibres of the sixth layer (Plate XII.

* In this figure the internal fibres (n') are seen twisting or bending over in an outward direction to become continuous with the external fibres (n'').

fig. 6) are the internal continuations of the twofold set of fibres forming the second external layer. They wind in two portions (o, n), in a spiral slightly vertical direction from right to left upwards, or from apex to base, so that they cross the fibres of the second layer at a somewhat acute angle. At the base they fold upon themselves in an outward direction, and form flattened bands ($n' n''$), which become continuous with corresponding bands belonging to the second layer (Plate XII. fig. 2, $f g$, $d d' e$). The amount of spiral made by the fibres of the sixth layer from the time they leave the apex until they reach the base, is rather under a turn and a half. Taken together, the fibres of the second and sixth layers embrace in their convolutions the fibres of the third, fourth, and fifth layers, they themselves being overlapped by the fibres of the first and seventh layers.

Seventh or last internal layer of the left ventricle (*Mammal*). The fibres of the seventh or last internal layer (Plate XII. figs. 7 & 8), as was stated when describing the first or superficial layer, form the carneæ columnæ and musculi papillares, especially the latter. When the two sets of fibres which constitute the superficial or first external layer are traced from above downwards, the anterior set, as was explained, is found *to enter the apex posteriorly* (Plate XII. fig. 10, d), to become continuous with the fibres *of the anterior musculus papillaris* (Plate XII. figs. 12 & 13, y) and those of the carneæ columnæ next to it—the posterior set *entering the apex anteriorly* (Plate XII. fig. 10, g), and becoming continuous with the fibres *of the posterior musculus papillaris* (Plate XII. figs. 12 & 13, x) and the adjoining carneæ columnæ. The musculi papillares and carneæ columnæ therefore occupy a variable position in the interior of the ventricle,—the anterior musculus papillaris winding in a spiral almost vertical direction from below upwards, and from right to left, from *the interior of the apex posteriorly* to occupy *an anterior position*; the posterior musculus papillaris winding in a corresponding direction from *the interior of the apex anteriorly*, and curving round to occupy *a posterior position*. The musculi papillares, as will be seen from this description, are not the simply vertical columns usually represented (Plate XII. fig. 14, x, y), but vertical *spiral* columns (Plate XII. fig. 13, x, y). The musculi papillares are seen to advantage in the ventricle of the giraffe, camel, lion, horse, ox, ass, deer, seal, and dog. They occur in various stages of development *.

Most commonly they appear as conical-shaped spiral bodies, which project into the ventricular cavity and extend, in moderate-sized hearts, from the extreme apex to within half an inch or so of the base. At the apex, where they may be said to originate, and where they project least into the cavity, their spiral nature is very distinct †.

Towards the base, where their spiral direction is less marked, and where they project most into the ventricular cavity, they terminate in free blunted extremities, which are obliquely cut from below upwards, and from within outwards. As the chordæ tendineæ connect the blunted extremities in question with the segments of the bicuspid valve,

* In the American elk (Plate XV. fig. 48, y) they are so rudimentary as scarcely to attract attention.

† To see the spiral course pursued by the musculi papillares, the ventricle should be opened anteriorly, the incision being carried not quite to the apex, as represented at Plate XII. fig. 13, x y.

and through them with the auriculo-ventricular tendinous ring, the musculi papillares may be said to be continuous at the apex and the base respectively. On some occasions the musculi papillares spring from the interior of the apex in two fascicular bundles, the fibres of each bundle radiating and rapidly increasing in number as the apex is receded from; in others they spring from several smaller fasciculi, the fibres of the fasciculi being arranged in two sets and remaining more or less distinct, so that each musculus papillaris has a bifid appearance (Plate XII. fig. 15, $x, y\, y'$). The musculi papillares are principally of use in regulating the spiral action of the bicuspid valve, which they do through the instrumentality of the chordæ tendineæ. They are also useful, from projecting into the ventricular cavity, in reducing the blood to a state of quiescence during the diastole. During the systole they act as spiral lavers, and scoop the blood out of the interior of the ventricle by communicating to it a gliding spiral movement.

Situated between the musculi papillares, and, in fact, occupying the spiral interspaces or hollows occasioned by their projecting into the ventricular cavity, are the carneæ columnæ. The carneæ columnæ, like the musculi papillares, proceed in a spiral nearly vertical direction from right to left upwards. They are more developed in some instances than in others. In the camel, red deer, and American elk they may be said to be altogether wanting (Plate XV. fig. 48). In the lion, leopard, horse, ox, and ass they are more or less rudimentary; while in the mysticetus, armadillo, giraffe, and sheep they attain a size which almost entitles them to rank with the musculi papillares themselves. In the human heart (Plate XII. fig. 15, z) the carneæ columnæ consist of irregularly shaped, rounded muscular bands, arranged so as to form an intricate network —some of the bands being attached at both extremities, others throughout their entire extent. A portion of the fibres of the carneæ columnæ are continuous with the fibres of the external layer at the base; others derive attachment from the fibrous ring surrounding the aorta, and from the auriculo-ventricular tendinous ring. Running between the carneæ columnæ and the musculi papillares are a series of delicate fibrous stays (Plate XII. figs. 13 & 14, r, and Plate XV. figs. 47 & 48, r), which hang loosely in the ventricular cavity. They vary in thickness with the size of the heart, and probably assist in coordinating the movements of the structures between which they are found. The spiral interspaces or hollows occupied by the carneæ columnæ, to which a passing allusion has been made, are two in number, a larger and a smaller. They both extend from the extreme apex to the extreme base. The larger groove or interspace proceeds from *the outward or lateral aspect of the apex*, and winds in an upward and inward or septal direction, until it reaches *the apex of the inner segment of the bicuspid valve*, where it bifurcates—the one portion conducting to the aortic orifice, with which it communicates, the other to the base of the segment, where it terminates. The smaller groove or interspace proceeds from *the septal side of the apex*, and winds in an upward and outward or lateral direction, until it reaches *the base of the outer segment of the bicuspid valve*, where it likewise terminates.

These grooves are important physiologically; for I find that in them the blood is arranged in spiral columns during the diastole, and that towards the end of the diastole and the beginning of the systole, it is made to advance in spiral waves from beneath on the segments of the bicuspid valve, and communicates to these structures a distinctly spiral upward movement, the amount of upward motion being regulated by the chordæ tendineæ to prevent retroversion and regurgitation. As the systole advances and the musculi papillares contract with the other portions of the ventricular wall, the segments of the valve are gradually drawn down by the chordæ tendineæ in an opposite direction to that by which they ascended and tightened upon the rapidly diminishing columns of blood, so that they form a spiral dependent cone, whose apex is directed towards the apex of the ventricle*.

Cast of the interior of the left ventricle (Mammal).

That the fibres of the seventh layer have a spiral direction and enclose a conical-shaped spiral cavity, may be readily ascertained by a reference to Plate XII. fig. 17, which is taken from a photograph of a wax cast of the interior of the left ventricle of the deer. On carefully examining the engraving in question, the cavity will be found to taper and twist towards the apex (*z*), and also, though to a less extent, towards the base (*b*). The diminution of the cavity towards the base is so slight that it might be overlooked, were it not that it renders the auriculo-ventricular orifice more easily closed than it would otherwise be.

Transverse sections of the left ventricle (Mammal).

The amount of spiral made by a cast of the left ventricular cavity, as ascertained by transverse sections, rather exceeds a turn and a half. This is proved by dividing the ventricle transversely into six unequal portions, by placing the sections in exactly the same positions, and by comparing the long axes of such portions of the cavity as are found in the sections with each other, and with the long axis of the cavity of the right ventricle. It is necessary to make the sections unequal, as the spiral formed by the cavity is much more rapid towards the apex than the base—the success of this demonstration depending on making the section in such positions as will intersect the spiral at every half turn of its progress. I have given views of five of the sections alluded to; and it will be observed that in the first two sections towards the base (Plate XV. figs. 49 & 50), the long axis of the cavity (*b*) of the left ventricle is at right angles to the long axis of the cavity of the right ventricle (*l*). In the third section (Plate XV. fig. 51), the long axis of the cavity (*b*) of the left ventricle is parallel with the long axis of the cavity of the right ventricle (*l*), showing that the left ventricular cavity has made half a spiral turn. In the fourth section (Plate XV. fig. 52), the long axis of the left

* For a detailed account of the action of the mitral and tricuspid valves, see paper by the author "On the Relations, Structure, and Function of the Valves of the Vascular System in Vertebrata," Transactions of the Royal Society of Edinburgh, vol. xxiii. p. 761.

ventricular cavity (*b*) is again at right angles to the long axis of the cavity of the right ventricle, making it evident that the cavity of the left ventricle has made a full spiral turn; and in the fifth section (Plate XV. fig. 53) the long axis of the left ventricular cavity is again parallel with the long axis of the cavity of the right ventricle, showing that the left ventricular cavity has made an additional half turn. If another section had been made at the extreme apex, probably an eighth of a turn more would have been obtained, as the spiral in this direction is very rapid. The object of the cavity twisting suddenly upon itself at the apex is obviously to protect the ventricular wall, where thinnest, from undue pressure; for it is plain that a fluid injected into a conical-shaped spiral and therefore tortuous cavity will not be transmitted to the apex with the same degree of force as it would if the cavity were not spiral.

Vertical section of the left ventricular wall (Mammal).

On making a vertical section of the left ventricle between the musculi papillares (Plate XII. figs. 13 & 14), the ventricular wall (*s*), like the ventricular cavity, is observed to form a double cone, the apices of which point towards the apex and base of the ventricle respectively, the bases, which are united in the upper portion of the middle third, corresponding with the thickest part of the ventricular wall. The varying degree of thickness in the ventricular wall is traceable to the fact, that the outermost and innermost layers extend further towards the apex and base than those which come next, and these, again, further than those which succeed them, and so on until the central layer is reached—this being of least extent, and confined indeed to about the middle third of the ventricle. Thus into the apicial portion of the ventricular wall, where thinnest, only one layer enters, viz. the superficial or first external*. Into a second portion, a little above the apex, two layers enter, the first and the seventh; into a third or higher portion four, viz. the first and the seventh, the second and the sixth; while into a fourth, or still higher portion, which corresponds with the upper part of the middle third, the whole seven layers enter. Tracing the thickness of the ventricular wall in an opposite direction, *i. e.* from the base towards the upper part of the middle third, the same changes present themselves, although in a less marked degree. Thus the extreme base consists of two layers, the continuations in fact of those forming the second part of the apex; the second portion of four, the third of six, and so on—an arrangement which accounts for the ventricular wall being thicker towards the base than towards the apex†.

Recapitulation of facts connected with the left ventricle (Mammal). Before leaving the left ventricle, it may be well to recapitulate briefly the points more particularly dwelt upon. They are the following:—

* This portion of the ventricular wall is formed by the external fibres turning round in a circular direction, to alter their direction and become continuous with the internal, the external and internal fibres, in virtue of their spiral direction, not crossing each other until removed an appreciable distance from the apex.

† The septum also tapers in two directions, more particularly when, as in the present demonstration, it is regarded as forming part of the left ventricle.

1st. It has been shown that the walls of the left ventricle, when the septum is included, are composed of four systems of spiral fibres, two external and two internal; the external systems running from left to right downwards from base to apex; the internal systems from right to left upwards from apex to base.

2ndly. That these spiral fibres are arranged in layers or strata, which increase in thickness from without inwards, and that the fibres composing them have each a different course, whereby they change their direction from the nearly vertical to the horizontal, and from the horizontal back again to the nearly vertical.

3rdly. That the fibres composing the strata alluded to are as nearly as may be of the same length, and enter the apex and issue from the auriculo-ventricular orifice at the base in two distinct parcels or bundles.

4thly. That the two sets of fibres forming the external layers are continuous at the apex and at the base with the two sets of fibres forming the internal layers, and give rise to twisted continuous loops, pointing to the apex and base respectively; the more superficial loops embracing in their convolutions the deeper or more central ones.

5thly. That the apex is opened into, and the apicial and basal orifices (on account of the double cone formed by the ventricular cavity) widened, by the removal of such strata as are found to the outside of the central stratum.

6thly. That the ventricular wall, like the ventricular cavity, tapers towards the apex and the base, the tapering towards the apex being very considerable, that towards the base being less appreciable.

7thly, and lastly. That the septum is of nearly the same thickness as the left ventricular wall, and must be dissected *pari passu* with it, if the left ventricle is to be considered complete in itself.

The right and left ventricles, septum, &c. considered relatively (Mammal).

By far the simplest way to regard the right ventricle is, to consider it as a segment of the left one—a view which is favoured both by the actual structure, and all that is at present known of the fœtal development of the organ. In works on embryology it is stated that at first the heart consists of a mass of nucleated cells; that by and by it assumes the form of *an elongated sac or dilated tube*; that about the fourth week *a septum begins to arise up internally, which proceeds from the right side of the apex and anterior wall of the cavity*, in the direction of the base, where the arterial bulb leads off; and that about the eighth week this interventricular septum is complete. It is further stated that the walls of the ventricles are, comparatively speaking, *very thick, the thickness of both being about the same*; but that on approaching the full period the left begins to be the thicker of the two—a change which was *à priori* to be expected, seeing that after birth the left ventricle has to perform nearly twice as much labour as the right.

Beginning therefore with the left or typical ventricle constructed as described, it

appears to me that in order to produce the right ventricle, and explain the relation existing between the right and left ventricles anteriorly, posteriorly, and septally, all that is necessary is to push in the anterior wall (Plate XVI. diag. 15, *a*) in an antero-posterior direction until it touches the posterior one (B), in imitation of the constructive process. As however, in pushing in the anterior wall until it touches the posterior one a double septum is produced which is unattached posteriorly, it is necessary, to complete the structure, to suppose the fibres forming the posterior border of the septal duplicature as coalescing with corresponding fibres of the posterior wall, until the central layer is reached (Plate XVI. diag. 17, K); whilst the fibres of the two halves of the duplicature itself pass through and are blended with each other to the same extent (E J). If the constructive process be so imitated, it will be seen that not only are two ventricles (C, D) produced, each of which has fibres peculiar to itself (I H, F E), but, what is remarkable, that these ventricles are united to each other posteriorly (K) and septally (E J) by a series of fibres, which are common to both, *i. e.* fibres which belong partly to the one ventricle and partly to the other, precisely as in the ventricles themselves. The fibres moreover of the left or principal ventricle form four systems of conical spirals, two external and two internal—the former winding from above downwards from left to right, to twist rapidly round in a whorl at the apex, where they are continuous with the two internal systems *, winding in an opposite direction, from below upwards and from right to left.

The fibres of the right ventricle, on the other hand, form only segments of spirals—they being continuous with each other not at one point as in the left ventricle, but throughout the track for the anterior coronary artery (A). That the foregoing arrangement approaches very closely to, if it is not identical with, that occurring in the ventricles of the adult heart, may be ascertained in various ways.

1st. When the right and left ventricles (Plate XIII. figs. 18, 21, & 24) are dissected from without inwards, the layers constituting the right ventricular wall ($f'f''$) gradually increase in thickness, and pass through the several changes in direction met with in the layers of the left ventricular wall ($d\ d'$), clearly showing that the right and left ventricles are constructed on the same type. As, however, the left ventricle, as will be shown presently, is the more complete of the two, it is more natural to suppose that the right ventricle is a segment of the left one, than the reverse.

2ndly. When both ventricles are dissected at the same time, the fibres forming the external layers posteriorly (Plate XIII. fig. 21) are for the most part common alike to the one ventricle and the other†; in other words, the fibres on the back part of the left ventricle (*f*) cross over the posterior coronary track (*j*), and pass on to the right

* The external and internal systems, as has been explained, are rendered continuous at the base by bending over until they meet each other. Similar remarks apply to the fibres at the base of the right ventricle.

† The fibres forming the left apex are peculiar to itself, and belong exclusively to the left ventricle. This distribution of the fibres is accounted for by the fold which I believe forms the right ventricle beginning fully half an inch above the apex in question.

ventricle ($f'f''$); whereas in front, with the exception of a large cross band at the base (Plate XIII. fig. 20, *n*), which is evidently for the purpose of binding the ventricles more securely together anteriorly, the fibres of the right and of the left ventricle respectively dip in (Plate XIII. fig. 23, *r*) at the anterior coronary track ($o\,o'$), as if altogether independent of each other.

3rdly. When the fibres on the anterior aspect which belong to one or other of the ventricles are traced into the septum, and the ventricles forcibly separated (Plate XIV. fig. 45) in a line corresponding with the course which the fibres peculiar to each ventricle naturally take, the right ventricle (*k*) claims, as its share of the partition alluded to, rather less than one-third of its entire breadth (*l*), the remaining two-thirds (*n*) going to the left ventricle (*m*). Why the right ventricle should claim less than a third of the septum is difficult to explain, unless it be that this portion of the septum, belonging as it does more particularly to the right ventricle, represents the right half of the septal fold atrophied to half its original dimensions (Plate XVI. diag. 16, K), in common with the other portions of the right ventricular wall (F)*. The right ventricular wall after birth, it will be remembered, is only half the thickness of the left (Plate XV. compare $c''d''$ with $c\,d$ of fig. 51). This view seems probable from the fact, that the septum in some places (Plate XV. fig. 50, $c'd'$) is nearly a third thicker than the left ventricular wall ($c\,d$) between the papillary muscles (x, y)—an excess in breadth which very nearly corresponds with what would be obtained when allowance is made for the right and left halves of the septal duplicature passing through each other, until the central layer in either is reached, and for the atrophy of the right half of the septal fold as suggested.

4thly. When the cut ends of the common fibres found on the left ventricle, *i. e.* those to the outside of the central layer (Plate XIII. fig. 25, *f*), are applied to the fibres forming the two-thirds of the septum (*g*) which belong to this ventricle, they are ascertained to agree in direction, and would, if united, give rise to four complete systems of conical spirals (two external and two internal), these conical spiral fibres being continuous with each other at the apex and also at the base; whereas the fibres of the right ventricle and its share of the septum, treated in the same way, although likewise continuous at the base and in the track for the anterior coronary artery, consist merely of spiral fragments (Plate XIV. fig. 35, $f\,g$, $d\,d'$), and represent only a part of a more complete system

* In this explanation I have supposed that the right ventricle and the right half of the septal fold have become atrophied to half their original bulk, in accordance with the law that structure and function are related to each other as cause and effect—the efforts required for maintaining the pulmonic circulation being probably about half those required for the maintenance of the systemic. The converse, however, of this explanation is equally true, and might be adopted with the same result as far as the comparative thickness of the ventricles is concerned. Thus, instead of supposing that the right ventricle and its half of the septal duplicature becomes atrophied, it might be assumed, in accordance with the same law, that the left ventricle and its half of the septal duplicature becomes hypertrophied to twice its original dimensions, the right ventricle and its share of the septum remaining stationary.

—a portion nipped off as it were from the perfect cone. (Compare $c''d''e'$ with $c\,d\,d'$ of fig. 50, Plate XV., and E F with H I of diag. 17, Plate XVI.)

5thly. When casts of the interior of the ventricles are taken, the left ventricular cavity (Plate XII. fig. 17), in accordance with the more perfect arrangement of the fibres forming the left ventricle, supplies a highly symmetrical double conical screw, the right ventricular cavity (Plate XII. fig. 16), although it has the same twist, furnishing only an incomplete portion.

6thly. When the so-called common fibres posteriorly (Plate XIII. fig. 21, $f\,f'$) are dissected, layer after layer, synchronously with the independent anterior fibres (Plate XIII. fig. 23, $p\,q$), both sets are seen to pass through the same changes in direction; in other words, they proceed from left to right downwards, gradually becoming more and more oblique as the central or transverse layer is reached.

7thly, and lastly. When the fibres of the septum (Plate XIII. figs. 19, 22, & 25, $g\,e$) are dissected, layer after layer, with the other portions of the ventricular walls (Plate XIII. figs. 18, 21, & 24, $f\,f'$), they are observed to pass through the same changes in direction; *i. e.* they pursue a spiral course from left to right downwards, becoming more and more oblique as the central layer (Plate XIII. fig. 28, $f\,g$, $d\,e$) is reached, after which they reverse their course and become more and more vertical in an inverse order. They consist moreover of three kinds: first, such as, properly speaking, belong to the right ventricle (Plate XV. fig. 45, l); secondly, such as belong more particularly to the left ventricle (n); and thirdly, such as belong partly to the one ventricle and partly to the other (Plate XV. fig. 50, e'). Thus, in dissecting the septum from the right side, the fibres first met with belong almost exclusively to the right ventricle. These fibres, if traced from below upwards like the other internal fibres of the right ventricle, proceed from right to left. Traced from above downwards, their direction is just the reverse, or from left to right; and it is important to note this circumstance, as the internal fibres of the right ventricle become mixed up on the septum, at no great depth from its surface, with fibres belonging exclusively to the left ventricle*, the direction of which is also from left to right downwards. There is therefore a portion of the septum in which the internal fibres of the right ventricle are mingled with the external fibres of the left, and where the two sets pass through each other as the fingers of the one hand might be passed between those of the other. The fibres found still deeper, and which in fact constitute the left two-thirds of the septum, belong exclusively to the left ventricle. These points may be readily established by dissection.

External layers of the right and left ventricles (Mammal).

Superficial or first external layer. If the ventricles are dissected together (Plate XIII. figs. 18 & 20), the fibres of the superficial or first external layer posteriorly (Plate XIII.

* The existence of these fibres in the right third of the septum induced me, when describing the left ventricle, to regard the septum as forming a part of its walls.

fig. 18) run in a spiral almost vertical direction* from left to right downwards, some of them proceeding to the left apex (d''), others to the right ventricle (f'); and if an incision be made through the right ventricular wall (Plate XIII. fig. 19), a little to the right of the posterior coronary track, and the breach dilated to expose the septum and the interior of the right ventricle, it will be seen that the fibres on the right side of the septum (g) follow a similar course. On the anterior aspect of the ventricles (Plate XIII. fig. 20) the fibres also pursue a spiral nearly vertical direction from left to right downwards†; but there is a great difference between them and the fibres on the posterior aspect.

On the posterior of the ventricles (Plate XIII. fig. 18) the fibres from the left auriculo-ventricular opening and the left ventricle generally ($d\,f$) cross the track for the posterior coronary artery (j), and are found also on the right ventricle ($f'\,f''$); hence the epithet common fibres; whereas on the anterior of the ventricles (Plate XIII. fig. 20) the fibres, with the exception of the cross band at the base already referred to (n), dip in at the anterior coronary track (o), to appear on the right third of the septum (Plate XIII. fig. 19, g, h), where they are continuous with fibres having a corresponding direction. The fibres occurring on the right side of the septum (Plate XIII. fig. 19, g, h), as well as those lining the interior of the right ventricle generally, are, for anything I can discover to the contrary, segmental portions lopped off or isolated by the primary notch or reduplication (Plate XVI. diag. 15, A ;. Plate XV. compare $k\,l$ with $m\,n$ of fig. 45) from the spiral nearly vertical fibres originally lining the interior of the left or typical ventricle. If this explanation be adopted, the great structural resemblance presented by the internal fibres of the right and left ventricles respectively is at once accounted for.

Septum Ventriculorum composed of two elements (Mammal). That two elements enter into the composition of the septum is probable for the following reasons:—

1st. If the right ventricle be detached a little to the right of the tracks for the anterior and posterior coronary arteries, and the septum dissected from the right side (Plate XII. figs. 1, 2, 3, & 4), many of the fibres ($g\,e$) at no great depth from the surface proceed without breach of continuity to the anterior wall (Plate XII. fig. 10, f) and apex (g) of the left ventricle, thus showing that they belong exclusively to the left ventricle; whereas a certain number of them, as has been stated, are ascertained to be continuous with the fibres on the outside of the right ventricular wall (Plate XIII. fig. 21, $f\,f''$), proving them to belong more particularly to the right ventricle.

* The superficial fibres from the right and left ventricles converge in the track for the posterior coronary artery in a manner resembling the letter V ($f\,f'\,j$)—an arrangement which is confined to the upper or basal third of the first layer. Ultimately these fibres curve round to enter the left apex anteriorly.

† In the superficial layers of the right ventricle anteriorly, the fibres at the root of the pulmonary artery interweave to a considerable extent, and are matted together. As a similar arrangement exists in the superficial layers of the left ventricle at the root of the aorta, it is just possible that the large vessels are thereby supplied with more secure points of attachment.

2ndly. The fibres of the right side of the septum, especially the right third of it, are densely matted together, and separate with greater difficulty than the fibres of the other portions of the septum and ventricular wall generally.

3rdly. The exact width of the septum (Plate XV. fig. 50, *c' d'*), as compared with the left ventricular wall (*c d*) between the musculi papillares (x, y), is in some parts nearly one-third greater—this increase in bulk affording a redundancy of material, which was to be anticipated, since the two halves of the septal fold (Plate XVI. diagram 16, H K) are supposed to have passed partially through, and become blended with each other (Plate XVI. diagram 17, E J).

4thly. Such of the fibres as are found near the centre of the right third of the septum cross each other slightly towards the base, and give rise to a curious Y-shaped arrangement at a point corresponding to the crossing which would be produced by the reduplication.

There are other arguments in favour of the septum being formed of two elements by a septal reduplication.

When, for example, the common fibres are dissected posteriorly, more or less interruption is experienced in their separation (particularly in the deeper layers) in a line corresponding with the track of the posterior coronary artery (Plate XV. fig. 54, *c c'*), where the fibres of the border of the reduplication (Plate XVI. diagram 16, G) are believed to have united with the fibres of the posterior wall (B).

The external fibres of the right ventricular wall moreover (Plate XIII. fig. 20, *d' f'*, and fig. 23, *p q*) enter the track for the anterior coronary artery (*o*) throughout its entire extent, for the purpose of appearing on the septum (Plate XIII. figs. 19 & 22, *g*), the track referred to corresponding with the rut which would be produced by the junction of the two halves of the septal duplicature (Plate XVI. diagram 16, A).

Lastly, the external fibres of the right ventricular wall enter the interior by simply bending or folding upon themselves (Plate XIV. figs. 34 & 35, *d e*, *f g*, and Plate XV. fig. 45, *k*, *o*, *l*)—an arrangement which presupposes a corresponding reduplication or folding in of the anterior wall at some period or other, and one which is altogether different from the arrangement of the external fibres of the left ventricle at the apex, where the fibres enter the interior in two divisions in a regular whorl (Plate XII. fig. 10, *g*, *d*, and Plate XVI. fig. 55, *e*, *f*).

Carneæ columnæ and musculi papillares of the right ventricle (Mammal). The carneæ columnæ of the right ventricle (Plate XIV. figs. 43 & 44) are in general better marked, and the musculi papillares more numerous than in the left—a modification traceable partly to the shape of the right ventricular cavity (Plate XV. fig. 49, *l*), and partly to the greater number of fixed points required for the attachments of the chordæ tendineæ distributed to the tricuspid valve.

In the right ventricle, as in the left, the carneæ columnæ pursue a spiral nearly vertical direction from right to left upwards, and are subject to great variation as regards size, number, and general appearance. The musculi papillares in this ventricle, although

usually consisting of two as in the left (Plate XIV. fig. 43, *p p'*), are not necessarily limited to this number. In the camel and American elk the carneæ columnæ are altogether wanting, the musculi papillares (two in number) being alone present. In the heart of the armadillo and red deer the carneæ columnæ are feebly developed, the musculi papillares being generally three in number and small. In the heart of the sheep the carneæ columnæ are more fully developed than in the preceding, the musculi papillares being sometimes two in number, and sometimes three (Plate XIII. fig. 19, *h*). In the heart of the pig, leopard, and calf (Plate XIV. fig. 43) the carneæ columnæ are still more strongly developed, and appear in the form of thick muscular spiral ridges (*o*), which slightly intersect each other and cross the floor of the ventricle (*r'*), the musculi papillares being sometimes two (*h h'*), sometimes three, and sometimes four in number. In the porpoise, dugong, mysticetus, and human heart (Plate XIV. fig. 44) the carneæ columnæ are more or less reticulated, particularly in the two latter, the musculi papillares varying from two to four (*h h' h'' h'''*). This increase in the number of papillary muscles in the right ventricle, which might at first sight seem to interfere with the bilateral distribution of the fibres in the primary or typical ventricle, is accounted for by the fact that in the right ventricle the muscular fasciculi, from which the papillary muscles spring, do not always coalesce as in the left ventricle, but remain permanently apart. The musculi papillares of the right ventricle are less distinctly spiral than those of the left, and are somewhat flattened to suit the concavo-convex shape of the right ventricular cavity. They occupy the septal wall posteriorly (*h'' h'''*), and the right ventricular wall anteriorly (*h h'*). In the right ventricle, as in the left, many of the fibres of the carneæ columnæ (Plate XIV. fig. 43, *o'*) are continuous at the base with corresponding external fibres (*d' f*), such of them as are not continuous, together with the musculi papillares, being rendered so by the intervention of the right auriculo-ventricular tendinous ring (Plate XIV. fig. 43, and Plate XV. fig. 46, *n'*), the chordæ tendineæ, and the segments of the tricuspid valve (Plate XIV. fig. 44, *i'*).

Muscular valve of the right ventricle of the bird, how formed?

Distinction founded thereupon. In the right ventricle of the bird, where the tricuspid valve of the mammal* is supplied by a fleshy one, the continuity of the external with the internal fibres at the base is complete. This valve, as has been stated, forms the distinguishing characteristic between the ventricles of the bird and mammal, and differs essentially in its structure from all the other valves of the heart.

It consists of a solitary fold of muscular substance (Plate XIV. figs. 38, 40, & 41, *i*), which extends from the edge and upper third of the septum posteriorly (Plate XIV.

* The tricuspid valve, as its name implies, consists of three leaves or segments. As, however, the anterior segment, or that nearest the pulmonary artery, is larger than the posterior and internal segments, from which it is divided by a deeper notch than divides the posterior and internal segments from each other, some investigators regard the valve as consisting of two portions only; and I am inclined to assent to this view, from the bilateral nature of the left ventricle, and from my conviction that the right ventricle and every thing pertaining to it is a segment of the left.

fig. 39, *g*) to the fleshy pons anteriorly (*e*). The fold opens towards the interior of the ventricle (Plate XIV. fig. 40, *i*), in a direction from above downwards (Plate XIV. fig. 41, *i*), and is deepest at the edge of the septum posteriorly (Plate XIV. figs. 39 & 40, *g*). As it gradually narrows anteriorly (*i*), it is somewhat triangular in shape, its dependent and free margin (*g*) describing a spiral which winds from behind forwards, and from below upwards. The valve, from its substance and structure, may be appropriately termed the musculo-spiral valve, and is seen to advantage in the right ventricle of the emu (Plate XIV. fig. 41), swan (Plate XIV. figs. 38 & 39), turkey (Plate XIV. fig. 40), capercailzie, and eagle. It is composed of fibres from all parts of the floor and lower third of the right ventricle interiorly (Plate XIV. fig. 40, *j*), and from the upper third of the left ventricle and septum posteriorly (Plate XIV. figs. 39 & 40, *g*). The fibres from the lower third of the right ventricle interiorly, are spread over a large surface, and pursue a more or less vertical and slightly spiral direction. They gradually detach themselves in two portions (Plate XIV. fig. 40, *h*, *j*) from the right ventricular wall, and converge towards the centre of its middle third, where they form a flattened spindle-shaped muscular band (Plate XIV. fig. 40, *h*). Arrived at this point and continuing their spiral course, they diverge or spread out to assist in forming the inner and free leaf (*i g*) of the muscular fold (Plate XIV. figs. 39, 40, & 41, *g*)—one portion bending over in graceful spiral curves (Plate XIV. fig. 40, *i*) in a direction from within outwards and from below upwards, to become continuous with the superficial or external fibres at the base, a second portion bending over in like manner (e'') to become continuous with certain fibres from the upper third of the second layer of the septum and left ventricle posteriorly, a third portion (e''') pursuing a similar course to unite with the fibres from the upper third of the third layer of the septum and from the left ventricle. If that surface of the dependent or free leaf of the valve which is directed towards the right ventricular wall be examined, a fourth portion (*g*) is found to be continuous with the fibres of the upper third of the fourth or transverse layer. The muscular valve of the bird may therefore be said to be composed of the fibres entering into the formation of the several layers of the right ventricular wall, (the ventricular wall in fact bifurcates or splits up towards its base,) the external layers forming the outer wall of the valve, the internal layers, which are slightly modified, forming the inner. It is to this splitting up of the right ventricular wall towards the base (Plate XIV. fig. 41, *k*) that its greater tenuity in this direction, as compared with the right ventricular wall of the mammal (Plate XV. fig. 49, *f*), is to be traced. If the muscular valve be regarded as an independent formation, which it can scarcely be, it will be best described as a structure composed of fibrous loops, these loops being of three kinds and directed towards the base—the first series consisting of spiral nearly vertical fibres forming a somewhat acute curve, the second series consisting of slightly oblique spiral fibres forming a larger or wider curve, and the third series consisting of still more oblique fibres and forming a still greater curve. As the fibres composing the different loops act directly upon each other during contraction, the object of the arrangement is obviously to supply a move-

able partition or septum which shall occlude the right auriculo-ventricular opening during the systole. The manner in which the several loops act is determined by their direction. Thus the more vertical ones, in virtue of their contracting from above downwards, have the effect of flattening or opening out the valvular fold, and in this way cause its dependent or free margin to approach the septum. The slightly oblique fibres, which contract partially from above downwards, but principally from before backwards, assist in this movement by diminishing the size of the right auriculo-ventricular orifice in an antero-posterior direction,—it remaining for the very oblique and transverse fibres, which contract from before backwards, and from without inwards, to complete the movement, by pressing the inner leaf of the fold directly against the septum—an act in which the blood plays an important part, from its position within the valve, this fluid, according to hydrostatic principles, distending equally in all directions and acting more immediately on the dependent or free margin of the valve, which is very thin and remarkably flexible. When a vertical section of the fold forming the valve of the bird is made, that portion of it which hangs free in the cavity is found to be somewhat conical in shape, the thickest part being directed towards the base, where it has to resist the greatest amount of pressure—the thinnest corresponding to its dependent and free margin, where it is applied to, and supported by, the septum. The upper border of the fold is finely rounded, and in this respect resembles the convex border which limits the right ventricle of the mammal towards the base.

The spindle-shaped muscular band (Plate XIV. fig. 40, *h*), which from its connexion may be said to command the upper (e'') and lower (j) portions of the right ventricle interiorly, is obviously for the purpose of coordinating the movements of the muscular valvular fold; and as its position and direction nearly correspond with the position and direction of the musculus papillaris situated on the right ventricular wall of the mammal (Plate XIV. fig. 44, h'), it is more than probable that it forms the homologue of this structure. Indeed this seems almost certain from the fact that, if the ventricles of the bird be opened anteriorly (Plate XIV. fig. 40), and the band referred to contrasted with the anterior musculus papillaris of the left ventricle (y), both are found to occupy a similar position. The fleshy band therefore may be said to be to the muscular valve of the right ventricle, what the anterior musculus papillaris and its chordæ tendineæ are to the segments of the mitral valve. Compared with the tricuspid valve of the mammal, the muscular valve of the right ventricle of the bird is of great strength. As, moreover, it applies itself with unerring precision to the septum, which is slightly prominent in its course, its efficiency is commensurate with its strength. The prominence on the septum alluded to is very slight, and might escape observation, were it not that immediately below it the septum is hollowed out to form a spiral groove of large dimensions ($e'' e''$). This groove, like the valve, runs in a spiral direction from behind forwards, and from below upwards, and, when the valve is applied to the septum during the systole, converts the right ventricular cavity into a spiral tunnel, through which the blood is forced, on its way to the pulmonary artery. Such of the fibres of the superficial or first external

layer of the right ventricle of the mammal as are not continuous with corresponding external fibres at the base arise in two divisions (Plate XIV. fig. 30, *d*, *f*),—the one from the fibrous ring surrounding the pulmonary artery (Plate XV. fig. 46, *k*) and aorta (*a*), and the anterior half of the fibrous ring surrounding the right auriculo-ventricular opening (*l*), together with a corresponding portion of the septum (*e*); the other from the posterior half of the fibrous ring (*n'*) surrounding the right auriculo-ventricular opening, the posterior half of the septum, and a limited portion of the left auriculo-ventricular tendinous ring posteriorly (*n*). In this layer, consequently, comparatively few of the fibres belonging to the left ventricle (Plate XIII. fig. 18, *f*) cross the posterior coronary groove (*j*) to become continuous with the fibres on the right (*f'*); and it is worthy of observation, that as the dissection advances the number of the so-called common fibres is augmented. This increase of the common fibres, which is gradual and follows a certain order, is referable to the source and direction of the fibres constituting the several layers. In the first layer, as has been explained, the common fibres proceed from a limited portion of the left auriculo-ventricular opening posteriorly; and as their direction is little removed from the vertical, few of them cross the posterior coronary groove to appear on the right. In the second layer, however, the common fibres proceed from the posterior and outer portion of the left auriculo-ventricular opening (Plate XIII. fig. 21, *f'*), and, their direction being more oblique, a considerable proportion cross the posterior coronary groove (*j*). In the third layer (Plate XIII. fig. 24, *f d'*) the direction is still more oblique, and a greater number of the fibres consequently cross the groove referred to. In the fourth layer the direction of the fibres is horizontal (Plate XIII. fig. 27, *f d*), and the fibres almost all cross the groove in question. In the last-mentioned layer the fibres may be said to emanate from the left auriculo-ventricular opening all round. In speaking, therefore, of the fibres which are common to both ventricles posteriorly, it will facilitate the comprehension of their arrangement, to say that they radiate from different portions of the left auriculo-ventricular opening at different levels, these levels corresponding with the depth of the layer involved.

Peculiarities of the right ventricle of the mammal—fleshy pons—infundibulum—bone of the heart, &c. In the right ventricle, as in the left, the layers increase in thickness from without inwards; but there is this difference: the layers of the right ventricle are comparatively thinner than those of the left, owing to the fibres constituting them being more delicate. The greater delicacy of the fibres of the right ventricle may be explained either by an arrest of growth after birth, or to their becoming subsequently atrophied. The fibres of the right ventricle, as a rule, form only curves or segments of spirals (Plate XIV. figs. 36 & 37), a certain number of them anteriorly (especially those of the internal layers*) bending over and uniting with corresponding fibres from the right side of the septum to form a fibrous archway (Plate XIII. figs. 18, 21, 24, & 27, *m*), which

* The fibres of the external layers which enter into the formation of the fleshy pons, arise in many instances from the root of the pulmonary artery and aorta, and from the anterior portion of the right auriculo-ventricular opening.

separates the right auriculo-ventricular opening (*l*) from that of the pulmonary artery (*k*). This fibrous archway has been appropriately denominated the fleshy pons, and is more or less spindle-shaped, from the fact of its forming the boundary between the auriculo-ventricular and pulmonic orifices, the former of which is oval, the latter circular. It varies in size according to the dimensions of the heart. In the sheep, calf (Plate XIV. fig. 43, *m*), hog, leopard, deer, and seal it is usually about half an inch in breadth at its narrowest portion, and rather less than a quarter of an inch in thickness; while in the giraffe, camel, and horse it increases to twice these dimensions.

Another peculiarity in the right ventricle of the mammal, to which a passing allusion is due, appears in the form of a conical-shaped projection (Plate XIV. fig. 37, *w*), the so-called infundibulum (CRUVEILHIER), or conus arteriosus (WOLFF *), situated at the upper and anterior portion (*p*) of the ventricle. This projection, which communicates above or at its summit with the pulmonary artery (*k*), has the effect of lengthening the right ventricle towards the base to the extent of half an inch or so in moderate-sized hearts, and in this way makes up the deficiency of the right ventricle towards the apex.

The vertical measurement of the right and left ventricles is consequently nearly equal. The conus arteriosus is composed externally † of fibres which arise more immediately from the fibrous ring surrounding the orifice of the pulmonary artery (Plate XV. fig. 46, *k*)—these fibres having a plicated or tortuous arrangement (*c*), similar to that which occurs in the superficial layer of the ventricle of the fish and reptile. As, however, the fibres alluded to are separable into layers (Plate XIV. figs. 30 & 31, *k*), and are continuous with the fibres of the external layers of the right ventricle generally, with which they correspond in direction (*f d*), they are not entitled to a separate description. The more internal portions of the conus arteriosus are composed of the internal layers of the right ventricle. An additional peculiarity in the right ventricle of the mammal consists in the existence, in a large number of quadrupeds, of a curiously shaped bone (Plate XIV. figs. 30 & 31, *c*) which is imbedded in the right side (*a*) of the fibro-cartilaginous ring surrounding the aortic orifice. The bone in question, on account of its being more fully developed in some instances than in others ‡, varies considerably as regards

* "This author drew a distinction between the conus arteriosus and the infundibulum, applying the former epithet to that portion of the ventricle from which the pulmonary artery springs, and which is prolonged upwards above the level of the rest of the ventricle. In the term infundibulum he included a larger portion of the ventricle, apparently that portion placed above a line drawn from the upper and right margin of the ventricle obliquely downwards to the anterior fissure. As the upper part of the right ventricle becomes gradually narrower, he supposed that it increases the velocity and impetus of the blood as it is drawn from the ventricle." (Acta Acad. Imper. Petropol. pro anno 1780, tom. v., vi. p. 209, 1784.)

† The arrangement of the fibres entering into the composition of the conus arteriosus interiorly is described at p. 479.

‡ BLUMENBACH (Comparative Anatomy, translated by Mr. LAWRENCE, p. 138) speaks of two bones as existing in the heart of the stag and the larger adult bisulca. According to Mr. W. S. SAVORY, two principal bones (a larger and a smaller), together with several irregular fragments, are found in the hearts of the larger ruminants. (Observations on the Structure and Connexions of the Valves of the Human Heart, 1851.) The author of the present paper has occasionally seen the fragments alluded to by Mr. SAVORY.

form. Usually it resembles the mould of a ploughshare; *i. e.* it is more or less triangular, and slightly bent or twisted upon itself to suit the curve of the aorta. That it performs no very important function, and is not necessary for the attachment of the fibres of the septum, is abundantly proved by its absence in a great number of instances. The *os cordis*, as it has been termed, is generally met with in the hearts of the horse, ox, sheep, and deer, and very rarely in man, the seal, pig, dog, hedgehog, hare, rabbit, and cat.

Lastly, the shape of the right ventricle is peculiar. Viewed vertically, it forms two irregular cones, a larger and inferior cone, and a smaller and superior one (the conus arteriosus). They spring from a common base and have widely separated apices. The bases of the cones correspond with a line drawn from the posterior portion of the right auriculo-ventricular opening (Plate XIV. fig. 31, *c*) to a point in the track for the anterior coronary artery, midway between the apex (*d*) of the right ventricle and the root of the pulmonary artery (*k*). The apex of the larger or inferior cone (Plate XIV. fig. 31) corresponds with the apex of the right ventricle (*d*), and the apex of the smaller or superior one with the root of the pulmonary artery (*k*). Viewed transversely or by means of transverse sections, the right ventricle is found to be concavo-convex (Plate XV. figs. 49, 50, & 51, *l*), its concavity being turned towards the convexity of the left (*b*); in other words, the right ventricle is as it were flattened out and applied to or round the left one. Viewed from before backwards, or in an antero-posterior direction, the right ventricle is found to be twisted upon itself (Plate XII. fig. 16, and Plate XIV. figs. 34, 35, 36, & 37), the two cones of which it is composed twisting in opposite directions—the larger or inferior cone (Plate XII. fig. 16, *l h m*, and Plate XIV. fig. 37, *p' q q'*) in a direction from left to right downwards, the smaller or superior one (Plate XII. fig. 16, *h k*, and Plate XIV. fig. 37, *p w k*) in a direction from right to left upwards.

Second external layer of the right and left ventricles (*Mammal*). When the superficial or first external layer, which in the mammal is comparatively thin, is removed, the second layer (Plate XIII. fig. 21), composed of fibres similarly arranged to those taken away, is exposed. The fibres of the second layer posteriorly proceed in a spiral direction from left to right downwards, and are for the most part* common to both ventricles (*f f''*); *i. e.* the fibres found on the left ventricle (*f*) cross the posterior coronary track (*j*), and are found also on the right ventricle (*f' f''*); while the fibres on the anterior aspect, which also proceed in a spiral direction from left to right downwards, with the exception of a broad band at the base, dip in at the anterior coronary track to appear on the right third of the septum, where they are continuous with fibres having a corresponding direction (Plate XIII. fig. 22, *g*). The fibres of the second layer differ from those of the first in being slightly fascicular, and in issuing from the auriculo-ventricular openings (Plate XIII. fig. 21, *b l*) and entering the left apex and anterior coronary groove more obliquely. They further differ in having a more oblique direction. The fibres of the second layer are arranged in two sets, the one of which proceeds from rather

* A few of the fibres of the second layer proceed to the left apex only.

more than the anterior half of the septum and right auriculo-ventricular opening (*l*), and from the root of the pulmonary artery (*k*); the other from rather less than the posterior half of the septum and right auriculo-ventricular opening (*l*), and from the posterior and outer half of the left auriculo-ventricular opening (*b*). The number of fibres, consequently, which proceed from the left ventricle (*f*) to cross the track of the posterior coronary artery (*j*), to become continuous with fibres having a similar direction on the right ventricle ($f'\,f''$), is greater in the second layer than in the first.

Second layer of the septum—curious Y-shaped arrangement of the fibres—fibres of the right ventricle continuous anteriorly and posteriorly. The direction of the fibres of the second layer of the septum (Plate XIII. fig. 22, *g*) corresponds with the direction of the fibres of the second layer on the anterior and posterior aspects ($f f'$, $d\,d'$) of the left ventricle, and with the direction of the fibres of the second layer of the right ventricle (Plate XIV. fig. 31, $f f'$, $d\,d'$). In addition, the fibres of this and the succeeding layer diverge from each other at the upper third of the septum, and occasion an arrangement resembling the letter Y, the one portion bending over to assist in the formation of the fleshy pons anteriorly, the other, curving round to become continuous with the fibres of the right ventricle, having a like direction posteriorly (Plate XIV. fig. 31, *f d*). The fibres of the right ventricle (Plate XIV. fig. 35, *f d*), as was shown, *are continuous in the track for the anterior coronary artery* (*o*) with fibres having a similar direction on the septum ($g\,e'$). They are likewise in many instances *continuous with fibres from the septum, in the track for the posterior coronary artery* (Plate XIII. apply *g e* to $f'\,d'$ of fig. 22), and form flattened rings. These points are best seen when the left ventricle is detached from the right, and the septum is dissected from the left side.

Third external layer of the right and left ventricles (Mammal). The fibres of the third layer (Plate XIII. fig. 24) resemble in their course and general configuration the fibres of the second layer, and proceed in a spiral direction from left to right downwards anteriorly, posteriorly ($f f'$, $d'\,d''$), and septally (Plate XIII. fig. 25, *g e*). They differ, however, from those of the second in being slightly more fascicular, in forming a thicker layer, and in having a direction which is almost transverse. The number of common fibres posteriorly (Plate XIII. fig. 24, $f f''$, $d'\,d''$) is greater in this layer than in the second, partly on account of their very oblique direction, and partly from their proceeding from the left auriculo-ventricular opening nearly all round. Such of the fibres as cross the posterior coronary track to appear on the right ventricle* posteriorly ($f'\,d''$), curve round in an anterior direction until they reach the track for the anterior coronary artery. Arrived here, they dip in at the anterior coronary groove (throughout its entire extent) to assist in forming the third layer of the septum (Plate XIII. fig. 25, *g e*). They also contribute to the Y-shaped arrangement of the fibres alluded to in layer two, the one arm or process giving off fibres to assist in building up the fleshy pons, the other giving off fibres to become continuous posteriorly with such as belong to the right

* In this layer, as in the last, a certain number of the fibres do not cross the posterior coronary groove, but proceed at once to the left apex.

ventricle. The fibres of the third layer (Plate XIII. fig. 24) are continuous with corresponding internal fibres at the base, and proceed from two sources—the one set from the posterior third of the septum and the posterior half and anterior third of the right and left auriculo-ventricular openings (*b l*), the other from the remaining anterior portions of the auriculo-ventricular openings and the anterior two-thirds of the septum.

Fourth or central layer of the right and left ventricles (*Mammal*). The fibres of the fourth layer (Plate XIII. fig. 27, *f d*), unlike the fibres of the other layers, run athwart the ventricles, or at right angles to an imaginary line drawn from the base to the apex. Their direction, which from this circumstance is more or less circular, is accounted for by the external fibres, which run in a spiral direction from left to right downwards, turning abruptly upon themselves in this layer (Plate XII. compare *f g*, *d e*, with *k* of fig. 4) to reverse their course and proceed in an opposite direction, viz. from right to left upwards (Plate XIII. fig. 27, *p q*). The fourth layer consequently forms the boundary between the external and internal layers in both ventricles*; and when it is removed the order of arrangement is reversed: the fibres, instead of proceeding from left to right downwards, becoming more and more oblique, proceed from right to left upwards, gradually returning to an imaginary vertical in an inverse order. The fibres of the right ventricle, it may be observed, pass through the several changes in direction referred to, more rapidly than those of the left; in other words, the fibres of the right ventricle, when the dissection is conducted from without inwards, change from the nearly vertical to the horizontal, and from the horizontal back again to the nearly vertical at comparatively slight depths from the surface, an arrangement evidently occasioned by the greater tenuity of the right ventricular fibres. The difference in the depths at which the layers of the right and left ventricles are found, introduces important changes in the appearance presented by the ventricles at different stages of the dissection. Thus, when the left ventricular wall is half dissected through posteriorly (Plate XV. fig. 54), the right ventricular one is quite dissected away†. I say posteriorly, because, as was explained, the left ventricular wall anteriorly is but little affected, owing to the manner in which the fibres common to both ventricles radiate from the latter. The fibres of the fourth layer proceed in flattened fascicular bands from the auriculo-ventricular orifices all round (Plate XIII. fig. 27, *b l*), and illustrate very well the comparative depths at which the layers of the right and left ventricles are found. On transverse section the outer half of the central layer of the left ventricle posteriorly is ascertained to be on the same level with the internal layers of the right ventricle. Such of the fibres of the fourth layer as are common to both ventricles proceed from left to right, and, having crossed the posterior coronary groove, curve round on the right ventricle until they reach the groove for the anterior coronary artery (Plate XIII. fig. 23, *o o'*), where they dip in (*r*) to traverse the septum (Plate XIII. fig. 28, *g e*) in an antero-posterior direction, and so

* The fourth layer of the right ventricle is represented at Plate XIII. fig. 28, *p q*.

† The right ventricle is only half the thickness of the left.

return to the posterior wall, where many of them are continuous with the fibres of the fourth layer of the right ventricle. The fibres which are not common, and which belong more particularly to the left ventricle, dip in at the posterior coronary groove to traverse the septum (Plate XV. fig. 54, *g*) in an opposite direction, or from behind forwards, where they are continuous with the fibres of the left ventricular wall.

Internal layers of the right ventricle (Mammal).

When the fourth or central layer of the right ventricle (Plate XIII. fig. 23, *p q*), which is on the same level with the more superficial portions of the central layer of the left one (Plate XIII. fig. 27, *f d*), is removed, there is no longer any continuity between the fibres of the right and left ventricles posteriorly; in other words, the common fibres, or those which pass from the one ventricle to the other, are dissected away, and the layers beyond or to the inside of the layer in question belong exclusively to one or other of the ventricles. This arrangement is apparently occasioned by the fibres of the posterior border of the septal duplicature (Plate XVI. diag. 16, G) passing through and becoming blended with those of the posterior wall, only until the central layer in either ventricle is reached (Plate XVI. diag. 17, K). The internal layers of the right ventricle (Plate XIII. figs. 26 & 29, and Plate XIV. figs. 32, 33, 36, & 37), as has been partially explained, are, according to my belief, segmented portions of similar layers isolated from the left or typical ventricle by the primary notch or reduplication. This hypothesis is, I may observe, countenanced by their possessing the general characters of the internal layers of the left ventricle without being so complete. The fibres of the internal layers of the right ventricle proceed in a spiral direction from right to left upwards (Plate XIV. fig. 32, $p\,p'$, $q\,q'$), the fibres of the several layers, when the dissection is conducted from without inwards, becoming more and more vertical (Plate XIV. fig. 33, $p\,p'$, $q\,q'$) as the interior is reached. They therefore pass through all the changes in direction through which those of the left ventricle pass—the fibres of the seventh or last internal layer crossing the fibres of the superficial or first external layer at an acute angle, the fibres of the second and sixth layers at a slightly obtuse angle, and the fibres of the third (Plate XIV. fig. 31) and fifth (Plate XIV. fig. 32) layers at a very obtuse angle. When, however, the spirals formed by the fibres of the internal layers of the right ventricle (Plate XIV. figs. 36 & 37) are examined or compared with the spirals formed by the fibres of the internal layers of the left ventricle (Plate XII. figs. 6 & 7), the segmentary nature of the former is at once apparent, these in no instance forming complete double conical spirals similar to those found in the left ventricle, but only spiral fragments, or at most flattened rings, such as would be obtained by isolating or detaching a portion of a perfect cone composed of the double conical spirals described.

Fifth layer of the right ventricle (Mammal). The fifth layer of the right ventricle (Plate XIV. fig. 32), from the fact of its being immediately to the inner side of the fourth central or transverse layer (Plate XIII. figs. 23 & 28, *p q*), is the first of the internal ones (Plate XIV. fig. 32, $p\,p'$, $q\,q'$). The fibres composing it proceed in a very

oblique spiral direction from behind forwards and from right to left upwards; and are aggregated into well-marked fascicular bundles which are continuous anteriorly (Plate XIV. fig. 36, *o*) and for the most part posteriorly with fibres having a like direction on the right third of the septum (*p'q'*). They are continuous also with the fibres of the third external layer at the base. As the common fibres do not extend to, or implicate the internal layers, the fibres of the right third of the septum readily become continuous with internal fibres on the posterior wall having a corresponding direction (Plate XIV. fig. 32, *p q*). The fibres of the fifth layer (Plate XIV. fig. 32) split up or bifurcate at a point corresponding with the fleshy pons, into the formation of which they enter, the one half bending over in a direction from without inwards (*m*) to become continuous with fibres from the septum having a similar direction—the other half curving round the infundibuliform portion of the right ventricle anteriorly (*p'*) to dip in at the track for the anterior coronary artery (Plate XIII. fig. 26, *o*), from which they emerge with the septal fibres referred to. The fibres of the fifth layer seldom make more than one turn of a spiral, many of them, from terminating at the root of the aorta and septum anteriorly, making less.

Sixth layer of right ventricle (*Mammal*). The fibres of the sixth layer of the right ventricle (Plate XIV. fig. 33) agree in their more important features with the fibres of the fifth layer; *i. e.* they proceed in a spiral direction from right to left, or from behind forwards and from below upwards (*p p'*, *q q'*), the fibres bifurcating (*m q'*) to assist in forming the fleshy pons (*m*) and the infundibulum (*q'*). The fibres of the sixth layer agree with those of the fifth in being continuous anteriorly (Plate XIV.* fig. 37, *w*) and posteriorly (Plate XIV. fig. 33, *p q*) with fibres having a similar direction on the septum (Plate XIV. fig. 37, *p'q'*), and with corresponding external fibres (the fibres of the second layer) at the base (Plate XIV. fig. 31, *ff'*, *d d'*). The fibres of the sixth layer differ from those of the fifth in pursuing a slightly more vertical direction, and in not being quite so fascicular. They form the homologues of the fibres of the second layer.

Seventh or last internal layer of right ventricle (*Mammal*). The fibres of the seventh or last internal layer of the right ventricle are those principally engaged in the formation of the carneæ columnæ and musculi papillares of the right ventricle, and have been already described.

The points established with reference to the right ventricle are these:—

1st. Many of the fibres entering into the formation of the right ventricular wall proceed from the left auriculo-ventricular opening, so that the right ventricle is to a certain extent dependent for its existence on the left.

2ndly. The external fibres of the right ventricle, with the exception of a broad band at the base, limited to the first two external layers, dip in at the track for the anterior coronary artery, to appear on the septum.

3rdly. Many of the fibres, especially of the deeper layers, are continuous anteriorly

* In Plate XIII. fig. 29, the fibres in question (*p q*) are seen dipping in at the anterior coronary track (*o o'*).

and posteriorly, and form flattened rings, which accommodate themselves to the shape of the ventricle.

The points in which the fibres of the right ventricle differ from those of the left are the following:—

1st. They are more delicate.

2ndly. They form segments of spirals and flattened rings, instead of double conical spirals.

3rdly. The external fibres enter the anterior coronary groove to become internal, not at one particular point, as in the apex of the left ventricle, but throughout its entire extent, the broad band at the base excepted.

4thly. The fibres of the right ventricle form a constriction anteriorly (the so-called fleshy pons). This constriction separates the pulmonary artery from the auriculo-ventricular opening, and does not exist in the left ventricle, unless the septal segment of the bicuspid valve is taken to represent it.

Vertical section of the right ventricle (Mammal).

When a vertical section of the right ventricle is made posteriorly (Plate XIV. figs. 43 & 44, *s*), it is found to taper in two directions, as in the left. It differs, however, from a similar section of the left ventricular wall in being comparatively much thicker towards the apex. This arises from the manner in which the right apex is formed, and is readily explained according to the segmentary process which is believed to separate the right apex from the left *in utero*. Into the extreme apex of the left or typical ventricle, as was shown, only one layer enters; whereas into successive portions of the left ventricular wall (at slight removes from the extreme apex) two, three, and four layers enter. But the right apex is known to be separated from the right side of the left apex at a part considerably above its extreme point, and where the left ventricular wall is somewhat thickened; so that probably two or even three layers enter into the construction of the right apex.

Transverse sections of the right and left ventricles (Mammal).

What has been said with reference to the difference in direction of the several external and internal layers of the right and left ventricles and septum, the common nature of the fibres posteriorly, and their independent nature anteriorly, the varying thickness of the right and left ventricular and septal walls, the conical shape of the right and left ventricular cavities, &c., is fully borne out by transverse sections. When a transverse section of the ventricles of the deer is made half an inch or so from the base (Plate XV. fig. 49), the following phenomena are observed:—

1st. The cut ends of the more vertical fibres exteriorly (*c c′ c″*) and interiorly (*d d′ d″*) are nearly equal in number, and are seen to the outside and inside of the deeper or more central fibres. The prevailing direction of the central fibres, on the septum (*e′*) and left ventricular wall (*e*), is more or less circular, owing probably to many of the

internal fibres changing their direction at this point to become external. On the right ventricular wall the fibres composing the more central layers are distinctly seen to cross each other (*f*).

2ndly. The fibres which are common to both ventricles posteriorly, can be traced passing from the left ventricle to the right (*g*).

3rdly. The large fascicular bundles of fibres which constitute the carneæ columnæ ($o\,o'\,o''$), and the free ends of the musculi papillares with their chordæ tendineæ attached (*x y*), may be recognized surrounding the ventricular cavities (*b l*).

4thly. The shape of the left ventricular cavity at this point is oval (*b*), and of the right cavity concavo-convex (*l*).

5thly. The thickness of the left ventricular wall (*c d*) and the septum ($c'd'$) is as nearly as possible equal, that of the right ventricular wall ($c''d''$) being only half the thickness of either of the former.

6thly. The thickness of the ventricular walls, as a whole, is not quite so great in this section as it is three-quarters of an inch nearer the apex (Plate XV. fig. 50).

In a transverse section rather less than an inch and a half from the base (Plate XV. fig. 50) the same phenomena are repeated, with the following slight differences:—

1st. The aggregate of the external fibres ($c\,c''$) is considerably less than the aggregate of the internal ones ($d\,d''$), out of which the carneæ columnæ (*o*) and musculi papillares (*x y*, $h\,h'$) spring—an arrangement necessitated by the external fibres requiring in this instance to crowd together, in order to accommodate themselves to the diminished calibre of the cone interiorly.

2ndly. The common fibres posteriorly (*g*) pass from the left to the right ventricle, and dip or bend in at the track for the anterior coronary artery, to become continuous with fibres having a similar direction on the septum (*m*).

3rdly. On account of the great preponderance of the internal fibres, and the projecting of the musculi papillares into the interior, the appearance of the ventricular cavities is considerably changed. Thus the left ventricular cavity (*b*) is triangular—the right ventricular one (*l*) being concavo-convex, and having two constrictions init caused by the anterior (h') and posterior (*h*) musculi papillares.

4thly. The thickness of the septum ($c'd'$) is nearly one-sixth greater than that of the left ventricular wall (*c d*), between the papillary muscles (*x y*), the thickness of the right ventricular wall ($c''d''$) being only half that of the latter.

5thly. The thickness of the ventricular walls is greater in this section than in any other.

When a transverse section is made about two inches and a quarter from the base (Plate XV. fig. 51), the thickness of the ventricular walls (*c d*, $c'd'$, $c''d''$) is found to have diminished slightly. The peculiarities of this section are these:—

1st. The preponderance of the internal ($d\,d''$) over the external ($c\,c''$) fibres, especially in the left ventricle, is more marked than in the two preceding sections.

2ndly. The circular nature of the more central fibres is also better defined ($e\,e'$), the fibres being observed to cross each other (*f*).

3rdly. The musculi papillares ($x\,y$) of the left ventricle (b) are very prominent, the left ventricular cavity (b) from this circumstance being more triangular in shape than in the second section (Plate XV. fig. 50).

4thly. The right ventricular cavity (l), from its proximity to the right apex, is moreover greatly reduced in size.

In a transverse section three and a half inches from the base, and fully half an inch from the left apex (Plate XV. fig. 52)—the right apex is now removed—the subjoined results are obtained:—

1st. The preponderance of the internal ($d\,d'$) over the external ($c\,c'$) fibres is still more marked, illustrating the necessity for the internal fibres overlapping and crowding on their appearance in the interior, more especially at the apex, where the cavity is greatly reduced.

2ndly. The circular nature of the more central fibres ($e\,e'$) is still better defined, many of the external fibres at this point reversing their direction to become internal.

3rdly. The musculi papillares are very prominent, the external fibres which have just entered the interior being seen to curve into them ($x\,y$).

4thly. The left ventricular cavity (b), from the comparatively large dimensions of the musculi papillares, is greatly diminished *; and the form of the left ventricular cavity is more or less bayonet-shaped.

In a transverse section a quarter of an inch from the extremity of the left apex (Plate XV. fig. 53), the peculiarities of the preceding section (fig. 52) are found exaggerated. Thus the quantity of the internal as compared with the external fibres is increased—the more central fibres ($e\,e'$), from the great number of external ones which at the apex change their direction to become internal, curving round in a regular whorl, many of them entering directly into the composition of the musculi papillares ($x\,y$); the left ventricular cavity is now all but closed.

Casts of the interior of the right and left ventricles (Mammal).

When casts of the interior of the ventricles are taken, the left ventricular cavity (Plate XII. fig. 17), as has been stated, yields a highly symmetrical conical screw whose spiral runs from left to right downwards, the right ventricular cavity (Plate XII. fig. 16) yielding a more unsymmetrical one—unsymmetrical in this sense, that it is flattened out and applied to or round the left. The amount of spiral made by the left ventricular cavity is rather over a turn and a half; that made by the right ventricular cavity rather under a turn.

* In this and the following section, the cavity of the left ventricle, towards the apex, would be at once obliterated by a slight approximation of the musculi papillares—this approximation of the papillary muscles being effected by the contraction of the spiral nearly circular fibres which constitute the apex. By this arrangement, the apex, which is the weak part of the ventricle, can be readily converted into a solid muscular wall capable of resisting any amount of pressure.

Shape of the right and left ventricular cavities, as shown by casts and transverse sections.

As it is difficult to obtain a correct idea of the shape of the ventricular cavities, a detailed description of them may prove not unacceptable. The left ventricular cavity in the fresh heart of the deer, at the extreme base (if the aortic opening is not included), is more or less circular in form (Plate XII. fig. 17, *b*). Half an inch or so from the base (Plate XV. fig. 49, *b*) it changes from the circular to the oval, and is slightly increased in size from the fact of the left ventricular cavity tapering from its middle third towards the base (Plate XII. fig. 17, *b*). In this portion of the left ventricular cavity (Plate XV. fig. 49), the chordæ tendineæ and the segments of the bicuspid valve hang loosely.

Receding from the base to the extent of fully an inch (Plate XV. fig. 50, *b*), the appearance of the left ventricular cavity again changes—the change in this instance being caused by the projection into it of the flattened oblique heads of the papillary muscles (x, y), which convert it from an oval shape into an irregularly triangular one (Plate XII. fig. 17, *w*).

Proceeding an inch or so nearer the apex, the left ventricular cavity (Plate XV. fig. 51, *b*) becomes smaller and more decidedly triangular, and is, from the prominence of the carneæ columnæ and musculi papillares (Plates XII. & XV. figs. 17 & 51, $x\,y$), somewhat bayonet-shaped. The bayonet-shaped appearance of the cavity becomes better defined as the extreme apex is reached (Plate XV. figs. 52 & 53, *b*), the cavity itself becoming smaller and smaller until it terminates in a point (Plate XII. fig. 17, *z*).

The right ventricular cavity (Plates XII. & XV. figs. 16 & 49, *l*), which is as it were applied to or round the left one (Plates XII. & XV. figs. 17 & 49), is also conical-shaped (Plate XII. fig. 16). It agrees with the left in having nearly the same vertical measurement, but differs from it in having a considerably greater antero-posterior measurement, and a decidedly less transverse one (Plate XV. compare *b* and *l* of fig. 49). Its shape at the extreme base, owing to the spindle-shaped constriction (fleshy pons) which separates it from the opening for the pulmonary artery, is oval * (Plate XV. fig. 46, *l*). Half an inch from the base it is concavo-convex (Plate XV. fig. 49, *l*), and, from the protruding of the carneæ columnæ and musculi papillares (*h*) at this point, more or less irregular. The chordæ tendineæ and the segments of the tricuspid valve hang loosely in this portion of the right ventricular cavity.

Receding from the base in the direction of the right apex an inch and a half or so (Plate XV. fig. 50), the shape of the right ventricular cavity (*l*) is still concavo-convex; it is moreover slightly diminished in its antero-posterior and transverse diameters, in conformity with its conical nature (Plate XII. fig. 16). Proceeding to within a quarter of an inch of the right apex, the right ventricular cavity (Plate XII. fig. 12, *m*, and Plate XV. fig. 51, *l*) is seen to maintain its concavo-convex shape, and to taper gradually until it terminates in a blunted extremity (Plate XII. fig. 16, *m*).

* In this description the infundibulum, or conus arteriosus, is regarded as projecting beyond what is commonly regarded as the base.

The comparative size of the right and left ventricular cavities has been the subject of considerable discussion, and is not likely soon to be set at rest, from the yielding nature of the ventricular parietes.

Inference deduced from a consideration of the Arrangement of the Fibres in the Ventricles of the Bird and Mammal.

Without presuming dogmatically to assert that the ultimate arrangement of the fibres of the ventricles of the bird and mammal is reducible to any known mathematical law, I cannot omit mentioning the fact that the arrangement in question can be so thoroughly imitated, even in its details, by certain mechanical contrivances about to be explained, that I would consider the present communication incomplete, were I not shortly to direct attention to them.

If a sheet of paper, parchment, or any flexible material * whose length is twice that of its breadth, be taken, and parallel lines drawn on either side of it in the direction of the length, to represent the course of the fibres (Plate XV. diag. 1), all that requires to be done, in order to convert it into a literal transcript of one-half of the left or typical ventricle, is to lay it out lengthwise across a table, and, catching it by the right-hand distant corner, to roll or turn in towards one's self a conical-shaped portion (C), and continue the rolling process in the direction of the opposite or oblique corner (U), until three and a half turns of the sheet have been made and a hollow cone produced, as shown at diagrams 4 & 5, Plate XVI. If the sheet be so manipulated, it will be found that every line in it is converted into a double conical spiral,—the one-half of the spiral being external to the other half, and running from base to apex and from left to right (Plate XVI. diag. 3, A B), precisely as in the left ventricle (Plate XII. compare with the direction of the fibres in figs. 1 & 9); the remaining half, which is internal, running from apex to base, and from right to left (Plate XVI. diag. 3, D E; Plate XII. compare with the direction of the fibres in figs. 7 & 8). Tracing the external spirals to the apex, they are seen to turn abruptly upon themselves at this point (Plate XVI. diagrams 3 & 4, C, diagrams 8 & 13, D; Plate XII. compare with apex of fig. 10), to reverse their direction and enter the interior, where they are continuous with the internal spirals (Plate XVI. diagrams 4 & 6, D E E′; Plate XII. compare with the fibres marked *k* of fig. 4); and if the corners of the sheet be folded out at the base, as has been done at Plate XVI. diagrams 3, 4, 5, & 6, and the internal spirals traced from the apex, or from below upwards (Plate XVI. diag. 3, H I J), they will be seen again to reverse their direction to become continuous with the external spirals (F G H),—an arrangement coinciding in

* Very beautiful transparent models of the left ventricle may be made, by employing sheets of net of a large pattern, with threads of wool drawn through the interstices at intervals of an inch or so. Sheets of this nature have been photographed to illustrate this part of the paper. Very convenient opaque models may be constructed by employing portions of newspapers, the reading of which represents the parallel lines. The author strongly recommends the use of these models, as a few minutes with such aids will throw more light on the course and direction of the fibres than hours of abstract reasoning without them.

the most perfect manner with the structure exhibited in the left ventricle (Plate XII. compare the fibres marked *ff'* of fig. 1 with those marked *o o' o''* of fig. 8). Nor does the coincidence stop here. If the cone composed of the spiral lines constructed as above (Plate XVI. diagram 4) be examined in a direction from without inwards, it will be seen that its walls on the one aspect consist of four distinct layers—two external (A A', B, compare with the direction of the fibres in Plate XII. figs. 1 & 3) and two internal (D, E E', compare with the direction of the fibres in Plate XII. figs. 5 & 8); and on the other (Plate XVI. diagram 5) of three—one external (B B', compare with the direction of the fibres in Plate XII. fig. 2), one internal (D D', compare with the direction of the fibres in Plate XII. fig. 6), and one central (C C' C'', compare with the direction of the fibres in Plate XII. fig. 4); and, what is remarkable, that the lines entering into the formation of the external layers run in different directions, and cross those forming the internal layers as in the ventricle itself (Plate XII., compare the direction of the external fibres in figs. 1, 2, & 3 with that of the internal fibres in figs. 5, 6, & 7). Another point of resemblance, deserving of particular attention, is the regional distribution of the layers themselves, that side of the cone which is composed of four layers (Plate XVI. diagram 4) showing how one layer enters into the formation of the extreme apex (C), two into the extreme base (A E') and the portion of the ventricular wall immediately above the apex (B D), three into the portion of the wall immediately below the base, and four into the upper and more central portion (B' D'),—the other or remaining side which is composed of three layers (Plate XVI. diagram 5), showing that one layer enters into the extreme apex (C), two into the extreme base (R), and three into the upper and central portion (C' C''). This arrangement proves, curiously enough, that the model ventricular wall, like the true one, tapers in two directions, viz. towards the base (Plate XII. compare with *s* of figs. 13, 14, & 15) and towards the apex (Plate XII. compare with *s'* of figs. 13, 14, & 15). There are other points of resemblance. If the marginal line* (Plate XVI. diagram 3, A B C D E) of the sheet rolled up as described (and which may be taken to represent the first and seventh layers of the left ventricle) be traced, it will be found to proceed in a spiral nearly vertical direction, and to run from the extreme base (A) to the extreme apex (C), at which point it reverses its course and enters the interior, after which it returns to the extreme base, making one turn and a half of a spiral in either direction, as in Plate XII. figs. 1 & 8. It in this manner embraces in its convolutions all the other lines, the initial letters of which are H M R V, in the same way that the first layer and its internal continuation, the seventh layer, overlaps or embraces all the deeper layers (Plate XII. *vide* figs. 2, 3, 4, 5, and 6), all of which are included within and embraced by figs. 1 and 7). If, on the other hand, a second line a little removed from the marginal line (Plate XV. diagram 1, F G H I J),

* I have chosen to speak of individual lines, as admitting of more precise description; but it would be equally correct to speak of the lines confined to any particular portion of the sheet, for it is the aggregate of the lines in certain portions of it which constitutes the layers. Thus the lines composing one-third of the sheet go to form the first external layer and the last internal (Plate XVI. diagram 4, A A' & E E').

and which when the sheet is rolled up represents two deeper layers (Plate XVI. diagram 3, F G H I J), be taken, it will be found to pursue a similar though slightly more oblique course, and to extend from the extreme base (F) to a point somewhat short of the extreme apex (H), where it turns abruptly upon itself at a much wider part of the cone, to change its direction and enter the interior; after which, like the other, it gradually regains the base (J),—an arrangement which proves that the apex of the artifical cone or ventricle is opened into or enlarged by the removal of successive lines, precisely in the same way that the apex of the left ventricle is opened into, or enlarged, by the removal of successive layers (Plate XII. compare *f g*, *e d*, of fig. 9 with *k l* of fig. 11). It further shows that many of the lines, like the fibres themselves, return to points not wide of those from which they started. If another line, about the middle of the sheet (Plates XV. & XVI. diagrams 1 & 3, K L M N O), be taken, and traced in the same manner, it will be seen to proceed from the extreme base K (Plate XII. compare with fibres marked *f* in fig. 3), and to turn upon itself or reverse its direction and enter the interior at a still wider portion of the cone M (Plate XII. compare with fibres marked *k* in fig. 3)—*i. e.* at a point still further removed from the original apex (C)—and so on until the opposite marginal line (Plates XV. & XVI. diagrams 1 & 3, U V W) is reached, which confines itself entirely to the base, or upper portion of the cone (Plate XII. compare with fibres marked *f g*, *d′ e*, fig. 4), and makes only one turn of a spiral. This disposition of the lines reveals what to me is a very interesting fact, viz. that certain lines, like certain layers, are confined to certain localities or regions. (The first layer of the left ventricle, as was pointed out, extends from the extreme base to the extreme apex (Plate XII. fig. 1), while the fourth layer does not quite extend to either (Plate XII. fig. 4). It further shows, what is scarcely less important, that, of the lines originally of the same length, some, from the fact of their winding very partially round the base or wider portion of the cone (Plate XVI. diagram 3, A) while they twist rapidly round the apex or narrow part (B C), make one turn and a half of a spiral, whereas others, which are confined to and wind round the base or wider portion of the cone, make only one turn of a spiral (Plate XVI. diagram 3, U V W)—an arrangement which, as I endeavoured to point out (see p. 456), prevails also in the left ventricle. When the interior of the cone, fashioned as recommended, is examined, the marginal lines are observed to twist out of the apex (Plate XVI. diagram 9, Y) and assume a more or less vertical direction (Plate XVI. diagram 6, E E′, and diagram 8, Y), in a manner which wonderfully accords with the direction of the fibres composing one or other of the musculi papillares (Plate XVI. compare Y of diagram 9 with *y* of figs. 12 & 13, Plate XII.).

Having constructed one half of the typical ventricle, it is necessary, to complete the other, to lay a second sheet upon the original one, at a slight angle, as represented in diagram 2, Plate XV.*, after which the sheets are rolled up (the one within the

* In the diagram adverted to, it will be observed I have represented on the corners of the sheets two imaginary papillary muscles (X, Y), for the purpose of showing the positions they occupy in the interior of the cone when the sheets are rolled up.

other), as was recommended when manipulating the single sheet. The additional sheet, which is thus made to pass through the same changes as the first, in nowise complicates the arrangement, and has the effect of doubling the layers, while it does not increase the number of spiral turns made by the lines composing them*. The object of having two sheets set at an angle is to ensure that the lines will enter the apex, and issue from the base, in two places, so as to render the cone bilaterally symmetrical, as it exists in the undissected left ventricle; for it is found that, if only one sheet is employed, the apex of the cone, from its being composed of spiral lines, is lopsided, or like the barrel of a pen cut slantingly (Plate XVI. diagram 6, D). This difficulty is at once obviated by the employment of the second sheet; for the one sheet being made by the angle of difference to wind from behind forwards (Plate XVI. diagram 10, C D), and the lines to enter the apex anteriorly (E), while the other winds from before backwards (A B), the lines entering the apex posteriorly (F), the apex, as every other portion of the cone, is rendered bilaterally symmetrical (Plate XVI. compare *a b e* with *c d f* of fig. 55; Plate XII. compare also *f g* and *e d* of figs. 9, 10, and *x y* of figs. 12, 13, & 14). When sheets adjusted as represented in Plate XV. diagram 2 are rolled up into a cone, as may be closely and conveniently imitated by placing the cone marked 5, Plate XVI., inside of that marked 4†, it will be found that a cone bilaterally symmetrical (Plate XVI. diagrams 7, 10, 11, 12 & 14), having a symmetrical apex (diagrams 7, 10, & 12, E F, diagram 11, X Y, and diagram 14, D K) with walls consisting of seven layers, each having a different direction (Plate XVI. diagram 4, A, B, D, E, and diagram 5, B C D), is at once produced. Of these layers three are external, three internal, and one central, precisely as in the left ventricle itself (Plate XII. compare with figs. from 1 to 8 inclusive).

If the foregoing arrangement were reduced to a principle and applied to the fibres of the left ventricle of the bird and mammal, it would be stated in something like the following terms.

By a simple process of *involution* and *evolution*, the external fibres become internal at the apex, and the internal ones external at the base; so that whether they be traced from without inwards, or from within outwards, they always return to points not wide of those from which they started. The fourfold set of fibres, viz. the two external sets and the two internal, being spirally arranged round a cone, and running in two diametrically opposite directions, it follows that, in order to involute and evolute, certain preliminary changes are necessary. Thus the two sets of external fibres, which wind from the base of the cone to the apex in a direction from left to right downwards, make

* Another effect produced by the two sheets being set at an angle and rolled within each other, which finds a counterpart in the ventricle itself, is the following. The lines composing the several anterior layers are relatively more vertical than those composing the posterior ones.

† Strictly speaking the sheet composing the one cone should be rolled within that composing the other, so that, when the unwinding of the cones takes place, alternate coils or layers from either cone are removed in constant succession.

smaller and smaller curves as they approach the apex, where they *involute or turn in;* whereas the two sets of internal ones, which wind from the apex of the cone to the base in a direction from right to left upwards, make larger and larger curves as they approach the base, where they *evolute or turn out.* The external and internal fibres have therefore different directions in different parts of their respective courses; and as the spirals formed by the external and internal sets cross and overlap at every half turn of their progress, we are in this way furnished not only with fibres having different degrees of obliquity, but also with different layers or strata of fibres.

Since the artificial ventricle, constructed as described, presents all the peculiarities of the typical or left ventricle, it is obvious that if, beginning a little above its apex, a portion of the anterior wall is pushed in (Plate XVI. diagram 15, A; Plate XV. compare with *m* of fig. 50, and with *o l*, *p n* of fig. 45) until it touches the posterior one (Plate XVI. diagram 15, B; Plate XV. compare with *g* of fig. 50), and allowance made for the passing through and blending of the lines posteriorly (Plate XVI. diagram 16, G) and septally (Plate XVI. diagrams 16, H K; also diagrams 17 & 18, H E), as recommended at page 464, and indicated by the division of the primary tube in the embryo, two ventricles (C & D) would be produced, resembling the true ventricles (compare with *b l* of figs. 49, 50, & 51, and *o p* of fig. 45, Plate XV. more particularly; also with *o* of fig. 23, Plate XIII.) as closely as the artificial single ventricle resembles the left or typical ventricle. This is so evident that further explanation is unnecessary.

With these remarks I finish the description of an avowedly difficult structure; and, in taking leave of the subject, I trust I may not be charged with forcing analogies and instituting resemblances where none exist. Having no theory to serve, my sole aim throughout the investigation has been the elucidation of truth; but where that is so cunningly, and, it may be added, so successfully concealed, it has often been exceedingly difficult to arrange the materials with which my numerous dissections supplied me, so as to preserve a sequence in the description while I at the same time contrasted the direction of the fibres composing the several layers with each other. The method employed in demonstrating the ventricles in the present instance, viz. by consecutive layers, while it is by far the most natural yet proposed, is, I believe, the best calculated to afford intelligible results; for as layers or strata of fibres unquestionably exist, and these intersect each other at various angles, and are found at different depths from the surface, it follows that all attempts to display in any individual heart, what can only be shown in a series of hearts, must prove abortive. The great advantage of conducting the dissection from without inwards by the removal of consecutive layers consists in its preserving the relation of the several layers to each other, and in showing how the fibres of each are continuous at the base and at the apex. This is well seen in the first seven figures of Plate XII., where the layers of the left ventricle are exposed posteriorly; for by placing fig. 7 within 6, figs. 7 & 6 within 5, and so on until all the figures are placed within fig. 1, not only are the relations of the several layers to each other maintained, but the ventricle is as it were rendered transparent, so that one may trace in imagina-

tion, or by the aid of diagrams 4 & 5, the fibres of figs. 8 & 7 crossing those of fig. 1, the fibres of fig. 6 crossing those of fig. 2, and those of fig. 5 crossing those of fig. 3. The minutely reticulated structure to which this disposition of the fibres gives rise, although very simple when the layers are regarded separately or apart from each other, becomes very perplexing when they are placed in apposition or as they occur in the undissected ventricle; and to the partial dissection of the layers perhaps more than to any other cause, is to be attributed that numerous class of complicated diagrams which represent the fibres of the ventricles as running in all directions without either law or order. In those diagrams that beautiful gradation in direction by which the fibres diverge from an imaginary vertical, and gradually return to it after having intersected each other in all directions, finds no place. In conclusion, the scheme of the course and direction of the fibres as summed up, while it greatly facilitates the comprehension of the general principle involved in the ultimate structure of the ventricles, harmonizes in the most perfect manner with all that is at present known of the heart's movements—those movements apparently so simple, and yet so difficult of analysis.

Explanation of the Plates.

In the engravings the same letters have been employed, as far as possible, to designate corresponding portions of the ventricles. The description of the figures in Plate XIII. from 22 to 29 inclusive does not follow in strictly numerical order. Thus the description of fig. 24 follows the description of fig. 22, and precedes that of figs. 25, 27, & 28; while the description of fig. 23 follows that of fig. 28, and precedes that of fig. 26. The object of this arrangement is to ensure that the description may be read as a connected narrative. In the figures of the right ventricle some of the layers have not been represented, from a desire to curtail the number of figures. This, however, can occasion no difficulty, as portions of the unrepresented layers may be seen in other figures. A portion of the second external layer of the right ventricle (one of the omitted layers) is seen at *f'* of fig. 22, while the greater portions of the two other unrepresented layers, viz. layers four and seven, are seen at Plate XIII. fig. 23, *p*, *q*, and Plate XIV. fig. 43, *o*, *h*.

PLATE XII.

Fig. 1. Left ventricle of the sheep's heart, seen posteriorly. Shows the superficial or first external layer. See pages 454, 455, & 456.

Fig. 2. Left ventricle of the sheep's heart, seen posteriorly. Shows the second layer. See pages 456 & 457.

Fig. 3. Left ventricle of the sheep's heart, seen posteriorly. Shows the third layer. See page 458.

Fig. 4. Left ventricle of the sheep's heart, seen posteriorly. Shows the fourth or transverse layer, which occupies a central position in the ventricular wall, and divides the external from the internal layers. The fibres of this layer are in the act of doubling or turning upon themselves. See pages 458 & 459.

Fig. 5. Left ventricle of the sheep's heart, seen posteriorly. Shows the fifth layer. See page 459.

Fig. 6. Left ventricle of the sheep's heart, seen posteriorly. Shows the sixth layer. See pages 459 & 460.

Figs. 7 & 8. Left ventricle of the sheep's heart, seen posteriorly. Show the seventh or last internal layer, the fibres composing which extend from the extreme base to the extreme apex. See pages 460, 461, & 462.

Fig. 9. Left ventricle of the sheep's heart, as seen posteriorly from above. Shows the course pursued by the two sets of fibres constituting the superficial or first external layer. See pages 454, 455, & 456.

f g. Posterior set of fibres of first layer, winding in a spiral direction from base to apex to enter the apex anteriorly.

e d. Anterior set of fibres of the first layer, winding in a spiral direction from base to apex to enter the apex posteriorly. As the convexity of the posterior set of fibres fits accurately into the concavity of the anterior set, they are linked or twisted into each other so as completely to close the apex and render it bilaterally symmetrical. The internal continuations of the major portions of the two sets of fibres forming the first layer are seen at *x* and *y* of figs. 12, 13, 14, & 15, where they appear as the anterior and posterior musculi papillares.

Fig. 10. Bird's-eye view of the apex of the left ventricle of the sheep's heart. Shows the two sets of fibres constituting the superficial or first external layer, separated from each other, and entering the left apex to become internal without breach of continuity. See pages 454, 455, & 456.

e d. The anterior set of fibres, curving or twisting into or round the posterior set.

f g. The posterior set of fibres, curving or twisting into or round the anterior set.

Fig. 11. Bird's-eye view of the apex of the left ventricle of the sheep's heart. Shows the appearance presented by the left apex when the two sets of fibres composing the first layer have been removed. See pages 456 & 457.

f g. Undissected portion of the anterior set of fibres of the first layer.

l. Anterior set of fibres of the second layer, preparing to enter the left apex posteriorly.

k. Posterior set of fibres of the second layer, preparing to enter the apex anteriorly.

Fig. 12. Transverse section of the left ventricle of the sheep's heart, half an inch above

the apex. Shows the anterior and posterior sets of fibres forming the seventh or last internal layer, twisting out of the interior of the left apex, in an opposite direction to that by which the anterior and posterior sets of fibres forming the first layer (with which they are continuous) entered. See pages 460, 461, & 462.

y. Anterior musculus papillaris, cut across.

x. Posterior musculus papillaris, cut across.

m. Right apex. Shows comparative absence of spiral twist in the fibres composing it.

Fig. 13. Left ventricle of the heart of a deer, opened anteriorly. Shows the anterior and posterior musculi papillares *in situ.* See pages 460, 461, & 462.

y. The anterior musculus papillaris, winding in a spiral nearly vertical direction from the apex to within a short distance of the base, where it terminates in a more or less flattened uneven head, the irregular surface being occasioned by muscular prominences which give off chordæ tendineæ to be inserted into the segments of the bicuspid valve.

x. The posterior musculus papillaris, twisting from behind the anterior one, and winding in a spiral nearly vertical direction from the apex to within a short distance of the base, where it terminates like the anterior, in a flattened uneven head.

s. Vertical section of the left ventricle, showing how the ventricular wall tapers towards the apex and the base.

r. Fibrous stay connecting the posterior musculus papillaris with the septal side of the left ventricular cavity.

Fig. 14. Left ventricle of the heifer's heart, opened laterally. Shows the musculi papillares and the bicuspid valve as usually displayed, the spiral twist peculiar to the musculi papillares being inadvertently destroyed. See pages 460 & 461.

v. Anterior or outer segment of the bicuspid valve, with the chordæ tendineæ (*t*) which proceed from each musculus papillaris terminating in it.

s s'. Vertical section of ventricular wall, tapering towards the apex and the base.

w. Termination of the spiral groove which forms one of the two hollows found between the spiral musculi papillares. This groove, or rather the cast taken from it, is seen throughout its entire extent at *z w b* of fig. 17.

r. Fibrous stays connecting the anterior and posterior musculi papillares with the septal side of the ventricular cavity.

z. Reticulated arrangement of the fibres lining the interior of the ventricle and forming the carneæ columnæ.

Fig. 15. Left ventricle of a human heart, opened laterally. Shows a tendency on the part of the fascicular bundles forming the musculi papillares to remain separate, also the highly developed nature of the carneæ columnæ. See page 461.

a. Aorta.

v. Posterior and inner segment of the bicuspid valve, attached by its chordæ tendineæ to the musculi papillares.

y y'. The anterior musculus papillaris, consisting in this instance of two portions, from the fibres composing it never having fully united.

x. Posterior musculus papillaris, likewise terminating in muscular processes.

z. Carneæ columnæ, forming a rich network which lines the interior of the ventricle.

s s'. Vertical section of the ventricular wall, showing how it tapers towards the base and the apex.

Fig. 16. Wax cast of the interior of the right ventricle of a deer's heart, seen anteriorly. Shows the spiral nature of the cavity. See page 482.

l. Right auriculo-ventricular opening.

k. Conical-shaped spiral infundibulum or conus arteriosus, with the spiral grooves occasioned by the projection of the carneæ columnæ into it. These grooves facilitate the passage of the blood towards the pulmonary artery during the systole.

h. Depression caused by the head of the right anterior musculus papillaris.

m. Right apex, the peculiarity of which consists in its being more blunted, and less distinctly spiral, than that of the left apex.

Fig. 17. Wax cast of the interior of the left ventricle of a deer's heart. Shows the peculiar spiral twist of the left ventricular cavity, and how, like the wall, it tapers towards the base and the apex. See pages 462 & 482.

b. Left auriculo-ventricular opening.

x. Spiral track of the posterior musculus papillaris.

y. Spiral track of the anterior musculus papillaris.

z w b. Projecting spiral ridge, corresponding to one of the spiral grooves or hollows found between the spiral musculi papillares.

z. Left apex, twisting rapidly upon itself and terminating in a point. The left ventricular cavity is widest at the upper part of its middle third (*w*), and the amount of spiral made by it rather exceeds a turn and a half.

Note.—Figs. 16 & 17 give the exact shape of the ventricular cavities, and consequently the precise form assumed by the blood, prior to the systole.

PLATE XIII.

Fig. 18. Right and left ventricles of the sheep's heart, seen posteriorly. Shows the superficial or first external layer of both ventricles. See pages 467 & 468.

Fig. 19. Right and left ventricles of the sheep's heart, seen posteriorly, the right ventricular wall being divided and separated to expose the septum. Shows how the direction of the fibres of the first layer of the septum corresponds with the direction of the fibres of the first layer of the right and left ventricles. See pages 468 & 469.

Fig. 20. Right and left ventricles of the sheep's heart, seen anteriorly. Shows the direction of the fibres of the superficial or first external layer of both ventricles, the track for the anterior coronary artery, &c. See page 468.

Fig. 21. Right and left ventricles of the sheep's heart, seen posteriorly. Shows the direction of the fibres of the second layer of the right and left ventricles. See pages 475 & 476.

Fig. 22. Right and left ventricles of the sheep's heart with the septum exposed, seen posteriorly. Shows how the fibres of the second layer of the septum correspond in direction with those of the second layer of the right and left ventricles generally. See page 476.

Fig. 24. Right and left ventricles of the sheep's heart, seen posteriorly. Shows the direction of the fibres of the third layer of the right and left ventricles. See pages 476 & 477.

Fig. 25. Right and left ventricles of the sheep's heart with the septum exposed, seen posteriorly. Shows how the direction of the fibres of the third layer of the septum corresponds with the direction of the fibres of the third layer of the right and left ventricles generally. See pages 476 & 477.

Fig. 27. Right and left ventricles of the sheep's heart, seen posteriorly. Shows the fourth or central layer of the left ventricle, and the fifth layer of the right one. The fourth layer of the right ventricle is seen at *p q* of fig. 23. See pages 477, 478, & 479.

Fig. 28. Right and left ventricles of the sheep's heart with the septum exposed, seen posteriorly. Shows that the direction of the fibres on the septum corresponds with that of the deeper fibres of the fourth or central layer of the left ventricle. See pages 477 & 478.

Fig. 23. Right and left ventricles of the sheep's heart, seen anteriorly. Shows the direction of the fibres of the fourth or central layer of the right ventricle (*p q*), as compared with the direction of the fibres of the undissected left ventricle (*d d''*). They run at nearly right angles. See pages 477 & 478.

Fig. 26. Right and left ventricles of the sheep's heart, seen anteriorly. Shows the direction of the fibres of the fifth layer of the right ventricle (*p q*) as compared with the direction of the fibres of the third layer of the left one (*d d'*). See pages 478 & 479.

Fig. 29. Right and left ventricles of the sheep's heart, seen anteriorly. Shows the direction of the fibres of the sixth layer of the right ventricle (*p q*) as compared with the direction of the fibres of the fourth layer of the left one (*d d'*). See page 479.

PLATE XIV.

Fig. 30. Right ventricle of the sheep's heart. Shows the bone of the heart (*c*) *in situ*, direction of the fibres of the first layer, &c. See pages 467, 468, 474, & 475.

Fig. 31. Right ventricle of the sheep's heart. Shows the bone of the heart *in situ*, the direction of the fibres of the third external layer, &c. See page 476.

Note.—The fourth or central layer of the right ventricle is seen at *p q* of fig. 23, Plate XIII.

Fig. 32. Right ventricle of the sheep's heart. Shows the direction of the fibres of the fifth layer of the right ventricle, fleshy pons, &c. See pages 478 & 479.

Fig. 33. Right ventricle of the sheep's heart. Shows the direction of the fibres of the sixth layer of the right ventricle. See page 479.

Fig. 34. Right ventricle of the sheep's heart, seen anteriorly and from the left side. Shows the first layer of the septum dissected from within, or from the left side, and how the fibres of the first external layer of the right ventricle bend or double upon themselves to become continuous with fibres having a similar direction on the septum. See page 468.

Note.—The second layer of the right ventricle is omitted. (See *f' d'* of fig. 22, Plate XIII.)

Fig. 35. Right ventricle of the sheep's heart, seen anteriorly and from the left side. Shows the fibres of the third layer of the septum dissected from within or from the left side, and how the fibres of the third layer of the right ventricle bend or double upon themselves, anteriorly and posteriorly, to become continuous with those on the septum having a similar direction. See page 476.

Note.—The fourth or central layer of the right ventricle is omitted. (See *p q* of fig. 23, Plate XIII.)

Fig. 36. Right ventricle of the sheep's heart, seen anteriorly and from the left side. Shows the fibres of the septum dissected from within or from the left side, as in the two preceding figures, also how the fibres of the fifth layer of the right ventricle are continuous on the septum, anteriorly and posteriorly. See pages 478 & 479.

Fig. 37. Right ventricle of the sheep's heart, seen anteriorly and from the left side. Shows the fibres of the septum dissected from within or from the left side, as in the three preceding figures, also how the fibres of the sixth layer are continuous on the septum, anteriorly and posteriorly. See page 479.

Note.—The seventh or last internal layer is omitted.

Fig. 38. Bird's-eye view of the base of the ventricles of the swan's heart. Shows the right and left auriculo-ventricular openings, with the muscular and bicuspid valves *in situ*. See pages 470, 471, & 472.

Fig. 39. The ventricles of the heart of the swan, seen from the right side. Shows the right side of the septum and the posterior portion of the muscular valve in position (g), also how the fibres from the upper third of the septum and left ventricle posteriorly enter into the formation of the inner half of the muscular fold. See pages 470, 471, & 472.

Fig. 40. Right and left ventricles of the turkey's heart, opened anteriorly. Shows the somewhat triangular shape of the muscular valve (*i*) of the right ventricle of the bird, with its spindle-shaped muscular band (*h*), as contrasted with the anterior and inner segment of the bicuspid valve (*v*) of the left ventricle, with its chordæ tendineæ and musculus papillaris (*y*). The spindle-shaped muscular band is to the muscular valve of the right ventricle of the bird what the anterior musculus papillaris and its chordæ tendineæ are to the anterior segment of the bicuspid valve of the left ventricle. See pages 470, 471, & 472.

Fig. 41. Transverse section of the ventricles of the heart of the emu, half an inch from the base, seen from below. Shows the shape of the right and left ventricular cavities, and the appearance presented by the muscular valve (*g i*) when viewed from beneath. See pages 470, 471, & 472.

Fig. 42. The heart of the dugong, seen anteriorly. Shows the peculiar plaited arrangement of the fibres of the ventricles, and the bifid or double apex. See page 448.

a. Aorta.

k. Pulmonary artery opened into, so as to expose the sigmoid or semilunar valves.

c. Left auricle opened into.

d d'. Plicated arrangement of the external fibres.

z. Plicated arrangement of the internal fibres.

i. Segment of the tricuspid valve.

p. Anterior musculus papillaris of the right ventricle.

y. Portion of the left ventricular wall and posterior musculus papillaris with chordæ tendineæ.

m. Right apex.

n. Left apex.

Fig. 43. Right and left ventricles of the heifer's heart, seen posteriorly, the right ventricle being opened into, to show the arrangement of the carneæ columnæ, musculi papillares, &c. See pages 469 & 470.

Fig. 44. Right and left ventricles of the human heart, seen posteriorly, the right ventricular wall being opened into, to show the distribution of the carneæ columnæ and musculi papillares. Shows the comparatively reticulated structure of the carneæ columnæ and the bifid nature of the musculi papillares, &c. See page 470.

PLATE XV.

Fig. 45. Right and left ventricles of the sheep's heart split up or separated from each other anteriorly. Shows how the fibres of the right and left ventricles are continuous with fibres having a similar direction on the septum. See page 466.

k. External fibres of the right ventricle becoming continuous, in the track for

the anterior coronary artery (*o o*), with fibres having a similar direction on the septum (*l*) by simply bending upon themselves.

n. Fibres of the septum passing from above downwards to become continuous in the track for the anterior coronary artery (*p p*) with the external fibres of the left ventricle (*m*), these fibres winding in a spiral direction from above downwards to enter the left apex, not by simply bending upon themselves, but by twisting rapidly round in a whorl.

Fig. 46. Base of the ventricles of the heifer's heart, as seen from above, posteriorly. Shows the relative position and shape of the auriculo-ventricular openings and the orifices of the large blood-vessels. It also shows how the external fibres curve out of the former all round. See pages 454, 470, 472, 473, & 483.

Fig. 47. Right and left ventricles of the heifer's heart laid open, seen posteriorly. Shows the position and shape of the musculi papillares and carneæ columnæ. See pages 460, 461, 469, & 470.

Fig. 48. Left ventricle of the heart of the American elk, inverted. Shows the complete absence of carneæ columnæ, and the undeveloped condition of the musculi papillares. See page 461.

Fig. 49. Transverse section of the right and left ventricles of the deer's heart, half an inch from the base. Shows the shape of portions of the right and left ventricular cavities, and how the right ventricular cavity curves round, or is applied to the left. See pages 480 & 481.

Fig. 50. Transverse section of the right and left ventricles of the deer's heart, rather less than an inch and a half from the base. Shows the same as last section, and in addition how the fibres of the right ventricle dip in at the anterior coronary groove (*m*) to become continuous with fibres having a similar direction on the septum (*e'*). See page 481.

Fig. 51. Transverse sections of the right and left ventricles of the deer's heart, two and a half inches from the base. Shows the same as the two preceding sections. See pages 481 & 482.

Fig. 52. Transverse section of the left ventricle of the deer's heart, three and a half inches from the base, and fully half an inch from the apex. See page 482.

Fig. 53. Transverse section of the left ventricle of the deer's heart, a quarter of an inch from the apex. See page 482.

Fig. 54. Left ventricle of the sheep's heart, seen posteriorly. Shows how the fibres composing the inner half of the central layer of the left ventricle (*f*) pass through the septum (*g*). See page 478.

Diagram 1, Plate XV. Sheet of net with threads of wool drawn through it at intervals to represent the fibres, laid out lengthwise or across the body. Shows how by folding in a portion of the sheet (B C) and rolling it obliquely upon itself in the direction of the arrow marked E, until three and a half turns have been

made, a cone is produced whose anterior wall consists of four layers (the posterior wall consists of three), the lines composing these layers having different directions, as represented at Plate XVI. diagrams 4 & 5, and at Plate XII. figures from 1 to 8 inclusive. See pages 484, 485, & 486.

Diagram 2, Plate XV. Double sheet of net, the one sheet (F G Y) being laid upon the other (A B X) at a slight angle. Shows how when the two sheets thus arranged are rolled up together the cone produced is bilaterally symmetrical, as might be imitated (Plate XVI.) by placing the cone marked diagram 5 within that marked diagram 4. It also shows how the lines composing the sheets enter the apex in a whorl in two distinct sets (Plate XVI. diagrams 7, 10, & 12, E F), after which they wind from the apex towards the base, likewise in two sets (Plate XVI. diagram 11, X Y). See pages 486 & 487.

G. Point which when the sheets are rolled into a cone corresponds to its apex, that portion of the uppermost sheet marked X assuming the position of the anterior musculus papillaris (Plate XVI. diagrams 8, 9, & 11, Y), that portion of the lower or undermost sheet marked X taking the place of the posterior musculus papillaris (Plate XVI. diagram 11, X).

PLATE XVI.

Diagram 3. Sheet of net with threads of wool drawn through it at wide intervals, rolled up in a cone as explained, the cone being placed upon its apex. Shows how lines originally of the same length make fewer turns of a spiral from winding round wider portions of the cone. It also shows how the apex of the cone may be enlarged by removing in succession the lines which make the greatest number of spiral turns, and which, curiously enough, overlap the lines which make the fewest number of turns. See pages 484, 485, & 486.

Note.—The lines in this diagram correspond with the lines in diagram 1, Plate XV., bearing similar letters.

Diagrams 4 & 5. Anterior and posterior views of a cone produced by rolling a sheet of net upon itself, similar to that figured in Plate XV. diagram 1. In the cone marked 5, that portion of the sheet which forms the base (R R) has been folded upon itself in an outward direction, to cause the internal lines to reverse their direction and become parallel with the external ones, with which they unite.

Note.—These diagrams show how by one portion of the sheet overlapping another several layers are produced, the lines composing these layers having different directions. They also show how the layers or overlappings are confined to different regions or localities, just as in the left ventricle, and satisfactorily account for the ventricular wall tapering towards the base and the apex respectively. See pages 484, 485, & 486.

A A′, diagram 4. Lines winding in a spiral nearly vertical direction from above downwards, and forming the first layer (Plate XII. fig. 1, compare with the fibres marked *f f′* and *d d′*). These lines are continuous at the base and the apex with the lines forming the seventh or corresponding internal layer E E′ (Plate XII. fig. 8, compare with the fibres marked *o* and *n*).

B B′, diagram 5. Lines winding in a spiral oblique direction from above downwards, and forming the second layer (Plate XII. fig. 2, compare with the fibres marked *f d′*). These lines are continuous at the apex and the base with the lines forming the sixth or corresponding internal layer D D′ (Plate XII. fig. 6, compare with the fibres marked *o n*).

B B′, diagram 4. Lines winding in a spiral very oblique direction from above downwards, and forming the third layer (Plate XII. fig. 3, compare with the fibres marked *f d′*). These lines are continuous at the apex and the base with the lines forming the fifth or corresponding internal layer D D′ (Plate XII. fig. 5, compare with the fibres marked *o n*).

C C′ C″, diagram 5. Lines winding in a circular or transverse direction and forming the fourth or central layer, which divides the three external from the three internal layers (Plate XII. fig. 4, compare with the fibres marked *f d′*). It is in this layer that the three external layers terminate, and the three internal ones (diagram 6, D E E′) begin.

Diagram 6. A cone similar to that figured at diagram 4, with the external layers uncoiled. Shows the fourth, central or transverse layer, and the three internal layers. See pages 484, 485, & 486.

Diagram 7. Symmetrical cone produced by the rolling of portions of two sheets within each other. Shows the double entrance of the lines at the apex, and how closely the position and shape of the musculi papillares may be imitated. The interior of this cone is seen at diagram 11, and should be compared (Plate XII.) with the interior of fig. 13. See pages 486 & 487.

A B. Portion of posterior sheet, the lines composing which enter the apex anteriorly at E, to represent the posterior musculus papillaris X (Plate XII. fig. 14, compare with *y*). See pages 486 & 487.

C D. Portion of anterior sheet, the lines composing which enter the apex posteriorly at F, to represent the anterior musculus papillaris Y (Plate XII. fig. 14, compare with *x*). See pages 486 & 487.

Diagram 8. Symmetrical cone produced by the rolling of two sheets of net within each other, similar to those figured at Plate XV. diagram 2. Shows the position and track of the anterior papillary muscle (Y). See pages 486 & 487.

Diagram 9. Interior of a single sheet of net rolled up as explained. Shows how the spiral almost vertical direction pursued by the anterior musculus papillaris (Y) may be imitated (Plate XII. fig. 13, compare with *y*). See page 484.

Diagram 10. Symmetrical cone produced by rolling two sheets of net within each other. Shows how the symmetry of the cone is maintained by the sheets winding from opposite points, the one, the anterior (A B), winding from before backwards, or from right to left, to enter the apex posteriorly (F), the other, the posterior (C D), winding from behind forwards, or from left to right, to enter the apex anteriorly (E) (Plate XVI. fig. 55, compare with *e f*; Plate XII. fig. 11, compare with *k l*). See pages 486 & 487.

Diagram 11. Interior of symmetrical cone, as seen at diagram 7, formed by the rolling of two sheets of net within each other. Shows how the position and shape of the spiral nearly vertical anterior (Y) and posterior (X) musculi papillares may be imitated (Plate XII. figs. 12 & 13, compare with *y x*). See pages 486 & 487.

Diagram 12. Two sheets of paper with parallel lines drawn upon them, rolled within each other, and then permitted to spring open (seen from above). Imitates the double entrance of the fibres at the left apex. Compare the direction of the lines on the sheet A B E with the direction (Plate XVI. fig. 55) of the fibres *a b e*, and also the direction of the lines in the sheet C D F with the direction of the fibres *c d f* of the same figure.

Diagram 13 represents the course pursued by a single fibre, how it winds from the base to the apex in one direction, and from the apex to the base in another, so as to return to the point from which it set out. It also shows how the fibre is continuous towards the apex and the base. See page 485.

A B C. External portion of the fibre winding from above downward, or from the base to the apex.

D. Point at which the fibre enters the apex and alters its direction.

E F. Internal portion of the fibre winding from below upward, or from the apex to the base, to return to the point A.

Diagram 14 represents the course pursued by two fibres, each of which is similar to that figured at diagram 13. In this diagram the external portion of one of the fibres (A B C) winds from behind forwards and enters the apex anteriorly (D), the external portion of the other fibre (G H I J) winding from before backwards and entering the apex posteriorly (K). The external portions of the fibres are therefore symmetrically disposed with reference to each other. Similar remarks apply to the internal continuations of these fibres (E G H, L A M), which are only partially seen. The external and internal portions of the fibres are continuous, and when seen from above appear to form complete circles. See pages 486 & 487.

Fig. 55. Sheep's heart separated into its bilateral elements. See pages 457, 486, & 487.

a b. Anterior fibres entering the left apex posteriorly (*e*).

c d. Posterior fibres entering the left apex anteriorly (*f*).

Diagram 15 shows how by pushing in the anterior wall (A) of the typical or left

ventricle, in imitation of the constructive process in the embryo, a double septum unattached posteriorly (B) is produced, this septum dividing the ventricle into two, a right or rudimentary ventricle (D), and a left or more complete one (C). It also shows how the fibres may be continuous or common to both ventricles posteriorly (B), while anteriorly (A) they dip in at the track for the anterior coronary artery and are to a certain extent independent of each other (Plate XV. fig. 50, compare with *g* and *m*). See pages 464, 465, 466, 467, & 488.

Diagram 16 shows how the posterior fold (G) of the septal duplicature, by passing through the posterior wall (B) until the central layer in either is reached, completely isolates the right ventricle (D) from the left (C), and how, by the atrophy of the right ventricular wall (F) and its share of the septum (K), after birth, to half their original dimensions, the right ventricle (F) is reduced to half the thickness of the left (I); while the septum (H K) is three times as thick as the right, and a third thicker than the left. See pages 465, 466, 467, & 488.

Diagram 17 shows how, by the partial absorption of the posterior fold of the septal duplicature (K), and the passing through and blending of the right half of the reduplication (E) with the left (H), the ventricles are more intimately united, and the septum reduced until it is only a sixth greater than the left ventricle,—an arrangement which very nearly corresponds to the measurement of the actual septum between the musculi papillares at the thickest part (Plate XV. fig. 50, compare $c' d'$ with $c d$). It also shows how the septum is composed of two elements, and how one portion of it (E E) belongs exclusively to the right ventricle, a second portion (H) to the left; while a third portion (J J) belongs partly to the one ventricle, and partly to the other. See pages 465, 466, & 467.

Diagram 18 shows how the right portion of the septum, as seen at E E of diagram 17, may become absorbed, so as to reduce the septum (H) to the thickness of the left ventricular wall (I) and half the thickness of the right (F) (Plate XV. figs. 49 & 51, compare $c d$ with $c' d'$, and $c d$, $c' d'$, with $c'' d''$).

XV. *On the Brain of a Bushwoman; and on the Brains of two Idiots of European Descent.* By JOHN MARSHALL, *F.R.S., Surgeon to University College Hospital.*

Received June 18,—Read June 18, 1863.

THE chief purpose of the present paper is to describe the *convolutions* of a Bushwoman's brain, and also those of the two smallest human idiot brains yet on record, belonging respectively to a microcephalic woman and boy of English parentage. But other points of interest, such as the weight, size, general form, and internal structure of these brains and their several parts, are likewise noticed; and, in the case of the idiots, such information is prefixed as could be collected concerning their feeble mental and bodily powers.

The attention which has recently been directed to the study of the cerebral anatomy of man, as compared with that of the quadrumanous animals, and the acknowledged scantiness of our information concerning the brain in the various races of mankind, induced me to request several medical friends residing in our colonies, to endeavour to procure certain specimens for me.

From my former pupil, Mr. JOHN EDWARD DYER, now practising in Cape Town, I received in April last, in part fulfilment of a promise made to me in the previous August, the entire head of a Bushwoman duly prepared according to my instructions. I most cordially acknowledge my obligations to him.

As a guarantee of the authenticity of the specimen, the entire head was to be sent with the brain in it. For this purpose, the neck was divided below the larynx; some strong spirit was injected into the carotid and vertebral arteries; the skull was trephined over each parietal bone, and the dura mater carefully slit open, so as to allow the spirit to percolate into the cranial cavity. The head was then put into a tin case, which was filled with spirit, hermetically closed by soldering, and despatched to England without delay. On opening the case on its arrival, no decomposition was apparent in any part of the preserved head; but the cerebellum was afterwards found to be somewhat softer than the cerebrum, owing probably to an accidental want of success in injecting the vertebral arteries, so that less spirit had been directly conveyed to the posterior parts of the encephalon.

Having secured, for further identification, a plaster cast of the entire head, and four photographic views of it, half the size of nature, viz. a front, back, and two profile views, I proceeded, by means of a longitudinal and other sections of the skull, to remove a sufficient portion of the left half of the cranium and dura mater to expose the corresponding side of the brain, still covered by the cerebral arachnoid and pia mater and

shrunken within the cranium, but having undergone no observable flattening or distortion.

The membranes being dissected off, a photograph was taken of this side of the head, with the brain and spinal cord *in situ*, so as to put on record the *bonâ fide* relationship between them. The brain being now removed with the medulla, and both cleared of their remaining membranes, nine photographs, the size of nature, were taken of it. These views included the base and vertex, the frontal and occipital aspects, both sides, a section showing the inner surface of the left half of the encephalon, the under surface of the left cerebral hemisphere, and, lastly, the left lateral ventricle and its contents. The right half of the encephalon was left undissected, to serve as a museum specimen, or for further investigation. The weight of this right half of the preserved brain being ascertained, the separated pieces of the left half, all of which had been preserved, were weighed in three portions, so as to give the respective weights of the left cerebral hemisphere, and of the left half of the pons, cerebellum and medulla oblongata. By adding all these weights together, the total weight of the preserved brain was found; and by allowing proportionate values for the several parts of its undivided right half, a sufficiently near approximation was obtained to the respective weights, in the preserved condition, of the entire cerebrum, cerebellum, and pons with the medulla oblongata. In proof of the satisfactory character of these weighings, it may be stated that the right and left halves of the brain differed in weight only 20 grains, the advantage being on the right side, in which, however, a part of the choroid plexus was necessarily included. The left half was probably, as has been observed in European brains by Dr. BOYD*, somewhat the heavier.

To determine from the preceding data the probable weight of the recent brain and its parts, two modes were had recourse to—the first depending on the ascertained loss of weight in brains preserved in spirit, the second having reference to the cubical capacity of the cranium measured by means of water. It cannot be assumed that such calculations are quite correct; but I have given the subject careful consideration, and have recorded the actual weights, taken from the preserved brain, with all other data which have been employed.

A plaster cast of the interior of the cranium, taken after putting its several pieces together, before the membranes were removed from the right half, assisted in determining the general form, dimensions, and relative position of the parts of the recent encephalon.

The fissures, lobes, and convolutions were studied on the preserved brain, and are illustrated in the several photographic views. They are compared, in the paper, with the same parts in the European brain, with the brain of the so-called Hottentot Venus†, who, there seems reason to believe, was of the Bosjes race, and with the brains of the higher Apes. The commissures, ventricles, and ganglionic masses were also examined and measured on the preserved brain, and are nearly all shown in the photographs.

* Philosophical Transactions, 1861, vol. cli. p. 261.

† GRATIOLET, 'Mém. sur les Plis Cérébraux de l'Homme,' &c. Paris. No date (1854?).

Various measurements of the Bushwoman's brain are tabulated at the end of this paper, with similar measurements taken from an average female English brain, the intracranial cast corresponding with which is in my possession. Furthermore it has been thought that additional interest would be given to the subject by introducing into the same Table the comparative measurements of a young Chimpanzee's brain, described by me elsewhere *, and also those of the two idiots' brains hereinafter investigated.

With regard to these idiots' brains, the larger one, that of the woman, has been temporarily placed at my disposal, for special description of the convolutions, by my friend Mr. R. T. GORE of Bath, who has given a general account of this brain to the Anthropological Society of London †. I have also had access to a cast of the interior of the cranium taken by Mr. GORE, from whom I have likewise received additional information relating to the woman herself.

The smaller of the two idiots' brains, and indeed the smallest yet described, that of the idiot boy, has been lent to me from the Anatomical Museum of University College, London, by the recommendation of Professor SHARPEY. It had been presented to that Museum by Professor JENNER, who supplied to the Catalogue a very full and circumstantial account of the appearances observed at the post-mortem examination ‡. When living, this idiot was under the observation of Dr. BEGLEY, of the Lunatic Asylum, Hanwell, who has kindly given me information concerning the mental manifestations of the boy during life. It is to be regretted that, of the skull of this idiot, nothing but the calvarium is preserved, so that I could only obtain a cast of the upper half of the interior of the skull. Unfortunately, though this includes the part which lodged the posterior lobes of the cerebrum, it scarcely extends to the posterior border of the cerebellum.

So far as was practicable, the same method of examination was followed with the two idiots' brains as with the Bushwoman's brain. Nine photographs of the Bushwoman's brain, nine of the idiot woman's brain, and eight of the idiot boy's were taken by Mr. HERBERT WATKINS. Lithographs from these accompany and illustrate this paper.

I. THE BRAIN OF THE BUSHWOMAN.

a. *General Account. The Face and Head.*

Mr. DYER states that the height of the Bushwoman was 5 feet, and her age doubtful; but she was certainly aged. It is fair to assume that she presented no remarkable departure in either direction from the ordinary intellectual and moral condition of her race. Her height being somewhat unusual (that of the Hottentot Venus was 4 feet 9 inches §), the question arose whether she was a Hottentot, not a Bushwoman proper.

* Nat. Hist. Review, July 1861.

† Anthropological Review, vol. i. 1863.

‡ I recently submitted these facts to the Anthropological Society of London, as will appear in the Anthropological Review for August 1863.

§ See WAILLY's figures, one-fourth the height of nature, in G. CUVIER's account of the Hottentot Venus, 'Hist. Nat. des Mammifères,' &c., par I. ST. HILAIRE et F. CUVIER. Paris, 1826.

But a friend, long resident at the Cape, who also knows Mr. DYER, tells me that no mistake between these two people is likely to be committed by a Colonist, their habits, mode of life, and language being quite distinct; and moreover, from a mere glance at the cast and photographic portraits, he unhesitatingly pronounced the head to be that of a genuine Bushwoman.

The colour of the skin is brownish black, with pale freckles on both cheeks; the hair, scattered in little tufts over the scalp, is mixed grey and black. There is no trace of moustaches or beard, and the hairs of the eyebrows and the eyelashes are very scanty. The skin of the face is much wrinkled, partly from the effect of the spirit, but also from old age. The eyebrows are not heavy. The distance between the inner angles of the eyelids is great, being equal to the width of the eyelids themselves. The conjunctivæ are slightly stained with pigment. The root of the nose is broad and remarkably flat; the nose itself is short, small, and also flattened, the nostrils being visible from the front. The cheeks are prominent and wide; there is great breadth opposite the angles of the lower jaws. The chin is square and somewhat prominent. The lower part of the face is only slightly prognathous; the mouth is large and projecting; the lips are thick, but rather straight in outline, the peculiar curves of the upper lip, as seen in the European, not being well pronounced. The ears are long, but tolerably flat to the head; the right one is equal in length to the vertical distance between the eyebrows and the mouth; the left one is shorter; their cartilaginous forms are well developed; the lobe is short and wide, and its posterior border glides without distinction into that of the rest of the ear; the external auditory meatus is smaller than in the European, and somewhat compressed from before backwards. The incisor and canine teeth are small; the upper ones are inserted somewhat obliquely, the lower ones nearly vertically. All the molars are wanting in both jaws, as is also the upper left premolar. The existing teeth are much worn, so as to appear short, the canines being quite level with the incisors, a proof of advanced age. The tongue is small, and the frænum scarcely distinguishable.

Considered as a whole, the face is characterized by the width and flatness of the cheeks, the extreme flatness and small size of the nose, the full mouth, and the moderate amount of prognathism. The general shape of the cranium, seen from above, is a long flattened ovoid—the greatest transverse diameter being placed a good way behind the ears, from which line the cranium is suddenly rounded backwards, but gradually narrowed off towards the forehead, the left half of which projects a little in advance of the right. The front, although narrow, is well elevated, so that the line of the vertex is evenly curved from before backwards—there being a total absence of the depressed forehead often observed in Negro heads.

Subjoined are a few measurements of the head, by which it will be seen that it is by no means small, as regarded from without:—

	inches.
Total height of the head and face	7·15
Extreme length of the head and face	7·25

	inches.
Extreme width at the parietal eminences	5·5
Extreme depth of the cranium, from beneath the occiput to the vertex	5·5
From one external auditory meatus to the other	12·5
From the root of the nose to the occipital protuberance	12
Circumference close above the ears	21

The proportions between the length and breadth of the cranium, which are as 100 to 76, show that it is not nearly so dolichocephalic as the Negro skull. In its thickness, however, it resembles the latter. The thinnest part on the median section is over the vertex, and measures 3 of an inch; passing backwards, the thickness of the parietal bone increases to ·35 of an inch, whilst the occipital protuberance exceeds ·5 of an inch; passing forwards, the average thickness of the frontal bone is about ·35 of an inch, increasing at the forehead itself to ·5 of an inch. The diploë is scarcely distinguishable, owing to the closeness of the bony texture. There is no appearance of frontal sinuses. The foramen magnum measures 1·4 inch in its antero-posterior diameter and 1·3 inch transversely; its anterior border corresponds with a line passing across ·375 of an inch behind the internal auditory meatuses. All the sutures at the top of the skull are closed; the line of the fronto-parietal is quite obliterated, that of the sagittal suture nearly so, whilst the lambdoidal suture can still be traced. The squamous part of the left temporal bone is also absolutely joined to the frontal and parietal. The petrosal and other ridges in the base of the cranium are prominently marked; the crista galli is large, and the sella turcica deep. The internal surface of the cranium is more strongly marked than usual by the convolutions, especially in the orbital, frontal, temporal, and occipital regions, but much less distinctly so along the vertex. The auditory meatuses and optic foramina are small; the cribriform lamellæ are of moderate extent; the other foramina are of average size. On the outer surface of the cranium the zygomatic arches, though of well-marked curvature, are slender; the mastoid processes are very small, and the styloid processes also slender.

The right half of the cranium, as defined by the falx, which, together with the tentorium, was left undisturbed, holds nearly exactly 17·5 oz. avoirdupois of water; so that the total capacity of the entire cranial cavity is about 35 oz. of water, which are equal to 60·64 cubic inches. Thus the actual capacity of the cranial cavity proved to be less than its external dimensions would have led one to anticipate. The capacity of the largest skull measured by MORTON* was 114 cubic inches. The largest examined by WAGNER† was 115 cubic inches; the smallest, that of an adult female, 55·3 inches. As measured in ounces of water, the capacity of the male Hottentot's skull has been given as high as 75 cubic inches; whilst that of the Negro's ranges from 69·3 to 60·5 cubic inches, that of the Malay's skull from 62·2 to 57·1 cubic inches, and that of the Hindoo's

* Crania Americana. Philadelphia, 1839.

† Vorstudien zu einer Morph. und Phys. des menschlichen Gehirns. Leipzig, 1860.

as low as 46·7 cubic inches *. Between all these examples, however, the stature having been neglected, comparisons, to say the least, must be inexact; and the height of the Bushwoman, a fair average one even for a European female, must not be forgotten in any estimate of the dimensions of her cranial cavity.

In the Bushwoman's head no evidence of arrested development exists, either in the character of the bones, or in the sutures, or in the features of the face, though a supposition of that kind has been entertained, yet not generally assented to, in regard to the so-called Hottentot Venus. Certain infantile characters are undoubtedly present in the cranium of the Bushwoman—such as the slight elevation of the nasal bones, the absence of the frontal sinuses, the small size of the mastoid processes, the slenderness of the styloid processes, and the markings on the inner surface of the cranium; but these characters should probably be regarded rather as belonging to sex or race than as indicative of any arrest of development in the individual, especially as the general proportions between the face and the cranium, the dolichocephalic form of the latter, the prominent cheek-bones, the square jaw, and the well-marked chin would lead to the opposite inference of a perfected individual development.

b. *Weights of the Encephalon and its parts.*

The entire encephalon of the Bushwoman, hardened in spirit and deprived of its membranes, weighed 21·77 oz. The loss of weight in specimens of brain preserved in a similar way I find to be from one-third to one-fourth, *i. e.*, as a mean, $\frac{7}{24}$ths of their original weight, on which calculation the recent Bushwoman's brain, deprived of its membranes, would weigh exactly 30·75 oz. With the membranes, the weight would be about 31·5 oz.

The capacity of the skull, as already stated, was equal to 35 oz. avoirdupois of water, or 60·64 cubic inches. If the brain, in its natural state, filled the cranial cavity as completely as water will afterwards, it would be easy, by taking the specific gravity of nervous substance as compared with water, to estimate the quantity of brain which once occupied any given skull; but the fact that this is not the case, especially in regard to the base of the brain, and the difficulty of determining the weight of the membranes, the amount of blood which the vessels may contain, and the quantity of cerebro-spinal fluid which fills the ventricles and all otherwise unoccupied spaces, render it impossible thus to arrive at so definite an estimate as in the other way.

Now the smallest healthy European female brain recorded by WAGNER weighed about 31·7 oz.†; and the smallest observed by Dr. REID, in the case of an aged woman, 32 oz.‡ These numbers, however, are unsatisfactory, as neither the heights nor the weights of the individuals are on record. But Dr. BOYD's valuable Tables§ supply this omission, and thus enable us to appreciate the comparative size of the Bushwoman's brain. Among 149 females between the ages of 60 and 70 years, the minimum weight of the

* TIEDEMANN and HUSCHKE, quoted by SCHAAFHAUSEN, MÜLLER's 'Archiv,' 1858.

† *Loc. cit.* ‡ London and Edinburgh Monthly Journal, &c., April 1843.

§ Philosophical Transactions, vol. cli. p. 251.

entire encephalon, as found by Dr. Boyd, was 32·5 oz.; but then the minimum stature in 148 women at the same period of life was only 4 feet 6 inches; whilst the average weight of the encephalon, in the same groups of individuals, was 42·96 oz., and the average height 5 feet $1\frac{1}{2}$ inch. Deducting the odd 2·96 oz. for the $1\frac{1}{2}$ inch over 5 feet, which is more than enough, the European female brain at that stature, and between 60 and 70 years of age, would weigh 40 oz.; or, again, by adding 7·5 oz. to the minimum weight for the difference between the minimum height and that of the Bushwoman, which would be about a proper allowance, we should again obtain a European average of 40 oz. for the brain of a female measuring 5 feet at the above-named age; whilst the weight of the Bushwoman's brain, including the membranes, was probably, as shown, not more than 31·5 oz.

Considering, therefore, on the one hand, the stature of the Bushwoman, and allowing, on the other, for her advanced age (the former justifying the expectation of a large brain, the latter of some comparative waste in that organ), the safe general conclusion is that her entire encephalon was, for her height, decidedly small.

In the absence of positive information, it is impossible to do more than speculate on the ratio between the weight of the entire brain and the body in the Bushwoman; but, considering her height, 90 lbs. would not be an exaggerated estimate for the latter quantity in health. The ratio in that case would be 1 to 45; whereas the mean proportions usually given for the European, dying from a sudden cause, are as 1 to 37.

The weight of the preserved encephalon being, as already stated, 21·77 oz., the left half of the preserved cerebellum weighed 1·22 oz., and the left half of the pons and medulla oblongata ·25 oz. Doubling these halves, we have 2·44 oz. for the total weight of the preserved cerebellum, and ·5 for the total weight of the preserved pons and medulla oblongata. Their joint weights, 2·94 oz., being deducted from the weight of the preserved encephalon above mentioned, gives 18·83 oz. for the total weight of the cerebrum. On these data, the weight of the cerebrum to the cerebellum is as 7·7 to 1. In the adult female European, according to Dr. Reid, the average ratio is 8·25 to 1; but by Dr. Boyd's Tables it is, between 60 and 70 years of age, also 7·7 to 1.

The actual weights above given, being increased in the proportion of 17 to 24, to allow for the loss by maceration in spirit, we arrive at a total weight of 3·44 oz. for the recent cerebellum, and of ·7 oz. for the recent pons and medulla oblongata; and lastly, by deducting their joint weight, viz. 4·14 oz. from 30·75 oz., the estimated weight of the entire fresh encephalon, we have 26·61 oz. as the weight of the recent cerebrum. Distributing the estimated weight of the membranes in relative quantities, we obtain 27·25 for the recent cerebrum, 3·45 for the cerebellum, and ·8 for the pons and medulla oblongata, enveloped in their respective shares of membranes, making, as above shown, 31·5 oz. for the entire encephalon. The average weights of the same parts in 134 European females, between the ages of 60 and 70, and measuring 5 feet $1\frac{1}{2}$ inch high, according to Dr. Boyd, are 37·13 oz. for the cerebrum, 4·68 for the cerebellum, and ·83 for the pons and medulla oblongata, making 42·64 oz. for the entire encephalon.

The ratio of the cerebrum to the body in the Bushwoman, assumed with a height of 5 feet to weigh 90 lbs., would therefore be as 1 to 52, whilst that of the cerebellum to the body would be as 1 to 418; whereas, allowing 6 lbs. additional weight (96 lbs.) to the average European females of 5 feet 1½ inch high, the corresponding ratios would be 1 to 41, and 1 to 328.

Without claiming for these numbers a perfect accuracy, and even subjecting them to certain small corrections, they support the statement that, in reference to the body, the cerebrum and cerebellum are both inferior in the Bushwoman as contrasted with the European aged female; and it will be seen that both organs are about equally defective, *i. e.* in a proportion of about ·78 to 1.

Judging from the restored figure of the Hottentot Venus's brain, the Bushwoman's brain was in its recent state only a very little smaller than it (see Plate XX. and its explanation).

c. *The general Form, Dimensions, and relative Position of the Parts of the Encephalon.*

The Bushwoman's brain, injected with spirit, and hardened within the cranium, had, as already stated, undergone very little change of form, although it had shrunk from the cranial walls, chiefly over the vertex, and slightly at either end. This subsidence of the brain was less marked before the veins passing from its upper surface into the longitudinal sinus were divided. Even when removed from the cranium and denuded of its membranes, the brain maintained its shape, and the relations of its several parts; but in describing these reference is made to the intracranial cast, and the dimensions of the organ are also given from that source.

When viewed from above, the Bushwoman's cerebrum (Plate XVII. fig. 1), like her cranium, presents a long and narrow ovoid form. The line of greatest width corresponds with the parietal eminences, and is placed rather far back, viz. at two-thirds the total length of the cerebrum from its anterior border, so that one-third only is behind those eminences. From this prominent parietal region the cerebrum slopes or falls away in all directions—very suddenly backwards, and rather so forwards, as far as the entrance of the Sylvian fissure, where, like the fœtal brain, it appears remarkably constricted, and then widens again a little (Plate XVII. fig. 2) at the outer angles of the frontal region, which is nevertheless decidedly narrow. The left hemisphere, as seen from above, is ·2 of an inch longer than the right, the increase being almost entirely behind. This relative greater length of one hemisphere backwards (usually the left, so far as I have observed) is very common in European brains.

Viewed laterally (Plate XVIII. fig. 3), the parietal region is salient; the vertex is low and flattened, its highest point being placed far back; the frontal region is shallow, but ends in a nearly upright anterior border, whilst the beak-like projection of its median portion next to the longitudinal fissure is very marked, and its outer corner projects over the entrance to the Sylvian fissure. The temporal lobe is narrow, the line from its point to the tip of the posterior lobe being very long; the curve formed by the under

border of the cerebrum, above the cerebellum, is slighter, and its direction more oblique upwards and backwards than in the European brain, owing apparently to a want of downward development of the occipital region, which is very shallow.

Viewed in front, the narrowness and want of depth of the frontal region, accompanied, however, by the singular projection of its outer angles and the depth of its beak-like projections, again strike the eye. Besides this, the orbital borders are strongly curved, the orbital surfaces deeply excavated, and the median beak-like portions very narrow and wedge-shaped; the angle formed by the meeting of the two orbital surfaces is smaller than in the European; and the tips of the temporal lobes are pointed and much incurved towards the middle line. Altogether this is an unfavourable, and indeed ape-like aspect of the Bushwoman's brain, the promise given by the better elevation of the frontal region of the skull being disappointed in consequence of the thickness of its walls.

Seen from behind, the comparative prominence of the parietal regions gives an angular outline to the cerebrum, in comparison with its usual fuller form. The deficiency of height at the vertex is very striking, as well as the upward inclination of the posterior border of the cerebrum, owing to the tapering of the posterior lobes.

On the base view (Plate XVII. fig. 2) the general form is again that of a long narrow cerebrum, the details concerning each region being such as have been already pointed out in speaking of the lateral, front, and hind views. The orbital surfaces are especially contracted, but have a square or human, and not a pointed or ape-like shape.

The median section of the cerebrum (Plate XVIII. fig. 4) again shows its low, flattened, elongated form. The portions of hemisphere in front, above, and behind the corpus callosum measure respectively 1·3, 1·25, and 1·9 inch; whereas in the European they measure 1·4, 1·6, and 2·2 inches.

Imaginary lines drawn from the centre of the medulla oblongata, where it intersects the pons, to the extreme occipital, frontal, parietal, and vertical points of the cerebrum, lines which I have elsewhere designated *cerebral radii**, measure, in the Bushwoman, respectively 34, 40, 35, and 41 tenths of an inch; whilst in a European female they measure 33, 43, 39, and 46 tenths. Accordingly the occipital radius, owing evidently to the length of the temporal lobe backwards, is slightly in excess, the frontal radius is a little defective, the parietal a little more so, whilst the *vertical radius is the most so*, as compared with the European brain.

The final result of these measurements, as well as of others given in Table I. at the end of this paper, and of the facts elicited by the examination of the several aspects of the brain, is to show that in this Bushwoman the cerebrum is small but long, defective in width, and especially so in *height*; that its outlines and surfaces are angular and flat, instead of rounded and full as in the European; that, of its several regions, the frontal, though long, is very narrow and *shallow*, much excavated below, and compressed laterally in a remarkable manner behind its angles, in front of the Sylvian

* Nat. Hist. Review, 1861, p. 304.

fissure; that the parietal region is *low*, though, relatively to the surrounding parts, prominent; that the occipital region is long, but narrow, and also remarkably defective in *height*; and, lastly, that the temporal region is long, though somewhat narrow.

As to the relations between the posterior lobes of the cerebrum and the cerebellum, the latter is entirely concealed by the former in the upper view of the brain (Plate XVII. fig. 1). In the base and lateral views (Plates XVII. & XVIII. figs. 2 & 3), the backward projection of the cerebrum beyond the cerebellum is equal to ·5 of an inch on the left side, and a little more than ·3 on the right, where the cerebral hemisphere is shorter. The actual amount of overlapping is therefore as great as in the European; but the relative overlap, as compared with the length of the cerebrum, of which it equals one-thirteenth part, is rather less, owing to the disproportionate length of the cerebrum in the Bushwoman. On the base view, less of the posterior part of the cerebrum is seen on each side of the cerebellum than usual. The cerebellum itself, judging from the intracranial casts, is more prominent at the sides, and proportionally wider and longer than in the European; but its outline is not so full and rounded, so that its actual bulk is smaller. It is, however, quite human and not ape-like in shape.

d. *The Fissures, Lobes, and Convolutions of the Cerebrum.*

The Fissures.—The *fissure of* SYLVIUS (Plate XVIII. fig. 3, *e–e*) in the Bushwoman's brain extends well backwards, but inclines more upwards than in the European brain, and its course is marked, soon after its commencement, by a peculiar horizontal step. It measures 3 inches in length on both sides; in the European brain it is 3·5 inches. On the left side, it sends off a branch near its summit, which nearly reaches the vertex. Its depth opposite the island of REIL is ·75 of an inch, instead of 1 inch (the usual depth). Its margins are not very closely adapted together, especially opposite the hinder border of the frontal lobe, which is here very defective. The fissure, indeed, is so patent that, without any separation of its margins, a portion of the island of REIL, or central lobe (C), though small, is distinctly visible. This condition recalls to mind the fœtal state of the human cerebrum*, but, so far as I am aware, is not present in any adult quadrumanous brain. The defect in the frontal lobe explains the remarkable constricted form of the Bushwoman's brain, already mentioned as existing at that point, a form which we may perhaps assume is a characteristic of the Bosjes brain, as it is equally present in the brain of the so-called Hottentot Venus, where it has also been noticed by GRATIOLET as a fœtal character†. Coupled with the infantile features noticeable in the Bushwoman's skull, this peculiarity becomes very interesting.

The *fissure of* ROLANDO (Plates XVII. & XVIII. figs. 1 & 3, *d–d*) commences 1·25 inch behind the tip of the temporal lobe, instead of 1·375 as in the European, just above the horizontal step of the Sylvian fissure, from which it is separated as usual. It terminates

* Compare TIEDEMANN's figures, Anatomie, &c. des Gehirns, &c. Nürnberg, 1816, taf. v.; and LEURET's plates 29 and 30, Anatomie Comparée du Système Nerveux, &c. Paris, 1837.

† *Op. cit.* plates 1 and 2, and p. 66.

considerably beyond the middle of the long axis of the cerebrum, nearly as far back as the line of greatest width of that organ; so that it passes proportionally further back than in the Hottentot Venus, or indeed than in the European, as illustrated in the outlines of the three brains given in Plate XX. figs. 7, 8, & 9. The left fissure reaches a little further back than the right.

The *external perpendicular fissures* (Plates XVII. & XX. figs. 1 & 9, *h*, *h*) can be traced as easily as in the Hottentot Venus (Plate XX. fig. 8), but are soon interrupted by the external connecting convolutions (α, β). Towards the sides, these fissures are certainly more easily followed than in the European—a circumstance which imparts a lower character to this part of the Bosjes brain; at the same time they are far more interrupted than in the Chimpanzee or Orang-outang (Plate XXIII. fig. 20). These short external perpendicular fissures join, as usual, the summits of the internal perpendicular fissures (Plate XVIII. fig. 4, *k*), and, together with the fissures of ROLANDO, divide the upper surface of the cerebrum into three regions. Supposing the total length of the hemispheres, as seen vertically, to be represented by 100, the region in front of the point of the V formed by the two fissures of ROLANDO is equal to 65; thence to the perpendicular fissures equals 17·5; and thence to the tips of the posterior lobes, also 17·5. The proportions in the European are 57, 23, and 20; in the Chimpanzee, 49, 28, and 23. Measured longitudinally over the vertex, the relative spaces occupied by these regions are, in the Bushwoman, as 60, 15, and 25; in the European, as 54, 23, and 23; in the Chimpanzee, as 46, 28, and 26. So that the fronto-parietal region in the Bushwoman appears lengthened backwards, with a proportionate want of development in the purely parietal and occipital regions.

The remarkable irregular and very deep fissure (Plate XVIII. fig. 3, *c–c*) always seen on the outer side of the frontal lobe (*antero-parietal*, HUXLEY) is very strongly marked. It commences about one inch behind the entrance of the fissure of SYLVIUS, with which it is more nearly continuous than in the European brain, except in the fœtal condition; passes more obliquely backwards than usual; and corresponds with the place of deficient width in this region already twice alluded to.

The *parallel fissure* (*f–f*) on the side of the temporal lobe is more tortuous on the left side than in the Hottentot Venus, though less so than in ordinary European brains. The *inferior temporal fissure* (*g–g*) is comparatively short and simple.

On the internal surface of the hemisphere, the *great fissure of the fronto-parietal lobe* (Plate XVIII. fig. 4, *i–i*), or *calloso-marginal fissure*, is twice interrupted by convolutional bridges, once (as usual) in front of the corpus callosum, and once (unusually) above the middle of that body. As in ordinary European brains, it reaches the surface of the hemisphere, well behind the hinder border of the corpus callosum.

The *internal perpendicular fissure* (Plates XVIII. & XIX. figs. 4 & 5, *k–k*) is more vertical than in the European, but much less so than in the Chimpanzee—the angle formed by this fissure and a base-line drawn through the corpus callosum being in the European 123°, in the Bushwoman 115°, and in the Chimpanzee 93°. As in the Euro-

pean brain, however, this fissure joins the fissure of the hippocampi below (Plate XIX. fig. 5), whilst in the Quadrumana it usually stops short of that fissure, owing to the development of a *superficial* connecting convolution in that situation.

The *fissure of the hippocampi* (Plates XVIII. & XIX. figs. 4 & 5, *l–l*, *m*) is nearly horizontal. Its outer or *calcarine portion* (*l–l*) ends in two shallow sulci, or notches, on the tip of the posterior lobe; it extends a little further forwards than is customary beneath the corpus callosum, but is separated, as usual, from the inner or *dentate portion* (*m*) of the fissure by a ridge of cerebral substance (*), which connects the convolution of the corpus callosum ($_{18}$) with the uncinate convolution ($_{19}$). The dentate fissure (*m*) is shallow. The *inferior middle temporal, parallel* or *collateral fissure* (Plate XIX. fig. 5, *n–n*) is very long and simple, and may be traced further upwards on to the hinder surface of the occipital lobe than usual.

On the whole, the fissures of the left hemisphere are rather more complex than those of the right, which would seem to be not only smaller but inferior in organization. This want of symmetry is itself, however, a mark of comparative elevation.

With a few exceptions, these primary fissures are somewhat more complex than those represented in GRATIOLET's figures of the brain of the Hottentot Venus; but nevertheless they are far more simple and more easily distinguished amongst the numerous secondary sulci than in the ordinary European brain. In this greater simplicity and definition of the fissures generally, in the slightly more vertical direction and step-like course of the Sylvian fissure, and in the decidedly more upright position of the internal perpendicular fissure, the Bushwoman's brain approaches somewhat the quadrumanous characters; but it deviates more widely from them by the special interruption of the external perpendicular fissure, by the greater length and inclination backwards of the fissure of ROLANDO, by the more marked want of symmetry on the two sides of the brain, and by the greater number and complexity of the secondary sulci.

The lobes.—Regarding the more important fissures as the true lines of subdivision between the cerebral lobes, we find that the frontal lobes (F), though contracted in width and depth, are proportionally long, that the parietal lobes (P), though high, are relatively contracted from before backwards, especially behind, that the occipital lobes (O) are very shallow from above downwards, and very pointed, that the temporal lobes (T) are long and narrow, and that the island of REIL, or central lobe (C), is small and partly visible at the entrance of the Sylvian fissure.

The convolutions and secondary sulci.—The *orbital* sulci and *convolutions* of the frontal lobe (Plate XVII. fig. 2) are certainly remarkably simple. The olfactory sulcus, which lodges the so-called olfactory nerve (*o*), has its ordinary length; but, owing to the shortness of the frontal lobe, it reaches to within half an inch of the tip of that lobe, whilst in the European it does not reach within one inch of that point. On the inner side of this sulcus is seen, as usual, the edge of the great marginal convolution ($_{17}$) of the inner surface. To its outer side, the deep triradiate sulcus, which cuts up the rest of the orbital surface into a posterior convolution ($_{1''}$) limiting the Sylvian fissure, an

internal convolution ($_{1'''}$) bounding the olfactory sulcus, and an external curved convolution ($_{1''''}$) forming the outer border of the frontal lobe, consists of three short simple curved branches, very like those found in the Ape, instead of the tortuous sulci seen in the European brain. The forms of the surrounding orbital convolutions themselves, including the proper supraorbital ($_{1'}$), are so broad and simple, that their subordinate divisions, which are so complex in the European brain, can hardly be said to exist.

The *frontal convolutions* (Plates XVII. & XVIII. figs. 1 & 3) are, as usual, arranged in three stages or rows, separated from each other by two deep secondary sulci. The *lower row* ($_{1-1}$) (le premier étage, GRATIOLET; infero-frontal, HUXLEY) is well defined, and intermediate in complexity between its condition in the Hottentot Venus and an average European brain. The *middle row* ($_{2-2'}$) (le second étage, GR.; medio-frontal, H.) resembles, in its simplicity of form and detail, that of the Hottentot Venus much more than that of the European brain, especially as seen from above. Posteriorly ($_{2'}$) it joins the upper end of the first ascending parietal convolution ($_{4}$), as in the Hottentot Venus, whilst in the European the continuity is usually interrupted by a secondary sulcus; but instead of being continuous, as in the former, in front of the Sylvian fissure, with the lower frontal, it is there separated from it, as in the latter. The *upper row* ($_{3-3'}$) of frontal convolutions (le troisième étage, GR.; supero-frontal, H.), in its proportion to the other two rows and its subordinate divisions, approaches nearer to the European type; but it is simpler, and in the upper view (Plate XVII. fig. 1) much narrower, narrower even than those figured in the Hottentot Venus. As usual, it joins ($_{3'}$) the first ascending parietal convolution ($_{4'}$), behind and above, in front of the fissure of ROLANDO. Along the border of the longitudinal fissure, where it is blended with the great marginal convolution, it is less frequently notched than in the European brain; so that one particular notch (in front of $_{3'}$) becomes very evident. In this respect the resemblance is very close to the Hottentot-Venus brain; but the left notch is further back than the right, instead of the reverse. In the European brain these notches are symmetrically placed on the two sides (see Plate XX.).

The *first*, or *anterior ascending parietal convolution* (Plates XVII. & XVIII. figs. 1 & 3, $_{4-4'}$) (premier pli ascendant, GR.; antero-parietal, H.) is larger and more pronounced in its form than in the Hottentot Venus; in its general mass it approaches the European character; but it has fewer secondary sulci, and, as already stated, joins anomalously the middle frontal row. It is broader on the right side than on the left. As usual, it forms the anterior border of the fissure of ROLANDO, joins at its lower end the supramarginal convolution ($_{4''-5''}$) which limits that fissure below, and at its upper end runs forwards into the upper row of frontal convolutions, and backwards around the upper end of the fissure of ROLANDO into the second or posterior ascending parietal convolution ($_{5}$). In the brain of the Hottentot Venus (see Plate XX.), this last-named connexion is almost concealed within the longitudinal fissure; whereas in the European brain it is superficial, owing to the greater upward development of this part of the brain. In the former case the hinder end of the fissure of ROLANDO loses itself in the longitudinal

fissure, whilst in the latter it stops short of that fissure. On the left side the Bushwoman's brain presents the European character, whilst on the right side it has the Hottentot-Venus character. As in the case of the deep notch in the upper frontal convolutions, the fissure of ROLANDO extends further back on the left hemisphere than on the right. In the Hottentot-Venus brain the reverse is the case. From all this it may be deduced that in the Bushwoman's brain the left frontal region is developed further backwards than the right, whilst in the Hottentot Venus the condition is reversed.

The *posterior ascending parietal convolution* (Plates XVII. & XVIII. figs. 1 & 3, 5–5) (deuxième pli ascendant, GR.; postero-parietal, H.) holds, as regards size and complexity of modelling, a position between the Hottentot and the European brain, but on the whole is nearer to the former than the latter (Plate XX.). But, as seen in the lateral view (Plate XVIII. fig. 3), its lower and posterior border is joined, as in the European brain, by a superficial connecting convolution (*) with the lobule (A–A) of the supramarginal convolution above and near the upper end of the Sylvian fissure, whilst in the Hottentot-Venus brain this is not the case. On the left hemisphere this posterior ascending convolution is broader and more complex in form than in the Hottentot Venus, or indeed than in many European brains; but on the right side it is quite simple and unusually narrow. It ends posteriorly, as is the rule, in its so-called "*lobule*" (5′–5′) (lobule du deuxième pli ascendant, GR.; postero-parietal lobule, H.), a triangular mass of secondary convolutions more or less variable in different brains, and even on the two sides of the same brain. In the Bushwoman's brain these lobules are, in point of size and complexity, intermediate between those of the Hottentot Venus and the ordinary European, but are nearer to the latter than the former. On the left side, however, the hinder border of the lobule is absolutely defined on the surface, owing to the deep position there within the external perpendicular fissure of the upper external connecting convolution (α), or first external "pli de passage" of GRATIOLET. In this point the Bushwoman's brain is more ape-like than even that of the Hottentot Venus (see Plate XX.).

The *supramarginal convolution* (Plate XVIII. fig. 3, 4″–5″) connects, as usual, the two ascending parietal convolutions below the lower end of the fissure of ROLANDO, and there forms the step-like border of the Sylvian fissure already alluded to, which is now seen to depend on the great downward development of the two ascending parietal convolutions. The anterior part of the supramarginal convolution, which overhangs the island of REIL, and is continued into the inferior frontal and adjacent orbital convolutions, is scantily developed, in correspondence with the open state of the Sylvian fissure, and the constricted form of the brain at this point.

The *central lobe, or island of* REIL (C), is small on both sides of the brain, but is somewhat larger on the right side than on the left; its total length is 1·75 inch on the left side and 1·5 inch on the right, whilst its average length, in an ordinary European brain, is from 1·75 inch to 2 inches. On the left side it is subdivided into three chief radiating convolutions, the middle one of which is partially subdivided by a slight sulcus; on the right side this middle portion is more deeply subdivided; so that there are four

radiating convolutions on both sides, but on the right side these appear larger than on the left. In the European brain these radiating convolutions again subdivide along their upper and outer borders, so as to appear still more numerous.

The hinder end of the *supramarginal* convolution expands, as usual, into its so-called "*lobule*" (Plates XVII. & XVIII. figs. 1 & 3, A–A), which is described by GRATIOLET as a part peculiar to Man—a statement undoubtedly true if the lobule be regarded merely as an expanded or highly developed portion of the supramarginal convolution itself, but not to be accepted as implying that the lobule is an entirely new part of the cerebrum, wholly unrepresented in the quadrumanous brain. Even as regarded in the former light, the condition of this lobule of the supramarginal convolution in the Bushwoman's brain is of special interest. It protrudes in her brain in the form of a large nearly quadrangular mass, situated exactly beneath the parietal eminence of the skull, and corresponding therefore with the line of greatest width of the cerebrum; it overhangs the upper part of the Sylvian fissure; it is connected in front, by a partially concealed convolution (*) already mentioned, with the posterior ascending convolution, above with the lobule of that convolution, and behind with the bent convolution ($_6$); and it is marked by several secondary sulci. In all these particulars it resembles the part in the European brain, but it is somewhat smaller. On the other hand, it is decidedly superior to that of the Hottentot-Venus brain, being larger and more complex, projecting more over the Sylvian fissure, and having a more superficial connexion with the posterior ascending convolution. Relatively it is one of the most developed parts of the Bushwoman's brain.

The *bent convolution* ($_6$–$_6$) (pli courbe, GR.), which limits the summit of the Sylvian fissure, is connected in front with the lobule of the supramarginal convolution, and, turning downwards, sinks in between the upper external temporal (Plate XVIII. fig. 3, $_7$) and middle temporal ($_8$) convolutions, on the left side, as is usual; whilst on the right it continues superficial, but does not join the superior temporal convolution as in the Hottentot-Venus brain; in the latter brain it is a simple though somewhat tortuous convolution, whilst in the European brain it is represented by two or more convolutional folds. It is decidedly defectively developed in the Bushwoman's brain.

The three rows of *external temporal convolutions* (Plates XVII. XVIII. & XIX. figs. 2, 3, & 5) are intermediate in character between the European and Hottentot condition. On the left side they are more complex than on the right, where they resemble more nearly those of the Hottentot Venus. The *upper temporal*, or *inframarginal convolution* ($_7$–$_7$) (pli temporal supérieur, GR.; antero-temporal, H.) is bent, and somewhat constricted, opposite the ends of the ascending parietal convolutions, at the peculiar horizontal step of the Sylvian fissure. It is separated from the bent convolution, as usual, by a secondary sulcus. As in the Hottentot Venus, it is proportionally wider than in the European brain. The *middle temporal convolution* ($_8$–$_8$) (pli temporal moyen, GR., medio-temporal, H.) is narrower, more tortuous, and slightly more complex than in the Hottentot brain, but not nearly so wide or so much intersected with secondary sulci as in the

European. The *lower temporal convolution* (9–9) (pli temporal inférieur, Gr.; postero-temporal, H.), which is not well defined along its upper border, also approaches the European rather than the Hottentot type, but is much less complicated than the former.

The three rows of *occipital convolutions* (Plates XVII. & XVIII. figs. 1 & 3, 10, 11, 12) (plis occipitaux, supérieur, moyen, et inférieur, Gr.; super-, medio-, and infero-occipital, H.), which in quadrumanous brains of moderate complexity (as in Cercopithecus) are simple and easily distinguishable, but which in the anthropoid Apes assume a puzzling complexity, become, as is well known, in the human brain so highly complicated and involved with the external connecting convolutions, that a detailed description of them is almost impossible. Considered generally, they are remarkably defective in total depth and in individual complexity, in the Bushwoman's brain. The vertical depth of the three rows, and of their connecting convolutions, in the European brain is 2·75 inches; in the Hottentot-Venus brain 2·25 inches; in the Bushwoman's brain only 2 inches. This deficiency affects all three rows of occipital convolutions, but is especially noticeable in the inferior row, along the lower border and extreme point of the occipital lobe. This is perhaps the most defective region of the Bushwoman's cerebrum.

Of the *external connecting convolutions* already mentioned, four in number, which in Man interrupt the external perpendicular fissure, the *upper one* (α) (premier pli de passage externe, Gr.; first external annectent, H.) is on the left side superficial behind, but sinks beneath the bent convolution in front. On the right side it is superficial throughout, so as speedily to obliterate the external perpendicular fissure, and is very tortuous. The three remaining connecting convolutions, viz. the *second* (β), *third* (γ), and *lowest* (δ) are remarkably simple and small, occupying so little space as, together with the deficiency in the bent and occipital convolutions, to account for the slight depth of the cerebrum in this region. They are all continuous posteriorly with the several rows of occipital convolutions, but are less directly connected anteriorly with the middle and inferior temporal convolutions than in the European, or even in the Hottentot-Venus brain, on the left side, whilst on the right side they are easily traceable into those convolutions.

On the inner surface of the hemisphere (Plate XVIII. fig. 4), the *great marginal convolution* (17–17) (pli de la zone externe, Gr.) is proportionally narrow. As usual, it is thicker opposite the beak-like prominence of the frontal lobe, where it is subdivided into two parallel convolutions by an unusually simple longitudinal secondary sulcus. It is intersected above the corpus callosum by transverse radiating sulci, which divide it into short secondary convolutions, arranged like the stones of an arch. In both hemispheres it is twice connected (*, *), in front of and above the middle of the corpus callosum, across the great fronto-parietal fissure (*i–i*), with the convolution of the corpus callosum.

The *convolution of the corpus callosum* (18–18) (pli du corps calleux, Gr.; circonvolution de l'ourlet, Foville; callosal, H.) presents its usual characters, turning round the posterior end of the corpus callosum (*c*), and becoming continuous beneath the cerebral

peduncle (Plate XIX. fig. 5), by a narrow ridge (*), with the middle internal or uncinate convolution ($_{19}$) of the temporal lobe. The callosal convolution is large, and its crested upper border is well marked posteriorly. The marginal convolution is relatively rather narrow, and its secondary convolutions are simple. Altogether these internal convolutions are less complex than in the European.

The *quadrilateral lobule* (Plates XVIII. & XIX. figs. 4 & 5, $_{18'}$) (lobule quadrilatère, FOVILLE; quadrate lobe, H.), the extension backwards and upwards of the callosal convolution, is very well marked; but it is short, and, in accordance with the more upright direction of the internal perpendicular fissure (already described), is not quite so much inclined backwards as in the European. It is smoother and narrower above than below, instead of being wider than, or of equal width as, in the European—a condition which coincides with the backward extension of the ascending parietal convolutions and the fissure of ROLANDO.

Of the three *internal temporal convolutions* (Plate XIX. fig. 5), the *upper* one, concealed in the dentate fissure, *m*, and corresponding with the corps godronné of authors (the dentate convolution, H.), is present, but very small, the fascia dentata being only just recognizable. The *middle internal temporal convolution* ($_{19}$–$_{19}$) (pli temporal moyen intérieur, GR.; uncinate, H.), continuous upwards by the narrow ridge (*) with the convolution of the corpus callosum, and ending forwards in the *unciform lobule* ($_{19'}$) (lobule de l'hippocampe, GR.; and *crochet* of VICQ D'AZYR), is relatively narrow and less prominent in the Bushwoman's brain, the crochet being particularly small. The *lower temporal convolution* ($_{9}$–$_{9}$), which in fact is the same as the lower external temporal, and is marked off from the middle one by the parallel fissure of GRATIOLET (*n*–*n*) (collateral fissure, H.), is very broad, though smooth. The secondary sulci of these convolutions are chiefly longitudinal, and more simple than in the European brain. The middle convolution ($_{19}$), that above the collateral fissure, is traceable round the very tip of the occipital lobe, and, curving upwards on its hinder aspect, joins the middle and lower occipital convolution; the lower one ($_{9}$) unites more directly with the lower occipital row.

The bottom of the calcarine portion (*l*–*l*) of the fissure of the hippocampi corresponds with the hippocampus minor in the posterior horn of the lateral ventricle. Deeply seated between its two short branches behind is a ridge of cerebral substance, representative of the *calcarine lobule* of FLOWER.

The triangular *occipital lobule* (Plates XVIII. & XIX. figs. 4 & 5, $_{25}$–$_{25}$) (lobule occipital, GR.) is of small size, and has its anterior and inferior margins more even, and its surface less complex than in the European brain. Owing to the less inclined position of the internal perpendicular fissure, this lobule approaches somewhat nearer to the vertex than is customary.

Lastly, the *lower internal connecting convolution* (Plate XIX. fig. 5) (pli de passage inférieur interne, GR.) which joins the lower and anterior angle of the occipital lobule with the adjacent part of the quadrate lobule, is represented, as usual, by a ridge *con-*

cealed (near *s*) within the perpendicular fissure, which accordingly joins superficially the fissure of the hippocampi. The *upper internal connecting convolution*, which in the Apes commonly crosses higher up between the occipital and quadrate lobules (opposite ζ), is, as customary in Man, completely absent; so that the development of these internal connecting convolutions, like that of the external ones, is perfectly normal.

From a general consideration of the above detailed convolutional characters of the Bushwoman's brain, the following conclusions may be announced.

1. All the primary convolutions which exist in the human brain, viz. the orbital convolutions, the three frontal rows, the two ascending parietal and the parietal lobule, the supramarginal with its lobule and the bent convolution, the three external temporal, the three occipital rows, those of the island of REIL, the marginal and callosal convolutions, the quadrate and occipital lobules, and the three internal temporal convolutions, are present in the Bushwoman's brain; but, as compared with the same parts in the ordinary European brain, they are smaller, and in all cases so much less complicated as to be far more easily recognized and distinguished amongst each other. This comparative simplicity of the Bushwoman's brain is of course an indication of structural inferiority, and indeed renders it a useful aid in the study of the more complex European form. On contrasting the several regions of the cerebrum, the primary convolutions of the upper frontal and outer parietal regions are, on the whole, the best developed; those of the middle and lower frontal regions, the temporal region, the central lobes, and the inner surface the next; whilst those of the orbital surface and occipital lobe are the least developed.

2. Of the connecting convolutions, those highly important and significant folds, the external connecting convolutions are, in comparison with those of the European brain, still more remarkably defective than the primary convolutions. All four of these convolutions are present; but all are characteristically short, narrow, and simple, instead of being complex, and occupying a large space; hence, though the external perpendicular fissure is soon filled up, the parietal and occipital lobes are more easily distinguishable from one another than in the European brain. The upper external connecting convolution on the left side does not superficially join the parietal lobule, but sinks beneath it and the bent convolution. Of the internal connecting convolutions the arrangement is normal.

The numerous secondary sulci and convolutions, which so complicate the larger ones in the European brains, are everywhere decidedly less developed in the Bushwoman—but especially so in the occipital and orbital regions, on the bent convolution, and on the external connecting convolutions. This is a further sign of structural inferiority.

3. Compared with the brain of the Hottentot Venus, as that is represented by GRATIOLET, the Bushwoman's brain is, in nearly all cases where comparison is possible, a little, though a very little, more advanced and complex in its convolutional development—the one exception being in regard to the size of the occipital and external connecting

convolutions, which are smaller in the Bushwoman. It is possible, however, that some of the apparent simplicity of the Hottentot-Venus brain may be due to the unavoidable loss of form and detail incidental to its long period of preservation, as compared with the more recent and comparatively uninjured Bushwoman's brain. This may account, for example, for the comparative breadth and smoothness of the upper frontal and of the middle and lower temporal convolutions in the figures of M. Gratiolet. Allowance being made for this, the resemblance between the convolutions of the two brains is very close, and serves to confirm the demonstration by that author of the relative simplicity of the Hottentot-Venus brain—a simplicity which he has only seen partially paralleled in normal European brains, but which, in my own more limited experience, I have never even seen approached in healthy brains.

4. Whilst, then, the difference between the Bushwoman's brain and the European brain, not merely as to size, but as to convolutional development, is very marked, that between the Bushwoman and the Hottentot Venus is very small; and indeed if we regard the relative general development of the convolutions as a gauge of proximity or separation, it is turned into a near resemblance; and since no suspicion either of idiocy or other defect exists as concerns the Bushwoman, this would go far towards proving that the inferiority in the cerebrum of the Hottentot Venus is not due, as has been suggested, to an arrest of development of a personal or individual kind, but that, whilst undoubtedly both brains show an infantile or fœtal leaning, this is to be attributed partly perhaps to sex, but in the main to the characterization of the race itself.

5. As regards the question of the symmetry of the convolutions, it may be said that, although it is certainly easier to compare those of the two hemispheres in the simpler brains of the Bushwoman and Hottentot Venus than in the more highly developed European brain, still a very cursory examination shows that in numerous particular points, already mentioned in our description, there is just as frequent an occurrence of asymmetry in the two former as in the latter, by which circumstance therefore they manifest a truly human character.

6. Although not only in size, but in every one of the signs of comparative inferiority manifested in the lower convolutional development of the Bushwoman's cerebrum, it leans as it were to the higher quadrumanous forms, yet, as regards the sum of its convolutional characters, judged of by the presence or absence, the individual and relative size and position, the comparative complexity or simplicity, and the symmetry or asymmetry of particular fissures and convolutions, there is a greater difference between it and the highest Ape's brain yet described, viz. the adult Orang's brain, than between it and the European brain (compare Plates XX. & XXIII. figs. 7, 9, & 20). This difference is, as one evidently would expect, especially marked in the regions peculiarly developed in Man, viz. in the anterior outer and upper parts of the frontal lobe, in the prominent part of the parietal lobe (that is, in the characteristically human supramarginal lobule), and in the regions of the external connecting convolutions, especially of the two upper external ones. It is almost needless to add that there is far less difference,

in convolutional development, between the Bushwoman's brain and the European brain, than between the lowest and highest quadrumanous brains. If, indeed, we disregard the general differences of size and complexity, and look only to those which have been considered as special peculiarities, such as the existence of the supramarginal lobule, and the joint relative development of the two upper connecting convolutions, there is less difference between the Bushwoman and the European than between the Chimpanzee and the Orang. But perhaps it is premature yet to decide this latter point. It is certain, however, that there is less difference in convolutional development between the Bushwoman and the highest Ape, than between the latter and the lowest quadrumanous animal.

7. Finally, the establishment of the conformable development of the brains of the Bushwoman and Hottentot Venus (herself believed by G. CUVIER to have been a Bushwoman of small stature) is a step gained in cerebral anatomy; and their common inferiority to the European brain may justify the expectation that future inquiries will show characteristic peculiarities *in degree* of convolutional development in the different leading races of mankind.

e. *Internal Structure of the Cerebrum and Cerebellum. The Commissures, Cavities, Ganglionic masses, and Laminæ, studied on the Preserved Brain.*

The cerebrum.—The general depth of the sulci in the Bushwoman's brain is rather more than half an inch; they are deepest on the parietal region, shallower in the frontal, and, with the exception of the posterior part of the fissure of the hippocampi, are shallowest on the occipital lobe. They are deeper on the outer than on the inner surface of the cerebrum; they are very shallow on the under surface near the tip of the temporal lobe, and also on the orbital surface of the frontal lobe. In these respects the Bushwoman's brain conforms to the usual conditions.

The average thickness of the grey matter is nearly $\frac{4}{30}$ths of an inch, the extremes being $\frac{2}{30}$ths and nearly $\frac{5}{30}$ths of an inch. The thickest grey matter is in the frontal and parietal regions, the thinnest at the tip of the occipital lobe, as usually found in both human and quadrumanous brains. The proportion of white matter in the centre of the hemisphere (see Plate XIX. fig. 6), which forms, on a horizontal section, the centrum ovale, appears smaller than in the European—a condition which coincides with the comparative narrowness of the brain. Both the grey and the white substance are darker than in the European, having a peculiar yellow tint. There was much pigment here and there in the membranes.

The corpus callosum (Plate XVIII. fig. 4, *c*) is long, but is wanting in general depth and in thickness at each end. As measured in the hardened brain, in thirtieths of an inch, its length, its greatest thickness, its least thickness, and its average thickness are represented by the numbers 78, 13, 5, and 6; whilst in the European the corresponding dimensions are 93, 16, 6, and 13; in the Chimpanzee I found them to be 51, 6, 2, and 4·5. The sectional area of the longitudinally divided corpus callosum is therefore in the

Bushwoman $\frac{468}{900}$ of a square inch, whilst in the European it amounts to $\frac{1209}{900}$, and in the Chimpanzee to $\frac{330}{900}$ of a square inch. Compared with the area of the internal surface of one hemisphere, the sectional area of the corpus callosum is in the Bushwoman's brain as 1 to 25, in the European as 1 to 12·5, and in the Chimpanzee as 1 to 28·5; so that the corpus callosum, thus estimated in proportion to the cerebrum, is in the Bushwoman only *half as large* as in the European, and not much larger proportionally than in the Chimpanzee. The anterior commissure (*a*) is also singularly small; the posterior commissure is very slender; whilst (probably an individual peculiarity only) there is no trace of the so-called soft commissure. On the whole, therefore, the system of transverse commissural fibres is defective; and as the size of the medulla oblongata, in proportion to the unusually narrow cerebrum, is larger even than in the European (so that the radiating system is probably not so much diminished), it would seem as if the relative deficiency of white substance within the hemispheres was owing in a great degree to the fewness of the transverse, as well, perhaps, as of other commissural systems of fibres. I have elsewhere pointed out the same condition in the Chimpanzee's brain; and it doubtless is associated, in the Bushwoman's brain, with its inferior bulk and less convoluted surface. The proportional size of the corpus callosum, thus considered, offers, I believe, a not inconvenient test of the relative perfection of any given normal brain of certain plan. Comparative anatomy supports this view. Of the longitudinal system of commissures, the fornix is thin, the tænia semicircularis slender, and the striæ longitudinales plainly visible.

The septum lucidum, with its intervening ventricle, is large, both in depth and extent from before backwards. The lateral ventricle in the left hemisphere (Plate XIX. fig. 6) proved to be a very large cavity; the body (*a*) measured 1·5 inch in length, the anterior cornu (*b*) between ·6 and ·7, the posterior cornu (*c*) 1·8, the descending cornu (*d*) 1·5; the corresponding numbers in an ordinary example of a European brain were 2·1, 1·4, 1·2, and 2·6. Comparing these dimensions with the total lengths of the two cerebra respectively, viz. 5·8 inches for the Bushwoman, and 6·5 for the European, we get the following proportions: ·25, ·11, ·31, and ·43 to 1 in the Bushwoman, and in the European ·32, ·21, ·184, and ·4 to 1. In the former, therefore, the body is short, the anterior cornu very short, the descending cornu long, and the posterior cornu very long, the proportion being as 5 to 3. The width and depth of the posterior cornu are as remarkable as its length; the width varies from ·3 of an inch, opposite the projection of the hippocampus minor, to upwards of ·4 in the wide recess behind that eminence; the depth of the cornu, at its deepest part, is ·4 of an inch. From the end of the posterior cornu to the extremity of the occipital lobe is ·5 of an inch, that is, a little more than $\frac{1}{11}$th of the total length of the brain, showing an unusual proximity of the posterior cornu to the apex of the posterior lobe. The hippocampus major (*e*) is narrow, it expands at its lower end, on the anterior border of which is a single prominence, but otherwise there is no trace of indentation. The hippocampus minor (*f*) is of large dimensions, projecting boldly into the middle of the posterior cornu, and subsiding

gradually backwards, being 1·1 inch in length, and ·4 inch in greatest breadth. The eminentia collateralis (*g*) is represented by a broad triangular and elevated surface between the two hippocampi.

On the method employed by Mr. FLOWER to determine the ratio between the antero-median and posterior portions of the cerebrum, measured forwards and backwards from the point of junction of the hippocampi, I find that the antero-median portion measures 3·75 inches, and the posterior 2·05 inches, showing a ratio of 100 to 54·6; whilst the ordinary ratio in the European is said by Mr. FLOWER to be 100 to 53; in the Chimpanzee it is as 100 to 52; so that in the Bushwoman the posterior region (thus measured on its under surface) is proportionally longer, or the antero-median region proportionally shorter than in the European or Chimpanzee's brain; but in other Quadrumana the posterior region is stated to be longer still (in Hapale 100 to 62).

As seen in the body of the ventricle, the corpus striatum (*h*) and optic thalamus (*a*) occupy about the same relative spaces from before backwards as in the European; but both, especially the optic thalamus, appear narrower from side to side. In an ordinary European brain, the exposed part of the corpus striatum measured ·9 inch long and ·5 inch wide; the optic thalamus 1·3 inch long and ·5 inch wide. In the Bushwoman the corpus striatum is ·9 inch long and ·3 inch wide, and the optic thalamus 1·2 inch by ·45 inch.

The corpora quadrigemina (Plate XVIII. fig. 4, *q*) are rather small, even in proportion to the brain; the anterior one, as usual, is the more prominent, and the posterior one the wider of the two. The corpora geniculata are both well marked, though of moderate size. The pineal body is small; its habenulæ are well developed. The corpora albicantia are prominently developed. The pituitary body is of moderate size.

The pons Varolii seems proportionally large: from its upper to its lower border it measures ·9 inch, whilst in the European brain it is about 1 inch; the mean thickness of its section is, in the Bushwoman's brain, ·8 inch; in the European, ·9 inch.

The medulla oblongata is relatively wide, and so are the cerebral peduncles. The medulla oblongata is ·85 inch wide at its widest part, offering the ratio of 1 to 6, instead of the ordinary ratio of 1 to 7, to the width of the cerebrum. Upon the medulla oblongata the anterior pyramids are well pronounced, and the corpora olivaria are narrow but prominent: the corpus dentatum within the latter is neatly defined and waved.

The cranial nerves generally appear small; the olfactory nerves, however, are well developed. The optic nerves, commissure, and tracks are small and flattened, even the nerves having an unusually thin oval section. The small size of the optic tracks and corpora quadrigemina is interesting in connexion with the defective development of the occipital lobes of the cerebrum, a part to which many of their fibres have been traced by GRATIOLET.

The cerebellum.—Every part of the cerebellum (Plates XVII. & XVIII. figs. 2, 3, 4, *Ce*) is present. The lateral portions or hemispheres are especially wide. On the upper surface the square lobes are not so square as usual, but are elongated laterally, and narrow

externally; the upper posterior lobes are also narrower and less curved than in the ordinary European brain; the superior vermiform process is long, but relatively narrow. On the under surface, the amygdalæ are not more than half the usual size, and are narrow and oblong, not broad and pyramidal in shape; the biventral lobes are, on the contrary, large, and, owing to the small size of the amygdalæ, are placed nearer to the middle line and have their laminæ more vertical; the slender lobes are also large and broad; lastly, the lower posterior lobes, separated from the upper posterior by the usual deep horizontal fissure, are likewise very broad. In the vallecula, the pyramid, uvule, and nodule are clearly defined, but somewhat narrow; the posterior velum is very well developed. The floccules, or subpeduncular lobes, are small, being both short and compressed. On the whole, therefore, the median parts of the cerebellum are relatively small, whilst its hemispheres are relatively large.

The transverse commissural, or middle peduncular fibres appear, even to the eye, proportionally more abundant than in the European, and likewise more so than in the Chimpanzee. A comparison of the sectional area occupied by these fibres in the pons, with the weight of the cerebellum, confirms this observation. The oval surface (Plate XXIII. fig. 23, *p*), which includes the ends of these transverse fibres, as divided in the median plane of the pons, is in the European ·95 inch long by ·65 inch wide, giving a sectional area of ·6175 square inch, which, as the total weight of the cerebellum is 4·68 oz., gives about ·13 square inch of surface to each ounce of cerebellum. In the Bushwoman, the corresponding oval surface (fig. 24, *p*) measures 1 inch by ·6 inch; its area equals therefore ·6 square inch, which, divided by the weight of the cerebellum, 3·45 oz., gives ·173 square inch of surface to each ounce of cerebellum. In the Chimpanzee the equivalent measurements are ·55 by ·4 inch; the area is ·22 square inch; the weight of the cerebellum is 2·02 oz., and the ratio of cut surface to each ounce of cerebellum is nearly ·11 square inch. This is an important point in which the brain of the Bushwoman does not stand intermediately between the European and the Ape, but surpasses even the European brain. The inferior peduncles, and also the superior peduncles of the cerebellum, appear relatively small; but no precise method of measurement could be adopted in regard to them.

It is difficult also to devise any satisfactory mode of determining the number of laminæ in any given cerebellum. If the superficial laminæ be counted, they are found to vary so in length, that, whilst some pass round the whole surface or border of the cerebellum, others disappear between adjoining laminæ at various points, so that no single line can be drawn over the surface which will cross the edges of all the superficial laminæ. On the other hand, if the deep as well as the superficial laminæ be counted, then it is difficult to determine how small or short a fold shall be considered a distinct lamina, some of them being very short. Furthermore, the larger ones are often slightly grooved along their edge, and might be reckoned or not as consisting of two. MALACARNE *, who gives the number of laminæ in the healthy cerebellum at as

* Neuro-encefalotomia. Pavia, 1791, p. 7.

high numbers as 600 and 780, evidently counted all the laminæ he could find. I have adopted the plan of counting the superficial laminæ only in the principal lobes and superior vermiform process; whereas in the smaller parts of the organ, and in the inferior vermiform process, all the laminæ have been counted. The following Tables show the results in the European, the Bushwoman, and the Chimpanzee, for the *left* half of the cerebellum.

Superficial laminæ only counted.

	Median portion.	Lateral parts.						Total laminæ in lateral parts.
	Superior vermiform process.	Square lobe.	Posterior superior lobe.	Amygdala.	Biventral lobe.	Slender lobe.	Posterior inferior lobe.	
European	18	21	13	9	6	5	16	70
Bushwoman	23	21	14	8	9	5	14	71
Chimpanzee	20	25	10	8	4	11	5	63

Superficial and deep laminæ counted.

	Median portion.					Lateral parts.	
	Inferior vermiform process.		viz. Pyramid and uvula.		Nodule.	Amygdala.	Floccule.
European	37	=	28	+	9	21	18
Bushwoman	33	=	24	+	9	20	17
Chimpanzee	34	=	29	+	5	22	17

As thus counted, the number of laminæ in the Bushwoman's cerebellum agrees very closely with that in the European, the differences being probably only such as might be met with between individuals of either race. The total number in the lateral parts or hemispheres is nearly identical. The differences between the upper and lower posterior lobes nearly compensate each other, as do those between the amygdalæ and biventral lobes; the square and slender lobes exactly agree. In the median portion the chief point of difference is found, viz. in the larger number of laminæ in the upper vermiform process of the Bushwoman; but then there is a smaller number in the pyramid and uvula of the lower vermiform process: the nodules coincide. It is worthy of note that, in the Bushwoman, the amygdala and floccule show but a slight defect in the number of their laminæ, although both those parts are so remarkably small. Indeed the total deficiency in weight, which has previously been shown to exist in the Bushwoman's cerebellum, depends essentially, not on the absence of any parts or laminæ, but on the narrowness of these latter; for they are obviously much finer than in the European brain.

In the Chimpanzee the square lobe occupies more of the upper surface of the hemisphere than in the European cerebellum, whilst the Bushwoman's cerebellum presents an intermediate condition. The upper posterior lobe is consequently straighter in the

Ape, and forms less of the outer border of the hemisphere; in this respect the Bushwoman also occupies an intermediate position. On the other hand, the amygdalæ, which are very small in the Bushwoman, are very large in the Ape; whilst the biventral lobes are large in the former and small in the latter. The slender lobes are very broad in both; the lower posterior lobes, broad in the Bushwoman, are very narrow in the Ape. The superior and inferior vermiform processes, especially the latter, are proportionally more marked in the Ape than in the Bushwoman, in whom they are smaller than in the European. The floccules, small in the Bushwoman, are well developed in the Ape. In none of these particulars, then, is the Bushwoman's cerebellum intermediate between that of the European and the Chimpanzee; nor is this the case in regard to the number of the laminæ, for there are fewer on the whole in that animal even than in the European, the particular excesses and deficiencies not appearing to be reducible to any rule. In accordance with the smaller bulk of this organ in the Ape, the laminæ themselves are very much finer even than in the Bushwoman.

In the Bushwoman the corpus dentatum is represented by a comparatively small oblong mass of grey matter, almost destitute of foldings, and having its internal white substance ill defined. Is this connected with the small size of the quadrigeminal bodies? In the Chimpanzee this body is long and narrow, but its foldings are distinct.

On the whole it may be said, judging from its transverse commissural fibres and its laminæ, that, with the exception of the amygdala and floccule, and the grey matter of the corpus dentatum, the cerebellum in the Bushwoman is very well developed, and that, as an organ, it is far more completely evolved than the cerebrum.

II. THE IDIOTS' BRAINS.

a. *General Account of the Idiots.*

The female idiot came of a healthy family, and died at the age of 42, of phthisis. Her height was about (probably below) 5 feet; her weight is unknown, but she was well proportioned, with shapely limbs, and small well-made hands and feet; she was never fat, and did not become emaciated until phthisis occurred. The general appearance of her microcephalic head, the form, size, and condition of the cranium, with other particulars, are described in Mr. GORE's paper*. From that source, and from information since supplied by him, it appears that her senses were perfect, including the appreciation of heat and cold. She had memory both of persons and things; she could say "child," "mamma," "morning," and "good" with tolerable distinctness; in the report of Mr. GORE's paper it is added, "but without connexion or clear meaning." In explaining this he writes, "I think she had *some*, though probably a very imperfect *knowledge* or *conception* of the meaning of the words she used; she certainly knew what was the meaning of 'good,' in relation to her own acts and conduct; she was, however, quite incapable of anything like conversation." It is stated that she could not count, and did

* Anthropological Review, vol. i. 1863.

not know the value of money. She was obedient to those around her, affectionate, and fond of carrying and nursing a doll. She never manifested any sexual propensity, though she menstruated regularly. She was not passionate, nor violent, but was susceptible of joy and fear. She could not feed herself with any degree of method or precision, nor could she dress herself; in walking her gait was unsteady and tottering, the heels not bearing with any firmness on the ground. As already stated, her articulation was imperfect. There is no reason to suppose that she had any sense of religion, or idea of futurity.

The idiot boy was born of healthy parents. They, however, were first cousins, and met with great vicissitudes of fortune, accompanied, as regards the mother, with other causes of grief. No ancestral relative, on either side, was known to have exhibited any mental defect; but a second child, also a boy, one year younger, was likewise idiotic, though able to walk and to talk pretty distinctly. Both children were born at their full time; the mother was not frightened when pregnant with either of them. The brother is also dead now *.

The boy whose brain we are about to describe did not notice persons or things till he was 6 months old, and then very little. He lived in London till he was 4 years of age, and was then sent into the country, but he could not be taught anything; he could not articulate, nor walk, nor feed himself, and was regarded as unimprovable.

When about 10 years of age he is described as having a remarkably small head, and a large face; he had a fine set of teeth, large eyes, prominent nose, receding forehead, and features resembling those of the male Aztec. His hands and arms were perfectly formed. He often put his hands into his mouth, like an infant; he was invariably fed, cleaned, and dressed by others. He smiled and cried; he could not talk, but uttered inarticulate sounds. Even at this age he was unable to walk, or even stand; and though he grew taller and stouter, he never gained strength to move about, but sat all day in a chair with a rail in front, to prevent him from sliding out. At the age of $10\frac{1}{2}$ years he weighed, with his clothes, 37 lbs., the weight of the clothes being 3 lbs. $8\frac{1}{4}$ oz. At the age of 11 he is said to have known persons, plucking at their garments, looking up in their face, laughing, and clapping his hands. By wriggling his chair about he contrived to move it a little way from its place; still he required to be dressed and fed, and could not handle anything. Subsequently he became irritable, fretful, and noisy, crying much, and striking the sides of his chair or bedstead. He never manifested any further signs of intelligence, emotion, or will, or any power of articulate speech. It is said that the head did not grow larger during the last two years of his life. He died at the age of 12 from spinal abscess, followed by abscess in the lung. At his death the body measured, from vertex to sole, $39\frac{1}{2}$ inches. Various measurements of the body and cranium, with other particulars, will be found in a paper in the 'Anthropological Review'†. His incapability of walking was a true accom-

* Dr. DOWN, of Earlswood Asylum, intends to publish an account of this boy's brain.

† For August 1863.

paniment of the idiotic condition, for it existed years antecedently to any spinal affection.

In neither the female nor the male idiot was there found any diseased state of the *substance* of the brain.

b. *Weights of the Idiots' Encephala and their parts.*

The recent brain of the female idiot, after removal of the membranes, weighed 10 oz. 5 grs. In the idiot boy the brain, with the membranes, weighed $8\frac{1}{2}$ oz. The normal weight of the female brain at 42 years of age would be about 42 oz., and that of a boy 12 years of age about 44 oz. Idiots' brains have already been noticed weighing 20·25 oz. (TODD *), 19·88 oz. (TIEDEMANN †), 13·125 oz. (OWEN ‡); and the lightest brain, not altered by disease, previously recorded (THEILE's case), weighed 10·6 oz.§

The weight of the body of the female idiot, seeing that she was of good proportions, about 5 feet high, but deficient in about 2 lbs. of brain, may be assumed to have been about 88 lbs. The idiot boy, at $10\frac{1}{2}$ years old, weighed $33\frac{1}{2}$ lbs. At his death, 19 months afterwards, his stature having increased though his growth was slow, for his ultimate height was only $39\frac{1}{2}$ inches, his weight, independent of the effects of disease, may be taken at 36 lbs.

In the idiot woman the weight of the brain, after preservation in spirit for many years, was found by Mr. GORE to be $7\frac{1}{2}$ oz., but, as subsequently weighed by myself, it proved to be 7 oz. 102 grs., or 7·23 oz. This weight was made up as follows:—cerebrum, 5·52 oz.; cerebellum, 1·41 oz.; pons and medulla oblongata, ·3 oz. Maintaining similar proportions for the several parts of the recent brain, which weighed 10 oz. 5 grs., the recent cerebrum would weigh 7·63 oz.; the cerebellum, 1·95 oz.; and the pons and medulla oblongata ·42 oz.

In the idiot boy the preserved brain weighed 5·1 oz., which total weight was thus composed:—cerebrum, 3·51 oz.; cerebellum, 1·35 oz.; pons and medulla oblongata, ·24 oz. Taking 8·5 as the ascertained weight of the recent brain, the recent cerebrum would weigh 5·85 oz.; the cerebellum, 2·25 oz.; and the pons and medulla oblongata ·4 oz.

The average weight of the cerebrum, cerebellum, and pons with the medulla oblongata, observed by Dr. BOYD in 94 females between the ages of 40 and 50, was 37·12 oz., 4·69 oz., and ·89 oz.; whereas the average weight of the same parts in 22 males between the ages of 7 and 14 was 40·36 oz., 4·84 oz., and ·76 oz.

Assuming 90 lbs. to be the weight of a healthy female 5 feet high, between 40 and 50 years of age, and 88 lbs. to have been the weight of the idiot woman; and again, taking 42 lbs. to be the weight of a healthy boy between 7 and 14 years of age, and 36 lbs. as the weight of the idiot boy, we have the following results:—

* Cyclop. Anat. and Phys. vol. iii. p. 719.

† Philosophical Transactions, vol. cxxvi. 1836, p. 502.

‡ Trans. Zool. Soc. vol. i. p. 343.

§ WAGNER's Vorstudien, *ut supra*, Th. 2.

	Female (Boyd) 40 to 50 years.	Idiot woman.	Boy (Boyd) 7 to 14 years.	Idiot boy.
Encephalon to body	1 to 33	1 to 140	1 to 14	1 to 67
Cerebrum to body	1 to 38	1 to 184	1 to 16	1 to 98
Cerebellum to body	1 to 307	1 to 722	1 to 140	1 to 256
Cerebrum to cerebellum ...	7·9 to 1	3·9 to 1	8·3 to 1	2·6 to 1

The relative amount of brain to body in both idiots, 1 to 140 in the woman, instead of 1 to 33, as in the healthy female at the same age, and 1 to 67 in the boy, instead of 1 to 14, is very small. Absolutely, as we have seen, the idiot boy had a brain smaller than the idiot woman's, in the ratio of 8·5 to 10; but his brain was more than twice as large, in proportion to his body, as that of the idiot woman was to hers. It must be observed, however, as is well shown in the Table, that the ratio of brain to body is far greater in the growing individual than in the adult; and, allowing for that, the proportion of brain to body in both idiots was somewhat less than one-fourth of what it would have been at corresponding ages in health. In neither case is the ratio between the idiotic and the healthy condition exactly as 1 to 4, being in the idiot woman 1 to 4·24, and in the idiot boy 1 to 4·78; so that, thus tested, instead of the boy's brain being twice as much developed as the woman's, the woman's was comparatively a little larger than the boy's.

Again, the idiot boy's cerebrum, absolutely smaller than the woman's in the ratio of 5·85 to 7·63, was, as compared with the idiot woman's, about twice the proportionate weight in reference to the body, the one being equal to $\frac{1}{98}$th part by weight of the body, the other being only $\frac{1}{184}$th. But if the two idiots be compared with the healthy condition in persons of corresponding age, then they are nearly equal in this respect; for, thus studied, the idiot woman's cerebrum is about 1 to 5 as compared with the ordinary proportion to the body, and the idiot boy's about 1 to 6, the actual ratios being, in the idiot woman's case as 1 to 4·84, in the boy's as 1 to 6·12. Hence a greater superiority is manifested, as regards the cerebrum, in the woman, than existed in reference to the entire encephalon. In her the cerebellum, as we shall next show, was below its due size.

Thus, the idiot boy's cerebellum, absolutely larger than the woman's in the ratio of 2·25 to 1·95, is, in proportion to his body, three times as large as the idiot woman's to her body—the boy's being $\frac{1}{256}$th part of his body, and the woman's only $\frac{1}{722}$nd part of hers; but, again allowing for the normal differences in the proportions between the cerebellum and the body at different ages and in the two sexes, this extreme inferiority of the idiot woman is somewhat, though not entirely redressed. For in the idiot woman the ratio of the cerebellum to the body, as compared with the healthy ratio, is nearly as 1 to 2 (actually 1 to 2·35), whilst in the idiot boy it is rather less than 1 to 2 (actually 1 to 1·8). In other words, the woman's cerebellum reached $\frac{10}{23}$rds of the natural standard, and the boy's $\frac{10}{18}$ths. Thus the idiot boy had not merely a cerebellum larger,

in proportion to his body, than the idiot woman, *i. e.* in the ratio of 3 to 1, but also larger in proportion to the healthy standard at the same age—the ratio in his favour then being, however, only as 4 to 3.

Lastly, as shown in the Table, in the idiot woman the ratio between the cerebrum and cerebellum is 3·9 to 1, in the idiot boy 2·6 to 1. Hence the idiot woman's cerebrum, in proportion to the cerebellum, is superior to the idiot boy's in the ratio of 3 to 2. But comparing these abnormal proportions with those observable in healthy persons of corresponding sex and age, the proportion of cerebrum to cerebellum in the idiot woman is to the natural proportion as 1 to 2, and in the idiot boy as about 1 to 3. In other words, the cerebrum in the woman, when compared with the cerebellum, has half the normal proportion, whilst in the boy it has only one-third. These figures show, not merely the exceedingly small relative size of the cerebrum in both idiots, but a marked superiority on the side of the woman; for whether we compare the actual ratios between the cerebrum and cerebellum in the woman and in the boy, or whether we contrast their respective ratios with the healthy standards, the superiority of the female's cerebrum, as compared with her cerebellum, is to the boy's, in either case, as 3 to 2.

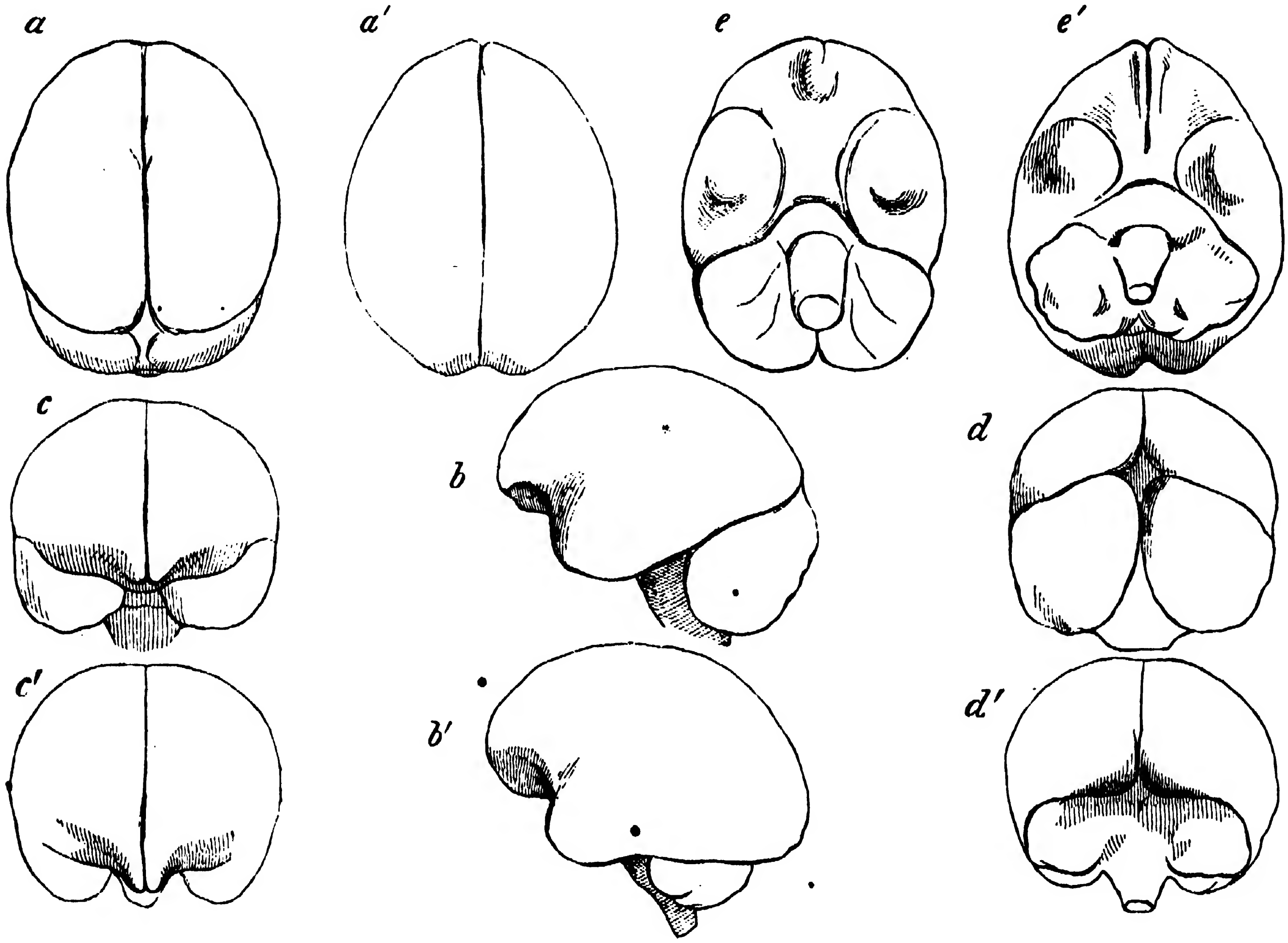
The general conclusions may thus be stated. These idiots fell remarkably short in both cerebral and cerebellar mass: in each the deficiency in cerebral mass was greater than in cerebellar: the idiot boy had more cerebellum than the idiot woman: the idiot woman had more cerebrum than the idiot boy.

c. *The general Form, Dimensions, and relative Positions of the Parts of the Idiots' Encephala.*

Judging from the intracranial cast, the general form of the cerebrum in the idiot woman, as compared with the normal human cerebrum, is, when seen from above, a short oval rather than a long ovoid,—the greatest transverse diameter being about the middle of the mass, and having a ratio to the length of 1 to 1·14 instead of 1 to 1·3. Far from being concealed, the cerebellum projects largely behind the cerebrum, and thus gives a long figure to the whole encephalon. Seen laterally, the entire brain has a low, contracted, globular outline, except of course below. The cerebellum appears to form about a fourth part of the mass, and projects beyond the cerebrum ·35 of an inch. In the base view, the relative preponderance of the cerebellum is again the most striking feature. The temporal portions of the cerebrum appear full and prominent; no part of the parietal region is seen on either side of them, and no part of the occipital either behind or at the sides of the cerebellum. The frontal region is singularly short, narrow, and pointed, instead of square, in front. The orbital surfaces are much excavated, the beak-like prominence of their median portions is well marked, and a slightly obtuse angle is formed by the meeting of their planes in the middle line.

In size and general form, the encephalon of the idiot woman, as represented by the intracranial cast, resembles at first sight that of the young Chimpanzee; but a nearer

examination reveals great differences in every aspect, as is illustrated in the subjoined outline figures.



a. Upper view of the intracranial cast of the cranium of the Idiot Woman.
a′. Ditto of a young Chimpanzee. *b, b′*. Left sides of the same two casts.
c, c′. Front views of the same. *d, d′*. Back views of the same. *e, e′*. Base views of the same.

N.B. All these figures are reduced to one-third of the proper linear dimensions.

Viewed from above (*a*), the general mass is more nearly oval, its greatest transverse diameter being opposite its centre, whilst in the Chimpanzee (*a′*) it is further back. The frontal region is not so pointed. The cerebrum does not project beyond the cerebellum, as in the Ape. Seen laterally (*b, b′*), when the idiot's brain is placed in its natural position, the base line of the encephalon slants obliquely upwards and forwards, whilst the Ape's is nearly horizontal. The vertex in the idiot is turned backwards and upwards, instead of directly upwards; the frontal lobe has a smaller beak-like projection, and in its general mass is deeper from its orbital margin upwards. The temporal lobe is much larger in all directions than in the Ape; the parietal lobe is less prominent; the occipital lobe is shorter. The cerebellum, in this view, appears very much larger than in the Ape, seeming to occupy an area more than twice as great. In front (*c, c′*), the general resemblance between the two casts is the most striking,—the differences

being, on the part of the idiot's brain, a slightly more elevated and broader frontal region; a shorter beak-like process; less excavated orbital surfaces, so that the angle formed by their meeting-point in the middle line is more obtuse; and larger, less incurved, and more widely separated temporal lobes. On the posterior aspect (*d*, *d'*) the casts, on the other hand, are very different,—the cerebrum of the idiot being of less width in the parietal region; attenuated in length, width, and depth in the occipital region; and having the cerebellum projecting at each side, and so large as to appear to tilt the cerebrum upwards, and to occupy in this aspect an area equal to one-third of the whole encephalon; whilst in the Ape the cerebellum is overhung by the cerebrum, and forms a mass not more than one-fourth of the visible part of the encephalon. Seen on the base (*e*, *e'*), the oval shape of the entire mass, the greater width and flatness of the frontal lobes, the size of and width between the temporal lobes, and the complete concealment of the posterior lobes of the brain by the voluminous cerebellum, distinguish the idiot's from the Ape's brain.

The preceding description, and the tabulated measurements of the brains, given at the end of this paper, show that in the idiot woman the temporal regions manifest the greatest relative size, whilst the parietal, occipital, and frontal are very small; whereas in the Chimpanzee's cerebrum the occipital lobes have a larger relative development; the frontal lobes stand next, whilst the temporo-parietal are defective.

The cerebrum of the idiot boy, as seen from above, the only view of which we have an intracranial cast, differs from that of the idiot woman in being at once narrower and somewhat angular in its outlines. The frontal region is more pointed (indeed, singularly so), the occipital region flatter, and the parietal regions longer and more compressed. The widest part corresponds with the centres of the parietal regions, and is somewhat behind the middle of the mass. The ratio between the width and length of the cerebrum is 1 to 1·23; so that the idiot boy's cerebrum is longer, in proportion to its width, than the idiot woman's—not from any actual superiority as to length, but rather owing to a deficiency in width of the whole cerebrum. As in the idiot woman, the cerebellum is not covered by the cerebrum behind; but probably it was not so much exposed. In shape the idiot boy's brain is so long and narrow as not to be comparable with the Ape's.

In the idiot woman the forms of the convolutions are scarcely traceable at any part of the intracranial cast, excepting some slight undulations about the frontal region. In the idiot boy they are remarkably distinct on the parietal regions, whilst the frontal and occipital regions are perfectly smooth. GRATIOLET regards this marking of the cranium by the convolutions as a sign of inferiority or degradation. This would appear to be, to a certain extent, an individual character, as it is not noticeable in the idiot woman.

d. *The Fissures, Lobes, and Convolutions of the Idiots' Cerebra.*

The Fissures.—The fissure of SYLVIUS (Plates XXI. & XXII. figs. 12 & 16, *e–e*) is not only absolutely, but relatively short, measuring 1·3 of an inch in the idiot woman, and 1·1 in the idiot boy. In both brains it is comparatively shallow. In the idiot woman its direction is nearly vertical; in the idiot boy it is more oblique, being pressed back by the large parietal lobe. Its posterior margin is well defined; but the anterior margin in the idiot boy, on the left side, is interrupted, and gives off a long branch towards the vertex. At the entrance of the fissure in the idiot woman is seen a slight elongated ridge, which runs transversely inwards and joins the feebly developed eminence (C) which represents the island of REIL; in the idiot boy this ridge is very narrow, and there is scarcely any insular eminence. In the idiot woman its upper end is simple on the left side, slightly bifurcated on the right; in the idiot boy it is more deeply bifurcated on the two sides.

The fissure of ROLANDO (Plates XXI. & XXII. figs. 10, 12, 14, & 16, *d–d*), in both these brains, is better marked on the right hemisphere. In the female idiot, on that side, it forms a simple sulcus, commencing, by a slight curve, in front of the Sylvian fissure, and then running, less obliquely than usual, backwards towards the longitudinal fissure. On the left side, the symmetry of this fissure is interfered with, in a remarkable manner, in both brains, by the upward intrusion of a triangular convolution, which appears to be the rudiment of the intended perfect anterior border of the fissure. In the idiot boy, on the right side, the anterior margin of this fissure is also irregular.

It may be noticed, then, thus early, in our examination of the cerebral surface of the idiots' brains, that they are at once distinguishable from the quadrumanous brains, and assert, even in their imperfect condition, their human character by an absence of symmetry in primary fissures and convolutions, so nearly symmetrical in the highest Apes.

The external perpendicular fissure (Plates XXI. & XXII. figs. 10 & 14, *h*) can be traced, in the case of the idiot woman, on each side for ·75 of an inch on to the surface of the hemisphere, defining accurately the parietal from the occipital lobe. It then divides into two deep sulci, the hinder one being the continuation of the fissure, which is not prolonged over the side of the hemisphere. In the idiot boy the external perpendicular fissure cannot be traced so far outwards from the longitudinal fissure, being sooner interrupted by the connecting convolutions.

The great parallel fissure (Plates XXI. & XXII. figs. 12 & 16, *f–f*) of the temporal lobe is well marked in the idiot woman's brain, running at first nearly vertically, and then backwards on the occipital lobe. It is of unequal extent on the two sides of the brain, reaching, on the right side, completely to the posterior border of the occipital lobe. It presents a curious wavy course. In the idiot boy this fissure is not quite so extensive, and ends, on both sides, further forward on the occipital lobe; on the left side it presents a simpler outline than on the right.

On the internal surface of the hemispheres, *the calloso-marginal or fronto-parietal*

fissure (Plates XXI. & XXII. figs. 13 & 17, *i–i*), in the idiot woman, curves as usual around the corpus callosum, but sooner reaches the upper border of the hemisphere, a little short of the hinder margin of that commissure. It is interrupted, as usual, by a convolution (*) just above the front of the corpus callosum. In the idiot boy's brain this fissure has a similar course, and is also interrupted (*) immediately in front of the corpus callosum; but it reaches the upper border of the hemisphere opposite the hinder end of the corpus callosum. This fissure is nearly symmetrical, in both idiots, on the two sides.

The internal perpendicular fissure (*k*) in the idiot woman is short and simple, and inclines backwards from a point about half an inch behind the corpus callosum; it joins the fissure of the hippocampi below, but a rudimentary connecting convolution exists in this situation. In the idiot boy this fissure has precisely the same arrangement.

The fissure of the hippocampi (Plates XXI. & XXII. figs. 13 & 17, *l–l*, *m*), in both brains, is less horizontal than usual. In the idiot woman its calcarine portion (fig. 13, *l*) terminates on the very point of the occipital lobe, in an open notch. On the right hemisphere, its anterior portion passes, as usual, a short distance beneath the cerebral peduncle; but on the left side it curves outwards on the under surface of the temporal lobe, and joins a deep sulcus which represents an irregular collateral fissure. In the idiot boy, on the left side, the fissure of the hippocampi is simple in outline and oblique in direction, reaches the tip of the occipital lobe, and extends forwards to the side of the cerebral peduncle. On the right side this fissure is represented by two deep parallel sulci (fig. 17, *l–l*), separated by a thin ridge (26) of convolutional substance. A further development of the convolutions above and below would have concealed this ridge, and left a single fissure.

The *collateral fissure* (Plate XXII. fig. 17, *n–n*) is normal but simple in both idiots, except, as above alluded to, on the left hemisphere in the idiot woman.

On comparing the fissures of the idiots' brains with those of the healthy human brain on the one hand, and those of the Chimpanzee on the other, the following points deserve notice. The Sylvian fissure is both shorter and much more vertical in the idiots' than in either the human or quadrumanous brain, owing evidently to the defective development of the fronto-parietal region of the cerebrum in its ordinary backward direction, and to the disproportionate size of the temporal region. In respect to the former region, the idiots' brains manifest a marked inferiority even to the Ape; for, whilst the masses of brain seen on the lateral aspect in the Chimpanzee, in front of and behind the Sylvian fissure, appear nearly equal, or even show a preponderance in front, in the idiots' brains the quantity in front is only about half the quantity behind. In the healthy human brain, the preponderance is decidedly in the fronto-parietal region. The fissure of Rolando, so complex and zigzag in the healthy human brain, is in the idiots' brains, even on the side where it is most clearly traceable, a simple oblique sulcus, a little curved at its outer end, where, like the Sylvian fissure, and for the same reason, it is

more vertical than in the healthy cerebrum. The angle formed by the two fissures of ROLANDO posteriorly is a little more acute than in the perfect brain, owing to the narrownes of the brain in front of them. These fissures are far more simple in the idiots than in the highest Apes.

The external perpendicular fissure is traceable further on the upper surface of the cerebrum than in the perfect state, but cannot be followed at all on the lateral aspects of the hemispheres as in the quadrumanous brain.

Assuming the total length of each cerebrum to be 100, the relative lengths of the three regions, in front of the fissure of ROLANDO, between it and the external perpendicular fissure, and behind that fissure, when seen from above are in the preserved brain of the idiot woman about 46, 30, and 24; in the idiot boy 38, 34, and 28. In the healthy brain, the proportionate dimensions of these regions are 54, 23, 23. Measured longitudinally over the vertex in the cranial casts, the same regions occupy the following relative spaces: in the idiot woman, 46, 29, 24; in the idiot boy, 42, 32, 26; in the healthy brain, 54, 23, 23. In both idiots the frontal region is therefore strikingly defective, the parietal region is proportionally increased, whilst the occipital region exceeds somewhat the healthy ratio. The frontal region is larger in the idiot woman, the parietal and occipital regions are larger in the boy. The preponderance of the parietal region in the boy is very remarkable. In the Chimpanzee, the corresponding spaces are 46, 28, and 26; so that the idiot woman presents nearly similar proportions; whilst in the idiot boy the frontal region still exhibits a marked deficiency; whilst the parietal region is in exactly corresponding excess, and the occipital region equal.

The parallel fissure is at first more vertical, and extends further back than in the perfect brain. The internal perpendicular fissure, short and simple in its course, still approaches the human type in its inclination backwards, contrasting very remarkably with its vertical direction in the Ape. The fissure of the hippocampi is less horizontal than in the perfect brain, slanting upwards behind, in accordance with the want of depth in the occipital lobe—in this respect approaching the quadrumanous character. In the idiot woman, on the left side, this fissure is anomalous in its junction with the collateral fissure, which is elsewhere regular.

The Lobes.—Considered as defined by the several fissures, and compared with the perfect brain, the frontal lobes (Plates XXI. & XXII. F), in both idiots, are remarkably contracted both in length, in width, and in vertical height, being very short, pointed, and shallow, especially in the idiot boy. The parietal lobe (P) is defective in the antero-posterior direction in the idiot woman, and in the transverse direction in the idiot boy; whilst it is comparatively long in the boy, and wide in the woman. The occipital lobe (O) is much and equally contracted in the two, especially in its vertical measurement. The temporal lobes (T) are larger proportionally than any other part of the cerebrum, and are fuller and rounder in the idiot woman than in the idiot boy. The central lobe, or island of REIL (C), lies on the surface, but is slightly developed in the idiot woman, and scarcely recognizable as an eminence in the idiot boy. As regards

mass, the temporo-parietal region predominates in both brains, whilst the occipital and, especially, the frontal are defective.

Contrasting the idiots' brains with the Orang-outang's or the Chimpanzee's brain, the frontal lobe is still small, especially in the idiot boy; the parietal lobe occupies, on the whole, proportionally a larger space, whilst the occipital lobe occupies less. The temporal lobes are relatively much larger and fuller, but shorter than in the Chimpanzee's brain. The central lobe is much less developed; for in the Chimpanzee this part, completely concealed as in Man, has five radiating convolutions; whilst in the idiot woman's brain it consists only of a slight smooth eminence; and in the idiot boy no very distinct elevation of the surface can be detected.

The Convolutions.—The *orbital convolutions* (Plates XXI. & XXII. figs. 11 & 15, ı′–ı′′′′) are remarkably simple, being only slightly marked off from one another, and very smooth. In the idiot woman's brain (Plate XXI. fig. 11), the sulci which lodged the olfactory nerves are very short and shallow, especially that (*o*) on the left hemisphere; they are represented in the idiot boy's brain (Plate XXII. fig. 15) by a slight linear depression only on the right hemisphere, and a still smaller depression on the left. The deep triradiate sulci which cut up the orbital surfaces in the perfect cerebrum in so complex a manner, are much simplified in both the idiots' brains; they are more developed in the woman's than in the boy's brain, and in both cases are more developed on the right than on the left hemisphere. On the right hemisphere of the female idiot brain alone is this sulcus distinctly triradiate; on the left hemisphere it is more irregular. In the idiot boy it is represented on the right by a shallow longitudinal sulcus, on the left by a slightly curved one. Accordingly the orbital surfaces are much less complex and smoother even than in the Ape; at the same time, with all their simplicity, the want of symmetry of the two sides is remarkable.

The three rows of *frontal convolutions* (Plates XXI. & XXII. figs. 10, 12, 14 & 16) can be distinguished on each side, above the orbital border, in both the idiots' brains. In the idiot woman (figs. 10 & 12), the *inferior row* (ı) is represented by a short, simple, horizontal convolution, which speedily joins the anterior ascending convolution behind and the middle frontal row in front. This *middle row* (₂) consists likewise of a single convolution bent once outwards. The *upper row* (₃), as usual, larger and more complex than the others, occupies half the frontal lobe, but is still remarkably simple in form. The upper frontal row, as ordinarily, blends with the upper end of the anterior ascending parietal convolution; the middle row is joined to that convolution near its lower end, by the same connecting ridge as the third row. In the idiot boy (figs. 14 & 16) the *three rows* can be distinguished, but are still more simple in their form and direction. The three rows are more equally developed, the inferior row being relatively well pronounced, giving a proportional breadth and squareness to that part only of the frontal lobe, not noticeable in the female brain. Easily discriminated in the idiots' brains, both as to their position and connexions, chiefly owing to their great simplicity, these frontal convolutions are singularly short and defective as compared with their wonder-

fully tortuous and complex character in the perfect brain. In comparison even with the Orang's (Plate XXIII. fig. 20) or Chimpanzee's brain, they are far more simple. They are rather better developed in the idiot woman than in the boy.

The *two ascending parietal convolutions* (Plates XXI. & XXII. figs. 10, 12, 14 & 16, 4—4' & 5—5) commence, in both idiots, very far forward on the cerebrum, just above the entrance to the Sylvian fissure, instead of, as in the perfect human brain and in the higher Apes, a little in front of the middle of that fissure; they thus resemble the condition found in Cercopithecus and other similar Quadrumana. In the perfect human brain these ascending convolutions, as pointed out by GRATIOLET, are interposed between an anterior and a posterior set of longitudinal ones, which occupy the rest of the cerebrum—the portions in front and behind the slanting line of the ascending convolutions being nearly equal, the occipital region preponderating slightly. On the other hand, in the idiots' brains the portion in front of these ascending convolutions, as seen either laterally or from above, is by far the smaller; indeed it is not a *fourth* as large as that behind. In the Orang and Chimpanzee it is about one-third. (Compare Plates XX. & XXIII. figs. 7, 18, 19 & 20.)

As usual, these ascending parietal convolutions arise from the supramarginal convolution, which unites them below the outer end of the fissure of ROLANDO. In the idiot woman, on the right side, where the fissure of ROLANDO is a simple sulcus, the two ascending parietal convolutions are also simple, forming two oblique ridges instead of the zigzag bands seen in the perfect human brain. On the left side, the anterior convolution is represented partly by the intrusive triangular mass (4') before spoken of, as if by an arrest of development. In the idiot boy it is curious that the same condition exists on the same side of the cerebrum, whilst on the right side the anterior ascending convolution is developing itself, as it were, into its more perfect but still simple form of an even oblique ridge. The symmetry between the two sides is thus again effectually destroyed in both brains. In both the idiots' brains, as usual, the *anterior ascending parietal convolution* (4—4') joins, or would have joined, at its upper end, the upper frontal row, whilst the *posterior convolution* (5—5) expands into its so-called *lobule* (5'—5'), which is proportionally large, has its customary lozenge-shape, and extends backwards to the external perpendicular fissure (*h*). In the idiot woman (Plate XXI. fig. 10), on the left side, this lobule is notched by a single deep sulcus running from the longitudinal fissure; on the right side it has this sulcus placed further forwards, and another slight triradiate one besides, but its surface is remarkably smooth in comparison with its complex form in the perfect brain, or even in the Orang (Plate XXIII. fig. 20) or Chimpanzee. In the idiot boy's brain (Plate XXII. fig. 14) the lobule of the posterior ascending parietal convolution is on both sides proportionally larger, and slightly more complex in form, in accordance with the greater development of the parietal region in the idiot boy. In both the idiots' brains the outer border and angles of the lobule are distinctly defined on the left hemisphere, whilst on the right hemisphere it blends at the outer border with the neighbouring convolutions—that is to say, with the lobule of the supramarginal convo-

lution (A), the bent convolution (6), and the second connecting convolution (β). This better definition of the lobule of the left side, it is worthy of remark, is not uncommon in perfect European brains. In the Bushwoman's brain, too (Plate XVII. fig. 1), described in the first part of this paper, the lobule in question is better defined on the left than on the right hemisphere. In the brains of the higher Apes (Plate XXIII. fig. 20) the parts are more nearly symmetrical; so that in this region again the idiots' brains manifest the human want of symmetry.

In both the idiots' brains the Sylvian fissure is so short that the *supramarginal convolution* (4''–5''), its so-called *lobule* (A), and the *bent convolution* (6) are all three necessarily very small, and, indeed, are represented only by a simple convolutional band, turning round the front and upper end of the nearly vertical or slightly oblique Sylvian fissure. Nothing can show more clearly the fundamental unity of these parts, especially of the supramarginal convolution and its so-called lobule; whilst the bent convolution is a sort of connecting convolution between the supramarginal and one of the temporal convolutions. Thus understood, the very short supramarginal convolution ultimately joins, in both idiots, the lower frontal row anteriorly. In the idiot woman it sends downwards and inwards a short process to the transverse smooth eminence, lying partly exposed at the entrance of the Sylvian fissure, which expands into the *rudimentary central lobe, or island of* REIL (C). In the idiot boy this process is a mere ridge, and the eminence itself is not distinctly recognizable. The so-called *convolutions* of the island of REIL, or central lobe, are absent, even in the idiot woman, whilst in the idiot boy a plain indistinctly elevated surface occupies its usual situation.

The supramarginal lobule (A), if defined to be a largely developed part of the convolution so named, overhanging the Sylvian fissure and helping to depress its hinder end, is certainly absent in both the idiots' brains; but it is doubtless really represented by the part of the supramarginal convolution just in front of the hindmost bifurcation of the upper end of the Sylvian fissure. The structure of this part of the idiots' brains is exceedingly simple, as simple indeed as in the Cercopithecus, far simpler than in the higher Apes (Plate XXIII. fig. 20). It is smaller and more simple in the idiot woman than in the idiot boy, is larger in both on the right than on the left side, and is largest on the right side in the idiot boy. In the idiot woman the supramarginal lobule is connected, on the right side, with the lobule of the posterior ascending parietal convolution, and also with the second external connecting convolution; on the left side only with the latter. In the idiot boy it is, on both sides, connected only with the latter, but by a larger and more tortuous band.

The bent convolution (Plates XXI. & XXII. figs. 10, 12, 14 & 16, 6), turning, as usual, behind the summit of the Sylvian fissure, is connected posteriorly, in both the idiots' brains, with the second connecting convolution, and joins below the upper external temporal or inframarginal convolution (Plates XXI. & XXII. figs. 12 & 14, 7), instead of being separated from that by a secondary sulcus and running between it and the middle external temporal (8); this peculiarity is found only in the simplest quadrumanous brains, and

indicates a very great degree of simplicity in the idiots' brains. The bent convolution itself, also, is very simple and symmetrical. The two correspond, in both brains, to the most prominent parts of the parietal regions, and therefore to the widest part of the cerebrum, lying beneath the parietal eminences of the skull; whereas in the perfect human brain it is the large and peculiar supramarginal lobule which occupies this post.

The temporal convolutions (Plates XXI. & XXII. figs. 12 & 16) in both the idiots' brains are simple in form, but large, and, indeed, in the female enormously developed. On both sides in the latter (Plate XXI. fig. 12) the *middle temporal* (8–8) is the largest, the *upper temporal* or *inframarginal* (7–7) is the next in size, whilst the *lower temporal* (9–9) is rather more moderate in size. In the idiot boy (Plate XXII. fig. 16), on both sides, the middle temporal is still the largest, but the difference is not so marked. In both brains there is a want of symmetry in the convolutions of the two sides, those of the right side being bounded by more tortuous furrows, and having a few secondary sulci; whilst on the left side they are nearly smooth. They are all continued backwards into the diminutive occipital lobe by simple, but, in the woman, serpentine connecting convolutions.

The occipital lobe is so small in both idiots, and so slightly marked by rudimentary sulci, that its three ordinary stages or rows (Plates XXI. & XXII. figs. 12, 14 & 16, 10, 11, 12) can scarcely be separately recognized; but it may be described as consisting of a shallow, smooth edge of cerebral substance, so blended on the outer side with the second, third, and lowest external connecting convolutions, and on its under side with the internal temporal, as almost to lose its identity in those aspects, and to appear like a mere narrow continuation backwards of the temporal lobe itself. It is defined only on its inner border, and for a short distance on its upper surface.

The upper external connecting convolution in the idiot woman (Plate XXI. fig. 10, α) does not completely bridge over the external perpendicular fissure on either side, but is more nearly superficial throughout on the right hemisphere; in the idiot boy (Plate XXII. fig. 14, α) this convolution dips downwards into the internal perpendicular fissure, becoming partly concealed on both sides. In the perfect brain (Plate XX. fig. 7) it is quite superficial throughout. In the female idiot, on the left side, *the second connecting convolution* (Plate XXI. fig. 10, β–β) is massive and double, one part ending in the bent convolution, and the other in the upper temporal; the *third* (γ) ends in the middle temporal, and the *lowest* one (δ) both in that and the lower temporal; on the right side the second, also double and very tortuous, runs by its upper part into the parietal lobule and the supramarginal lobule, and by its lower part into the upper temporal, whilst the third and fourth are blended, and end in the middle and lower temporal. In the idiot boy (Plate XXII. fig. 14), on the left side, the second connecting convolution (β–β), very broad and tortuous, runs into the bent convolution, the upper temporal, and the middle temporal; the third (γ) and fourth (δ) connecting convolutions join the middle and lower temporal; on the right side they are more tortuous, and serve to connect the same parts, the second one also joining the parietal lobule. In both idiots the second (β), third,

and lowest external connecting convolutions, though broad and superficial (the second being especially massive), are so simple, in comparison with their singular complexity and extraordinary tortuosity in the perfect brain, as to form mere bands between the temporal and occipital lobes. Nevertheless this part of the brain, more deeply furrowed in the woman than in the boy, but larger from before backwards in the boy than in the woman, is relatively well developed, and has perfectly human characters; for all four connecting convolutions exist, instead of there being defects in one or other of the two upper ones, as is found in the higher Apes. They are also, as seen above, singularly asymmetrical.

On the inner surface of the hemisphere *the marginal convolution* (Plates XXI. & XXII. figs. 13 & 17, $_{17}$–$_{17}$) pursues its usual course, and terminates just behind the upper ends of the two ascending parietal convolutions. Like these convolutions, and owing also to a defective development backwards of the frontal lobe, it does not extend so far back as in the perfect brain, but, becoming very narrow, stops, in the idiot woman, on the left side (Plate XXI. fig. 13) at a point opposite the hinder border of the corpus callosum (*c*), on the right side somewhat short of that point. In the idiot boy (Plate XXII. fig. 17) it is on both sides arrested at a point a little in front of the hinder border of the corpus callosum. In the idiot woman, instead of the numerous radiating secondary sulci which in the perfect state cut up this convolution into little quadrangular lobes, only two or three such sulci exist, passing horizontally forwards in front of the corpus callosum; below the anterior border of the corpus callosum is another rudimentary longitudinal sulcus, and a little depression indicative of a second; above the corpus callosum this marginal convolution is joined, as usual (*), to the convolution of the corpus callosum ($_{18}$–$_{18}$), and then becomes very wide, and faintly marked with a slight depression. In the idiot boy's brain the same description would suffice. In both brains its forms and subdivisions, simple as they are, imitate closely its general features in the perfect state, but are wanting in complexity. As compared with its condition in the Apes, it is less uniform in width, and far less frequently subdivided by secondary sulci.

The convolution of the corpus callosum (Plates XXI. & XXII. figs. 13 & 17, $_{18}$–$_{18}$) occupies its customary position around that commissure, but differs from its condition in the perfect brain by the absolute smoothness of its surface and the absence of the peculiar crest-like upper margin posteriorly. It is equally smooth in both the idiots' brains. Its surface is more complex in the higher Apes. In the idiots' brains the marginal convolution ($_{17}$) greatly exceeds in width the subjacent convolution of the corpus callosum ($_{18}$), especially in the region in front of and below the corpus callosum—a character well marked also in the perfect European brain—whereas in the Ape the proportionate space occupied by the two convolutions is nearly equal. The connecting bridge of convolutional substance (*) passing from one to the other in both the idiots' brains, on both hemispheres, is usually present also in the human brain, but not in the brains of the higher Apes.

The quadrilateral lobule in both the idiots' brains (Plates XXI. & XXII. figs. 13

& 17, $_{18'}$–$_{18'}$), the prolongation upwards and backwards of the callosal convolution, is, owing to the deficient development of itself and of the portions of the cerebrum in front and behind it, less compressed than in the perfect state, so that it has a roundish and not angular outline, and forms a perfectly smooth, bent or knee-like, and not quadrate lobule, ascending in front of the internal perpendicular fissure (*k*). Though thus rudimentary and smooth, it has the oblique direction backwards characteristic of this part in the human brain, and not the nearly vertical position which it exhibits in the Apes. Indeed the angle which its posterior border, bounding the internal perpendicular fissure, forms in both idiots with a base line passing through the corpus callosum is nearly 145°, *i. e.* greater than in the perfect brain, owing probably to the very scanty development of the occipital lobule behind it.

The triangular occipital lobule (Plates XXI. & XXII. figs. 13 & 17, $_{25}$), like the quadrilateral lobule, is so feebly developed in both the idiots' brains, that it does not appear to fit closely in between the parts in front and behind it, but forms a mere ridge of cerebral substance, widening as it passes from below upwards and backwards to the tip of the occipital lobe, interposed between the internal perpendicular fissure (*k*) and the posterior part (*l*) of the fissure of the hippocampi. This simple, smooth, but slightly flexuous ridge takes the place then of the triangular and complexly convoluted lobule seen in the perfect brain. This condition coincides with the simplicity of the upper external connecting convolution, with the region of which this occipital lobule corresponds. It is far less developed than in the Apes.

On the under surface of the idiots' brains, the convolution of the corpus callosum (Plate XXII. fig. 17, $_{18}$) is, as usual, continued beneath the cerebral peduncle by a ridge of cerebral matter (*) into the middle internal temporal convolution, or uncinate convolution ($_{19}$). This connecting ridge is proportionally wider than in the perfect cerebrum; it is said by Gratiolet to be peculiar to the human brain; it certainly does not exist in every Chimpanzee's brain, though it is met with again in lower Quadrumana.

Of the *internal temporal convolutions*, the *upper*, or *dentate convolution*, is very narrow, and on neither idiot's brain can the fascia dentata be traced. In the idiot woman, however, the parts are much damaged here, and in the idiot boy somewhat injured also. The *middle internal temporal convolution* (Plate XXII. fig. 17, $_{19}$), ending anteriorly in the *unciform lobule* ($_{19'}$), is proportionally well developed. On the right hemisphere of the idiot woman's brain, the unciform termination, or crochet, is well marked; on the left hemisphere this convolution is much narrower, and the crochet scarcely recognizable. In the boy this convolution is very broad on both sides, and the crochet neatly defined, though small. The *lower internal temporal convolution* ($_{9}$–$_{9}$), which is the same as the lower external one, is also well marked. On both sides these two last-named convolutions are proportionally more simple than in the perfect human brain, or even than in the higher Apes. They are continued, as usual, backwards into the occipital lobe, and, owing to the imperfect development of that lobe, seem to extend almost to its tip. They are broader in the idiot boy.

It has already been stated that within the double fissure of the hippocampi, in the right hemisphere, in the idiot boy's brain (Plate XXII. fig. 17, *l*–*l*), there is present a projecting ridge of cerebral substance (*м*), which appears to be the analogue of the *calcarine lobule*, described by Mr. FLOWER in the brains of Cercopithecus, Macacus, and Cebus, but which is absent in the highest and lowest Quadrumana. It is not present on the left side of the idiot boy's brain, nor on either side in the idiot woman's brain. In the perfect human brain, I have sometimes found it as a superficial ridge extending along the posterior two-thirds of the fissure, sometimes as a well-marked concealed ridge; sometimes it is altogether absent. It is continuous backwards with the lowest occipital convolutions.

The *lower internal connecting* convolution (Plate XII. fig. 17, *s*) is feebly represented in both idiots, being, as usual, concealed in the idiot woman, but superficial in the idiot boy; the upper one is absent, as usual, in both idiots.

In the preceding account of the cerebral convolutions in the idiots' brains, constant comparisons have been made between them and the perfect brain, between the brains of the one and the other idiot, and between both and the brains of the higher Apes. Notwithstanding, it is necessary to state some further general conclusions from the facts above recorded.

1. In the first place it is obvious that the idiots' cerebra are not merely diminutive brains possessing every convolution, both primary and secondary, proper to the perfect human cerebrum, each having its natural shape, proportion and position, though on a diminished scale; but, on the contrary, that they are profoundly modified in their convolutional forms, which are not merely smaller in bulk, but are fewer in number, of simpler shape, and different in proportion and position, as compared with those of the perfect cerebrum.

2. Nevertheless all the primary and connecting convolutions belonging to the perfect cerebrum are represented by definite corresponding parts in these brains, mostly by actual convolutional foldings of the cerebral substance of a comparatively more simple kind, but sometimes by scarcely convoluted, or even by entirely smooth though slightly elevated portions of the cerebral substance.

3. The parts which can be easily detected as actual convolutions in the idiots' brains are the three frontal rows, the two ascending parietal convolutions, with the lobule of the posterior one, the supramarginal and bent convolutions, the external and internal temporal convolutions, the marginal and callosal convolutions on the inner surface, with the quadrilateral and occipital lobules, and all the connecting convolutions proper to the human cerebrum. The parts which are less easily distinguished are the orbital convolutions and, especially, the three rows of occipital convolutions. The central lobe, or island of REIL, is distinguishable, as a distinct smooth eminence, in the idiot woman, but only as a smooth indistinctly elevated surface in the idiot boy. In neither does there exist such an expansion of the supramarginal convolution as would form a prominent supramarginal lobule, a part so characteristically human.

On the whole, the temporal convolutions, in both brains, are the boldest and best marked; then the convolutions of the parietal lobes, especially in the idiot boy; next stand the connecting convolutions and frontal rows, and those of the inner surface; afterwards the orbital and occipital convolutions; and lastly the island of REIL.

4. On contrasting the idiots' brains with one another, the convolutions generally are seen to be decidedly more developed in the idiot woman than in the idiot boy—the marked exception being in the parietal region of the latter, where the lobule of the posterior ascending parietal convolution, the supramarginal convolution on the left side, the bent convolution, and the adjacent second external connecting convolution are more fully developed.

5. Agreeably to the opinions already expressed by other anatomists in regard to similar examples, the condition of the cerebra in these two idiots is neither the result of atrophy, nor of a mere arrest of *growth*, but consists essentially in an imperfect evolution of the cerebral hemispheres or their parts, dependent on an arrest of *development* (*agénésie*, *asthénie-génie*) occurring at some stage or other of their metamorphosis from a simpler to a higher form.

6. On comparing the condition of the cerebral convolutions of these brains with the representations of the brains of two fœtuses at about $6\frac{1}{2}$ and 7 months, published by LEURET and GRATIOLET*, it would appear that in both idiots the convolutions are more complex than in the former, but less so than in the latter fœtus. From this, one might hastily suppose that in both idiots the development of the convolutions, and indeed of the entire cerebra, had been arrested in the latter part of the seventh month of intra-uterine life; those of the idiot boy a little earlier than those of the idiot woman.

But on further reflection such a supposition does not appear to be tenable, and it is not supported by facts. It necessarily assumes that, up to a certain period of development, the evolution of all the parts of the cerebrum had been normal in rate and in character; whereas, in the first place, there is nothing at present to show why that rate may not, in such cases as these, be more or less retarded, so that any given stage is attained at a much later period than usual, and the ultimate condition of development be reached perhaps some time after birth; and, in the second place, there is evidence in the brains themselves, of such a disproportionate development of parts as to prove that the normal character of the evolutional changes has been profoundly disturbed at some period or other, by at least one *local* departure from, or interference with the regular mode and order of development.

A comparison of the size of the cerebellum and cerebrum in the idiots' brains, and in the brain of a fœtus at the seventh month, shows most strikingly that the development of the former organ had continued to progress long after the latter had experienced its final arrest; but, what is more essential to the present inquiry, even within the idiots' cerebra themselves there is proof that all the parts are not equally and normally developed.

* *Op. cit.* pl. 30. figs. 1, 2, 3; pl. 31. figs. 1, 2.

In the two fœtal brains represented by LEURET and GRATIOLET, already alluded to, the parts in front of the fissure of ROLANDO, comprising the frontal lobe and the so-called anterior ascending parietal convolution (which latter should, I think, be associated with the frontal region itself), form a far larger proportion of the entire cerebrum than they do in the idiots' brains. In the brain of the human fœtus between the fourth and fifth months (Plate XXIII. figs. 21 & 22), in which the fissure of ROLANDO (*d–d*), the great parallel temporal fissure (*f*), and the perpendicular fissure (*h*) are clearly traceable, the same fact is well illustrated. In fœtal brains at still earlier periods* the same thing is observable, whilst at later periods than the seventh month the parts in front of the fissure of ROLANDO become still longer in proportion to those behind it. Indeed, in the normal course of development, there is no period at which the frontal part seems, as it were, to stand still, or retrograde relatively to the rest of the cerebrum; but after once the fissure of ROLANDO is formed, there is a variable but progressive relative increase of the parts in front of that fissure. It is certain, therefore, that the frontal lobes of the idiots' cerebra are not proportionally developed in comparison with the temporo-parietal regions. The same appears to me to be true likewise of the occipital lobe; but we may confine the argument here to the defective state of the frontal lobes.

Fully to appreciate the importance of the diminutive size of these last-named lobes, we must take into account certain facts to be hereafter stated in detail, regarding the internal structure of the cerebral hemispheres. The corpora striata in the idiots' brains are very small; not merely absolutely, but also relatively to the size of the optic thalami, the ordinary proportions between these two ganglia being actually reversed, the former being usually much larger than the latter, whilst in the idiots' brains they are much smaller. Since in a series of normally developed fœtuses the corpora striata, at all periods, form larger masses than the optic thalami, we have further evidence, within the idiots' brains themselves, of the fact already announced of an irregular and disproportionate development of their parts. There is, indeed, an obvious correspondence between the diminutive size of the corpora striata and that of the frontal lobes; whilst the relatively larger optic thalami are associated with a larger growth of the hinder portion, especially of the temporo-parietal regions.

The conclusions which we would draw from the preceding facts are these:—First. Instead of the idiots' cerebra having been uniformly and normally developed up to a certain date (say the latter part of the seventh month), and having then been subjected to a general cessation of development, they have experienced an inequality or irregularity of evolution in certain of their parts. Secondly. Whilst all parts have been more or less arrested, the frontal and occipital lobes have suffered more than the temporal and parietal. Thirdly. Whilst both the large ganglia at the base of the cerebrum (those cores or nuclei of the cerebral hemispheres, the corpora striata and optic thalami) have participated in this disturbance of the ordinary course and degree of evolution,

* See LEURET, *loc. cit.* pl. 29, from fig. 10; and also TIEDEMANN, Anat. Bildungsgeschichte des Gehirns. Nürnberg, 1816.

the corpora striata have been more especially involved. Fourthly. The original vice of formation, in all probability, affected these two pairs of ganglia primarily; and this entailed, as a necessary consequence, an interrupted, irregular, defective, and perhaps retarded evolution of the convolutions of the hemispheres themselves. Fifthly. The primitive starting-point of the future idiotic condition dates from a period far earlier than that at which all further evolution ceases; and in fact, as regards the optic thalami and, especially, the corpora striata, probably from a very early period of development indeed. This conclusion is obviously more acceptable to the physiologist (because more consistent with the radical deficiency in cerebral power manifested by idiots) than the supposition that the idiotic state should be due to a sudden arrest of a previously normal development at some later period of fœtal life. Sixthly. The anatomical connexion which, by the comparison of these idiots' brains with healthy fœtal brains, has been shown to exist (in human brains, at least) between the development of the corpora striata and the frontal lobes, and the optic thalami and the temporal and parietal lobes, has a considerable general interest, and probably has a physiological significance which may hereafter throw light on the functions of the convolutions of those several parts. Lastly. The deficiency in the corpora striata and the associated frontal lobes becomes particularly interesting when we reflect on the special connexion of those ganglia with the anterior or motor columns of the cord, and on their probable intimate concern in the execution of voluntary movements, *i. e.* in the mechanical expression by the body of those numerous acts which are the outward exponents of that important psychical faculty commonly designated "the will." Now, it is the inadequate performance or entire abrogation of those acts, whether locomotive, manipulative, or articulate, which constitutes one of the most striking characteristics of the idiotic state.

7. It is impossible, in the present state of our knowledge, to determine the interesting question whether some parts of the idiots' cerebra had undergone, after the general arrest of ordinary morphological changes, further local development, as the result of use or ordinary training.

8. There are, however, certain evident grounds for inferring that, after the cessation in these cerebra of all further evolutional changes, they experienced an increase of size, or a mere growth of their several parts. Thus the idiots' cerebra are considerably larger than fœtal cerebra in which the convolutional development is at a similar stage; whilst the individual convolutions themselves, the same in number, are necessarily broader and deeper. Again, from Dr. Boyd's observations, it appears that in a certain number of fœtuses prematurely born, with an average height of 14 inches for males and 13·5 for females, whose brains would about correspond with the idiots' degree of convolutional development, the average weight of the cerebrum in the former was 5·33 oz., and in the latter 4·42 oz.; whereas, as we have seen, the idiot boy's cerebrum weighed 5·85 oz., and the idiot woman's 7·63 oz. The greatest difference is in the case of the woman, who lived to the adult age; whilst the boy, it must be remembered, died at the age of 12.

9. It has been shown that the temporal region preponderates in the idiot woman,

and the parietal in the idiot boy; the frontal lobe is also relatively a little larger in the woman. There can be no doubt also that the emotions, intelligence, and voluntary power of the woman were in advance of those of the boy; but at present it would be premature to attribute too much importance to these probably individual anatomical differences, or to endeavour to associate them with peculiarities of psychological endowment.

10. On contrasting the cerebral convolutions of the two idiots' brains with those of a female and male idiot, each four years of age, represented by LEURET *, there appears a very close and remarkable resemblance between them. There is the same paucity, simplicity, and breadth of the convolutions, the same deficiency in the frontal lobes, though in one of them (the second referred to in the foot-note) to a less degree. The details of the convolutions are also nearly similar; but in some slight particulars they are superior to those of the idiot woman, and especially so to those of the boy. For example, in both, the anterior ascending parietal convolution has passed beyond the stage of an intrusive convolution to that of an oblique smooth ridge of cerebral substance. There are also more numerous secondary sulci in most regions of the cerebrum, and the convolutions themselves are somewhat more tortuous.

11. Lastly, on comparing the convolutions of the idiots' cerebra with those of the Orang and Chimpanzee, they appear, in the human idiots, to be fewer in number than in the Apes, because, although the primary foldings correspond in each, they are individually less complex, broader, and smoother in the former than in the latter. In this respect the idiots' brains are even more simple than the brain of the Gibbon, and approach that of the Baboon (Cynocephalus) and Sapajou (Ateles) †.

As special and interesting results of this general simplicity of the primary convolutions, are the absence, as in the quadrumanous brains, of such a development of the supramarginal convolution as to constitute its so-called lobule, and the partial concealment of the upper external connecting convolution, as well as the imperfect development of the anterior ascending parietal convolution, and the extreme simplicity of the bent convolution. Of these, the non-development of a distinct supramarginal lobule is the most interesting defect, since it indicates the late appearance in the brain of a part whose presence is regarded by GRATIOLET as peculiarly characteristic of Man.

On the other hand, the points of special difference between the idiots' and the quadrumanous brains, both general and particular, are even more numerous. First, as a general difference, there is a remarkable want of symmetry even in these imperfectly developed cerebra, as if already preparations were being made to establish that higher and almost exclusively human character; this point has been so frequently exemplified in the previous descriptions that we may refer to them for abundant illustration of it. Secondly, the special differences, which likewise exhibit the decidedly human character, are the superficial position of all four of the external connecting convolutions; the consequent speedy interruption of the external perpendicular fissure, and complete obliteration of its posterior border or operculum; the concealed position of the lower internal

* *Op. cit.* pl. 24. fig. 4; pl. 32. figs. 1 to 5.

† GRATIOLET, pl. 4. figs. 1 & 2; pl. 9. figs. 1 & 4; pl. 10. figs. 1 & 2.

connecting convolution, and the absence of the upper one; and lastly, the great breadth of the connecting ridge which joins the callosal and uncinate convolutions.

Although, therefore, so defective in developmental detail, these microcephalic cerebra are still human, and differ as much from the Ape's cerebrum, or constitute as little an intermediate step towards it, as any other bodily defect in man is found to differ from a truly quadrumanous form, or manifest a serial approximation to it. Just as in a case of webbed human fingers the digits are still human and not gorilla-like, and just as in the deformity named talipes valgus, though the foot is inverted and the weight of the body is supported on its outer border, still the member is human and not ape-like, so these brains, though simplified by defect, possess characteristics which distinguish them as imperfectly human yet not quadrumanous. The community of plan observable in the brains of all the Primates, including Man himself, necessitates a general conformity to that plan, even in these defective human brains; but the special marks of human divergence from that plan have already been set upon them at some very early, probably at the earliest moment of their development.

e. *Internal Structure of the Idiots' Cerebra and Cerebella. The Commissures, Cavities, Ganglionic Masses, and Laminæ.*

Excluding the great fissures, the depth of the sulci (or, in other words, the prominence of the convolutions) in the idiots' brains is greatest in the lateral temporal region, next in the frontal and parietal regions, and least in the occipital region, where many of the sulci are mere marks or notches. The average depth of the sulci is, in the idiot woman's brain, ·5 inch, in the idiot boy's ·4 inch. In the idiot boy the thickness of the grey matter in the recent brain varied between $\frac{6}{30}$ths and $\frac{3}{30}$ths of an inch; in the preserved brain it varies from $\frac{5}{30}$ths to $\frac{2}{30}$ths of an inch, averaging $\frac{3}{30}$ths. In the idiot woman it ranges between $\frac{6}{30}$ths and $\frac{2}{30}$ths, averaging about $\frac{3}{30}$ths; in both brains it is, as usual, thickest in the frontal and parietal regions, and thinnest in the occipital. Unlike what is found in the Chimpanzee's brain, the quantity of white matter in proportion to the grey is very large, in accordance, as we shall find, with the relatively full development of the transverse commissural system of fibres.

The corpus callosum, in both the idiots' brains, is proportionally shorter though thicker than in the perfect human brain, but it is relatively longer than in the Chimpanzee. In the recent brain of the idiot boy it measures ·75 inch long. Its length, greatest thickness, least thickness, and average thickness in the preserved brain of the idiot woman are 46, 13, 6, and 7 thirtieths of an inch; in the idiot boy 41, 7, 4, and 5 thirtieths of an inch. The sectional area of this part in the idiot woman is about $\frac{322}{900}$ths of an inch, in the idiot boy only $\frac{205}{900}$ths of an inch. Comparing these numbers with the area of the internal surface of one cerebral hemisphere, we find that in the idiot woman the ratio is as 1 to 13·3, in the idiot boy as 1 to 14·5, whereas in the perfect human brain the ratio is 1 to 12·5, and in the Ape as 1 to 28·5; so that in respect of the transverse commissural fibres of the corpus callosum, the idiots' brains, diminutive as they are, are truly human in their structure. The anterior and posterior commissures likewise are

well developed in both brains; the soft commissure is very large in both. Of the longitudinal commissures the fornix is proportionally large; the tænia semicircularis and striæ longitudinales, very distinct in the boy, are also easily traceable in the woman. The septum lucidum and middle ventricle are small in both the idiots' brains, the interval between the fornix and the corpus callosum being narrow. The lateral ventricles in both brains, in accordance with the restricted development of the frontal and occipital regions, are comparatively short and wide cavities. The general direction of the body of the ventricle is not parallel with the median line, but divergent outwards and backwards—a condition owing evidently to the optic thalamus being proportionally large, or at any rate well developed, whereas the corpus striatum is relatively very small, the eminence formed by it within the ventricle being short and narrow, whilst the comparatively large optic thalamus carries the back part of the body of the ventricle outwards. This part of the ventricle in the idiot woman's brain measures 8·5 thirtieths of an inch; the anterior cornu 4, the posterior cornu 8, and the descending cornu 9 thirtieths. In the idiot boy the same parts measure respectively 6, 2·5, 7·5, and 8 thirtieths of an inch; the body of the ventricle is therefore in both cases shorter than in the perfect brain, being shorter in the idiot woman than in the idiot boy, the length of whose parietal region has already been noticed; the anterior cornu is also proportionally short, particularly so in the idiot boy; the posterior cornu is likewise short but wide, directed almost immediately backwards in the idiot boy, but divergent outwards in a remarkable manner in the idiot woman. The thickness of cerebral substance, between the end of the posterior cornu and the hinder edge of the cerebrum, is disproportionally great, being ·4 inch in the idiot woman and ·7 in the idiot boy. In the idiot woman the divergent direction of the point of the cornu makes it impossible to compare its relative proximity to the apex of the posterior lobe with that observed in ordinary brains; in the idiot boy the distance from the point of the cornu to the end of the posterior lobe, viz. ·7 inch, is more than one-fifth of the total length of the brain, whereas the ordinary proportion is about one-eighth. The descending cornu, in correspondence with the size of the temporal lobe, is well and equally developed in both the idiots' brains. The hippocampus major is large and wide in form, and terminates in front in an expanded and slightly sulcated extremity. The hippocampus minor is very wide, though short, in both brains; in the idiot boy it is directed backwards, in accordance with the direction of the posterior horn; whilst in the idiot woman it branches off outwards, in harmony with the divergent course of that cavity. The length of the hippocampus minor in the idiot woman is ·8 inch, its greatest breadth ·35; in the idiot boy the length is ·5 inch, and the greatest breadth ·3. The eminentia collateralis is distinctly visible in both brains.

Measuring off the antero-median from the posterior portion of the cerebrum on Mr. Flower's method, the ratio between the former and the latter is, in the idiot woman, 100 to 62·2, in the idiot boy 100 to 61·9; so that the occipital region, as thus estimated, *i. e.* in its inferior layers, is proportionally longer in the idiots' brains than in the perfect brain, or even than in the Chimpanzee's brain, in which the proportions are

100 to 53 and 100 to 52 respectively. This apparent relative increase in the posterior portion of the brain should rather be interpreted as due to a comparatively undeveloped condition of the antero-median region.

It has already been mentioned that the corpora striata are in both idiots' brains very small, whilst the optic thalami are relatively large. The part of the corpus striatum seen in the left lateral ventricle is, in the idiot woman, ·35 inch long and ·15 inch wide; in the idiot boy ·25 inch long and ·15 inch wide. The visible portion of the optic thalamus in the woman is ·8 inch long by ·3 inch wide, in the idiot boy ·65 inch long by ·4 inch wide. It has been previously noted that the large size of the optic thalamus corresponds with the great development of the temporo-parietal region in both the idiots' brains, and the diminutive corpus striatum with the wasted form of the frontal region; but besides the mere relative size of the two ganglionic masses, another condition doubtless concurs to produce this correspondence, viz. the relative number of the radiating fibres which spring from them to branch out into the corresponding parts of the hemisphere. The corpora quadrigemina are quite of proportional size to the rest of the brain in both idiots, the superior one being larger as usual. The corpora geniculata are likewise well developed in both. In the idiot woman the corpora albicantia are separate but not prominent; in the idiot boy the corpora albicantia are broad and somewhat fused together. The pineal gland is proportionally large in the idiot woman; in the idiot boy it was not found, having probably been destroyed. The habenulæ could not be traced in either brain. The pituitary body in the idiot woman is very large; in the idiot boy it has not been preserved, but the infundibulum is present and large. The cerebral peduncles in the idiot woman appear long and narrow, and the interval between them is deep; in the idiot boy, on the other hand, they are broad and flat, and the interval between them is shallow. This may be due to elongation of the woman's brain occurring as the effect of long suspension in spirit after complete removal of the membranes.

In the preserved brains the pons Varolii is broader from before backwards in the idiot woman than in the idiot boy, measuring in the former nearly ·6 inch deep, but only ·5 inch thick, in the latter only ·4 inch deep and ·5 inch thick: this appears remarkable, as the cerebellum of the boy is larger than the cerebellum of the woman.

The medulla oblongata is large in both idiots: in the woman its greatest width is ·7 inch, in the boy ·6 inch; its proportions to the entire width of the brain are as 1 to 5·14 in the former and 1 to 5·3 in the latter, instead of 1 to 7, the normal ratio. The olivary bodies are large, but especially so in the idiot boy; the grey matter in their interior is folded, as usual. In other respects the medulla is normal.

The cranial nerves, so far as can be judged from those which remain in the preparations, are of full size, and, indeed, large in proportion to the size of the cerebral mass.

The Cerebellum (Plates XXI. & XXII. figs. 10–16, *Ce*).—As already shown, this organ in both idiots, though more highly developed than the cerebrum, is still small, especially in the idiot woman. All its parts, however, are present, though more or less reduced in size and in complexity of structure.

In the idiot woman the hemispheres are proportionally large and well shaped, whilst the median portion is not quite so well developed. In the idiot boy the hemispheres are relatively larger, projecting laterally, and preponderating still more over the median portion.

In the idiot woman the square lobe is normal in shape and proportions, the posterior superior lobe is narrow and shallow, the amygdala and biventral lobe are strikingly large; the slender lobe is very small, the posterior superior lobe is, on the contrary, well developed; the pyramid and uvula are prominent, but narrow; the nodule is small; the posterior velum is exceedingly thick; the floccule is very small. In the idiot boy the square lobe is of ordinary shape and proportions, the posterior superior lobe is wide and deep; the amygdala, contrary to its condition in the idiot woman, is small, and partly concealed beneath the biventral lobe, which is large; the slender lobe is of moderate size, and the posterior inferior lobe again large; the pyramid, uvula, and nodule are small; the posterior velum, as in the idiot woman, is exceedingly thick, suggesting the idea that it usually becomes attenuated as development advances; the floccules are small, but not so small as in the female idiot.

The middle peduncular (or transverse commissural) fibres are defective, not merely in comparison with the perfect organ, but also in relation to the size of the idiots' cerebella, especially considering the great development of their hemispheres. The oval surface occupied by the divided ends of these fibres, as seen on a median section through the pons, is shorter and much smaller than usual. In the idiot woman (Plate XXIII. fig. 25, *p*) it measures ·6 of an inch long by ·325 inch wide, its sectional area being therefore ·195 square inch, whilst in the idiot boy (fig. 26, *p*) the dimensions of the same part are ·45 inch by ·28 inch, and the sectional area ·126 square inch. The idiot woman's recent cerebellum having weighed 1·95 oz., and the idiot boy's 2·25 oz., there would be to each ounce weight of that organ ·1 square inch of commissure in the woman, and only ·05 square inch in the idiot boy. The healthy proportion, as shown at a previous page, is ·13 square inch. In this point of view, then, the cerebellum is very defective in both idiots, but especially in the idiot boy; which is remarkable anatomically, since in him this organ is so much larger than in the woman. The imperfect gait and feeble power of control over the muscles generally, always associated with true idiocy, were noticeable in the case of both idiots; and this might appear to be in part explicable, on the ordinary hypothesis that the cerebellum is concerned in coordinating the voluntary muscular movements, by the obvious deficiency in the bulk of that organ in both these cases; but, on the other hand, the fact that the cerebellum is larger in the idiot boy, though his powers of locomotion were altogether absent, and though he could not handle anything, nor articulate any words, seems contradictory and inexplicable. Under these circumstances it is at least interesting to find that one of the cerebellar commissural systems of fibres is so much more deficient in him than in the idiot woman; and whether we adopt the view that the office of the cerebellum is directly to coordinate the muscular movements, or that it indirectly aids in this coordination, by registering the various muscular sensa-

tions indicative of the manifold conditions of the muscles, such a deficiency of these commissural fibres may be equally supposed to interfere with the functions of the organ. In both idiots the inferior and superior peduncles likewise appear small; but no exact computations could well be made concerning these parts.

On counting the total number of laminæ in the cerebellum of an idiot, MALACARNE found that they numbered only 324*, whilst in a healthy cerebellum there were from 700 to 780. In another case he found 362 laminæ, 240 on the upper surface, and 122 on the under surface†. It appears that he afterwards met with a third example, in which a similar deficiency in the number of the laminæ was likewise noted‡.

For reasons already stated in describing the cerebellum of the Bushwoman, I have limited myself to counting the superficial laminæ only of certain parts, and the deep and superficial ones of others. The following are the results.

Superficial laminæ only counted.

	Median portion.	Lateral parts.						Total laminæ in lateral parts.
	Superior vermiform process.	Square lobe.	Posterior superior lobe.	Amygdala.	Biventral lobe.	Slender lobe.	Posterior inferior lobe.	
Healthy brain......	18	21	13	9	6	5	16	70
Idiot woman	15	14	9	7	7	4	12	53
Idiot boy	19	14	11	5	7	4	12	53

Superficial and deep laminæ counted.

	Median portion.					Lateral parts.	
	Inferior vermiform process.		viz. Pyramid and uvula.		Nodule.	Amygdala.	Floccule.
Healthy brain......	37	=	28	+	9	21	18
Idiot woman	25	=	17	+	8	18	8
Idiot boy	20	=	15	+	5	11	11

The preceding numbers confirm the interesting observations made by MALACARNE eighty years ago; and it is to be noted that when both the superficial and deep laminæ are counted, the difference between the healthy and idiots' cerebellum is greater, and approaches more nearly the proportions shown by MALACARNE's numbers. The total number of superficial laminæ counted is the same in both idiots, and their distribution in the several lobes is equal, with the exception of the posterior superior lobe, which has two more laminæ in the idiot boy, and of the amygdala, which has two less. As compared with the healthy cerebellum, the greatest deficiency is in the square lobe; the

* *Op. cit.* p. 7. † P. 226.

‡ C. BONNET, Palingénésie Philosophique, part 2, chap. iv. vol. vii. of his collected works in 4to.

next in the two posterior lobes, and the least in the amygdala. A loss in the slender lobe is balanced by a gain in the biventral. In the median parts the whole of the laminæ are counted, and there is a more marked deficiency, as compared with the healthy condition, especially in the idiot boy. The floccule, however, presents an exception to this, being more complicated in him than in the woman. The laminæ of the cerebellum in both idiots are not only fewer in number, but are shorter and narrower than in the healthy cerebellum.

The corpus dentatum is represented merely by an elongated streak, differing in colour and consistence from the surrounding medullary substance, but presenting no indented outline, and no white mass in its interior.

In a fœtal brain the convolutions of whose *cerebrum* correspond with the idiot condition, the *cerebellum* is strikingly small, and would measure transversely not more than 1·5 inch, whereas in the idiot woman its transverse diameter is 3·15 inches, and in the idiot boy 3 inches. At the same period of development the laminæ visible on its under surface are only twenty in number, whilst in the same parts in the idiot woman there are thirty, and in the idiot boy twenty-eight. It is obvious, therefore, that in these idiots the cerebellum was not arrested in its development cotemporaneously with the cerebrum, but continued to increase greatly in size, and to undergo further changes of an evolutional character long after the cerebrum had ceased to be affected in the latter way. At the same time, from the small relative size of the cerebellum to the body in both idiots as compared with the healthy standard at corresponding ages, from the undoubted paucity, shortness, and narrowness of its laminæ, from the deficiency in its transverse commissural fibres, and from the ill-defined condition of its corpus dentatum, it had at length, as well as the cerebrum, succumbed from want of developmental energy. In other words, though more evolved than the idiots' cerebra, the cerebella are still imperfectly developed. The date of their final arrest of development is uncertain; it was probably retarded to a period several years after birth; it may be assumed to have corresponded with the condition attained by the healthy cerebellum at about the end of the second year of life.

POSTSCRIPT. (August 6, 1863.)

Since the preceding paper was written I have had opportunities of inspecting the exterior of two idiots' brains in the Museum of St. Bartholomew's Hospital, and of examining, besides a wax model and two drawings of an idiot's brain, a most interesting series of models in wax of human fœtal brains, from the second month to the full term, in the Anatomical Museum at Guy's Hospital.

a. Of the two idiots' brains preserved at St. Bartholomew's, the smaller one (Catalogue, Series A. 123) is that mentioned by Professor OWEN as weighing 13 oz. 2 dr. avoirdupois*. It was the brain of a male idiot, aged 22. The larger brain (Series A. 121) was that of a female idiot; also aged 22.

* See p. 527.

In both cerebra the fissures and convolutions are readily comparable with those of the idiots' brains of which an account is given in this paper, and so resemble them as to suggest a similarity in their mode of evolution. Like them, too, they present individual peculiarities, and a disproportional development of their component parts. In a few points they are more advanced than even the idiot woman's brain described above. An examination of them serves to confirm in every particular the explanations ventured upon in the preceding pages. The fissures present nothing remarkable; but the convolutions require some notice.

In the smaller or male brain (A. 123) the orbital convolutions are extremely simple, the triradiate sulcus being only a linear mark; the frontal are very simple also; both ascending parietals are precsnt on the right side, whilst on the left only the posterior one exists, but the intrusive convolution is seen largely developed, forming nearly a complete anterior ascending convolution; the parietal lobules are plain, short, and wide; the supramarginal and bent convolutions are simple, and there is no developed supramarginal lobule; the temporal are short and thick; the occipital are simple; the external connecting are very short.

In the larger or female brain (A. 121) the orbital convolutions are not quite so rudimentary; the frontal are large and few; the two ascending parietals are present on both sides, but are plain narrow bands; the parietal lobules are large, the left one being more defined than the right; the supramarginal and bent convolutions are large, and so well formed as to remind one of the condition of the parts seen in the Orang's brain, but there is no overhanging supramarginal lobule; the temporal are very large, and especially long; the occipital are broad, shallow, and coalesced; the external connecting are short and simple.

The ridge leading to the island of REIL is prominent and exposed in the female brain, but not so much so in the other. Both brains exhibit in special details a decided want of symmetry.

It is interesting to find that the variable conditions of the anterior ascending parietal convolution establish the correctness of the explanation given above of the ultimate conversion of the intrusive convolution into that gyrus. It must not escape notice that, in the smaller or male brain, this intrusive convolution is again present on the left side, as in our idiot woman and idiot boy. The two brains, moreover, differ, not only in the parietal convolutions, but also in the orbital region, in the shape of the temporal lobes, and in other respects to be presently mentioned. There is evidence of a defective and therefore disproportional size of the frontal lobe in both brains, in the fact that, taking the total length of the cerebrum as 100, the part in front of the fissures of ROLANDO, the part between them and the perpendicular fissures, and the part behind the latter, as seen from above, measure in the smaller or male brain 29, 42, and 29, and in the larger or female brain 40, 35, and 25. The frontal region, in front of the fissure of ROLANDO, as compared with the parts lying behind that fissure, is therefore in the former as 29 to 71, and in the latter as 40 to 60. In both idiots the frontal region

is therefore defective, but unequally so in the two. It is scarcely possible to doubt that dissection would establish the fact of a coincident want of development in the corpora striata of these brains.

In both these idiots also the cerebellum is, comparatively to the cerebrum, enormously developed. In the smaller brain, only one-third of it is covered by the cerebrum; in the larger brain, the two parts reach back to exactly the same level; in both the laminæ appear to be few, only twenty being distinguishable on the under surface of one hemisphere in each.

The skull of the male idiot is thin, and appears to be much marked on its internal surface with the convolutions; that of the female idiot is thicker, and not so much marked in that way.

b. Assuming the accuracy of the dates assigned to the several models of fœtal brains in the Museum at Guy's, the effect would be to place the period at which the idiots' cerebra reached their ultimate stage of evolution, at from one month to even six weeks later than that given in the preceding paper. But it is notoriously difficult to determine the age of any given fœtus. An examination of the entire series shows that, if the dates be correct, the cerebrum does not, either in size, or in the condition of development of its convolutions, always attain exactly the same point at exactly the same period of intra-uterine life.

The condition of the convolutions in these models confirms the history above given of the conversion of the intrusive convolution into the anterior ascending parietal; for the change is traceable through a certain number of the fœtal brains. It also supports the views expressed as to the early arrest of the evolution of the corpora striata, and of the special effect of this on the development of the frontal lobes; for, with certain fluctuations, the corpora striata, where shown in the models, are always larger than the optic thalami; and the proportions of the frontal to the hinder regions of the cerebrum, as marked off by the fissures of ROLANDO, vary, from the first appearance of this fissure to the full term of development, between the ratios of 37 to 63 and 58 to 42. Lastly, these models show that idiot brains must grow a little after they have ceased to be further evolved; for the convolutions, and indeed the cerebral hemispheres themselves, are broader and larger in the idiot brains than in the models of brains of equally forward convolutional development. It is certainly true that, taking the four idiots' brains, viz. the two hereinbefore described and the two in the Museum at St. Bartholomew's, their respective sizes and their degrees of evolution correspond; but this does not disprove the occurrence of a growth in them after the cessation of development, an event shown to occur on other grounds.

The model and drawings of the idiot's brain at Guy's also confirm all our previous notions; and indeed it may be concluded that the idiotic condition is produced in all cases by conformable influences, affecting the cerebrum in slightly different degrees in different examples.

TABLE I.—Measurements of the parts of the Encephalon in the European, in the Bushwoman, in two idiots of European descent, and in the Chimpanzee, given in tenths of an English inch, with the ratios of those measurements to the European measurements regarded as units. The measurements of the Cerebrum and Cerebellum are taken by aid of the intracranial casts. In the case of the idiot boy, the cast of the skull being defective, only a few such measurements could be obtained; but in a second column are given the corresponding measurements taken from the preserved brain. The measurements of the Medulla oblongata, Corpus callosum, Corpus striatum, Optic thalamus, and Pons Varolii are taken from the preserved brains.

							Ratios of the several measurements.				
	European.	Bush-woman.	Idiot woman.	Idiot boy.		Chim-panzee.	European.	Bush-woman.	Idiot woman.	Idiot boy.	Chim-panzee.
Cerebrum.											
a. Extreme breadth	50	51	36	32	31	37	1	1·02	·72	·64	·74
b. Extreme length	65	66	41	39·5	34	44	1	1·01	·63	·61	·68
c. Extreme height	45	41	31		14*	29	1	·91	·68	......	·65
d. Length of orbital surface	23	23	15		9	15	1	1	·65	......	·65
e. Extreme depth of the frontal lobe	35	31·5	20·75		11*	20	1	·9	·59	......	·57
e'. Extreme width of the frontal lobe	42	38	29	23	18	31	1	·9	·69	·54	·73
f. From the point of the middle lobe to the hinder end of the brain	48	49	35		28	34	1	1·02	·72	......	·7
g. Cerebral radius, occipital	33	34	25	......		23	1	1·03	·75	......	·7
h. " frontal	43	40	27	......		29	1	·93	·62	......	·67
i. " parietal	39	35	25	......		26	1	·9	·64	......	·66
j. " vertical	46	41	29	......		29	1	·89	·63	......	·63
k. Projection of the cerebrum beyond the cerebellum	6	5	−3·5		−3·5	5	1	·83	"	......	·83
Cerebellum.											
l. Extreme breadth	36	41	31·5		30	30	1	1·1	·87	......	·73
m. Extreme length	24	25	21		14	16	1	1·04	·87	......	·66
n. Extreme depth	14	14	20·5		12	8	1	1	1·46	......	·57
Medulla oblongata.											
o. Extreme breadth	7·1	8·5	7	6		......	1	1·26	·98	·7	
Corpus callosum.											
p. Length	31	26	15·3	13·6		17	1	·84	·5	·44	·55
q. Average thickness	4·3	2	2·3	1·6		1·5	1	·46	·53	·37	·35
Corpus striatum.											
r. Length of visible part	9	9	3·5	2·5		8	1	1	·39	·28	·9
s. Width of visible part	5	3	1·5	1·5		3	1	·6	·3	·3	·6
Optic thalamus.											
t. Length of visible part	13	12	8	6·5		10	1	·92	·61	·5	·76
u. Width of visible part	5	4·5	3	4		4·5	1	·9	·6	·8	·9
Pons Varolii.											
v. From upper to lower border	10	9	6	4		6	1	·9	·6	·4	·6
w. Thickness	9	8	5	5		6·5	1	·9	·5	·5	·72

* These measurements are especially diminished by the extreme flattening of the preserved brain.

TABLE II.—Ratios between the Dimensions of different parts of the Encephalon, in the European, in the Bushwoman, in the two Idiots, and in the Chimpanzee.

		European.	Bushwoman.	Idiot woman.	Idiot boy.	Chimpanzee.
Cerebrum	*a* to *b*	1 to 1·3	1 to 1·29	1 to 1·14	1 to 1·23*	1 to 1·19
	a to *c*	1 to ·9	1 to ·82	1 to ·86	1 to ·45	1 to ·78
	c to *b*	1 to 1·44	1 to 1·61	1 to 1·32	1 to 2·43	1 to 1·52
	d to *f*	1 to 2·09	1 to 2·13	1 to 2·3	1 to 3·1	1 to 2·26
Cerebellum	*m* to *l*	1 to 1·5	1 to 1·64	1 to 1·5	1 to 2·14	1 to 1·87
	m to *n*	1 to ·58	1 to ·56	1 to ·97	1 to ·86	1 to ·5
	n to *l*	1 to 2·57	1 to 2·91	1 to 1·53	1 to 2·5	1 to 3·75
Cerebrum and Cerebellum	*m* to *b*	1 to 2·7	1 to 2·64	1 to 1·95	1 to 2·43	1 to 2·75
	n to *c*	1 to 3·21	1 to 2·91	1 to 1·51	1 to 1·16	1 to 3·62
	l to *a*	1 to 1·39	1 to 1·42	1 to 1·14	1 to 1·03	1 to 1·23
Medulla oblongata and Cerebrum	*o* to *a*	1 to 7	1 to 6	1 to 5·14	1 to 5·3*	1 to 5·7

EXPLANATION OF THE PLATES.

With the exception of figures 7, 8, and 20, which are taken from GRATIOLET, and of figure 23, which is merely a tracing from nature, the illustrations to this paper have been lithographed from Mr. HERBERT WATKINS'S photographs, aided by reference to the objects themselves. All the figures agree in size with the preserved brains, excepting figure 9, which, as well as figures 7 and 8, taken from GRATIOLET, is reduced to three-fourths of the proper linear dimensions. The references to the cerebral convolutions are alike in all the Plates; and a common explanation of these is appended to the general description. References to a few other details are given under each Plate.

PLATE XVII.

Fig. 1. Upper surface of the preserved brain of the Bushwoman.

Fig. 2. Base of the same brain: *o*, olfactory nerve; *a*, corpora albicantia.

PLATE XVIII.

Fig. 3. Left side of the same brain.

Fig. 4. Vertical median section of the same brain, showing the inner surface of the left hemisphere of the cerebrum, with the cut surfaces of the cerebral peduncle, the corpus callosum, the cerebellum, pons Varolii (*p*), medulla oblongata, and other parts: *a*, anterior commissure; *q*, corpora quadrigemina.

* These ratios are calculated from measurements taken on the intracranial cast; the rest of the ratios, in the case of the idiot boy, are taken from measurements of the preserved brain.

PLATE XIX.

Fig. 5. The left hemisphere of the same brain, detached from its peduncle, showing the under surface with the inner surface foreshortened.

Fig. 6. Horizontal dissection of the same hemisphere, showing the left lateral ventricle laid open: *a*, body of the ventricle and optic thalamus; *b*, anterior cornu; *c*, posterior cornu; *d*, commencement of middle cornu; *e*, hippocampus major; *f*, hippocampus minor; *g*, eminentia collateralis; *h*, corpus striatum.

PLATE XX.

Fig. 7. Convolutions of the upper surface of a European brain, shown in outline (from Gratiolet).

Fig. 8. Corresponding view of the brain of the Hottentot Venus (from Gratiolet).

Fig. 9. Corresponding view of the brain of the Bushwoman. To suit the size of the page, these comparative views of the upper cerebral convolutions in the European, the Hottentot Venus, and the Bushwoman, are reduced to three-fourths of their proper linear dimensions. But it must be observed that the originals of figures 7 and 8 are of the full size of the recent or restored brains; whilst the original figure of the Bushwoman's brain (figure 1 of this memoir) is of the size of that organ after it had become shrunk from maceration in spirit. Judging from the measurements of the interior of the Bushwoman's skull, this reduced figure of her brain should be about $\frac{1}{10}$th of an inch shorter than that of the brain of the Hottentot Venus.

PLATE XXI.

Fig. 10. Upper surface of the preserved brain of a female idiot, aged 42 years.

Fig. 11. Under surface, or base, of the same brain: *o*, olfactory sulcus.

Fig. 12. Left side of the same brain.

Fig. 13. Vertical median section of the same brain, showing the inner surface of the left hemisphere of the cerebrum, with the cut surfaces of the corpus callosum (*c*), cerebral peduncle, cerebellum, pons Varolii (*p*), medulla oblongata, and other parts.

PLATE XXII.

Fig. 14. Upper surface of the preserved brain of a male idiot, aged 12 years.

Fig. 15. Under surface, or base, of the same brain: *o*, olfactory sulcus.

Fig. 16. Left side of the same brain.

Fig. 17. Right hemisphere of the same brain, detached from its peduncle, showing the internal and under surfaces. The frontal region is turned to the left hand of the observer. The section of the corpus callosum (*c*) is also shown.

PLATE XXIII.

Fig. 18. Convolutions of the upper surface of the brain of the female idiot.

Fig. 19. Convolutions of the upper surface of the brain of the male idiot.

N.B. In both of these figures a few additional markings, not represented in figures 10 and 14, have been put in from careful examinations of the brains themselves.

Fig. 20. Convolutions of the upper surface of the brain of an Orang-outang (after GRATIOLET'S restored figure).

Fig. 21. Outer surface of the left hemisphere of the cerebrum of a human fœtus, at probably between the fourth and the fifth month.

Fig. 22. Median vertical section of the same fœtal cerebrum, showing the inner surface of the same hemisphere.

Fig. 23. Vertical median section of the medulla oblongata and pons Varolii of a preserved European brain, to show the area (*p*) occupied by the divided transverse fibres of the pons.

Fig. 24. The same parts from the brain of the Bushwoman.

Fig. 25. The same parts from the brain of the female idiot.

Fig. 26. The same parts from the brain of the male idiot.

References to the Lobes, Fissures, and Convolutions.

The names here given to the fissures and convolutions are, for the most part, founded on M. GRATIOLET'S nomenclature.

Ce. The cerebellum.

Lobes.

C. Median lobe, or Island of REIL.
F. Frontal.
P. Parietal.
O. Occipital.
T. Temporal.

Fissures.

c–c. Antero-parietal (HUXLEY).
d–d. ROLANDO'S.
e–e. Sylvian.
f–f. Parallel.
g–g. Inferior temporal.
h, h. External perpendicular.
i–i. Fronto-parietal (*calloso-marginal*, HUXLEY).
k–k. Internal perpendicular.

l–l, m. Hippocampal.

l–l. Outer or calcarine portion (HUXLEY).

m. Inner or dentate portion (HUXLEY).

n–n. Inferior middle temporal or great collateral.

Convolutions.

1′. Supraorbital.

1″. Posterior orbital.

1″′. Internal orbital.

1″″. External orbital.

1–1. Lower frontal.

2–2′. Middle frontal.

3–3′. Upper frontal.

4–4′. Anterior ascending parietal.

5–5. Posterior ascending parietal.

5′–5′. Lobule of the posterior ascending parietal.

4″–5″. Supramarginal.

A–A. Lobule of the supramarginal.

6–6. Bent, or angular.

7–7. Upper external temporal, or inframarginal.

8–8. Middle external temporal.

9–9. Lower external temporal, which is the same as the lower internal temporal.

10–10. Upper occipital.

11–11. Middle occipital.

12–12. Lower occipital.

α–α. First or upper external connecting.

β–β. Second external connecting.

γ–γ. Third external connecting.

δ–δ. Fourth or lowest external connecting.

17–17. Great marginal.

18–18. Callosal.

18′–18′. Quadrilateral lobule.

19–19. Middle internal temporal or uncinate.

19′. Unciform lobule or crochet.

20. Dentate (not shown in any figure).

ε. Place of lower internal connecting, here (as usual) concealed.

ζ. Place of upper internal connecting (present only in Quadrumana).

25–25. Occipital lobule.

26. Calcarine (FLOWER). Shown only in figure 17.

XVI. *A Second Memoir on Skew Surfaces, otherwise Scrolls.* *By* A. Cayley, *F.R.S.*

Received April 29,—Read May 26, 1864.

The principal object of the present memoir is to establish the different kinds of skew surfaces of the fourth order, or Quartic Scrolls; but, as preliminary thereto, there are some general researches connected with those in my former memoir "On Skew Surfaces, otherwise Scrolls"*, and I also reproduce the theory (which may be considered as a known one) of cubic scrolls; there are also some concluding remarks which relate to the general theory. As regards quartic scrolls, I remark that M. Chasles, in a footnote to his paper, "Description des courbes de tous les ordres situées sur les surfaces réglées du troisième et du quatrième ordres"†, states, "les surfaces réglées du quatrième ordre .... admettent *quatorze* espèces." This does not agree with my results, since I find only eight species of quartic scrolls; the developable surface or "torse" is perhaps included as a "surface réglée;" but as there is only one species of quartic torse, the deficiency is not to be thus accounted for. My enumeration appears to me complete, but it is possible that there are subforms which M. Chasles has reckoned as distinct species.

On the Degeneracy of a Scroll, Article Nos. 1 to 5.

1. A scroll considered as arising from any geometrical construction, for instance one of the scrolls $S(m, n, p)$, $S(m^2, n)$, $S(m^3)$ considered in my former memoir, or say in general the scroll S, may break up into two or more inferior scrolls S', S'',..; but as long as S', S'',.. are proper scrolls (not torses, and *à fortiori* not cones or planes), no one of these can be considered, apart from the others, as the result of the geometrical construction, and we can only say that the scroll S given by the construction is the aggregate of the scrolls S', S'',..; and the like when we have the scrolls S', S'',..., each repeated any number of times, or say when $S=S'^{\alpha}S''^{\beta}\ldots$ Suppose however that the scrolls S', S'',.. are any one or more of them a torse or torses—or, to make at once the most general supposition, say that we have $S=\Sigma S'$, where Σ is a torse, or aggregate of torses ($\Sigma=\Sigma'^{\alpha}\Sigma''^{\beta}\ldots$), and S' is a proper scroll or aggregate of proper scrolls; then, although it is not obligatory to do so, we may without impropriety throw aside the torse-factor Σ, and consider the original scroll S as degenerating into the scroll S', and as suffering a reduction in order accordingly.

2. As an illustration, consider the scroll $S(m, n, p)$ generated by a line which meets three directrix curves of the orders m, n, p respectively; and assume that the curves

* Philosophical Transactions, vol. cliii. (1863), pp. 453–483.

† Comptes Rendus, t. liii. (1861), see p. 888.

m, n, p are each of them situate on the same scroll Σ, the curve m meeting each generating line of Σ in α points, the curve n each generating line in β points, and the curve p each generating line in γ points. Each generating line of Σ is $\alpha\beta\gamma$ times a generating line of S, and we have $S=\Sigma^{\alpha\beta\gamma}S'$, where S' may be a proper scroll; it is however to be noticed that if the curves m, n, p any two of them intersect, S' will itself break up and contain certain cone-factors, as will presently appear. And if Σ, instead of being a proper scroll, be a torse, then we may consider S as degenerating into S', the reduction in order being of course $=\alpha\beta\gamma\times$ order of Σ.

3. But this is not the only way in which the scroll $S(m, n, p)$ may degenerate; for suppose that two of the directrix curves, say n and p, intersect, then the lines from the point of intersection to the curve m form a cone of the order m which will present itself as a factor of S; and generally if the curves n and p intersect in α points, the curves p and m in β points, and the curves m and n in γ points, then we have α cones each of the order m, β cones each of the order n, and γ cones each of the order p, or say $S=CS'$, where C is the aggregate of the cone-factors; and the scroll S degenerates into S', the reduction in order being $=\alpha m+\beta n+\gamma p$. It is hardly necessary to remark that if a point of intersection of two of the curves is a multiple point on either or each of the curves, it is, in reckoning the number of intersections of the two curves, to be taken account of according to its multiplicity in the ordinary manner.

4. There is yet another case to be considered: suppose that the curves n and p lie on a cone, and that the curve m passes through the vertex of this cone; this cone, repeated a certain number of times, is part of the locus, or we have $S=C^{\theta}S'$, so that the scroll S degenerates into S', the reduction in order being $=\theta\times$ order of cone. If, to fix the ideas, the curves n and p are respectively the complete intersections of the cone by two surfaces of the orders g, h respectively (this implies $n=gk$, $p=hk$, if k be the order of the cone), which surfaces do not pass through the vertex of the cone, and if, moreover, the vertex of the cone be an a-tuple point on the curve m, then $\theta=agh$, and the reduction in order is $=aghk$.

5. The foregoing causes of reduction, or some of them, may exist simultaneously; it would require a further examination to see whether the aggregate reduction is in all cases the sum of the separate reductions. But the aggregate reduction once ascertained, then writing $S(m, n, p)$ for the order of the reduced scroll, we shall have

$$S(m, n, p)=2mnp-\text{Reduction}.$$

In particular, in the case above referred to, where the curves n and p, p and m, m and n meet in α, β, γ points respectively, but there is no other cause of reduction,

$$S(m, n, p)=2mnp-\alpha m-\beta n-\gamma p,$$

which is a formula which will be made use of.

The foregoing investigations apply, *mutatis mutandis*, to the scrolls $S(m^2, n)$, $S(m^3)$; but I do not at present enter into the development of them in regard to these scrolls.

Scrolls with two directrix lines, Article Nos. 6 to 11.

6. Consider now a scroll having two directrix lines: it may be assumed that these do not intersect; for if they did, then any generating line, *quà* line meeting the two directrix lines, would either lie in the plane of the two lines, or else would pass through their point of intersection; that is, the scroll would break up into the plane of the two lines, considered as the locus of the tangents of a plane curve, and into a cone having for its vertex the point of intersection of the two lines. Each generating line meets any plane section of the scroll in the point where such generating line meets the plane of the section; the plane section constitutes a third directrix; or the scrolls in question are all included in the form $S(1, 1, m)$, where m is a plane curve. The order of the scroll $S(1, 1, m)$ is in general $=2m$; but if the one line meets the curve α times, that is, in an α-tuple point of the curve, and the other line meets the curve β times, that is, in a β-tuple point of the curve, then by the general formula (*ante*, No. 5) the order of the scroll is $=2m-\alpha-\beta$; and in particular if $\alpha+\beta=m$, then the order is $=m$.

7. We may *without loss of generality* attend only to the last-mentioned case. To show how this is, suppose for a moment that the two lines do not either of them meet the curve; the scroll is then of the order $2m$. Call the point in which each line meets the plane of the curve the foot of this line, then the line joining the two feet meets the curve in m points; and it is in respect of each of these points a generating line of the scroll; that is, it is an m-tuple generating line: the section of the scroll by the plane of the curve m is in fact this line counting m times, and the curve m; $m+m=2m$, the order of the scroll. And in like manner the section by any plane through the m-tuple line is this line counting m times, and a curve of the order m not meeting either of the directrix lines. But the section by any other plane is a curve of the order $2m$ meeting each of the directrix lines in a point which is an m-tuple point of the section (each directrix line is in fact an m-tuple line of the scroll); and by considering, in place of the particular section m, this general section, we have the scroll of the order $2m$ in the form $S(1, 1, 2m)$, where the two directrix lines each meet the section m times; so that the order is $4m-m-m=2m$.

8. And so in general, m being a plane curve, when the scroll $S(1, 1, m)$ is of an order superior to m, say $=m+k$, this only means that the section chosen for the directrix curve m is not the complete section by the plane of such curve, but that the line joining the feet of the two directrix lines is a k-tuple generating line of the scroll, and that the complete section is made up of this line counting k times and of the curve m. So that taking, not the section through the multiple generating line, but the general section, for the plane directrix curve, the only case to be considered is that in which the section is a proper curve of an order equal to that of the scroll; or, what is the same thing, we have only to consider the scrolls $S(1, 1, m)$ for which the order is depressed from $2m$ to m in consequence of the directrix lines meeting the plane section α times and β times, that is, in an α-tuple point and a β-tuple point respectively, where $\alpha+\beta=m$.

9. It is clear that in the case in question the directrix lines are an α-tuple line and a β-tuple line respectively. The generation is as follows: Scroll S(1, 1, m) of the order m; the curve m being a plane curve of the order m having an α-tuple point and a β-tuple point, where $\alpha+\beta=m$: the directrix lines, say 1 and 1′, pass through these points respectively, and they do not intersect each other. The generating lines pass through the directrix lines 1 and 1′ and the curve m, and we have thence the scroll S(1, 1, m). Taking at pleasure any point on the curve m, we can through this point draw a *single* line meeting each of the directrix lines 1, 1′; that is, the curve m is a simple curve on the scroll. Taking at pleasure a point on the directrix line 1, and making this the vertex of a cone standing on the curve m, this cone has an α-tuple line (the line 1) and a β-tuple line (the line joining the vertex with the foot of the line 1′); the line 1′ meets this cone in the foot of the line 1′, counting β times, and besides in $m-\beta$, $=\alpha$ points; the lines joining the vertex with the last-mentioned points respectively (or, what is the same thing, the lines, other than the β-tuple line, in which the plane through the vertex and the line 1′ meets the cone) are the α generating lines through the assumed point on the line 1; and the line 1 is thus an α-tuple line of the scroll. And in like manner, through an assumed point of the directrix line 1′, we construct β generating lines of the scroll; and the line 1′ is a β-tuple line of the scroll.

10. The scroll S(1, 1, m) now in question has not in general any multiple generating line; in fact a multiple generating line would imply a corresponding multiple point on the section m; and this section, assumed to be a curve having an α-tuple point and a β-tuple point, has not in general any other multiple point. But it *may* have other multiple points; and if there is, for example, a γ-tuple point, then the line from this point which meets the two directrix lines counts γ times, or it is a γ-tuple generating line; and so for all the multiple points of m other than the α-tuple point and the β-tuple point which correspond to the directrix lines respectively. It is to be noticed that the multiplicity γ of any such multiple generating line is at most equal to the smallest of the two numbers α and β; for suppose $\gamma>\alpha$, then, since $\alpha+\beta=m$, we should have $\gamma+\beta>m$, and the line joining the γ-tuple point and the β-tuple point would meet the curve m in $\gamma+\beta$ points, which is absurd. In the case of several multiple lines, there are other conditions of inequality preventing self-contradictory results *.

11. The general section is a curve of the order m, having an α-tuple point and a β-tuple point corresponding to the directrix lines respectively, and a γ-tuple point, &c. ... corresponding to the other multiple points (if any). A section through the directrix line 1 is in general made up of this line, counting α times, and of β generating lines passing through one and the same point of the directrix line 1′; if the section pass

* Suppose, for example (see next paragraph of the text), that there were a γ-tuple generating line and a δ-tuple generating line lying *in plano* with the line 1; these lines counting as $(\gamma+\delta)$ lines, must be included among the β generating lines through the plane in question; this implies that $\gamma+\delta \ngtr \beta$, a conclusion which must be obtainable from consideration of the curve m irrespectively of the scroll.

also through a γ-tuple generating line, then, of the β generating lines in question, γ (which, as has been seen, is $\not>\beta$) unite together in the γ-tuple generating line; and so for the sections through the directrix line 1'. The general section through a γ-tuple generating line is this line counting γ times, and a curve of the order $m-\gamma$, which has an $(\alpha-\gamma)$tuple point at its intersection with the directrix line 1, and a $(\beta-\gamma)$tuple point at its intersection with the directrix line 1'; it has a δ-tuple point, &c... at its intersections with the other multiple generating lines, if any.

Scrolls with a twofold directrix line, Article Nos. 12 to 16.

12. But there is a case included indeed as a limiting one in the foregoing general case, but which must be specially considered; viz. the two directrix lines 1 and 1' may coincide, giving rise to a twofold directrix line. To show how this is, I return for the moment to the case of the scroll S(1, 1, m) with two distinct directrix lines 1 and 1', and, to fix the ideas, I suppose that the directrix lines do not either of them meet the curve m, so that the order of the scroll is $=2m$. Through the line 1 imagine the series of planes A, B, C, ... meeting the line 1' in the points a', b', c' ..; the generating lines through the point a' are the lines in the plane A to the points in which this plane meets the curve m; the generating lines through the point b' are the lines in the plane B to the points where this plane meets the curve m; and so for the generating lines through the points c', d' .... And it is clear that the points a', b', c', .. correspond homographically with the planes A, B, C, ... This gives immediately the construction for the case where the two directrix lines come to coincide. In fact, on the twofold directrix line $1=1'$ take the series of points a, b, c.., and through the same line, corresponding homographically to these points, the series of planes A, B, C, ..; the generating lines through the point a are the lines through this point, in the plane A, to the points in which this plane meets the curve m; and so for the entire series of points b, c, .. of the line $1=1'$; the resulting scroll, which I will designate as the scroll S $(\overline{1,1},m)$, remains of the order $=2m$. If there is given a point of the curve m, then the plane through this point and the directrix line is the plane A; and the point a is then also given by the homographic correspondence of the series of planes and points, and the generating line through the given point on the curve m is the line joining this point with the point a.

13. We may say that, in regard to any point a of the line 1, the corresponding plane A is the plane of approach of the coincident line 1'; and that in regard to the same point a and to any plane through it, the trace on that plane of the plane of approach is the line of approach of 1'; that is, we may consider that the coincident directrix line 1' meets the plane through a in a consecutive point on the line of approach. In particular if the point a be the foot of the directrix line 1 (that is, the point where this line meets the plane of the curve m), and the plane through a be the plane of the curve m, then the intersection of the last-mentioned plane by the plane A which corresponds to the point a is the line of approach, and the foot of the coincident directrix line 1' is

the consecutive point to a along the line of approach. The expression "the line of approach," used absolutely, has always the signification just explained, viz. it is the intersection of the plane of the curve m by the plane corresponding to the foot of the directrix line.

14. Suppose now that the line 1 meets the curve m, or, more generally, meets it α times, that is, in an α-tuple point; it might at first sight appear that the coincident line 1′ should also be considered as meeting the curve α times, and that the resulting scroll should be of the order $2m-\alpha-\alpha=2m-2\alpha$. But this is not the case; so long as the direction of the line of approach is arbitrary, the line 1′ must be considered as a line indefinitely near to the line 1, but nevertheless as a line not meeting the curve at all; and the order of the scroll is thus $=2m-\alpha$. If, however, the line of approach is the tangent to a branch through the α-tuple point—that is, if the plane corresponding to the α-tuple point meet the plane of the curve in such tangent, then the coincident line 1′ is to be considered as meeting the curve m in a consecutive point on such branch, and the order of the scroll is $=2m-\alpha-1$. And so if at the multiple point there are β branches having a common tangent, then the coincident line 1′ is to be considered as meeting the curve m in a consecutive point along each of such branches, or say in a consecutive β-tuple point along the branch, and the order of the scroll sinks to $2m-\alpha-\beta$. The point spoken of as the α-tuple point is, it should be observed, more than an α-tuple point with a β-fold tangent; it is really a point of union of an α-tuple point and a β-tuple point, or say a united $\alpha(+\beta)$tuple point, equivalent to

$$\tfrac{1}{2}\alpha(\alpha-1)+\tfrac{1}{2}\beta(\beta-1)$$

double points or nodes; and the case is precisely analogous to that of the scroll S(1, 1, m), where the two directrix lines pass through an α-tuple point and a β-tuple point of the curve m respectively. It may be added that if at the multiple point in question, besides the β branches having a common tangent, there are γ branches having a common tangent, then the point is, so to speak, a united $\alpha(+\beta, +\gamma)$tuple point equivalent to $\tfrac{1}{2}\alpha(\alpha-1)+\tfrac{1}{2}\beta(\beta-1)+\tfrac{1}{2}\gamma(\gamma-1)$ double points or nodes; but the order of the scroll is still $=2m-\alpha-\beta$.

15. In the same way as the scrolls S(1, 1, m) are all included in the case where the order of the scroll, instead of being $=2m$, is $=m$, so the scrolls S($\overline{1, 1}$, m) are all included in the case where the order of the scroll, instead of being $=2m$, is $=m$. That is, we may suppose that the curve m has a united $\alpha(+\beta)$tuple point $(\alpha+\beta=m)$, and may take the directrix line to pass through this point, and the line of approach to be the common tangent of the β branches; and this being so, the order of the scroll will be $2m-\alpha-\beta, =m$. It may be added that if the curve m has, besides the $\alpha(+\beta)$tuple point, a γ-tuple point, then the scroll will have a γ-tuple generating line, and so for the other multiple points of the curve m.

16. We may, in the same way as for the scroll S(1, 1, m), consider the different sections of the scroll S($\overline{1, 1}$, m) of the order m. The general section is a curve of the order m, having an $\alpha(+\beta)$tuple point at the intersection with the directrix line, and a

γ-tuple point, &c. corresponding to the multiple generating lines, if any. A section through the directrix line is in general made up of this line counting α times, and of β generating lines through the point which corresponds to the plane of the section; if the section pass also through a γ-tuple generating line ($\gamma \not> \beta$, in the same way as for the scroll S(1, 1, m)), then, of the β generating lines, γ unite together in the γ-tuple generating line. The general section through a γ-tuple generating line breaks up into this line counting γ times, and a curve of the order $m-\gamma$, which has on the directrix line an $\alpha-\gamma(+\beta-\gamma)$tuple point and a δ-tuple point, &c. at its intersections with the other multiple generating lines, if any.

Equation of the Scroll S(1, 1, m) *of the Order* m, Article Nos. 17 & 18.

17. Taking for the equations of the directrix lines ($x=0$, $y=0$) and ($z=0$, $w=0$), and supposing that these are respectively an α-tuple line and a β-tuple line on the scroll $\alpha+\beta=m$, it is obvious that the equation of the scroll is

$$(*\between x, y)^{\alpha}(z, w)^{\beta}=0.$$

In fact starting with this equation, if we consider the section by a plane through the line ($x=0$, $y=0$), say the plane $y=\lambda x$, then the equation gives

$$x^{\alpha}(*\between 1, \lambda)^{\alpha}(z, w)^{\beta}=0;$$

that is, the section is made up of the line ($x=0$, $y=0$) reckoned α times, and of β other lines in the plane $y=\lambda x$; and the like for the section by any plane through the line ($z=0$, $w=0$), say the plane $z=\nu w$. Hence the assumed equation represents a scroll of the order m, having the two lines for an α-tuple line and a β-tuple line respectively, and conversely such scroll has an equation of the assumed form.

Case of a γ-tuple generating line.

18. The multiple generating line meets each of the lines ($x=0$, $y=0$) and ($z=0$, $w=0$); and we may take for the equations of the multiple generating line $x+y=0$, $z+w=0$. This being so, the foregoing equation of the scroll may be expressed in the form

$$(\dagger\between x, y)^{\alpha}(z, z+w)^{\beta}=0,$$

or say

$$(\mathrm{U}, \mathrm{V}, \mathrm{W}, \ldots)(z, z+w)^{\beta}=0,$$

where U, V, W, ... are functions of the form $(*\between x, y)^{\alpha}$. Hence ($\gamma \not> \alpha$ or β), if the functions U, V, W, ... contain respectively the factors $(x+y)^{\gamma}$, $(x+y)^{\gamma-1}$, $(x+y)^{\gamma-2}$, ..., the equation will be of the form

$$(*\between x+y, z+w)^{\gamma}=0$$

(the coefficients being functions of x, y, z and $z+w$, or, what is the same thing, x, y, z, w, of the order $\alpha+\beta-\gamma$), and the scroll will therefore have the line $x+y=0$, $z+w=0$ as a γ-tuple generating line.

Equation of the Scroll $S(\overline{1,1}, m)$ *of the Order* m, **Article Nos. 19 to 24.**

19. We may take $x=0$, $y=0$ for the equations of the twofold directrix line, $z=0$ for the equation of the plane of the curve m (an arbitrary plane section of the scroll). Then $(\alpha+\beta=m)$, if the curve m have at the point $(x=0, y=0)$, or foot of the directrix line, an $\alpha(+\beta)$tuple point, and if moreover we have $y=0$ for the equation of the common tangent of the β branches (viz. if the plane $y=0$, instead of being an arbitrary plane through the directrix line, be the plane through this line and the common tangent of the β branches), the equation of the curve m will be of the form

$$\Sigma(yw)^{\beta'}(*\between x, y)^{\alpha+\beta-2\beta'}=0,$$

where the summation extends to all integer values of β' from 0 to β, both inclusive.

20. Taking $y=\lambda x$ for the equation of any plane through the directrix line, then the corresponding point on the directrix line will be the intersection of this line ($x=0$, $y=0$) by the plane $z=\theta w$, where $\theta=\dfrac{a\lambda+b}{c\lambda+d}$; the foot of the directrix line is given by the value $\theta=0$, or $\lambda=-\dfrac{b}{a}$, and the equation of the line of approach is therefore $y=-\dfrac{b}{a}x$; this should coincide with the line $y=0$, which is the common tangent of the β branches; that is, we must have $b=0$; I retain, however, for the moment the general value of b.

21. The equations of a generating line will be

$$y=\lambda x,\ z=\theta w-px;$$

and then taking X, Y, (Z=0) and W for the coordinates of the point of intersection with the curve m, we have

$$\mathrm{Y}=\lambda\mathrm{X},\ 0=\theta\mathrm{W}-p\mathrm{X},$$

$$\Sigma(\mathrm{YW})^{\beta'}(*\between \mathrm{X}, \mathrm{Y})^{\alpha+\beta-2\beta'}=0,$$

and thence

$$\Sigma\left(\frac{\lambda p}{\theta}\right)^{\beta'}(*\between 1, \lambda)^{\alpha+\beta-2\beta'}=0,$$

or, what is the same thing,

$$\Sigma\theta^{-\beta'}(\lambda p)^{\beta'}(*\between 1, \lambda)^{\alpha+\beta-2\beta'}=0;$$

which equation, substituting therein for θ its value in terms of λ, gives the parameter p which enters into the equations of the generating line; or, what is the same thing, the equation of the scroll is obtained by eliminating λ, θ, p from the equation just mentioned and the equations

$$y=\lambda x,\ z=\theta w-px,\ \theta=\frac{a\lambda+b}{c\lambda+d}.$$

22. These last three equations give

$$\lambda=\frac{y}{x},\ \theta=\frac{ay+bx}{cy+dx},\ p=\frac{\theta w-z}{x}=\frac{(ay+bx)w-(cy+dx)z}{x};$$

and substituting these values, we find for the equation of the scroll

$$\Sigma(ay+bx)^{\beta-\beta'}y^{\beta'}[(ay+bx)w-(cy+dx)z]^{\beta'}(*\between x, y)^{\alpha+\beta-2\beta'}=0,$$

which is of the order $\alpha+2\beta$, $=2m-\alpha$, so that the $\alpha(+\beta)$tuple point, in the case actually under consideration, produces only a reduction $=\alpha$. If however the line of approach coincides with the tangent of the β branches, then $b=0$; the factor y^β divides out, and the equation is

$$\Sigma(ayw-cyz-dxz)^{\beta'}(*\between x, y)^{\alpha+\beta-2\beta'}=0,$$

which is of the order $\alpha+\beta$, $=m$, so that here the reduction caused by the $\alpha(+\beta)$tuple point is $=\alpha+\beta$. We may without loss of generality substitute ax for $cy+dx$, and then, putting also $a=1$, we find that when the equation of the curve m is as before

$$\Sigma(yw)^{\beta'}(*\between x, y)^{\alpha+\beta-2\beta'}=0,$$

but the plane through the directrix line ($x=0$, $y=0$), and the point on this line, are respectively given by the equations $x=\lambda y$, $z=\lambda w$, the equation of the scroll is

$$\Sigma(yw-xz)^{\beta'}(*\between x, y)^{\alpha+\beta-2\beta'}=0.$$

23. The result may be verified by considering the section by any plane $y=\lambda x$ through the directrix line. Substituting for y this value, we find

$$x^\alpha\Sigma x^{\beta-\beta'}(\lambda w-z)^{\beta'}(*\between 1, \lambda)^{\alpha+\beta-\beta'}=0,$$

which is of the form

$$x^\alpha(\dagger\between x, \lambda w-z)^\beta=0;$$

so that the section is made up of the directrix line ($x=0$, $y=0$) reckoned α times and of β lines in the plane $y-\lambda x=0$, the intersections of the plane $y-\lambda x=0$ by planes such as $z=\lambda w-px$.

Case of a γ-tuple generating line.

24. The equation of the scroll may be written

$$(\mathrm{U}, \mathrm{V}, \mathrm{W}, \ldots\between 1, yw-xz)^\beta=0,$$

where U, V, W, ... are functions of x, y of the forms

$$(*\between x, y)^m,\ (*\between x, y)^{m-2},\ (*\between x, y)^{m-4}, \ldots$$

Assuming that these contain respectively the factors

$$(y-\varkappa x)^\gamma,\ (y-\varkappa x)^{\gamma-1},\ (y-\varkappa x)^{\gamma-2}\ldots,$$

where $\gamma \not> \frac{1}{2}m$, then the equation takes the form

$$(\mathrm{U}', \mathrm{V}', \mathrm{W}'\ldots\between y-\varkappa x,\ w(y-\varkappa x)+x(\varkappa w-z))^\gamma=0,$$

where the coefficients U′, V′, W′, ... are functions of x, y, z, w of the orders $m-\gamma$, $m-\gamma-1$, $m-\gamma-2$, ...; or, what is the same thing, the equation is

$$(\mathrm{U}'', \mathrm{V}'', \mathrm{W}'', \ldots\between y-\varkappa x, \varkappa w-z)^\gamma=0,$$

where U″, V″, W″, ... are functions of x, y, z, w of the order $m-\gamma$. The scroll has thus the γ-tuple generating line

$$y-\varkappa x=0,\ \varkappa w-z=0.$$

Cubic Scrolls, Article Nos. 25 to 35.

25. In the case of a cubic scroll there is necessarily a nodal* line; in fact for the m-thic scroll there is a nodal curve which is of the order $m-2$ at least, and of the order $\frac{1}{2}(m-1)(m-2)$ at most, and which for $m=3$ is therefore a right line. And moreover we see at once that every cubic surface having a nodal line is a scroll; in fact any plane whatever through the nodal line meets the surface in this line counting as 2 lines, and in a curve of the order 1, that is, a line; there are consequently on the surface an infinity of lines, or the surface is a scroll. We have therefore to examine the cubic surfaces which have a nodal line.

26. Let the equations of the nodal line be $x=0$, $y=0$; then the equation of the surface is

$$\mathrm{U}z+\mathrm{V}w+\mathrm{Q}=0,$$

where U, V, Q are functions of (x, y) of the orders 2, 2, 3 respectively. Suppose first that U, V have no common factor, then we may write

$$\mathrm{Q}=(\alpha x+\beta y)\mathrm{U}+(\gamma x+\delta y)\mathrm{V};$$

and substituting this value, and changing the values of z and w, the equation of the surface is of the form

$$\mathrm{U}z+\mathrm{V}w=0,$$

or, what is the same thing,

$$(*\between x, y)^2(z, w)=0;$$

so that, besides the nodal directrix line ($x=0$, $y=0$), the scroll has the simple directrix line ($z=0$, $w=0$): it is clear that the section by any plane whatever is a cubic curve having a node at the foot of the nodal directrix line ($x=0$, $y=0$), and passing through the foot of the simple directrix line ($z=0$, $w=0$); that is, it is a cubic scroll of the kind S(1, 1, 3); and since for $m=3$ the only partition $m=\alpha+\beta$ is $m=2+1$, there is only one kind of cubic scroll S(1, 1, 3), and we may say *simpliciter* that the scroll in question is the cubic scroll S(1, 1, 3).

27. If however the functions U, V have a common factor, say $(\lambda x+\mu y)$, then $z\mathrm{U}+w\mathrm{V}$ will contain this same factor, and the remaining factor will be of the form

$$z(\alpha x+\beta y)+w(\gamma x+\delta y),\ =y(\beta z+\delta w)+x(\alpha z+\gamma w),$$

or, changing the values of z and w, the remaining factor will be of the form $yw-xz$, and the equation of the scroll thus is

$$(\lambda x+\mu y)(yw-xz)+(*\between x, y)^3=0,$$

where it is clear that the section by any plane whatever is a cubic curve having a node at the foot of the directrix line $x=0$, $y=0$. The scroll is thus a cubic scroll of the form S($\overline{1, 1}$, 3), viz. it is the scroll of the kind where the section is a cubic curve with a 2(+1)tuple point (ordinary double point, or node), the line of approach being one of the two tangents at the node; and since for $m=3$ the only partition $m=\alpha+\beta$ is

* The nodal line of a cubic scroll is of course a double line, and in regard to these scrolls the epithets 'nodal' and 'double' may be used indifferently.

$m=2+1$, there is only one kind of cubic scroll $(\overline{1,1}, 3)$, and we may say *simpliciter* that the scroll in question is the cubic scroll $S(\overline{1,1}, 3)$. The conclusion therefore is that for cubic scrolls we have only the two kinds, $S(1, 1, 3)$ and $S(\overline{1,1}, 3)$. The foregoing equations of these scrolls admit however of simplification; and I will further consider the two kinds respectively.

The Cubic Scroll S(1, 1, 3).

28. Starting from the equation

$$(*\between x, y)^2(z, w)=0,$$

or, writing it at full length,

$$z(a, b, c\between x, y)^2+w(a', b', c'\between x, y)^2=0,$$

we may find θ_1, θ_2 so that

$$(a, b, c\between x, y)^2+\theta_1(a', b', c'\between x, y)^2=(p_1x+q_1y)^2,$$
$$(a, b, c\between x, y)^2+\theta_2(a', b', c'\between x, y)^2=(p_2x+q_2y)^2,$$

θ_1 and θ_2 being unequal, since by hypothesis $(a, b, c\between x, y)^2$ and $(a', b', c'\between x, y)^2$ have no common factor. This gives

$$(a, b, c\between x, y)^2=\alpha(p_1x+q_1y)^2+\beta(p_2x+q_2y)^2,$$
$$(a', b', c'\between x, y)^2=\gamma(p_1x+q_1y)^2+\delta(p_2x+q_2y)^2;$$

or the equation becomes

$$(\alpha z+\gamma w)(p_1x+q_1y)^2+(\beta z+\delta w)(p_2x+q_2y)^2=0;$$

or changing the values of (x, y) and of (z, w), the equation is

$$x^2z+y^2w=0,$$

which may be considered as the canonical form of the equation. It may be noticed that the Hessian of the form is x^2y^2.

29. We may of course establish the theory of the surface from the equation $x^2z+y^2w=0$; the equation is satisfied by $x=\lambda y$, $w=-\lambda^2z$, which are the equations of a line meeting the line $(x=0, y=0)$ (1) and the line $(z=0, w=0)$ (1'). The generating line meets also any plane section of the surface; in fact, if the equation of the plane of the section be $\alpha x+\beta y+\gamma z+\delta w=0$, then we have at once

$$x : y : z : w=\delta\lambda^3-\gamma\lambda : \delta\lambda^2-\gamma : \alpha\lambda+\beta : -\alpha\lambda^3-\beta\lambda^2$$

for the coordinates of the point of intersection.

30. The form of the equation shows that there are on the line 1 two points, viz. the points $(x=0, y=0, z=0)$ and $(x=0, y=0, w=0)$, through each of which there passes a pair of coincident generating lines: calling these A and B, then, if the coincident lines through A meet the line 1' in C, and the coincident lines through B meet the line 1' in D, it is easy to see that $x=0$, $y=0$, $z=0$, and $w=0$ will denote the equations of the planes BAC, BAD, BCD, and ACD respectively.

31. We obtain also the following construction: take a cubic curve having a node, and from any point K on the curve draw to the curve the tangents Kp, Kq; through the points of contact draw at pleasure the lines pAC and qBD; through the node draw a line meeting these two lines in the points A, B respectively, this will be the line 1; and through the point K a line meeting the same two lines in the points C and D respectively, this will be the line 1′; and, the equations $x=0$, $y=0$, $z=0$, $w=0$ denoting as above, the equation of the surface will be $x^2z+y^2w=0$.

The points A and B are cuspidal points on the nodal line; any section of the scroll by a plane through one of these points is a cubic curve having at the point in question a cusp.

32. It is to be noticed however that the cuspidal points are not of necessity real; if for x, y we write $x+\iota y$, $x-\iota y$, and in like manner $z+\iota w$, $z-\iota w$ for z, w, then the equation takes the form

$$(x^2-y^2)z-2xyw=0,$$

which is a cubic scroll S(1, 1, 3) with the cuspidal points imaginary.

In the last-mentioned case the nodal line is throughout its whole length crunodal; in the case first considered, where the equation is $x^2z+y^2w=0$, the nodal line is for that part of its length for which z, w have opposite signs, crunodal; and for the remainder of its length, or where z, w have the same sign, acnodal. There are two different forms, according as the line is for the portion intermediate between the cuspidal points crunodal and for the extramediate portions acnodal, or as it is for the intermediate portion acnodal and for the extramediate portions crunodal.

Cubic Scroll S($\overline{1, 1}$, 3).

33. Starting from the equation

$$(\lambda x+\mu y)(yw-xz)+(*\between x, y)^3=0,$$

then putting $w-\mu z$ for w and λz for z, this may be written

$$(\lambda x+\mu y)\{yw-z(\lambda x+\mu y)\}+(*\between \lambda x+\mu y, y)^3=0,$$

or, what is the same thing,

$$x(yw-xz)+(*\between x, y)^3=0;$$

and then, if $(*\between x, y)^3=(\alpha, \beta, \gamma, \delta\between x, y)^3$, this may be written

$$x\{y(w+\beta x+\gamma y)-x(z-\alpha x)\}+\delta y^3=0;$$

or changing the values of w and z, we have

$$x(yw-xz)+y^3=0$$

for the equation of the scroll S($\overline{1, 1}$, 3)*.

* It is somewhat more convenient to change the sign of z, and take $x(yw+xz)+y^3=0$ as the canonical form.

34. The Hessian of the form is x^4, and it thus appears that the plane $x=0$ is a determinate plane through the double line. But $y=0$ is not a determinate plane; in fact, if for y we write $y+\lambda x$, the equation is

$$-x^2z+xw(y+\lambda x)+(y+\lambda x)^3=0,$$

that is,

$$-x^2(z-\lambda w-3\lambda^2y-\lambda^3x)+xy(w+3\lambda x)+y^3=0,$$

which, changing z and w, is still of the form $x(yw-xz)+y^3=0$.

The planes $z=0$, $w=0$ will alter with the plane $y=0$, but they are not determined even when the plane $y=0$ is determined; in fact we may, without altering the equation, change w, z into $w+\theta y$, $z+\theta x$ respectively.

35. In the equation $x(yw-xz)+y^3=0$, writing $y=\lambda x$, we find for the equations of a generating line, $y=\lambda x$, $z=\lambda w+\lambda^3x$. Considering the section by the plane $\alpha x+\beta y+\gamma z+\delta w=0$, we have

$$x:y:z:w=-\gamma\lambda-\delta:-\gamma\lambda^2-\delta\lambda:-\delta\lambda^3+\beta\lambda^2+\alpha\lambda:\gamma\lambda^3+\beta\lambda+\alpha$$

for the coordinates of the point where the generating line meets the section.

The generating line meets the nodal line at the intersection of the nodal line by the plane $z=\lambda w$; that is, the points $z=\lambda w$ on the nodal line correspond to the planes $y=\lambda x$ through the nodal line. In particular the point $w=0$ on the nodal line corresponds to the plane $x=0$ through the nodal line: the point $\gamma z+\delta w=0$ on the nodal line (that is, the point where this line is met by the plane $\alpha x+\beta y+\gamma z+\delta w=0$) corresponds to the plane $\gamma x+\delta y=0$ through the nodal line; the intersections of the plane $\alpha x+\beta y+\gamma z+\delta w=0$ by this plane $\gamma x+\delta y=0$, and by the plane $x=0$, are the tangents of the section at the node.

Quartic Scrolls, Article Nos. 36 to 50.

36. We may consider, first, the quartic scrolls S(1, 1, 4). The section is a quartic curve having an α-tuple point and a β-tuple point, where $\alpha+\beta=4$; that is, we have $\alpha=2$, $\beta=2$, a quartic with two nodes (double points), or else $\alpha=3$, $\beta=1$, a quartic with a triple point. But the case $\alpha=2$, $\beta=2$ gives rise to two species: viz., in general the quartic has only the two double points, and we have then a scroll with two nodal (2-tuple) directrix lines, and without any nodal generator; the section *may* however have a third double point, and the scroll has then a nodal (double) generator. For the case $\alpha=3$, $\beta=1$, the section admits of no further singularity, and we have a quartic scroll with a triple directrix line and a single directrix line.

37. Next for the quartic scrolls S($\overline{1,1}$, 4). The section is here a quartic curve with an $\alpha(+\beta)$tuple point, where $\alpha+\beta=4$; that is, $\alpha=2$, $\beta=2$, or else $\alpha=3$, $\beta=1$. In the former case the section has a 2(+2)tuple point, that is, a double point where the two branches have a common tangent—otherwise, two coincident double points: say the curve has a *tacnode*; the line of approach is the tangent at the tacnode. We have here a scroll with a twofold double line; there are however two cases: viz., in general

the section has, besides the tacnode, no other double point; that is, the scroll has no nodal generator: the section *may* however have a third double point, and the scroll has then a nodal (double) generator. In the case $\alpha=3$, $\beta=1$ the section has a triple point, and the line of approach is the tangent at one of the branches at the triple point; the scroll has a twofold, say a $3(+1)$tuple directrix line: as the section admits of no further singularity, this is the only case. The foregoing enumeration gives three species of quartic scrolls $S(1, 1, 4)$, and three species of quartic scrolls $S(\overline{1, 1}, 4)$, together six species, viz. these are as follows:—

Quartic Scroll, First Species, $S(1_2, 1_2, 4)$, *with two double directrix lines, and without a nodal generator.*

38. Taking ($x=0$, $y=0$) and ($z=0$, $w=0$) for the equations of the two directrix lines respectively, the equation of the scroll is

$$(*\between x, y)^2(z, w)^2=0.$$

Quartic Scroll, Second Species, $S'(1_2, 1_2, 4)$, *with two double directrix lines, and with a double generator.*

39. This is in fact a specialized form of the first species, the difference being that there is a nodal (double) generator. Supposing as before that the equations of the directrix lines are ($x=0$, $y=0$) and ($z=0$, $w=0$) respectively; let the equations of the nodal generator be ($x+y=0$, $z+w=0$); then, observing that for the first species the equation may be written $(*\between x, y)^2(z, z+w)^2=0$, it is clear that if the terms in z^2 and $z(z+w)$ are divisible by $(x+y)^2$ and $(x+y)$ respectively, the surface will have as a new double line the line ($x+y=0$, $z+w=0$), which will be a double generator; and we thus arrive at the equation of the second species of quartic scrolls, viz. this is

$$((x+y)^2, (x+y)(x, y), (x, y)^2\between z, z+w)^2=0.$$

Quartic Scroll, Third Species, $S(1_3, 1, 4)$, *with a triple directrix line and a single directrix line.*

40. Taking ($x=0$, $y=0$) for the equations of the triple directrix line, and ($z=0$, $w=0$) for the equations of the single directrix line, the equation is

$$(*\between x, y)^3(z, w)=0.$$

Quartic Scroll, Fourth Species, $S(\overline{1_2, 1_2}, 4)$, *with a twofold* ($2(+2)$tuple) *directrix line, and without a nodal generator.*

41. Taking ($x=0$, $y=0$) for the equations of the directrix line, $z=0$ for that of a plane section of the scroll, $y=0$ for the equation of a plane through the tangent at the tacnode of the section, and supposing (see *ante*, No. 22) that the plane through the directrix line and the corresponding point on this line are respectively given by the equations $x=\lambda y$ and $z=\lambda w$, the equation of the scroll is

$$(yw-xz)^2+(yw-xz)(x, y)^2+(x, y)^4=0.$$

Quartic Scroll, Fifth Species, $S'(\overline{1_2, 1_2}, 4)$, *with a twofold* (2(+2)tuple) *generating line, and with a double generator.*

42. Let the equations of the double generator be $x+y=0$, $z+w=0$; then the line in question must be a double line on the surface represented by the last-mentioned equation, and this will be the case if only the second and third terms contain the factors $(x+y)$ and $(x+y)^2$ respectively. The equation for the fifth species consequently is

$$(yw-xz)^2+2(yw-xz)(x+y)(x, y)+(x+y)^2(x, y)^2=0.$$

Quartic Scroll, Sixth Species, $S(\overline{1_3, 1}, 4)$, *with a twofold* (3(+1)tuple) *generating line.*

43. Taking $(x=0, y=0)$ for the equations of the directrix line, $z=0$ for the equation of a plane section, and assuming that the plane $y=0$ passes through the tangent which is the line of approach, and that the plane through the directrix line and the corresponding point on this line are respectively given by the equations $x=\lambda y$ and $z=\lambda w$, the equation of the scroll is

$$(yw-xz)(x, y)^2+(x, y)^4=0.$$

I refrain on the present occasion from a more particular discussion of the foregoing six species of quartic scrolls. I establish two other species, as follows:—

Quartic Scroll, Seventh Species, S(1, 2, 2), *with nodal directrix line, and nodal directrix conic which meet, and with a simple directrix conic which meets the nodal conic in two points.*

44. We see, *à priori*, that the scroll generated as above will be of the order 4, that is, a quartic scroll. In fact using the formula (*ante*, No. 5),

$$\text{Order} = 2mnp-\alpha m-\beta n-\gamma p,$$

we have here

$$\begin{array}{lll} \text{Nodal conic,} & m=2, & \alpha=0, \\ \text{Simple conic,} & n=2, & \beta=1, \\ \text{Line} \quad , & p=1, & \gamma=2, \end{array}$$

and hence

$$\text{Order} = 8-2-2, =4.$$

45. Take $(x=0, y=0)$ for the equations of the directrix line, $z=0$ for the equation of the plane of the simple conic, $w=0$ for that of the plane of the nodal conic; since the conics intersect in two points, they lie on a quadric surface, say the surface $U=0$; the equations of the simple conic thus are $z=0$, $U=0$; those of the nodal conic are $w=0$, $U=0$. The directrix line $x=0$, $y=0$ meets the nodal conic; that is, U must vanish identically for $x=0$, $y=0$, $w=0$; and this will be the case if only the term in z^2 is wanting; that is, we must have

$$U=(a, b, 0, d, f, g, h, l, m, n \between x, y, z, w)^2.$$

But we may in the first instance omit the condition in question, and write

$$U=(a, b, c, d, f, g, h, l, m, n \between x, y, z, w)^2;$$

this would lead to a sextic instead of a quartic scroll.

46. The equations of a generating line (since it meets the directrix line $x=0$, $y=0$) may be taken to be

$$x=\alpha y,\ z=\beta\left(y-\frac{w}{\theta}\right);$$

the condition in order to the intersection of the generating line with the nodal conic is at once found to be

$$a\alpha^2+2h\alpha+b+2\beta(f+g\alpha)+c\beta^2=0,$$

and that for its intersection with the simple conic

$$a\alpha^2+2h\alpha+b+2\theta(m+l\alpha)+d\theta^2=0;$$

and writing the equations of the generating line in the form

$$\alpha=\frac{y}{x},\quad \theta=\frac{\beta w}{\beta y-z},$$

the elimination of α, β, θ from these four equations gives the required equation of the scroll. Writing for a moment

$$\begin{aligned}\Theta&=a\alpha^2+2h\alpha+\beta,\\ \mathrm{F}&=g\alpha+f\\ \mathrm{M}&=l\alpha+m\end{aligned}$$

we find

$$\begin{aligned}&c\beta^2+2\mathrm{F}\beta+\Theta=0,\\ &(\Theta y^2+2\mathrm{M}yw+dw^2)\beta^2-2(\Theta yz+\mathrm{M}wz)\beta+\Theta z^2=0;\end{aligned}$$

or, introducing at this place the condition $c=0$, the first equation gives $\beta=-\frac{\Theta}{2\mathrm{F}}$, and we thence obtain

$$\Theta(\Theta y^2+2\mathrm{M}yw+dw^2)+4\mathrm{F}(\Theta yz+\mathrm{M}wz)+4\mathrm{F}^2z^2=0,$$

or, what is the same thing,

$$(\Theta y+2\mathrm{F}z)^2+2\mathrm{M}w(\Theta y+2\mathrm{F}z)+\Theta dw^2=0;$$

whence, observing that we have

$$\Theta=\frac{ax^2+2hxy+by^2}{y^2},\quad \mathrm{F}=\frac{gx+fy}{y},\quad \mathrm{M}=\frac{lx+my}{y},$$

the equation of the scroll is

$$\begin{aligned}&(ax^2+2hxy+by^2+2gzx+2fyz)^2\\ &+2(ax^2+2hxy+by^2+2gzx+2fyz)(lx+my)w\\ &+(ax^2+2hxy+by^2)dw^2=0.\end{aligned}$$

And we see from the equation that the surface contains the line ($x=0$, $y=0$) as a double line, the conic

$$w=0,\quad ax^2+2hxy+by^2+2gzx+2fyz=0$$

as a double curve, also the conic

$$z=0,\quad ax^2+2hxy+by^2+2lxw+2myw+dw^2=0$$

as a simple curve on the surface,—the complete intersection by the plane $z=0$ being in fact the last-mentioned conic, and the pair of lines

$$z=0, \quad ax^2+2hxy+by^2=0.$$

Quartic Scroll, Eighth Species, S(1, 3^2), *with a directrix line, and a directrix skew cubic met twice by each generating line.*

47. We see, *à priori*, that the scroll is of the order 4, that is, a quartic scroll; in fact for the scroll S(1, m^2) the order is $=[m]^2+\mathrm{M}$ (first memoir, p. 457), and we have here $m=3$, $\mathrm{M}=h-\frac{1}{2}[m]^2=1-3=-2$; that is, order $=6-2$, $=4$.

48. The equations of the cubic curve may be taken to be

$$\left\| \begin{matrix} x, & y, & z \\ y, & z, & w \end{matrix} \right\| =0,$$

or, what is the same thing,

$$xz-y^2=0, \quad xw-yz=0, \quad yw-z^2=0;$$

those of the directrix line may be represented by

$$\alpha x+\beta y+\gamma z+\delta w=0,$$
$$\alpha' x+\beta' y+\gamma' z+\delta' w=0;$$

or, what is the same thing, if

$$\beta\gamma'-\beta'\gamma=a, \quad \alpha\delta'-\alpha'\delta=f,$$
$$\gamma\alpha'-\gamma'\alpha=b, \quad \beta\delta'-\beta'\delta=g,$$
$$\alpha\beta'-\alpha'\beta=c, \quad \gamma\delta'-\gamma'\delta=h$$

(and therefore identically $af+bg+ch=0$), the line is defined by means of its "six coordinates" (a, b, c, f, g, h).

49. The equations of the cubic curve are satisfied by writing therein

$$x:y:z:w=1:t:t^2:t^3,$$

and therefore the coordinates of any two points on the curve may be represented by $(1, \theta, \theta^2, \theta^3)$ and $(1, \phi, \phi^2, \phi^3)$; hence, if x, y, z, w are the coordinates of a point in the line joining the last mentioned two points, we have

$$x:y:z:w=l+m:l\theta+m\phi:l\theta^2+m\phi^2:l\theta^3+m\phi^3,$$

which equations, treating therein l, m as indeterminate parameters, give the equations of the line in question. And putting moreover

$$p=yw-z^2, \quad q=yz-xw, \quad r=xz-y^2,$$

we have identically

$$p:q:r=\theta\phi:-(\theta+\phi):1.$$

50. In order that the line in question may meet the directrix line, we must have

$$l(\alpha+\beta\theta+\gamma\theta^2+\delta\theta^3)+m(\alpha+\beta\phi+\gamma\phi^2+\delta\phi^3)=0,$$
$$l(\alpha'+\beta'\theta+\gamma'\theta^2+\delta'\theta^3)+m(\alpha'+\beta'\phi+\gamma'\phi^2+\delta'\phi^3)=0;$$

that is, eliminating l and m, we must have

$$\begin{vmatrix} \alpha+\beta\theta+\gamma\theta^2+\delta\theta^3, & \alpha+\beta\phi+\gamma\phi^2+\delta\phi^3 \\ \alpha'+\beta'\theta+\gamma'\theta^2+\delta'\theta^3, & \alpha'+\beta'\phi+\gamma'\phi^2+\delta'\phi^3 \end{vmatrix}=0,$$

or, developing,

$$(\alpha\beta'-\alpha'\beta)(\phi-\theta)+(\alpha\gamma'-\alpha'\gamma)(\phi^2-\theta^2)+(\alpha\delta'-\alpha'\delta)(\phi^3-\theta^3)$$
$$+(\beta\gamma'-\beta'\gamma)(\theta\phi^2-\theta^2\phi)+(\beta\delta'-\beta'\delta)(\theta\phi^3-\theta^3\phi)+(\gamma\delta'-\gamma'\delta)(\theta^2\phi^3-\theta^3\phi^2)=0;$$

the several terms in (θ, ϕ), each divided by $\overline{\phi-\theta}$, give respectively

$$1,\ \phi+\theta,\ (\phi+\theta)^2-\phi\theta,\ \theta\phi,\ \theta\phi(\phi+\theta),\ \theta^2\phi^2,$$

which are equal to

$$(r^2,\ -qr,\ q^2-pr,\ pr,\ -pq,\ p^2);$$

hence replacing also $\alpha\beta'-\alpha'\beta$, &c. by their values c, &c., we find

$$(c,\ -b, f,\ a,\ g,\ h)(r^2,\ -qr,\ q^2-pr,\ pr,\ -pq,\ p^2)=0,$$

or, what is the same thing,

$$(h, f,\ c,\ b,\ a-f,\ -g \between p,\ q,\ r)^2=0,$$

where the coefficients (a, b, c, f, g, h) satisfy the relation $af+bg+ch=0$; p, q, r stand respectively for

$$yw-z^2,\quad yz-xw,\quad xz-y^2.$$

Writing for greater convenience

$$(h, f,\ c,\ b,\ a-f,\ -g)=(\mathrm{a},\ \mathrm{b},\ \mathrm{c},\ 2\mathrm{f},\ 2\mathrm{g},\ 2\mathrm{h}),$$

or, what is the same thing,

$$(a, b, c, f, g, h)=(\mathrm{b}+2\mathrm{g},\ 2\mathrm{f},\ \mathrm{c},\ \mathrm{b},\ -2\mathrm{h},\ \mathrm{a}),$$

then we have

$$af+bg+ch=\mathrm{ac}+\mathrm{b}^2+2\mathrm{bg}-4\mathrm{fh}=0.$$

And hence finally we have for the equation of the scroll $S(1, 3^2)$,

$$(\mathrm{a},\ \mathrm{b},\ \mathrm{c},\ \mathrm{f},\ \mathrm{g},\ \mathrm{h} \between yw-z^2,\quad yz-xw,\quad xz-y^2)^2=0,$$

where the coefficients satisfy the relation

$$\mathrm{ac}+\mathrm{b}^2+2\mathrm{bg}-4\mathrm{fh}=0.$$

The equations of the directrix cubic are of course

$$yw-z^2=0,\quad yz-xw=0,\quad xz-y^2=0;$$

and the directrix line is given by its six coordinates,

$$(\mathrm{b}+2\mathrm{g},\ 2\mathrm{f},\ \mathrm{c},\ \mathrm{b},\ -2\mathrm{h},\ \mathrm{a})$$

On the general Theory of Scrolls, Article Nos. 51 to 53.

51. I annex in conclusion the following considerations on the general theory of scrolls. Consider a scroll of the nth order; the intersection by an arbitrary plane, say the plane $w=0$, is a curve of the nth order $(*\between x, y, z)^n=0$; any point $(x, y, z, 0)$ where (x, y, z) satisfy the foregoing equation, is the foot of a generating line; and we may

imagine this generating line determined by means of the coordinates (X, Y, Z, W), given functions of (x, y, z) of a point on the line. This being so, the "six coordinates," say (p, q, r, s, t, u), of the line are

$$\begin{matrix} \mathrm{X}, & \mathrm{Y}, & \mathrm{Z}, & \mathrm{W} \\ x, & y, & z, & 0 \end{matrix}$$

viz.

$$\begin{aligned} p&=\mathrm{Y}z-\mathrm{Z}y, & s&=-\mathrm{W}x, \\ q&=\mathrm{Z}x-\mathrm{X}z, & t&=-\mathrm{W}y, \\ r&=\mathrm{X}y-\mathrm{Y}x, & u&=-\mathrm{W}z; \end{aligned}$$

or, writing for greater convenience $-v$ in the place of W, the six coordinates of the line are p, q, r, vx, vy, vz, where p, q, r are functions of (x, y, z), connected by the relation $px+qy+vz=0$; and v is also a function of (x, y, z).

52. Consider the intersection of the surface by an arbitrary line the six coordinates whereof are (A, B, C, F, G, H); then for the generating lines which meet this line we have

$$v(\mathrm{A}x+\mathrm{B}y+\mathrm{C}z)+\mathrm{F}p+\mathrm{G}q+\mathrm{H}r=0.$$

And this equation, together with the equation $(*\between x, y, z)^n=0$, determines (x, y, z), the coordinates of the foot of a generating line which meets the arbitrary line (A, B, C, F, G, H). Since the order of the scroll is equal n, the number of such generating lines should be $=n$, that is, there should be n relevant intersections of the two curves,

$$\begin{aligned} &v(\mathrm{A}x+\mathrm{B}y+\mathrm{C}z)+\mathrm{F}p+\mathrm{G}q+\mathrm{H}r=0, \\ &(*\between x, y, z)^n=0. \end{aligned}$$

But if (p, q, r, vx, vy, vz) are each of the order k, the number of actual intersections is $=kn$, which is too many by $(k-1)n$.

53. Suppose that the curves

$$p=0,\ q=0,\ r=0,\ vx=0,\ vy=0,\ vz=0,$$

or say the curves

$$p=0,\ q=0,\ r=0,\ v=0$$

have in common θ intersections, and let these be points of the multiplicities $\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_\theta$ on the curve $(*\between x, y, z)^n=0$ (viz. according as the curve does not pass through any one of the intersections in question, or passes once, twice, &c. through such intersection, we have for that intersection $\alpha_1=0, 1, 2$, &c., as the case may be, and so for the other intersections); then the kn points of intersection include the $\alpha_1+\alpha_2 \ldots . +\alpha_\theta$, or say the $\Sigma\alpha$ intersections; but these, being independent of the line (A, B, C, F, G, H) under consideration, are irrelevant points, and the number of relevant points of intersection is $kn-\Sigma\alpha$; that is, if we have $\Sigma\alpha=(k-1)n$, then the scroll in question, viz. the scroll generated by a line which meets the plane $w=0$ in the curve $(*\between x, y, z)^n=0$, and which has for its six coordinates (p, q, r, vx, vy, vz), will be a scroll of the nth order.

XVI. *Algebraical Researches, containing a disquisition on* NEWTON'S *Rule for the Discovery of Imaginary Roots, and an allied Rule applicable to a particular class of Equations, together with a complete invariantive determination of the character of the Roots of the General Equation of the fifth Degree, &c.* *By* J. J. SYLVESTER, *M.A., F.R.S., Correspondent of the Institute of France, Foreign Member of the Royal Society of Naples, etc. etc., Professor of Mathematics at the Royal Military Academy, Woolwich.*

Received April 6,—Read April 7, 1864.

Turns them to shapes and gives to airy nothing
A local habitation and a name.

(1) THIS memoir in its present form is of the nature of a trilogy; it is divided into three parts, of which each has its action complete within itself, but the same general cycle of ideas pervades all three, and weaves them into a sort of complex unity. In the first is established the validity of NEWTON'S rule for finding an inferior limit to the number of imaginary roots of algebraical equations as far as the fifth degree inclusive. In the second is obtained a rule for assigning a like limit applicable to equations of the form $\Sigma(ax+b)^m=0$, m being any positive integer, and the coefficients a, b real. In the third are determined the absolute invariantive criteria for fixing unequivocally the character of the roots of an equation of the fifth degree, that is to say, for ascertaining the exact number of real and imaginary roots which it contains. This last part has been added since the original paper was presented to the Society. It has grown out of a foot-note appended to the second, itself an independent offshoot from the first part, but may be studied in a great measure independently of what precedes, and constitutes, in the author's opinion, by far the most valuable portion of the memoir, containing as it does a complete solution of one of the most interesting and fruitful algebraical questions which has ever yet engaged the attention of mathematicians (1). I propose in a subsequent addition to the memoir to resume and extend some of the investigations which incidentally arise in this part. The foot-notes are numbered and lettered for facility of reference, and will be found in many instances of equal value with the matter in the text, to which they serve as a kind of free running accompaniment and commentary.

(1) I owe my thanks to my eminent friend Professor DE MORGAN for bringing under my notice, in a marked manner, the original question from which all the rest has proceeded. As all roads are said to lead to Rome, so I find, in my own case at least, that all algebraical inquiries sooner or later end at that Capitol of Modern Algebra over whose shining portal is inscribed "Theory of Invariants."

PART I.—ON NEWTON'S RULE FOR THE DISCOVERY OF IMAGINARY ROOTS.

(2) In the 'Arithmetica Universalis,' in the first chapter on equations, NEWTON has given a rule for discovering an inferior limit to the number of imaginary roots in an equation of any degree, without proof or indication of the method by which he arrived at it, or the evidence upon which it rests([2]). MACLAURIN, in vol. xxxiv. p. 104, and vol. xxxvi. p. 59 of the Philosophical Transactions, CAMPBELL([3]) in vol. xxxviii. p. 515 of the same, and other authors of reputation have sought in vain for a demonstration of this marvellous and mysterious rule([4]). Unwilling to rest my belief in it on mere empirical evidence, I

([2]) It appears to be the prevalent belief among mathematicians who have considered the question, that NEWTON was not in possession of other than empirical evidence in support of his rule.

([3]) CAMPBELL's memoir is rather on an analogous rule to NEWTON's than on the rule itself, to which he refers only by way of comparison with his own. In it the same singular error of reasoning is committed as in the notes of the French edition of the 'Arithmetica,' viz. of assuming, without a shadow of proof, that if each of a set of criteria indicates the existence of some imaginary roots, a succession of sets of such criteria must indicate the existence of at least as many distinct imaginary pairs of roots as there are such sets (see par. at foot of p. 528, Phil. Trans., vol. xxxv.)—much as if, supposing a number of dogs to be making a point in the same field, the existence could be assumed of as many birds as pointers.

([4]) Mr. ARCHIBALD SMITH has obligingly called my attention to WARING's treatment of the question of NEWTON's rule in the 'Meditationes Analyticæ.' On superficial examination the reader might be induced to suppose that in part 9, p. 68, ed. 1782, WARING had deduced a proof of the rule from the preceding propositions; but on looking into the case will find that there is not the slightest vestige of proof, the rule being stated, but without any demonstration whatever being either adduced or alleged. In fact, on turning to the preface of this (the last) edition of the 'Meditationes,' the reader will find at p. 11 an explicit avowal of the demonstration being wanting. After referring in order to CAMPBELL's, MACLAURIN's, and NEWTON's rules, as well as his own, for discovering the existence of impossible roots, he adds these words:

"At omnes hæ regulæ prædictæ perraro invenerunt verum numerum impossibilium radicum in æquationibus multarum dimensionum *et adhuc demonstratione egent*; vulgares enim demonstrationes solummodo probant impossibiles radices in data æquatione contineri, non vero quod *saltem tot sunt quot invenit regula*."

"Vera resolutio problematis est perdifficilis et valde laboriosa; cognitum est radices ex possibilitate per æqualitatem transire ad impossibilitatem; ergo in generali resolutione hujusce problematis necesse est invenire casum in quo radices datæ æquationis evadunt æquales; resolutio autem hujus casus valde laboriosa est; et consequenter resolutio generalis prædicti problematis magis *erit* laboriosa."

Written in Latin, and when the proper language of algebra was yet unformed, it is frequently a work of much labour to follow WARING's demonstrations and deductions, and to distinguish his assertions from his proofs. I find he agrees with the opinion expressed by myself, that NEWTON's rule will *not* "pene," as stated by NEWTON, but only "perraro," give the true number of imaginary roots. Like myself, too, in the body of the memoir WARING has given theorems of probability in connexion with rules of this kind, but without any clue to his method of arriving at them. Their correctness may legitimately be doubted.

[Since the above was sent to press, I have been enabled to ascertain that the great name of EULER is to be added to the long list of those who have fallen into error in their treatment of this question: see Institutiones Calculi Differentialis, vol. ii. cap. xiii. He says (p. 555, edition of Prony), "videndum est utrum hæc duo criteria (meaning NEWTON's criteria of imaginariness) sint contigua necne; priori casu numerus radicum imaginarium non augebitur; posteriori vero *quia criteria litteras prorsus diversas involvunt*, unumquodque binas radices imaginarias monstrabit."

The force of the supposed argument is contained in the words in italics. It is sufficiently met by the question, why or how the conclusion follows from them? Moreover the letters of two non-contiguous criteria are *not* necessarily *prorsus diversæ*; for two criteria with but a single other intervening between them will contain one letter in common.]

have investigated and obtained a demonstration of its truth as far as the fifth degree inclusive, which, although presenting only a small instalment of the desired result, I am induced to offer for insertion in the Transactions in the hope of exciting renewed attention to a subject so intimately bound up with the fundamental principles of algebra.

Before commencing the inquiry I ought to state that, in addition to the rule for detecting the existence of a certain number of imaginary roots, NEWTON has given a remarkable subsidiary method for dividing this number into two parts, representing respectively how many of the positive and how many of the negative roots indicated by DESCARTES's rule are, so to say, absorbed, and thereby obtains two distinct limits to the number of positive and the number of negative roots separately: of the grounds of this method, as far as I am aware, no one has even attempted an explanation, nor do I propose here to enter upon it; the rule, as I treat it, may be stated, not in NEWTON's own words, but most simply as follows:—

If the literal parts of the coefficients of an equation affected with the usual binomial coefficients be a, b, c, d, e ... h, k, l, *and if we form the successive criteria* b^2-ac; c^2-bd; d^2-ce; ...; k^2-hl, *or, which is the same thing differently expressed, if we write down the determinants*(5) *of all the successive quadratic derivatives of the given equation, then as many sequences as there are of negative signs in the arithmetical values of these criteria, so many pairs of imaginary roots at least there will be in the given equation.* If we choose to consider a^2 and l^2 also as criteria, appearing at the beginning and end of the series, then we may vary the expression of the rule by saying that there will be at least as many imaginary roots as there are variations of sign in the complete series so formed.

It will, however, be found more convenient for our present purpose to confine the designation of criteria to the determinants above alluded to.

(3) I shall deal with the homogeneous equation $f(x, y)=0$ so that the question of the reality of the roots is that of the reality of the ratios $\frac{x}{y}$ or $\frac{y}{x}$. It is obvious, from known principles, that f cannot have fewer imaginary roots than exist in $\frac{d}{dx}f$ or $\frac{d}{dy}f$(6), or, more generally, than in $\left(\frac{d}{dx}+\lambda\frac{d}{dy}\right)f$; from which it immediately follows(7) that if f have all its roots real, and the quadratic derivatives of f be called Q_1, Q_2, ... Q_{n-1}, and the coeffi-

(5) To avoid the possibility of misapprehension, I state here once for all, that in the *discriminant* of a form of any degree I suppose the sign to be so taken as to render *positive* the term which is a power of the product of the first and last coefficients; and it may be well to remember that with this definition the number of real roots in any equation $\equiv 0$ or 1 to modulus 4 when the discriminant is positive, and $\equiv 2$ or 3 when the discriminant is negative; whereas the Determinant of a Quadratic form is to be taken in the same sense as that in which it is used by GAUSS, and is the same for such form as the Discriminant with the sign changed.

(6) This rule I find merges in the following more general and symmetrical one. Let f, φ be any two quantics in x, y; call the Jacobian of f, φ J; then the difference between the number of real roots in f and the like number in φ, taken positively and augmented by unity, cannot exceed the number of real roots in J. When φ is made equal to y, this theorem recurs to the familiar one alluded to in the text.

(7) By operating upon f successively with *any* $(n-2)$ distinct factors each of the form $\left(\frac{d}{dx}+\lambda_i\frac{d}{dy}\right)$.

cients of any function F of two degrees lower than f, whose roots are also *all* real, be $p_1, p_2, \ldots p_{n-1}$, the quadratic function $p_1Q_1+p_2Q_2+\ldots+p_{n-1}Q_{n-1}$ must have its roots real, *i. e.* its discriminant must be positive: a particular consequence of this is, that by causing F to consist successively of the single terms $x^{n-2}, x^{n-3}y, \ldots xy^{n-3}, y^{n-2}$, we see that the determinants of $Q_1, Q_2, \ldots Q_{n-1}$ must each of them be positive; or, in other words, if any of the Newtonian criteria of an equation are negative, it must have *some* imaginary roots, which is all that MACLAURIN, CAMPBELL, and others have succeeded in proving.

(4) The labour of proof of the cases hereinafter considered will be much lightened by the following rule of induction, viz., granting NEWTON's rule to be true for the degree $n-1$, it must be true for all those cases appertaining to the degree n in which the series of the signs of the criteria does not commence with $-+$ and end with $+-$: to prove this, we have only to remember that f must have at least as many imaginary roots as $\frac{df}{dx}$ or $\frac{df}{dy}$, and that the criterion-series corresponding to $\frac{df}{dx}$ and to $\frac{df}{dy}$ will be found by cutting off from the series of f one term to the right and left respectively(8). If, now, the series for f begins with $++$ or $--$ or $+-$, the number of negative *sequences* is the same as when the left-hand sign is removed; so that it is only necessary to prove that the number of imaginary roots in f is not less than the number of negative sequences in $\frac{df}{dx}$; but this, by hypothesis, is not greater than the number of pairs of imaginary roots in $\frac{df}{dx}$, and, *à fortiori*, not greater than the number of such in f. In like manner, if the two *last* criteria of f are not $+-$, it may be shown that the truth of the rule for such form of f is implied in what is supposed to be known to be true for $\frac{df}{dy}$.

We may therefore limit our attention, as we ascend in the scale of proof, to those forms of f in which the criterion-series begins with $-+$ and ends with $+-$. Accordingly, since the rule is a truism for $n=2$, it is at once proved, by virtue of the above considerations, for $n=3$(9).

(8) For
$$\frac{d}{dx}(a, b, \ldots k, l \between x, y)^n = n(a, b, \ldots k \between x, y)^{n-1},$$
and
$$\frac{d}{dy}(a, b, \ldots k, l \between x, y)^n = n(b, \ldots k, l \between x, y)^{n-1}.$$

(9) The theorem for the case of cubic equations may be also proved directly as follows:

Writing the equation $ax^3+3bx^2y+3cxy^2+dy^3=0$, the two criteria are $L=b^2-ac$, $M=c^2-bd$; and the discriminant is $a^2d^2+4ac^3+4db^3-3b^2c^2-6abcd=\Delta$.

1. Let L and M be of opposite signs, so that one and only one of them is negative. Then
$$\Delta=(ad-bc)^2-4(b^2-ac)(c^2-bd)=(ad-bc)^2-4LM,$$
and is therefore positive.

2. Let L and M be both negative. The equation may evidently, by writing x and y for ax, dy, be brought under the form
$$x^3+3\epsilon x^2y+3\eta xy^2+y^3=0,$$
with the conditions $\epsilon^2<\eta$, $\eta^2<\epsilon$; from which we may deduce that ϵ and η are both positive, and $\epsilon\eta<1$ and >0.

If all the criteria are zero, it is evident that, whatever n may be, all the roots are real. In every other case we shall find that *zero* may be made positive or negative at will. Thus in the case before us, if the two criteria are $0+$ or $0-$, there will be a pair of imaginary roots, as the first may be read as $-+$ and the second as $+-$.

To prove this, we have only to observe that in either case $\frac{df}{dx}$ will have two equal roots; so that f will be of the form $(ax+by)^3+cy^3$, which obviously, for any real values of a, b, c, has only one real root.

(5) We may now pass to the case of $n=4$, and excluding for the moment the consideration of *zeros*, limit our attention to the criterion series $-+-$.

Let $ax^4+4bx^3y+6cx^2y^2+4dxy^3+ey^4=0$ be the equation for which the signs of the criteria b^2-ac, c^2-bd, d^2-ce are $-+-$. Call these criteria L, M, N respectively. It has to be proved that all four roots are imaginary, since there are two distinct negative sequences, each sequence consisting of a single $-$. Let x become $x+\epsilon y$ [10], where ϵ is an infinitesimal quantity, and transformed into one between u and y; then we have obviously,

$$\delta a=0,\quad \delta b=a\epsilon,\quad \delta c=2b\epsilon,\quad \delta d=3c\epsilon,\quad \delta e=4d\epsilon,$$
$$\delta\mathrm{L}=2b\delta b-a\delta c=0,\quad \delta\mathrm{M}=2c\delta c-b\delta d-d\delta b=(bc-ad)\epsilon,$$
$$\delta^2\mathrm{M}=(b\delta c+c\delta b-a\delta d)\epsilon=2(b^2-ac)\epsilon^2=2\mathrm{L}\epsilon^2;$$

so that δ^2M is essentially negative, since L is so.

Hence, by continually augmenting x by an infinitesimal variation, we may, leaving L unaltered, so choose the sign of ϵ as to decrease M: nor can this process stop when $bc-ad$ becomes zero, by reason that δ^2M is *negative*. Hence we may reduce M to zero. Now,

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Also we have

$$\begin{aligned}\Delta&=1+4(\epsilon^3+\eta^3)-6\epsilon\eta-3\epsilon^2\eta^2\\&>1+4(\epsilon+\eta)\epsilon\eta-6\epsilon\eta-3\epsilon^2\eta^2\\&>1-6\epsilon\eta+8(\epsilon\eta)^{\frac{3}{2}}-3\epsilon^2\eta^2;\end{aligned}$$

or, writing $\epsilon\eta=q^2$,

$$\begin{aligned}\Delta&>1-6q^2+8q^3-3q^4,\\&>(1-q)^3(1+3q);\end{aligned}$$

but $1>q>0$. Hence Δ is positive.

Hence in either case two of the roots of the cubic are impossible. Or the same thing may be shown more immediately from the identities

$$a^2\Delta=(a^2d+2b^3-3abc)^2+4(ac-b^2)^3,$$
$$d^2\Delta=(ad^2+2c^3-3bcd)^2+4(bd-c^2)^3,$$

so that Δ must be positive, and therefore two roots imaginary, if either $bd>c^2$ or $ca>b^2$. It may be noticed that the square and cube in these identities are semi-invariants, being in the first of them unaffected by the change of x into $x+hy$, and in the second by the change of y into $y+hx$.

[10] This method of infinitesimal substitution is that which I applied in my memoir "On the Theory of Forms," in the Cambridge and Dublin Mathematical Journal, to obtain the partial differential equations to every possible species of invariants (including covariants and contravariants) of forms, or systems of forms, with a single set or various sets of variables, proceeding upon the pregnant principle that every finite linear substitution may be regarded as the result of an indefinite number of *simple* and *separate* infinitesimal variations impressed upon the variables. M. Aronhold has erroneously ascribed to others the priority of the publication of these equations.

in the course of this reduction, either N retains its sign or changes it; and if the latter is the case, N must have passed through zero. If when M becomes zero N is still negative, the criteria of the linearly transformed equation become $-0-$; and it may be noticed that its first, middle, and last coefficients must have the same sign, by virtue of the negativity of the two last criteria, and the second and fourth the same signs, by virtue of the zero middle criterion; consequently the equation will take the form

$$(\lambda^2+e^4)x^4\pm 4e^3\varepsilon x^3y+6e^2\varepsilon^2x^2y^2\pm 4e\varepsilon^3xy^3+(\mu^2+\varepsilon^4)y^4=0,$$

or

$$\lambda^2x^4+\mu^2y^4+(ex\pm\varepsilon y)^4=0,$$

which obviously has all its roots impossible. This being true of the transformed equation, will also, on the suppositions made, be equally so of the original equation.

Let us next suppose that N changes its sign either at the instant when, or before M becomes zero. If M and N both become zero together, so that the criteria of the transformed equation bear the signs $-$ 0 0, calling the transformed equation $F=0$, $\frac{dF}{dy}$ will have all its roots equal, and F will therefore be of the form $(ax+by)^4+kx^4$, with the condition $(a^3b)^2-(a^4+k)(a^2b^2)<0$.

Hence k is positive, and consequently $F=0$ has all its roots imaginary; and the same, as before, must hold good of the original equation $f=0$.

It remains then only to consider the case when N becomes zero before M vanishes. When this is the case, as soon as N is reduced to zero, in lieu of the substitution of $x+\varepsilon y$ for x, we must leave x unaltered, and continue substituting $y+\varepsilon x$ for y. We thus start from the sequence $-+0$; N will then always remain zero, and we must either come to the series $-$ 0 0, which we know, from what has been shown above, corresponds to four imaginary roots, or to the sequence $0+0$, which I shall proceed to consider.

Since the first and last coefficients must have the same sign, we may, by giving either variable a proper multiple(11), make these two coefficients alike, and with the first,

(11) (a) The form $(1, e, e^2, e, 1\between x, y)^4$ may be regarded as a new and, for many purposes, useful canonical form of a binary quartic. It may be made to comprise within its sphere of representation all forms corresponding to two or four imaginary factors, but excludes the case of four real factors. The ordinary canonical form $(1, 0, 6m, 0, 1\between x, y)^4$ comprises within its spheres of representation those forms for which the factors are all real or all imaginary, but, so far as real transformations are concerned, excludes the case of two real and two imaginary factors [that case is met by the form $1, 0, 6m, 0, -1\between x, y)^4$], as may easily be established either by decomposing the form first named into its factors, or by the consideration that its discriminant Δ is $(1-9m^2)^2$, and is therefore always positive; whereas if a form which it is used to represent have two real and two unreal factors, its discriminant is negative. If now the determinant of transformation be D, and the discriminant corresponding thereto be called Δ', we have $\Delta'=D^6\Delta$, showing that D^2 is negative, and the transformation therefore unreal.

(b) The *reality* of m for each of these cases (usually assumed without proof) may be demonstrated as follows: Calling the cubic invariant and the discriminant of any cubic form T, D, we shall have, using the *ordinary* canonical form, $\frac{(m-m^3)^2}{(1-9m^2)^3}=\frac{T^2}{D}$, showing that when D is positive, which is the case of four real or unreal factors, there will

second, and third, as well as the third, fourth, and fifth coefficients form geometrical series; hence it is obvious that the transformed equation may be reduced to one or the other of the two following forms, viz.

$$x^4+4ex^3y+6e^2x^2y^2-4exy^3+y^4=0, \quad \ldots \ldots \ldots \ldots \quad (a)$$

or

$$x^4+4ex^3y+6e^2x^2y^2+4exy^3+y^4=0, \quad \ldots \ldots \ldots \ldots \quad (b)$$

with the condition in the latter case that e^4-e^2 is positive, *i. e.* $e^2>1$.

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be one real value of m, and when D is negative, a real value of im. The former case possesses over the latter a striking distinction, which is that *all* the roots of m will be real; for, as I have shown elsewhere, if m is one root the complete system of roots will be $\pm m$, $\pm\frac{1-2m}{1+3m}$, $\pm\frac{1+2m}{1-3m}$: in the latter case the reality of the two values $\pm im$ does not seem necessarily to imply the reality of the other 4 values of the system.

(c) Analogy suggests the establishment of an analogous canonical form or forms for ternary cubics, of which, as is well known and is even dimly foreshadowed in NEWTON's Enumeration of Lines of the Third Order, the theory runs closely parallel to that of binary quartics. This will be effected by assuming the form

$$F(x, y, z)=\Sigma x^3+3e\Sigma x^2y+6gxyz,$$

and assuming g so as to make the discriminants of

$$\frac{dF}{dx}, \frac{dF}{dy}, \frac{dF}{dz}$$

all zero. This gives rise to a quadratic equation in g, of which the roots are $g=e$, $g=2e^2-e$. When $g=e$, I find

$$S=e(1-e)^3, \quad T=(1-e)^4(1+4e-8e^2), \quad \Delta=T^2+64S^3=(1+8e)(1-e)^8.$$

When $g=2e^2-e$, I find $\Delta=(1-e)^i(1-4e)^j(1+2e)^k$, where i, j, k are integers to be determined. These forms will, I think, be found important in the future perspective discussion of curves of the third degree. Whilst I yield to no one in admiration of the surpassing genius with which NEWTON has handled these curves, I cannot withhold the expression of my opinion that every theory of forms in which invariants are ignored must labour under an inherent imperfection, and that NEWTON, from want of acquaintance with the indelible characters which their invariants stamp upon curves, has in the parallel which he has drawn between the generation by shadows of all conics from a common type, and of all cubic curves from a limited number of forms, either himself fallen into error of conception, or at least used language which could scarcely fail to lead others into such error. For no species whatever of cubic curve can be formed for which an infinite number of individuals cannot be found which defy linear or perspective transformation into each other; whereas all conics proper may be propagated as shadows from a single individual. It should be noticed in connexion with this subject, that the *indelible* characters of quartic binary, and cubic ternary forms are two in number, viz. the value of $\frac{s^3}{t^2}$ (where s, t are the two fundamental invariants in either case) and the *sign* of t. The indelibility of the sign of s being implied in the invariability of the value of $\frac{s^3}{t^2}$, does not constitute a distinct character. Of course all symmetrical invariants have an invariable sign; but this is not the case with skew invariants, as *ex. gr.* M. HERMITE's octodecimal invariant of a binary quintic, which will change its sign with that of the determinant of transformation.

(d) Whilst upon this subject of invariants, I may allow myself to make a remark bearing upon what will be noticed further on in the text about a case of equality between roots not necessarily being a mark of transition from real to imaginary roots. If a, b, c, d being the roots of a binary quartic we form a secondary cubic, of which the roots are $(a-b)(c-d)$, $(a-c)(d-b)$, $(a-d)(b-c)$, it may be easily shown that two of these quantities become equal, or, in other words, the roots of the original equation mark out a harmonic group of points when t (the cubinvariant) is zero. Notwithstanding which a change of sign in t will not command a change of character in the above three roots of the secondary (nor consequently of the original equation), because it is not an odd but an even power of t, viz. t^2, which enters into the discriminant of the secondary.

It must be remembered that we know, from the form of the criteria-series to the derivatives in respect to either x or y (indifferently), that the equation must have *some* imaginary roots; and the question therefore lies between its having two or four. If the discriminant is negative, the former will be the case, if positive, the latter. I shall show that in each equation the discriminant is positive.

Let s, t represent in general the quartic invariants, then we have to show that s^3-27t^2 is positive.

In case (a), $s=1+4e^2+3e^4$
$=(1+e^2)(1+3e^2)$

$$t=\begin{vmatrix} 1 & e & e^2 \\ e & e^2 & -e \\ e^2 & -e & 1 \end{vmatrix} \begin{aligned} &=e^2-e^4-\ e^4-e^2-\ {}^5-e^2 \\ &=\ -e^2-2e^4-e^6 \\ &=\ -e^2(1+e^2)^2, \end{aligned}$$

so that

$$s^3-27t^2=(1-e^2)^3\{(1+3e^2)^3-27e^4(1+e^2)\}=(1+e^2)^3(1+9e^2),$$

and is positive.

In case (b),

$$s=(1-4e^2+3e^4)=(1-e^2)(1-3e^2)$$

$$t=\begin{vmatrix} 1 & e & e^2 \\ e & e^2 & e \\ e^2 & e & 1 \end{vmatrix} \begin{aligned} &=e^2+e^4+e^4-e^2-e^6-e^2 \\ &=-e^2+2e^4-e^6=-e^2(1-e^2)^2, \end{aligned}$$

and

$$\begin{aligned} s^3-27t^2&=(1-e^2)^3\big((1-3e^2)^3-27e^4(1-e^2)\big) \\ &=(1-e^2)^3(1-9e^2). \end{aligned}$$

The above can only be negative when e^2 lies between 1 and $\frac{1}{9}$; but in the case supposed $e>1$. Hence the discriminant is positive, and the roots are all imaginary([12]). Thus, then, the theorem is established for $n=4$, as well as for the cases where the criteria are zero (as will have been observed in the course of the demonstration), as for those where they are *plus* or *minus*; and it should be observed that the demonstration proceeds upon our being able to show that the quartic, in the case where it resists reduction to the case of the cubic, viz. where the criteria are negative at the two extremes and positive in the middle, may by real linear transformations be changed into a form where either the middle criterion is zero and the two extremes negative, or the two extremes zero, and the middle one positive.

([12]) The reader conversant only with ordinary algebra may easily verify this result. For writing $\frac{x}{y}+\frac{y}{x}=z$, the equation becomes $z^2+4ez+6e^2-2=0$, and this will have its roots impossible unless $4e^2>6e^2-2$, or $2e^2-2$ negative, which it cannot be, since $e^2>1$, and consequently $x:y$ has all its roots impossible. Moreover the same conclusion would (as before shown) hold good unless e^2 lay between 1 and $\frac{1}{9}$; for on making $z=2$, the function above written in z becomes $2+8e+6e^2$, or $2(1+e)(1+3e)$; and making $z=-2$, it becomes $2-8e+6e^2$, or $2(1-e)(1-3e)$, which two quantities evidently have both positive signs unless e lies between 1 and $\frac{1}{3}$, or between -1 and $-\frac{1}{3}$; so that the first and third Sturmian functions are (except on that supposition) respectively positive and negative for $z=2$, and also for $z=-2$, showing that no root of z can lie between 2 and -2, and consequently that all the roots of $x:y$ remain impossible.

Observation.—To make the foregoing demonstration quite exact, it should be noticed that when the criteria L, M, N have been brought to the form $-+0$, and the series of substitutions of $y+\iota x$ for y has set in, we have

$$N=0, \quad \delta N=0, \quad \delta M=(cd-be)\iota, \quad \delta^2 M=N\iota=0, \quad \delta^3 M=0.$$

Consequently if $cd-be$ should become *zero*, we can no longer go on decreasing M. But as soon as $cd-be=0$, since we have also $d^2=ce$, b, c, d, e come to be in geometrical progression, and the transformed equation takes the form

$$ax^4+4\omega x^3y+6\omega^2x^2y^2+4\omega^3xy^3+\omega^4x^4=0,$$

with the condition $\omega^2-a\omega^2$ negative, or $a>1$. Hence we have $q^2x^4+(x+\omega y)^4=0$, which obviously has all its roots impossible (13).

(6) We may now pass on to equations of the fifth degree, in which the case resisting induction will be that where the criterion-series bears the signs

$$- + + -.$$

Let the criteria be called L, M, N, P, so that writing the equation

$$ax^5+5bx^4y+10cx^3y^2+10dx^2y^3+5exy^4+fy^5=0,$$

$$L=b^2-ac, \quad M=c^2-bd, \quad N=d^2-ce, \quad P=e^2-df,$$

and writing for x, $x+\iota y$, we have, as before,

$$\delta L=0, \quad \delta M=(bc-ad)\iota, \quad \delta^2 M=L\iota^2,$$

so that M may be continually diminished.

If M becomes zero before either N or P changes its sign, the criterion-series for the transformed equation becomes $- \ 0 + -$, and for its derivative in respect to x, the series is $0 + -$, which proves the existence of four imaginary roots in the transformed, and consequently also in the given equation. In like manner, if N becomes zero before M or P have changed their signs, the criterion-series becomes $- + 0 -$, which obviously leads to the same result. So likewise the same inference may be drawn if L and M, or M and N, or L, M, N become zeros all at the same time, and we have only to consider the case when, L and M retaining their signs, N becomes zero. At this moment the order of the substitutions must be reversed, and for y must be written $y+\iota x$; we shall then have

$$P=0, \quad \delta P=0, \quad \delta N=(de-cf)\iota \ldots\ldots;$$

(13) From the first and third criteria it follows that in the form $(a, b, c, d, e \between x, y)^4$, a, c, e have the same sign and may be regarded as all positive; so that writing $a-\frac{b^2}{c}=h^2$, $e-\frac{d^2}{c}=k^2$, the form becomes $h^2x^2+F+k^2y^2$, where

$$F=\frac{b^2}{c}x^4+4bx^3y+bcx^2y^2+4dxy^3+\frac{d^2}{c}y^4,$$

and consequently the given form will have all its roots imaginary when this is true for F, so that we might have proceeded at once to deal with the forms marked (*a*), (*b*) at p. 585; but as the method of homographic transformation by infinitesimal substitutions appears to be necessary in passing to the corresponding forms in the case of the fifth degree, and as in treating that case reference is made to what appears above, I have thought that no object would be gained by altering the text.

and reasoning as in the preceding case for $n=4$ (with the sole difference, that if δN vanishes by virtue of $d\epsilon - cf$ vanishing, we should have $P=0$, $N=0$, and the criterion-series $- + 0\ 0$, which at once indicates the existence of four imaginary roots), we see that there remains only to consider the case where the criterion-series takes the form $0 + + 0$. It is scarcely necessary to observe that all the criteria can never vanish simultaneously; for that would indicate the equality of all the roots in the transformed, and therefore in the given equation, whose own criteria, contrary to hypothesis, would also be all zero. The zero values of the two extreme criteria indicates that the three first and the three last literal parts of the coefficients are in geometrical progression, from which it will immediately be seen that the equation to be considered may be thrown (by substituting in lieu of x and y suitable multiples of x and y, which will not affect the characters of the criteria) into the convenient form

$$x^5+5\epsilon x^4y+10\epsilon^2x^3y^2+10\eta^2x^2y^3+5\eta xy^4+y^5=0,$$

with the two conditions $\epsilon^4-\epsilon\eta^2$ *positive*, $\eta^4-\eta\epsilon^2$ *positive.*

The form of the criterion-series, apocopated from either end, shows that two of the roots must be imaginary; and consequently, in order to establish the existence of two imaginary pairs of roots, it is only necessary to show that the discriminant of the above equation, subject to the above conditions, must remain always positive. That discriminant I proceed to determine; but as a guide to the form under which it is to be expressed, the following observation is important. Let us take the more general form

$$ax^5+bx^4y+cx^3y^2+dx^2y^3+exy^4+fy^5=0,$$

where

$$a=1,\quad b=\lambda\epsilon,\quad c=\mu\epsilon^2,\quad d=\mu\eta^2,\quad e=\lambda\eta,\quad f=1,$$

λ, μ being any numerical quantities.

The discriminant will evidently be a symmetrical function of ϵ and ϵ.

Let $a^pb^qc^rd^se^t$ be the literal part of any term in the discriminant. By the *law of weight* we must have

$$q+2r+3s+4t=5\times4=20.$$

But in the equation before us, $a^pb^qc^rd^se^t$ (to a numerical factor *près*) is $\epsilon^{q+2r}\eta^{2s+t}$, and

$$\begin{aligned}(q+2r)-(2s+t)&=(q+2r+3s+4t)-5(s+t)\\&=5(4-s+t).\end{aligned}$$

Hence the difference between the indices of ϵ and η in each term is a multiple of 5, and consequently, since the discriminant is a symmetrical function in ϵ and η, it will be a rational integral function of $\epsilon^5+\eta^5$ and $\epsilon\eta$. Moreover, as no such term as c^4d^4 can figure in the discriminant, which, as we know, must in all cases contain one or the other of the two final and of the two initial coefficients, we see that no term can be of higher than the 14th degree in ϵ, η, nor yet so high, for the only terms that could be of that degree would be bc^3d^3e; but making a and f each zero in the original form, it becomes obvious

that all the terms free from a and f contain b^2e^2 as a factor(14). Hence, in fact, the discriminant will be only of the twelfth degree in ε, η, and being therefore of only the second degree in $\varepsilon^5+\eta^5$, will admit of comparatively easy treatment.

(7) Before proceeding to the calculation of this discriminant, it will be useful to investigate, as a Lemma ancillary to the subsequent discussion, under what conditions four of the roots of the supposed equation will become imaginary when $\varepsilon=\eta$.

In this case writing $\frac{x}{y}+\frac{y}{x}=z$, the equation

$$\frac{1}{x+1}(1, \varepsilon, \varepsilon^2, \varepsilon^2, \varepsilon, 1 \between x, y)^5=0$$

becomes

$$z^2-2-z+1+5\varepsilon(z-1)+10\varepsilon^2=z^2+(5\varepsilon-1)z+10\varepsilon^2-5\varepsilon-1=0,$$

or say $fz=0$.

The determinant of $f(z)$ is thus $(5\varepsilon-1)^2-40\varepsilon^2+20\varepsilon+y$, i. e. $5(1-\varepsilon)(1+3\varepsilon)$; and all the roots of z, and consequently of (x, y), will be impossible, unless z lies between 1 and $-\frac{1}{3}$.

Now
$$f(2)=1+5\varepsilon+10\varepsilon^2,$$
$$f'(2)=3+5\varepsilon;$$

so that when z has any real roots, *i. e.* when ε lies between 1 and $-\frac{1}{3}$, $f(2)$, $f'(2)$ are both positive, and the Sturmian functions are of the signs $+++$.

Again,
$$f(-2)=5-15\varepsilon+10\varepsilon^2=5(1-\varepsilon)(1-2\varepsilon),$$
$$f'(-2)=-5+5\varepsilon;$$

so that, on the same supposition as before, the Sturmian functions are $\pm-+$, viz.

$$+-+ \text{ when } \tfrac{1}{2}>\varepsilon>-\tfrac{1}{3},$$
$$--+ \text{ when } 1>\varepsilon>\tfrac{1}{2}.$$

In the former case two real roots, in the latter one real root of z lies between 2, -2. Hence in the former case no real roots of z lie between the limits ∞, 2, and the limits -2, $-\infty$, and in the latter case one real root lies between those limits. Hence x, y will have four imaginary roots, unless ε lies between 1 and $\frac{1}{2}$, and two such roots in every other case.

Thus the discriminant of $(1, \varepsilon, \varepsilon^2, \eta^2, \eta, 1 \between x, y)^5$, when $\varepsilon=\eta$, is negative when ε lies between 1 and $\frac{1}{2}$, but for every other value of ε is positive, save that it vanishes when

$$\varepsilon=1, \text{ or } \varepsilon=\tfrac{1}{2}\text{(15)}, \text{ or } \varepsilon=-\tfrac{1}{3}.$$

(8) I now proceed to calculate the discriminant of the form

$$x^5+5\varepsilon x^4y+10\varepsilon^2x^3y^2+10\eta^2x^2y^3+5\eta xy^4+y^5$$

(14) For the discriminant of $xy\phi(x, y)$ = the discriminant of $\phi(x, y)$ multiplied by the square of the product of the resultant of (x, ϕ) and of (y, ϕ).

(15) When $\varepsilon=\frac{1}{2}$ the discriminant of $f(z)$ does not vanish, but $z=-2$ satisfies the equation in z, and consequently $\frac{x}{y}$ has two equal roots -1, so that the discriminant of the original equation vanishes.

for general values of ϵ, η. This will be accomplished most expeditiously by taking the resultant of the two derivatives of the above form, say U and V, where

$$U=x^4+4\epsilon x^3y+6\epsilon^2x^2y^2+4\eta^2xy^3+\eta y^4,$$
$$V=\epsilon x^4+4\epsilon^2x^3y+6\eta^2x^2y^2+4\eta xy^3+y^4;$$

so that

$$\epsilon U-V \quad =6(\epsilon^3-\eta^2)x^2y^2+4(\epsilon\eta^2-\eta)xy^3+(\epsilon\eta-1)y^4=y^2P,$$
$$-U+\eta V=(\epsilon\eta-1)x^4+4(\eta\epsilon^2-\epsilon)x^3y+6(\eta^3-\epsilon^2)x^2y^2=x^2Q.$$

Hence

$$\textit{Resultant} \text{ of } (U, V)=\frac{1}{(\epsilon\eta-1)^4}\times \textit{Resultant} \text{ of } (y^2P, x^2Q)=\textit{Resultant} \text{ of } (P, Q);$$

where

$$P=6(\epsilon^3-\eta^2)x^2+4(\epsilon\eta^2-\eta)xy+(\epsilon\eta-1)y^2,$$
$$Q=(\epsilon\eta-1)x^2+4(\eta\epsilon^2-\epsilon)xy+6(\eta^3-\epsilon^2)y^2.$$

Hence, calling Δ the discriminant of the original form, we obtain by the well-known formula for the resultant of two binary quadratics, writing for the moment

$$P=(B, 4\eta A, A \between x, y)^2, \quad Q=(A, 4\epsilon A, B' \between x, y)^2,$$
$$\Delta=(4\epsilon A^2-4\eta AB')(4\eta A^2-4\epsilon AB)+(A^2-BB')^2$$
$$=(1-16\epsilon\eta)A^4+16(\epsilon^2B+\eta^2B')A^3-16\epsilon\eta BB'A^2-2BB'A^2+B^2B'^2.$$

Hence writing $\epsilon\eta=q$, $\epsilon^5+\eta^5=S$,

$$\Delta=(1-16q)(q-1)^4+96(S-2q^2)(q-1)^3-72(8q+1)(q^3+q^2-S)(q-1)^2$$
$$+36^2(q^3+q^2-S)^2.$$

Let $S-q^2-q^3=\sigma$, $q-1=p$, so that

$$S-2q^2=\sigma-q^2+q^3=\sigma+(p+1)^2p.$$

Then

$$\Delta=36^2\sigma^2+72(8p+9)p^2\sigma+96p^3\sigma+96(p+1)^2p^4-(16p+15)p$$
$$=1296\sigma^2+(648p^2+672p^3)\sigma+96p^6+176p^5+81p^4,$$
$$=\tfrac{1}{9}\{108\sigma+27p^2+28p^3)^2+729p^4+1584p^5+864p^6-(27p^2+28p^3)^2\},$$

or

$$9\Delta=(108\sigma+27p^2+28p^3)^2+72p^5+80p^6.$$

(9) Hence we see at once that Δ can be negative only when p lies between 0 and $-\frac{9}{10}$, *i. e.* when $\epsilon\eta$ (which is $p+1$) lies between 1 and $\frac{1}{10}$. Accordingly when Δ is negative, ϵ and η must be both positive or both negative. The latter supposition may easily be disproved as follows: treating the equation $\Delta=0$ as a quadratic equation in σ, in order that Δ may be capable of becoming negative, its discriminant in respect to σ must be negative, and its value when $\sigma=-\infty$ is positive. Now

$$S=\epsilon^5+\eta^5, \quad p+1=\epsilon\eta, \quad \sigma=S-(p+1)^2-(p+1)^3;$$

so that when ϵ and η are real we have

$$S>2(p+1)^{\frac{5}{2}}\,(^{16}), \text{ i. e. } \sigma>-(p+1)^2+2(p+1)^{\frac{5}{2}}-(p+1)^3$$

(16) It is of course understood that $(p+1)^{\frac{5}{2}}$ is to be taken *positive.*

when ε, η are both positive, and

$$S<-2(p+1)^{\frac{5}{3}}(^{16\,\text{bis}}),\ \text{i. e. } \sigma<(p+1)^2-(p+1)^3-2(p+1)^{\frac{5}{3}}$$

when ε, η are both negative.

If now we substitute $(p+1)^2+(p+1)^3-2(p+1)^{\frac{5}{3}}$ for σ in Δ, I say that the resulting value will be positive whatever positive value be given to $(p+1)$; in fact, if we write $p=\nu^3-1$, and make $\sigma=-\nu^4+2\nu^5-\nu^6$, so that Δ becomes a function of the twelfth degree in ν, this function is what the discriminant of the equation in x, y becomes when we have $\varepsilon=\eta=\nu$; but in the antecedent Lemma it has been shown that this discriminant is only negative when the two equal quantities ε or η, or, which is the same thing, when ν lies between 1 and $\frac{1}{2}$; hence Δ is positive when ν is negative, and consequently when

$$\sigma=(p+1)^2+(p+1)^3-2(p+1)^{\frac{5}{3}}.$$

Thus Δ, a quadratic function in σ, and its discriminant are respectively $+$ and $-$ for this value of σ, as well as for $\sigma=-\infty$. Hence no real root of σ lies between such value of σ and $-\infty$, and consequently Δ must be always positive when ε and η are both negative. Hence, if Δ is negative, we must have $1>\varepsilon\eta>\frac{1}{10}$; $\varepsilon>0$; $\eta>0$. But our *criteria* give

$$\varepsilon^4-\varepsilon\eta^2>0,\quad \eta^4-\eta\varepsilon^2>0,$$

which, when $\varepsilon>0$, $\eta>0$, imply $\varepsilon^3>\eta^2$, $\eta^3>\varepsilon^2$, and consequently $\varepsilon\eta>1$, which is in contradiction to the inequality $1>\varepsilon\eta$. Hence when these criteria are satisfied the determinant is necessarily *positive*, and all the roots are imaginary, which completes the proof of NEWTON'S rule for equations of the fifth degree.

(10) It follows as a corollary to the Lemma employed in the preceding investigation, that if in Δ we write $\sigma=-(\nu^2-\nu^3)^2$ and $p=\nu^3-1$, and distinguish this particular value by the symbol (Δ), then (Δ) ought to break up into the product of odd powers of $\nu-1$, $\nu-\frac{1}{2}$ of some even power of $(\nu+\frac{1}{3})$, and of a factor incapable of changing its sign, and remaining always positive. This may be easily verified; for dividing (Δ) by $(\nu-1)^4$, we obtain

$$1296\nu^8(648(\nu+1)^2+24(\nu^2-1)(\nu+1)^2)\nu^4+96(\nu^2-1)^2(\nu+1)^4+176(\nu^2-1)(\nu+1)^4+81(\nu+1)^4;$$

and collecting the terms $1296\nu^8-648\nu^6(\nu+1)^2+81(\nu+1)^4$ whose sum contains the factor $(\nu-1)$, we have

$$\begin{aligned}\frac{(\Delta)}{(\nu-1)^5}= &\ 648(\nu^7+\nu^6+\nu^5+\nu^4+\nu^3+\nu^2+\nu+1)\\ &-1296(\nu^6+\nu^5+\nu^4+\nu^3+\nu^2+\nu+1)\\ &-\ 648(\nu^5+\nu^4+\nu^3+\nu^2+\nu+1)\\ &+\ 81(\nu^3+5\nu^2+11\nu+15)\\ &-\ 24(\nu^7+3\nu^6+3\nu^5+\nu^4)\\ &+\ 96(\nu^7+5\nu^6+9\nu^5+5\nu^4-5\nu^3-9\nu^2-5\nu-1)\\ &+\ 176(\nu^5+5\nu^4+10\nu^3+10\nu^2+5\nu+1)\\ =&\ 720\nu^7-240\nu^6-328\nu^5+40\nu^4+65\nu^3+5\nu^2-5\nu-1.\end{aligned}$$

Hence

$$\begin{aligned}(\Delta)&=(\nu-1)^5(2\nu-1)^3\{90\nu^4+105\nu^3+49\nu^2+11\nu+1\}\\ &=(\nu-1)^5(2\nu-1)^3(3\nu+1)^2\{10\nu^2+5\nu+1\};\end{aligned}$$

($^{16\,\text{bis}}$) It is of course understood that $(p+1)^{\frac{5}{3}}$ is to be taken *positive*.

showing, agreeably with what was seen in the Lemma, that the discriminant of

$$(1, \epsilon, \epsilon^2, \epsilon^2, \epsilon, 1)\!\!\!\bigr)\!\!(x, y)^4$$

vanishes then, and then only, when

$$\epsilon=1, \text{ or } \epsilon=\tfrac{1}{3}, \text{ or } \epsilon=-\tfrac{1}{3},$$

but does not *change its sign*, except as ϵ passes through the limits 1 and $\frac{1}{3}$, and only within those limits can become negative(17).

(11) Although the theory of the possibility of the roots of $(1, \epsilon, \epsilon^2, \eta^2, \eta, 1)\!\!\!\bigr)\!\!(x, y)^5=0$ has now been completely investigated, so far as is necessary for the proof of NEWTON'S theorem applied to equations of the fifth degree, it will be found that the labour will not be ill spent of considering more closely the real nature of the criteria which separate the case of one pair from that of two pairs of impossible roots in the above equation. NEWTON'S *criteria* being constructed so as to cover every possible case for equations of every degree, will always be found to fit loosely, so to speak, upon each case treated *per se*; so that more precise conditions can be assigned in each particular case than those which are furnished by his rule. So, *ex. gr.*, it may be remembered that in the equation $(1, e, e^2, e, 1)\!\!\!\bigr)\!\!(x, y)^4=0$, NEWTON'S rule implies only that when $e>1$, the roots are all impossible; but we have found further that unless $1>e>\frac{1}{3}$ (a much closer condition), the same thing takes place.

It is obvious from what has been demonstrated above, that if we treat p and σ, which are respectively $\epsilon\eta-1$ and $\epsilon^5+\eta^5-\epsilon^2\eta^2-\epsilon^3\eta^3$, as the abscissa and ordinate of a variable point in a plane, the curve $\Delta=0$, i. e. $(108\sigma+27p^2+28p^3)^2+72p^5+80p^6=0$ will be the line of demarcation between those values of ϵ, η which correspond to one pair, and those which correspond to two pairs of imaginary roots.

For all values of ϵ, η corresponding to internal points of the curve Δ there will be two imaginary and three distinct real roots; for all such as correspond to external points there will be four imaginary roots, and for points *on* the curve two imaginary and two equal roots.

The curve Δ is a curve of the 6th degree whose form will presently be discussed. But there is an important remark to be made in the first instance. Not all the points

(17) In *general* the case of equal roots of an equation is the state of transition of two real roots into imaginary, or *vice versâ*. But we see by the above instance that this is not necessarily the case *always*, for Δ vanishes on making $\epsilon=-\frac{1}{3}$, and two roots become equal without any change in the nature of the roots when ϵ passes from being greater to being less than $-\frac{1}{3}$. In such case, however, there is a sort of unstable equilibrium in the form of the equation, by which I mean that the effect of any general infinitesimal change performed upon the coefficients of the equation would be either to cause the real roots in the neighbourhood of $\epsilon=-\frac{1}{3}$ to disappear by the factor $(\epsilon+\frac{1}{3})^2$ becoming superseded by a quadratic function of ϵ with impossible roots, or else a region in the neighbourhood of $\epsilon=-\frac{1}{3}$ would reappear, for which the equation would acquire two real roots, owing to $(\epsilon+\frac{1}{3})^2$ becoming superseded by a quadratic function of ϵ with real roots, in which case there would be two values in the neighbourhood of $\epsilon=-\frac{1}{3}$, for *each* of which there would be a pair of equal roots in the equation. The above is probably the first instance distinctly noticed of this singular obliteration of the usual effect upon real and imaginary roots of a passage through equality, owing to the appearance of a square factor in the discriminant.

within the curve Δ will correspond to *real* values of ε, η. In order that these quantities may be real, we must have

$$\varepsilon^6+\eta^6>2(\varepsilon\eta)^{\frac{3}{2}},$$

$$\text{i. e. } \sigma+q^2+q^3>2q^{\frac{3}{2}}, \text{ where } q=p+1,$$

or

$$\sigma^2+2(q^2+q^3)\sigma+q^4-4q^5+q^6>0.$$

Writing this inequality under the form $R>0$, we see that the curve $R=0$ will represent a second sextic curve intersecting the former. Δ may be called the curve of the discriminant or *discriminatrix*, and will be a close curve, and R the curve of equal parameters or *equatrix*, and will consist of a single infinite branch. All points on the latter correspond to equal values of ε, η, those on one side of it to real values of ε, η, and those on the other side of it to conjugate values of the form $\lambda+i\mu$, $\lambda-i\mu$ respectively. Thus the area confined within the curve Δ will be divided into two portions by the equatrix, and it is impossible to shut one's eyes to the inquiry as to the meaning of the variable point lying in that portion which gives conjugate values to ε, η. It becomes clear by analogy that some kind of distinction must be capable of being drawn between the nature of the roots of the equation $(1, \varepsilon, \varepsilon^2, \eta^2, \eta, 1\between x, y)^5=0$ when ε, η are conjugate, in some sense similar or parallel to that which we know to exist between them when ε, η are real; and obviously this inference cannot be confined to equations of the particular form and degree of that above written; in a word, equations whose coefficients are not real but conjugate, must have roots of two kinds, one analogous to the real, the other to the imaginary roots of equations with real coefficients. This inference will be justified in the sequel; but in the meanwhile it will be desirable to complete the investigation of the special equation under consideration, by a discussion of the forms and relations of the two curves Δ and R. These curves we know *à priori*, from what has been already demonstrated, can only meet in the three points corresponding to

$$\varepsilon=\eta=1, \quad \varepsilon=\eta=\tfrac{1}{2}, \quad \varepsilon=\eta=-\tfrac{1}{3};$$

and since $p=\varepsilon\eta-1$, the abscissæ of these three points will be 0, $-\frac{3}{4}$, $-\frac{8}{9}$.

Moreover the 3rd point will be distinguished from the other two by the circumstance that Δ does not change its sign as p passes through the value $-\frac{8}{9}$. Consequently the two curves must touch each other at this point.

Since when $\Delta=0$ p lies between 0 and $-\frac{9}{10}$, the curve Δ is confined to the negative side of the axis of σ. It is also confined to the negative side of the axis of p.

For between the limits $p=0$, $p=-\frac{9}{10}$,

$$648p^4+672p^3, \text{ i. e. } 24(27p^4+28p^3) \text{ is obviously positive,}$$

and

$$96p^6+176p^5+81p^4=\frac{p^4}{6}\{(24p+22)^2+2\} \text{ is always positive.}$$

Hence the two values of σ are both negative throughout the extent of the curve Δ.

Thus $\varepsilon^5+\eta^5-\varepsilon^3\eta^2-\varepsilon^2\eta^3$ being negative, $\varepsilon^3-\eta^3$ and $\eta^2-\varepsilon^2$ have the same signs when ε, η

are *real*, as should be the case; for in order that Δ may be capable of vanishing, $\epsilon(\epsilon^2-\eta^3)$ and $\eta(\eta^2-\epsilon^3)$ must, by NEWTON's rule, be *both* negative, which could not be the case if either ϵ or η were negative; so that $\epsilon^2-\eta^3$ and $\eta^2-\epsilon^3$ must have the same signs, in fact each must be negative.

The curve Δ under consideration has a multiple point of the 4th order of multiplicity at the origin, where it is touched by the axis of p. Its distance from the axis for the extreme value of p, viz. $p=-\frac{9}{10}$, is $\frac{27}{2000}$.

It has three real maxima and minima, two belonging to its upper portion and one to the lower portion at the points, for which p has the *approximate* values $-\frac{9}{16}$, $-\frac{15}{32}$, and $-\frac{7}{8}$ (18).

The curve R, i. e. $\sigma=\left((p+1)\pm(p+1)^{\frac{3}{2}}\right)^2$, has the values 0 and -4 at the origin, a cusp at its extremity corresponding to $p=-1$, where both of its branches meet and touch the axis of p, and a negative maximum in its upper branch at the point where $p=-\frac{5}{9}$.

At all points within the curve R, ϵ and η are conjugate, and for the points outside real. Its lower branch will meet and touch the lower portion of Δ at the point where $p=-\frac{8}{9}$, and its upper branch will intersect and pass out of the upper branch of Δ at the point where $p=-\frac{3}{4}$. The only part of the area Δ therefore which corresponds to real values of ϵ, η, is that which is included between the upper segment of Δ and the upper branch of R, and extends only from $p=0$ to $p=-\frac{3}{4}$, i. e. from $\epsilon\eta=1$ to $\epsilon\eta=\frac{3}{4}$. Hence we may easily find an inferior limit to the values of ϵ and η when the equation (ϵ, η) has two real roots; for we have in that case ϵ, η, $\eta^2-\epsilon^3$, $\epsilon^2-\eta^3$ all positive. Hence

$$\eta^5>\epsilon^3\eta^3>q^3, \quad \eta^5<\epsilon^2\eta^2<q^2.$$

Consequently ϵ, η must each of them always lie between $q^{\frac{2}{5}}$, $q^{\frac{3}{5}}$; and since the least value of q is $\frac{1}{4}$, ϵ, η must each be always greater than $(\frac{1}{4})^{\frac{4}{5}}$, i. e. than $\cdot 33499$ (19).

(18) The large numbers which enter into Δ may be usefully reduced, and the equation $\Delta=0$ made more manageable, by aid of the simple substitutions $\sigma=-\frac{27v}{64}$, $p=-\frac{9u}{4}$. The equation $\Delta=0$ then becomes

$$(v-3u^2+7u^3)^2=2u^5-5u^6,$$

whose maxima and minima will be given by the equation

$$(v-3u^2+7u^3)(-6u+21u^2)=5u^4-15u^5;$$

which, making $1-3u=\omega$, becomes

$$270\omega^3-46\omega^2-9\omega+1=0,$$

whose roots are all real, and are one just a little greater than $-\frac{1}{5}$, another a little less than $\frac{1}{4}$, and the third a very little less than $\frac{1}{11}$ respectively; whence $p=\frac{3}{4}(\omega-1)$ will have the approximate values given in the text.

(19) $\epsilon:\eta$ will have a maximum value, which can be found by writing $\delta\epsilon:\delta\eta::\epsilon:\eta$; and consequently, remembering that $q=p+1$, $S=\epsilon^5+\eta^5$, $\sigma=S-q^2-q^3$,

$$\delta S:\delta q::5S:2q,$$

and therefore

$$\delta\sigma:\delta p::5\sigma+q^2-q^3:2q::5\sigma+p(p+1)^2:2(p+1).$$

Substituting the values of $\delta\sigma:\delta p$ in $\delta\Delta=0$, and combining the result with the equation $\Delta=0$, p and σ may be found by the solution of a numerical equation of the 5th degree, and then ϵ and η may be found by the solution

There is a third curve not undeserving of notice, of only the 3rd degree, which embodies the joint effect of the two middle criteria (the two extremes being supposed to be each zero) in the two cases where NEWTON'S rule will prove all the roots of the equation under consideration to be impossible. These criteria are $c_1=s^4-s\eta^3$, $c_2=\eta^4-\eta s^3$. But

$$c_1\eta^4+c_2s^4=q(2q^3-S)=q(2q^3-q^3-q^2-\sigma)=q(q^3-q^2-\sigma),$$

which for all values of q on the positive side of the line $p=-1$ (i. e. $q=0$) will have the same sign as $q^3-q^2-\sigma$, which we may call K[20]; and K positive will evidently imply that c_1, c_2 are one or both of them positive. The whole plane will be divided by the curve K into an *upper* region (commencing at $\sigma=\infty$), for which K is negative, and a lower region, in which K is positive. For any point of the curve K, $\sigma=q^3-q^2$, which within the limits of q with which we are concerned, viz. those within which Δ lies, is negative; for any point of the curve R, the smaller absolute value of σ is

$$-q^3-q^2+2q^{\frac{5}{2}}=q^3-q^2+2(q^{\frac{5}{2}}-q^3),$$

which $<q^3-q^2$ within the limits in question. So that, remembering that each of these values of σ is negative, we see that the portion of the area Δ corresponding to real values of s, η will be completely above the curve K, *i. e.* in the negative region of K, and that accordingly Δ for *real values* of s, η can never vanish when K is positive, as should be the case. This remark does not, however, apply to the conjugate region of Δ; for the curve K will *pass through*[21] the lower or conjugate portion of the area Δ.

(12) I may now say a few words on the signification of that portion of Δ in which s and η are conjugate imaginary quantities.

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of a quadratic and the extraction of 5th roots. To find the maxima and minima values of s and η themselves exactly would lead to the solution of an equation of a degree quite unmanageable.

But we may first find the greatest maximum and least minimum values of S, i. e. $s^5+\eta^5$, by making $\delta\sigma=(2q+3q^2)\delta q$ in $\delta\Delta=0$, which leads to an equation (I forget whether) of the 3rd or 5th degree (it is one of the two): calling this maximum and minimum m, μ respectively, and naming ρ (which of course must exceed unity) the greatest quotient of $\frac{s}{\eta}$ or $\frac{\eta}{s}$, we shall have

$$\sqrt[5]{\frac{\rho^5}{1+\rho^5}m}>s\,;\quad \eta>\sqrt[5]{\frac{1}{1+\rho^5}\mu}.$$

These limits will be tolerably near to the absolute maximum and minimum values of s or η. It may be noticed that we know, from what has gone before, that ρ can never exceed $\left(\frac{1}{q}\right)^{\frac{1}{5}}$; and consequently ρ^5 cannot exceed 4, since q is always $>\frac{1}{4}$.

(20) I call K the Indicatrix, as exhibiting the joint effect of the *indicia* or criteria of the Rule.

(21) This may easily be verified; for at the point $p=-\frac{3}{4}$ it will be found that the ordinate in K and the lower ordinate in Δ are equal, and at the point $p=-\frac{9}{10}$ the lower ordinate in Δ is $-\frac{27}{2000}$, and in K is $-\frac{18}{2000}$; which shows that the curve K entering the area Δ when at the lower half of the curve, at a point where $p=-\frac{3}{4}$, must pass through its upper contour in order to cut the line $p=-\frac{9}{10}$ as it does above the point where Δ is touched by that line.

The curve K has its negative maximum at the point $q=\frac{2}{3}$, i. e. $p=-\frac{1}{3}$. It passes through the origin, and begins with sweeping under the curve Δ, which it enters exactly under the point where R quits Δ, and passes

In general, let

$$(a+i\alpha,\ b+i\beta,\ c+i\gamma,\ \ldots\ldots,\ c-i\gamma,\ b-i\beta,\ a-i\alpha \between x,\ y)^n=0$$

be an equation in which all the coefficients, reckoning simultaneously from the two ends, are conjugate to one another, and the central coefficient, if there is one, which can only be when n is even, *real.*

Let $\frac{x}{y}=p+iq$ satisfy this equation. Then evidently $\frac{y}{x}=p-iq$ will also satisfy it; or, which is the same thing, $\frac{x}{y}=\frac{p+iq}{p^2+q^2}$ will satisfy it.

Now either this root will be identical with the former one, or a distinct root; in the former case we must have $p^2+q^2=1$, and the root will be of the form $\cos\alpha+i\sin\alpha$; in the second case p^2+q^2 will differ from unity, and there will be a pair of imaginary roots of the form $\rho(\cos\alpha+i\sin\alpha)$, $\frac{1}{\rho}(\cos\alpha+i\sin\alpha)$, in which the real parts ρ, $\frac{1}{\rho}$ are reciprocal to one another, and the directive parts $e^{-i\alpha}$ identical. Moreover, if we write the given equation under the form $U+iV=0$, and suppose, as can always be done, that U and V have been divested of any algebraical common factor, it may easily be shown that the equation so prepared, and which may be called a Conjugate Equation *proper*, can have no real roots and no *pairs of imaginary roots* in the sense in which that term is employed in the theory of equations with real coefficients; but the distinction between *simple* or solitary and *twin* or associated roots reappears in the theory of conjugate equations, under a different form. It will of course be understood that the class of simple roots for which the modulus is unity is quite as general as that of twin roots, for each of which the modulus may be anything different from unity, just as in the ordinary theory the case of real is quite as general as that of imaginary roots, although the former may be represented by points on a fixed straight line, whilst the points representing the latter may be anywhere in the plane, this liberty of displacement being balanced, so to say, by the constraint of coupling. The general geometrical representation of the roots of a real equation is a system of points in a line, and a system of pairs of points at equal distances on opposite sides of the line. So the general geometrical representation of the roots of a conjugate equation will be system of points in the circumference of a circle to

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through Δ at a point very close indeed to the horizontal extremity of Δ. It may be noticed that when $p=-\frac{3}{4}$, the smaller ordinates of R and Δ are each $-\frac{1}{64}$, the ordinate of K and the larger ordinate of Δ being each $-\frac{3}{64}$.

I have found the points of contact of K with Δ by actually substituting q^3-q^2, i. e. $p(p+1)^2$ for σ in $\Delta=0$. This gives the equation

$$2064p^4+7352p^3+9823p^2+5832p+1296=0,$$

one factor of which is $4p+3$, dividing out which we have

$$516p^3+1451p^2+1368p+432=0.$$

The Newtonian criterion applied to the three first coefficients of the above gives $-1362\frac{1}{4}$, showing that two of the roots are impossible; the remaining real root I find to be $\cdot 8946$, &c. It does not appear to be a rational number.

radius unity, and of points situated in pairs in the same radii at reciprocal distances from the centre. In a word, in each case we may say that the roots can be geometrically represented by points on a circle, and pairs of points electrical images of each other in respect to the circle, but the radius of the circle in the one case will be infinity, in the other unity. Conjugate like real equations will have all their invariants of an even degree real, and those of an odd degree will be pure imaginaries, or real quantities affected with the multiplier i. Their morphological derivatives (covariants, contravariants, &c.) will be also conjugate forms. The whole doctrine of equations, as regards the separation of real from imaginary roots, and the determination of the limits within which the former lie, will reproduce itself with suitable modifications in the theory of conjugate equations, in which simple, on the one hand, and coupled or twin roots, on the other, will correspond respectively as analogues to the real and imaginary roots of the ordinary theory. Thus the following theorem may be demonstrated without difficulty, viz., in any conjugate equation the number of coupled roots is congruent to 0 in respect to the modulus 4 when the discriminant is positive, and to 2 in respect to the same modulus when the discriminant is negative[22]. We see now how to interpret the

[22] [a] A very simple linear transformation shows the immediate connexion between the solitary and associated roots of conjugate with the real and paired imaginary roots of ordinary equations. For if $f(x, y)=0$ be a conjugate equation, writing

$$y=v+iu, \quad x=v-iu,$$

$f(x, y)$ becomes $F(u, v)$, a real form in u, v.

When u, v are real, we have

$$\frac{y}{x}=\frac{v+iu}{v-iu}=\cos\left(\tan^{-1}\frac{v}{u}\right)+i\sin\left(\tan^{-1}\frac{v}{u}\right);$$

when $\frac{v}{u}=c\pm i\gamma$, the two values correspond to

$$\frac{y}{x}=\frac{c+i\gamma+i}{c+i\gamma-i}, \quad \left(\frac{y}{x}\right)'=\frac{c-i\gamma+i}{c-i\gamma-i}.$$

Thus

$$\frac{y}{x}:\left(\frac{y}{x}\right)'::c^2+(\gamma+i)^2:c^2+(\gamma-i)^2;$$

also

$$\frac{y}{x}\times\left(\frac{y}{x}\right)'=\frac{c^2-1+\gamma^2+2ci}{c^2-1+\gamma^2-2ci},$$

of which the modulus is obviously unity.

[b] Now it is known that if t be the number of real, and τ of imaginary roots in the real form, $(u, v)^n$, its discriminant, bears the sign $(-)^{\frac{t(t-1)}{2}}$. Hence the sign of the discriminant of the conjugate form $(x, y)^n$ (since the determinant of $v+iu$, $v-iu$ is $2i$) will be $(-)^q$, where

$$q=\frac{n(n-1)}{2}+\frac{t(t-1)}{2}=\frac{(t+\tau)(t-1+\tau)+t(t-1)}{2}=t(t-1)+t\tau+\frac{\tau(\tau-1)}{2}.$$

Hence since τ and $t(t-1)$ are both even, $(-)^q=(-)^{\frac{\tau(\tau-1)}{2}}$, and the sign of the discriminant of a conjugate form is + or − according as the number of imaginary roots does or does not contain 4 as a factor.

It must be remembered that the sign of the discriminant is not in general the same as that of the *zeta* or squared product of differences of the roots. The sign of the *zeta* for real equations follows precisely the same law as the sign of the *discriminant* for conjugate ones.

effect of the variable point whose coordinates are $\epsilon^5+\eta^5$ and $\epsilon\eta$ lying within the area Δ, in that portion of it for which ϵ, η became imaginary; viz. it is that in such case the equation (ϵ, η), which then becomes of a conjugate form, will have three simple and two twin roots; and thus the unity of the interpretation is restored if we choose, as we very well may, to extend the use of these terms to the real roots and the paired imaginary roots of ordinary equations. We may neglect the curve of reality R altogether, and affirm that all over the area Δ, ϵ, η will have such values as will give rise to three simple and two coupled roots.

(13) That part of the theorem of NEWTON which had received a demonstration from MACLAURIN and CAMPBELL in the generalized form in which I have enunciated it in this paper, may be easily extended to the case of conjugate equations. It will, as applied to them, read thus: If the $(n-1)$ quadratic derivatives of a conjugate form of the nth degree, all whose roots are simple, be multiplied respectively by the coefficients of any other conjugate form, all whose roots are also *simple*, of the degree $(n-2)$, and the sum of these products be taken as a new quadratic form, the discriminant of this latter must be positive, or, which is the same thing, its determinant must be negative.

(14) So much for the case of $n=5$. If we were to proceed to the consideration of equations of the 6th degree, *two* cases of resistance would present themselves in the demonstration of NEWTON's rule, viz. one in which the signs of the criteria are $-+++-$, the other $-+-+-$. In the latter it would only be necessary to show that the discriminant is necessarily negative, since we know from the derivatives that the equation must have four imaginary roots, and the choice would lie between the alternatives of there being four or six. In the former case the derivatives only indicate the necessary existence of two real roots, and it would become requisite to prove that there must be four or six—an alternative which depends not on the sign of one function of the coefficients, but on the nature of the signs of two such functions given by STURM's or any equivalent theorem. It would thus become requisite to prove that two functions of the coefficients, say L, M, could not *both* be negative; and this might be shown by demonstrating the existence of two quantities, L′, M′, other functions of the coefficients incapable of assuming any but the positive sign such that $L'L+M'M$ would be necessarily positive.

PART II.—ON THE LIMIT TO THE NUMBER OF REAL ROOTS IN EQUATIONS OF THE FORM $\Sigma(ax+b)^n$.

(15) I shall now proceed to the consideration of a theorem relating to a particular class of ordinary equations, which occurred to me in the course of and in connexion with the preceding investigations. The theorem itself, but unaccompanied by proof, has appeared in the 'Comptes Rendus' of the Academy for the month of March 1864.

Both as regards its nature and the processes involved in the proof, it stands in close relation to NEWTON's rule, my study of which in fact led me to its discovery. It will therefore take its place most appropriately in this paper.

Certain preliminary properties of circulation introducing some new notions of polarity must be first established, by way of Lemmas to the proof in question.

By a *type* let us understand a succession of symbols of any subject matter whatever susceptible of receiving the signs $+-$, or any suchlike indications of opposite polarity.

Let $a, b, c, \ldots l, k, l$ be any such type, where the *elements* $a, b, c, \ldots$ may be regarded either as points in a line or rays in a pencil affected respectively with the signs of $+$ and $-$.

Then by a *per-rotatory* circulation of such type, I mean the act of passing from the first element to the second, from the second to the third, &c., from the last but one to the last, and from the last to the first.

By a *trans-rotatory* circulation of the same, I mean the act of passing from the first to the second, the second to the third, &c., from the last but one to the last, and from the last to the first, *with its sign reversed*.

A type considered subject to per-rotatory circulation may be termed a Per-rotatory Type; one subject to the other sort of circulation, a Trans-rotatory Type.

If a, b, c, d, e be a per-rotatory type, its direct *phases* are

$$\begin{array}{ccccc}
a, & b, & c, & d, & e, \\
b, & c, & d, & e, & a, \\
c, & d, & e, & a, & b, \\
d, & e, & a, & b, & c, \\
e, & a, & b, & c, & d,
\end{array}$$

and its retrograde phases

$$\begin{array}{ccccc}
a, & e, & d, & c, & b, \\
e, & d, & c, & b, & a, \\
d, & c, & b, & a, & e, \\
c, & b, & a, & e, & d, \\
b, & a, & e, & d, & c.
\end{array}$$

If, on the other hand, a, b, c, d, e be a trans-rotatory type, its direct *phases* will be

$$\begin{array}{ccccc}
a, & b, & c, & d, & \bar{e}, \\
b, & c, & d, & \bar{e}, & \bar{a}, \\
c, & d, & \bar{e}, & \bar{a}, & b, \\
d, & \bar{e}, & \bar{a}, & \bar{b}, & \bar{c}, \\
\bar{e}, & \bar{a}, & \bar{b}, & \bar{c}, & \bar{d},
\end{array}$$

and its retrograde phases

$$\begin{array}{ccccc}
a, & \bar{e}, & \bar{d}, & \bar{c}, & \bar{b}, \\
\bar{e}, & \bar{d}, & \bar{c}, & \bar{b}, & \bar{a}, \\
\bar{d}, & \bar{c}, & \bar{b}, & \bar{a}, & e, \\
\bar{c}, & \bar{b}, & \bar{a}, & e, & d, \\
\bar{b}, & \bar{a}, & e, & d, & c,
\end{array}$$

where the sign (—) is, for greater convenience of writing, placed over instead of before the elements which it affects; and so on in general a type of n elements, whether per-rotatory or trans-rotatory, will admit of n direct and n retrograde phases.

If we count the number of variations of sign in the circulations of any phase of a per-rotatory type, this number will be the same for all the phases, and will be an even number; this even number may be termed the variation-index of the type.

So, again, if whatever be the original signs of the element in a trans-rotatory type, we count the number of variations in the circulation of any of its phases, this number also will be constant and will be odd, and this odd number may then be termed the variation-index of the type.

(16) Let any phase be taken of a per-rotatory type, and out of such phase let any element be *suppressed*; then we obtain a type one degree lower in the elements, which, if we please, we may consider as a trans-rotatory type, and such trans-rotatory type may be termed a derivative of the original per-rotatory one.

In like manner any phase being taken of a trans-rotatory type, one element may be suppressed, and the reduced type treated as a per-rotatory one, and termed a derivative of the original trans-rotatory one.

We may now enunciate the following important general proposition, viz.

Any trans-rotatory type or any per-rotatory type whose variation-index is different from zero being given, a per-rotatory derivative of the one and a trans-rotatory derivative of the other may be found such that the variation-index of the derived types in either case shall be less by a unit than the variation-index of the types from which they are derived.

Case (1). Let the given type be per-rotatory. Then by hypothesis, since it has some variations, we may find a phase of it beginning with $+$ and ending with $-$, by which I mean beginning with an element that is positive and ending with one that is negative. This gives rise to two sub-cases.

T, the phase in question, will be $+\ldots\ldots+-$

Θ, the phase in question, will be $+\ldots\ldots--$.

In either sub-case let the last sign be suppressed, and the result treated as a trans-rotatory type; then T, Θ become respectively T$'$, Θ', where

$$T' \text{ is } +\ldots\ldots+$$

and

$$\Theta' \text{ is } +\ldots\ldots-$$

and evidently the variation-index of T $-$ variation-index of T$'=$ number of changes of sign in $+-+$ less changes of sign in $+-=2-1=1$; and again variation-index of $\Theta-$ variation-index of $\Theta'=$ number of changes of sign in $--+$ less changes of sign in $--=1-0=1$. Hence the theorem is proved for the case where the given type is per-rotatory.

Case (2). Let the given type be *trans-rotatory*.

Then, again, there must either be a phase of the form P, or one of the form Φ, where

P represents a *continual succession* of signs of the same name as $+ + \ldots +$ or $- - \ldots -$, and Φ represents a succession beginning with one sign as $+$ and ending with one or more signs $-$, or else beginning with $-$ and ending with a succession of signs $+$. Essentially, then, as a change of signs throughout a whole succession does not affect the variation-index, we may suppose

$$P = + \ldots\ldots + +,$$
$$\Phi = - \ldots - + \ldots +,$$

the signs intervening between the two expressed signs $-$ in Φ being filled up in any manner whatever, and those between the two signs $+$ with signs exclusively $+$.

Let now that phase of Φ be taken which commences with the first sign of the final succession of $+$. Then Φ becomes

$$(\Phi) = + \ldots + + \ldots +,$$

which is of the form

$$+ \ldots\ldots + +,$$

so that P is only a particular case of (Φ).* If the last sign in (Φ) be suppressed and the result treated as a per-rotatory type be called $(\Phi)'$, so that $(\Phi)' = + \ldots\ldots +$, we have variation-index in $(\Phi)-$ variation-index in $(\Phi)' =$ changes of sign in $-+$ less changes of sign in $++ = 1 - 0 = 1$.

Hence the proposition is established for both cases.

(17) The theorem to which this Lemma-proposition is to be applied concerns equations of the form

$$\epsilon_1 u_1^m + \epsilon_2 u_2^m + 0 \ldots + \epsilon_n u_n^m = 0,$$

where $u_1, u_2, \ldots, u_n$ are any linear functions of x, y; m is any positive integer, and $\epsilon_1, \epsilon_2, \ldots \epsilon_n$ are each respectively and separately, either *plus unity* or *minus unity*.

Such an equation for convenience of reference may be termed a superlinear equation, and the function equated to zero a superlinear function.

Every superlinear function may be conceived as having attached to it a pencil of rays constructed in a manner about to be explained.

1. We may conceive the function to be prepared in such a manner, that supposing $ax+by$ to be any one of the n linear elements u, every b shall be positive. If m is even, this can be effected by writing when required for $ax+by$, $-ax-by$ without further change. If m is odd, we may write when required $-ax-by$ in place of $ax+by$, changing at the same time the factor ϵ, which appertains to $(ax-by)^m$ from $+1$ to -1, or *vice versâ*, from -1 to $+1$.

Now take in a plane any two axes of coordinates $O\xi$, $O\eta$, and consider a, b as the ξ and η coordinates of a point. All the n points thus obtained, on account of every b being positive, will lie on the same side of the axis $O\eta$, and thus the entire n linear functions will be represented by a pencil of n rays, the two extreme rays of which make an angle less than two right angles with each other; but each term of the superlinear function contains, besides $(ax+by)^m$, a definite multiple $+1$, or -1, and we must accordingly, to

completely express such term, conceive every ray affected with a distinct sign + or —. A pencil thus drawn with its rays so polarized will give a complete representation of any given superlinear function, and may be called its type-pencil([23]).

I am now able to state the following proposition:

(18) *The number of real roots in a superlinear equation cannot exceed the variation-index of its type pencil, regarded as a per-rotatory type, if the degree of the equation be even, and as a trans-rotatory type if the degree of the equation be odd.* I prove this inductively as follows.

1. Suppose the theorem to be true when the variation-index of the type-pencil is not greater than the even number ν, and consider an equation of the odd degree $(2i+1)$, for which the type-pencil viewed as trans-rotatory has the variation-index $\nu+1$.

Let a *phase* of this type be taken, say corresponding to the rays $\varrho_n, \varrho_{n-1} \ldots \varrho_2, \varrho_1$, such that the per-rotatory type obtained by striking out the term ϱ_1 has the variation-index ν (as we know may be done by virtue of the Lemma).

Take for new axes $O\xi'$, $O\eta'$, when $O\xi'$ coincides with ϱ_1; then it is clear that the pencil $\varrho_n, \varrho_{n-1} \ldots \varrho_2, \varrho_1$ will still serve as a type-pencil to the given function, the only change being that some of the rays, namely those that did lie on one side of ϱ_1, have been inverted in direction and changed in sign (corresponding to a change in the coefficient a, b, accompanied with a change in the sign of the corresponding ϵ), whilst the rays on the other side of ϱ_1 have been left unaltered.

The points (a_1, b_1), $(a_2, b_2) \ldots (a_n, b_n)$ corresponding to the rays $\varrho_1, \varrho_2, \ldots \varrho_n$ will, with respect to the new axes, change their values, becoming converted into $(\alpha_1, 0)$, (α_2, β_2), $(\alpha_3, \beta_3), \ldots (\alpha_n, \beta_n)$, where $\beta_2, \beta_3, \ldots \beta_n$ will still all be positive, the angle between ϱ_1 and ϱ_n being the same as between the two extreme rays in the original figure of the type-pencil, and the superlinear equation may now be written in the form

$$F(u, v)=\epsilon_1(\alpha_1 u)^{2i+1}+\epsilon_2(\alpha_2 u+\beta_2 v)^{2i+1}+\epsilon_3(\alpha_3 u+\beta_3 v)^{2i+1}+\epsilon_n(\alpha_n u+\beta_n v)^{2i+1}=0,$$

where u, v are real linear functions of x, y.

([23]) Let a circle be imagined pierced by a pencil containing any number of rays protracted in both directions, say in the opposite points a, α; b, β; c, γ; d, δ; and let these points, taken in order of natural succession from left to right, or right to left, be $a, b, c, d, \alpha, \beta, \gamma, \delta$. Then, commencing with any point c, a *complete* circulation will be represented by the succession of transits

$$c \text{ to } d, \quad d \text{ to } \alpha, \quad \alpha \text{ to } \beta, \quad \beta \text{ to } \gamma, \quad \gamma \text{ to } \delta, \quad \delta \text{ to } a, \quad a \text{ to } b, \quad b \text{ to } c.$$

But whether $\alpha, \beta, \gamma, \delta$ bear respectively the same signs or signs contrary to those of a, b, c, d, the transit between any two points β to γ will be of the same nature, as regards continuance or change of sign, as the transit from b to c, and thus we see that the complete cycle or total revolution above indicated is only a reduplication of, and may be fully designated by the hemicyclic succession c to d, d to α, α to β, β to γ, for which the number of variations therefore will be the same as for any similar succession obtained by commencing with any other element in the original system of points instead of c. If the opposite points bear like signs, the above succession of transits may be indicated by the order c, d, a, b, c; if they bear contrary signs by the order $c, d, \bar{a}, \bar{b}, \bar{c}$, and thus it is that the idea arises of the two kinds of so-called circulation, but which are in fact only more or less disguised species of semicirculation.

Let the derivative of this function be taken in regard to v, and we have

$$\frac{1}{2i+1}\mathrm{F}'(u,v)=\beta_2\varepsilon_2(\alpha_2u+\beta_2v)^{2i}+\beta_3\varepsilon_3(\alpha_3u+\beta_3v)^{2i}\ldots+\beta_n\varepsilon_n(\alpha_nu+\beta_nv)^{2i},$$

where $\beta_2\varepsilon_2$, $\beta_3\varepsilon_3\ldots\beta_n\varepsilon_n$ have the same signs as $\varepsilon_2, \varepsilon_3, \ldots \varepsilon_n$ respectively.

Now the pencil-type of $\mathrm{F}'(u, v)$ will be the per-rotatory type $\varrho_n, \varrho_{n-1}, \ldots \varrho_2$, of which by construction the variation-index is ν. Hence by hypothesis $\mathrm{F}'(u, v)$ has not more than ν real roots, i. e. at least $2i-\nu$ imaginary roots. Hence $\mathrm{F}(u, v)$ has at least that number of imaginary roots, i. e. at most $(2i+1)-(2i-\nu)$, i. e. $\nu+1$ real roots. Hence if the theorem is true for ν an even number, it is true for $\nu+1$.

In like manner let us proceed to show that when it is true for ν an odd number, it would remain true for $\nu+1$.

The reasoning will be precisely similar to that followed in the antecedent case. We must find a phase of the *per-rotatory* type $\varrho_n, \varrho_{n-1}, \ldots \varrho_2, \varrho_1$ having the variation-index ν such that the trans-rotatory reduced type $\varrho_n, \varrho_{n-1}, \ldots \varrho_2$ shall have the variation-index $\nu-1$; the new pencil will still continue to be a type-pencil of the given superlinear function, the change of direction in the bunch of rays one on side of ϱ_1 being now unaccompanied with change of sign, such change corresponding to $\varepsilon(ax+by)^{2i}$ becoming changed into $\varepsilon(-ax-by)^{2i}$ without ε undergoing a change of sign.

As before, the axes of coordinates are transformed from ξ, η into ξ', η', and we obtain

$$\mathrm{F}(u, v)=\varepsilon_1(\alpha_1u)^{2i}+\varepsilon_2(\alpha_2u+\beta_2v)^{2i}+\ldots+\varepsilon_n(\alpha_nu+\beta_nv)^{2n+1},$$

$$\frac{1}{2i}\mathrm{F}'(u, v)=\beta_2\varepsilon_2(\alpha_2u+\beta_2v)^{2i-1}+\ldots+\beta_n\varepsilon_n(\alpha_nu+\beta_nv)^{2i}.$$

for which the type-pencil is the trans-rotatory type $\varrho_n, \varrho_{n-1}, \ldots \varrho_2$, of which by construction the variation-index is $\nu-1$, so that its number of imaginary roots is $2i-(\nu-1)$, and consequently the number of real roots of $\mathrm{F}(u, v)$ will be $\nu+1$.

Thus, then, if the theorem be true for ν, whether ν be even or odd, it will be true for $\nu+1$.

But when $\nu=0$, the superlinear function becomes a sum of even powers of linear functions of x, y, all taken with the same sign, of which the number of roots is evidently 0. Hence, being true for this case, the proposition is true universally.

It will be noticed that the algebraical part (as distinguished from the purely polar-tactic part of the above demonstration) depends on the same principle of which such abundant use has been made in the former part of this dissertation, viz. that the number of imaginary roots in any ordinary algebraical equation in x cannot be increased when we operate any homographic substitution upon x, and take the derivative of the equation thus transformed in lieu of the original([24]).

([24]) For greater clearness I present in an inverted order of arrangement a summary of the foregoing argument.

By an ith derivative of $f(x, y)$ is meant any derived form

$$\left(\lambda_1\frac{d}{dx}+\mu_1\frac{d}{dy}\right)\left(\lambda_2\frac{d}{dx}+\mu_2\frac{d}{dy}\right)\ldots\left(\lambda_i\frac{d}{dx}+\mu_i\frac{d}{dy}\right)f(x, y),$$

(19) The proposition above established leads immediately to the theorem and corollary following, viz.

THEOREM. If $c_1, c_2, \dots c_n$ be a series of ascending or descending magnitudes, and m any positive integer, the equation

$$\lambda_1(x+c_1)^m+\lambda_2(x+c_2)^m+\dots+\lambda_n(x+c_n)^m=0$$

cannot have more real roots than there are changes of sign in the sequence $\lambda_1, \lambda_2, \dots \lambda_n, (-)^m\lambda_1$.

For obviously $(1, c_1)$, $(1, c_2), \dots (1, c_n)$ will be points corresponding to rays within a semirevolution, and therefore forming a type-pencil.

Corollary. If the above equation be transformed by any real homographic substitution into the form

$$\mu_1(y+\gamma_1)^m+\mu_2(y+\gamma_2)^m+\dots+\mu_n(y+\gamma_n)^m=0,$$

where $\gamma_1, \gamma_2, \dots \gamma_n$ are taken in ascending or descending order, the number of changes of sign in the series $\mu_1, \mu_2, \dots \mu_n, (-)^m\mu$ is *invariable*(25); for the effect of any such formation will be to leave the type-pencil unaltered except in its *phase*.

(20) If we look to the undeveloped form of the superlinear function

$$S=\varepsilon_1 u_1^m+\varepsilon_2 u_2^m+\dots+\varepsilon_n u_n^m,$$

and are supposed to possess no knowledge of the coefficients which enter into the linear elements u, we may still draw some general inferences as to the limit of the number of real roots in $S=0$. Thus if the number of positive units ε is j, and of the negative units k, and j is not greater than k, it is obvious that, whatever may be the form of the type-pencil to S, its variation-index cannot be more than $2j$ when m is even, nor more than $2j+1$ when m is odd; for the arrangement the most favourable to the largeness of the number of the real roots is that where every two rays with the signs belong-

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the λ, μ quantities being any real quantities whatever. Then I say—

1. If T is the type-pencil (per-rotatory or trans-rotatory) of any superlinear form F, every derivative of T of the contrary name is the type-pencil of some first derivative of F, as shown in art. (18).

2. A derivative of T of contrary name may be found such that its variation-index shall be less by a unit than that of T itself, as shown in art. (16).

3. Hence if i is the variation-index of the type-pencil of F, an ith derivative of F may be found such that its variation-index shall be zero, and consequently having no real roots.

Hence, finally, since the number of real roots of any rational integral homogeneous function in x, y cannot exceed by more than i the number of the real roots in any of its ith derivatives, F cannot have more real roots than there are units in the variation-index of its type-pencil.

The subtle point of the argument, it will be noticed, lies in forming the conception of the variation-index to a trans-rotatory pencil, in which the singular phenomenon occurs of a reversal of *relative polarity* in passing from the last ray to the first, whereas in a per-rotatory pencil any ray indifferently may be regarded as the initial ray, no such reversal in that case taking place.

(25) It may be noticed that, contrariwise, the limit to the number of real roots given by NEWTON's criteria is *not* an invariant; it fluctuates with the homographic transformations operated upon the equation; and a question suggests itself as to the maximum value the number of imaginaries indicated by the rule can attain. I presume this maximum is not in all cases necessarily the actual number of the imaginary roots possessed by the equation.

ing to the j group of ϵ are separated by one or more of the rays with a contrary sign to themselves. Thus it appears that when only the units $\epsilon_1, \epsilon_2, \ldots \epsilon_n$ are given, we may impose a maximum upon the number of real roots in the superlinear equation; this limit may be called the *absolute maximum*, being the double of the inferior number of like signs in the series $\epsilon_1, \epsilon_2, \ldots \epsilon_n$ when the degree is even, and one more than such double when the degree is odd [26].

The *specific maximum*, on the other hand, will depend on the form of the type-pencil, and cannot be ascertained until the coefficients of the linear elements are given. It can never exceed, but may be less than the absolute maximum. It may, indeed, be easily proved that *in general* the specific maximum will be less than the absolute maximum. Thus, by way of example, suppose the degree to be even, and the inferior number of like signs to be 2; the absolute maximum number of real roots will be four, but the specific maximum will more generally be only two. For let the number of linear terms in the superlinear function be $2+n$, n being 2 or any greater number; and first, to fix the ideas, suppose $n=2$. The type-pencil, which is to be read per-rotatorily, consists of four rays, say a, b, c, d, following each other in uninterrupted circular order, of which two are to bear positive and two negative signs. If the two negative signs fall on a, c or on b, d, the variation-index will be 4, but in the other four cases of incidence such index will be only 2. Consequently the chance is 2 to 1 [27] that the specific maximum, which may be 4, is not greater than 2; and consequently the chance that there will be four real roots in the equation will be only a chance (too difficult to be calculated, but which is a function of the degree of the equation) *of* the chance $\frac{1}{3}$ that there will be as many as four real roots in the equation $u_1^n+u_2^n-u_3^n-u_4^n=0$, where u_1, u_2, u_3, u_4 are

[26] [a] If a superlinear form of an odd degree contains an odd number of terms, say $2k+1$, the greatest value of the *inferior* number of like signs is k, and the extreme limit to the number of real roots will be $2k+1$.

If it contain an even number of terms, say $2k$, the greatest value of the inferior index is k; but for this particular case it will readily be seen that a limit may be assigned to the variation-index closer than that given by the rule in the text; in fact the variation-index cannot in that case exceed $2k-1$, which will therefore be the extreme limit to the number of real roots. Now suppose the canonizant of an odd-degreed function of x, y to have all its roots real, then it may be expressed by a superlinear form of which the number of terms will be $2i+1$ or $2i$, according as the degree is $4i+1$ or $4i-1$. In the one case the number of real roots cannot exceed $2i+1$, in the other $2i-1$. Hence the following somewhat curious theorem:

[b] *If the canonizant of an odd-degreed quantic in* x, y, *of the degree* $4i\pm1$, *has* no *imaginary roots, the quantic itself must have at least* i *pairs of imaginary roots.* From the fact that when the roots of the canonizant of a quintic are all real there must be one pair at least of imaginary roots, we can infer that when the discriminant of a quintic is positive and that of its canonizant is negative, the equation has one real and four imaginary roots. This observation has led to a long train of reflections, which will be found embodied in the 3rd part of the memoir.

[27] This, in fact, is identical in substance with the noted problem of determining the chance that two straight lines drawn on a black board will cross. Mr. CAYLEY, of whom it may be so truly said, whether the matter he takes in hand be great or small, "nihil tetigit quod non ornavit," suggests the following independent proof of this. Taking unity as the length of the contour, fixing the extremity of one of the lines, and calling s the distance of its other end from it measured on the contour, the chance of the second line crossing this is easily seen to be $2s(1-s)$, which, integrated between $s=0$, $s=1$, gives $\frac{1}{3}$, as before obtained.

unknown linear functions of x: thus we are entitled to say that *in general* the number of real roots in such an equation is *not* the maximum four, but a less number. This remark is of importance, as showing that on this subject it is possible to speak with scientific certainty, and on other than empirical grounds, of what may *in general* be expected to take place. Thus we find NEWTON declaring twice over in the chapter quoted, that *in general* his rule will give not merely the maximum, but the actual number of the imaginary roots in an equation. I am strongly inclined to doubt the truth of this assertion; but it is important to be satisfied by analogy that such an assertion may rest on a scientific and demonstrative basis, and not on the utterly fallacious foundation of arithmetical empiricism([28]).

([28]) A few additional words on this question of probability may not be unacceptable. In order to meet the case of the degree of the superlinear form or equation being odd as well as even, let it be supposed known under the form

$$\Sigma_n^1 \lambda_i (x+c_i)^m,$$

the values of the quantities c_1 being supposed to be left wholly indeterminate, and only the signs of the quantities λ to be given. Let ω be the inferior number of like signs in the λ series, meaning thereby that the number of signs of one sort is ω, and of the other sort ω, or more than ω.

Let the probability of the specific maximum of real roots being $2k$ when m is even, be represented by p_{2k}, and of its being $2k+1$ when m is odd by π_{2k+1}; also let s_{2k}, σ_{2k+1} represent the number of cases when ω and n are given which correspond to the specific maximum being $2k$, $2k+1$ respectively. Suppose $\omega=1$, then obviously, when m is even, we have $s_2=n$, $p_2=1$. But when n is odd $\sigma_1=2$ (for when either extreme element alone is negative the trans-rotatory cycle has the variation-index unity), and $\sigma_3=n-2$, so that

$$\pi_1=\frac{2}{n}, \quad \pi_3=\frac{n-2}{n}$$

Again, suppose $\omega=2$, m being even; then obviously s_2 is the number of contiguous duads in a cycle of n elements, and s_4 is the remaining number of duads; hence

$$s_2=n, \; s_4=n\frac{n-1}{2}-n=n\frac{n-3}{2};$$

so that

$$p_2=\frac{2}{n-1}, \quad p_4=\frac{n-3}{n-1}.$$

2nd. Suppose $\omega=2$, m being odd, so that σ_1, σ_3, σ_5 will have to be separately estimated. To fix the ideas, let the λ series be termed a, b, c, d, e, f, g, in which two of the elements are supposed of one sign, say negative, and the rest of the opposite sign, say positive; then the only dispositions of sign which correspond to the specific maximum being 1 are those in which a, b or else f, g are both negative. Hence $\sigma_1=2$. Again, the dispositions of sign which make the specific maximum equal to 3 are those in which a, g are both negative, those in which a and c, d, e, or f are negative, those in which g and e, d, c, or b are negative, and, finally, those in which any two contiguous elements except the a and g are negative. Hence $\sigma_3=1+2(n-3)+(n-3)=3n-8$; and it should be observed that this result cannot be prejudiced in its generality by the supposition of any of the components of σ_3 becoming negative, since $\omega=2$ implies that n is at least 4. Hence, finally,

$$\sigma_5=\frac{n^2-n}{2}-(3n-8)-2=\frac{n^2-7n+12}{2}=\frac{(n-3)(n-4)}{2};$$

so that

$$\pi_1=\frac{4}{n^2-n}, \quad \pi_3=\frac{6n-20}{n^2-n}, \quad \pi_5=\frac{n^2-7n+16}{n^2-n}$$

This example serves to show how much more difficult is the computation of the respective probabilities when m is odd than when m is even, owing to the break of continuity in the cycle of readings on passing from the last to the first term.

NOTES TO SECTION II.

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On the probability of the specific superior limit to the number of real roots in a superlinear equation equalling any assigned integer.

(21) The question comes to that of determining the probability of a per-rotatory or trans-rotatory pencil with a definite number of rays of each kind possessing a given variation-index.

Since the foot note below was written, a method has occurred to me of obtaining the probability in question in general terms, as follows.

1. For a *per-rotatory* pencil of μ positive and ν negative rays. Let $[\mu, \nu, g]$ be the probability of the rays being so disposed as to give rise to $2g$ variations of sign in making a complete revolution. Then there will be g distinct groups of positive, and g of negative rays. The number of partitions with permutations of the parcels *inter se* of μ elements in g parcels is $\frac{(\mu-1)(\mu-2)..(\mu-g+1)}{1.2..(g-1)}$, and of ν elements into g parcels is $\frac{(\nu-1)(\nu-2)..(\nu-g+1)}{1.2..(g-1)}$.

If we combine each parcel with each in every possible way, and then imagine the combined parcels let into a circle containing $m+n$ places and shifted round in the circle through a complete revolution, we shall obtain

$$(\mu+\nu)\times\frac{(\mu-1)(\mu-2)..(\mu-g+1)1}{1.2..(g-1)}\cdot\frac{(\nu-1)(\nu-2)..(\nu-g+1)}{1.2..(g-1)}$$

arrangements; but on examination it will be found that every arrangement so produced will be repeated g times; moreover it is obvious that no other arrangement giving rise to g groups of each sort can be found. Hence the true number of distinct groupings of the sort in question is

$$\frac{(\mu+\nu)}{g}\cdot\frac{(\mu-1)(\mu-2)..(\mu-g+1)}{1.2..(g-1)}\cdot\frac{(\nu-1)(\nu-2)..(\nu-g+1)}{1.2..(g-1)}.$$

It seems hardly worth while to pursue this subject in greater detail. I will only notice that when m is even the chance of the specific maximum attaining the absolute maximum, *i. e.* becoming 2ω, will depend on the proportion of the ways in which in a cycle of n elements ω of them may be marked with a distinctive sign in such a way that no two of such signs shall come together. Accordingly I find by a computation of no great difficulty (understanding πx to mean $1.2.3...x$),

$$s_{2\omega}=\frac{n\pi(n-\omega-1)}{\pi\omega\pi(n-2\omega)},$$

and hence, since the total number of combinations of n elements ω and ω together is $\frac{\pi(n)}{\pi\omega\pi(n-\omega)}$, I deduce

$$p_{2\omega}=\frac{\pi(n-\omega)\pi(n-\omega-1)}{\pi(n-1)\pi(n-2\omega)}.$$

Thus when n has its minimum value, viz. 2ω, $p_{2\omega}=\frac{\pi\omega\pi(\omega-1)}{\pi(\omega-1)}$, and becomes very small as ω increases. When again n increases towards infinity $p_{2\omega}$ approaches indefinitely near to unity, and the chance approaches near to certainty of the specific not beoming less than the absolute maximum of real roots.

And the total number of arrangements, which is the number of ways in which μ things can be distributed over $(\mu+\nu)$ places, is $\frac{\pi(\mu+\nu)}{\pi\mu\pi\nu}$. Hence we obtain

$$[\mu,\nu,g]=\frac{\pi\mu\pi\nu}{\pi(\mu+\nu-1)}\left\{\frac{(\mu-1)(\mu-2)..(\mu-g+1)\times(\nu-1)(\nu-2)..(\nu-g+1)}{1.2.(g-1)(1.2..g)}\right\}$$

$$=\frac{\pi\mu\pi(\mu-1)\pi\nu\pi(\nu-1)}{\pi g\pi(g-1)\pi(\mu-g)\pi(\nu-g)\pi(\mu+\nu-1)}.$$

[Throughout these investigations $\pi(x)$ is used in the same sense as Πx, to signify the factorial $1.2.3\ldots x$.]

If there should appear any obscurity in the statement of the method by which has been obtained the number of distinct distributions of the μ, ν elements into g groups of each, the reader is referred to the equation in differences obtained further on in this Note, by which all doubt of the correctness of the result will be removed.

(22) For a *trans-rotatory* pencil of rays, to ascertain the probability of the variation-index being $2g+1$.

Imagine a circular arrangement of μ positive elements and ν negative elements containing 2γ variations.

Let this circle be supposed opened out at any point and the variations of the open pencil so formed to be reckoned according to the trans-rotatory law, which is that in passing from one extremity to the other a change is to be seen as a variation, and a variation as a change. If the break is made between two negative or between two positive elements, the number of variations obviously becomes *increased* by one unit; but if between a positive and a negative element, that number becomes decreased by one unit. The number of these latter intervals is 2γ, and of the former $\mu+\nu-2\gamma$.

Hence the probability of the index becoming $2\gamma+1$ is $\frac{\mu+\nu-2\gamma}{\mu+\nu}$, and of its becoming $2\gamma-1$ is $\frac{2\gamma}{\mu+\nu}$.

If, then, we denote the probability to be calculated by $[\mu,\nu,g+\frac{1}{2}]$, it is obvious that we shall have

$$[\mu,\nu,g+\tfrac{1}{2}]=\frac{\mu+\nu-2g}{\mu+\nu}[\mu,\nu,g]+\frac{2(g+1)}{\mu+\nu}[\mu,\nu,g+1].$$

But by the formula previously obtained it will easily be seen that

$$[\mu,\nu,g+1]=\frac{(\mu-g)(\nu-g)}{g(g+1)}[\mu,\nu,g].$$

Hence

$$[\mu,\nu,g+\tfrac{1}{2}]=\frac{[\mu,\nu,g]}{\mu+\nu}\left\{(\mu+\nu-2g)+\frac{2(\mu-g)(\nu-g)}{g}\right\}$$

$$=\left(\frac{2\mu\nu}{g(\mu+\nu)}-1\right)[\mu,\nu,g] \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (*)$$

$$=2\frac{(\pi\mu)^2(\pi\nu)^2}{\pi(g-1)\pi(g+1)\pi(\mu+\nu)\pi(\mu-g)\pi(\nu-g)}-\frac{\pi\mu\pi(\mu-1)\pi\nu\pi(\nu-1)}{\pi(g-1)\pi g\pi(\mu+\nu-1)\pi(\mu-g)\pi(\nu-g)}.$$

When $g=0$ the above expression fails; but reverting to the equation from which it is derived, we obtain

$$(\mu, \nu, \tfrac{1}{2})=\frac{2}{\mu+\nu}[\mu, \nu, 1]=\frac{2\pi\mu\pi\nu}{\pi(\mu+\nu)}.$$

(23) These combined results admit of an easy corroboration, for

$$\Sigma^0_\infty(\mu, \nu, g+\tfrac{1}{2})=1, \text{ and } \Sigma^0_\infty(\mu, \nu, g)=1.$$

Hence the equation marked * gives

$$1=[\mu, \nu, \tfrac{1}{2}]+\frac{2\mu\nu}{\mu+\nu}\Sigma\frac{[\mu, \nu, g]}{g}-1.$$

Hence we ought to have

$$\frac{\pi\mu\pi\nu}{\pi(\mu+\nu)}+\frac{\mu\nu}{\mu+\nu}\Sigma\frac{[\mu, \nu, g]}{g}=1, \text{ i. e. } 1+\Sigma\frac{\pi\mu\pi\nu}{\pi(\mu-g)\pi g\pi(\nu-g)\pi g}=\frac{\pi(\mu+\nu)}{\pi\mu\pi\nu};$$

which is true, since the left-hand side of the equation is $1+\mu\nu+\mu\frac{\mu-1}{2}\cdot\nu\frac{\nu-1}{2}+\ldots$, which is obviously the coefficient of x^ν in $(1+x)^\mu(x+1)^\nu$, i. e. in $(1+x)^{\mu+\nu}$.

(24) If we wish to find the chance of the specific superior limit becoming equal to the absolute superior limit, we must write g in the above formulæ equal to ν, that one of the two quantities μ, ν which is not greater than the other, and we shall obtain

$$[\mu, \nu, \nu] \quad =\frac{\pi\mu\pi(\mu-1)}{\pi(\mu+\nu-1)\pi(\mu-\nu)},$$

$$[\mu, \nu, \nu+\tfrac{1}{2}]=\frac{\pi\mu\pi(\mu-1)}{\pi(\mu+\nu)\pi(\mu-\nu-1)};$$

so that, in fact, $[\mu, \nu, \nu+\frac{1}{2}]=[\mu, \nu+1, \nu+1]$, which relation may also be obtained by *à priori* considerations.

(25) With reference to the remark made concerning the mode of obtaining the value of $[\mu, \nu, g]$, I proceed to show how it may be obtained directly by the integration of an equation in differences, and by a method analogous in idea to that by which $[\mu, \nu, g+\frac{1}{2}]$ was made to depend on $[\mu, \nu, g]$. For as in that case we conceived an open pencil to be closed and then reopened, so we may imagine one of the rays to be withdrawn and then reinserted. In this way, observing that the effect of introducing a negative sign into a circle of μ positive and n negative signs consisting of ν distinct groups of each is to produce no change in the number of the groups if inserted between two negative signs, but to increase that number by unity if inserted between two positive signs, we may infer that the probability of ν becoming $\nu+1$, in consequence of such insertion, is $\frac{\mu-\nu}{\mu+\nu}$, and of ν remaining unaltered, is $\frac{n+\nu}{\mu+n}$.

Hence we obtain the equation in differences,

$$[\mu, \nu, g]=\frac{\nu-1+g}{\mu+\nu-1}[\mu, \nu-1, g]+\frac{\mu-g+1}{\mu+\nu-1}[\mu, \nu-1, g-1],$$

in which μ may be considered constant, and ν and g to vary.

The integral must satisfy the further condition that $[\mu, 1, g]$ shall be unity when g is 1, and zero for all values of g greater than 1.

Assume the value of $[\mu, 1, g]$ obtained by the method given in art. (21). This obviously satisfies the initial conditions corresponding to $g=1$. Moreover we may easily deduce from it the equalities

$$[\mu, \nu-1, g-1]=\frac{(g-1)g}{(\mu-g+1)(\nu-g)}[\mu, \nu-1, g], \text{ and } [\mu, \nu, g]=\frac{(\nu-1)\nu}{(\mu+\nu-1)(\nu-g)}[\mu, \nu-1, g].$$

Hence the equation in differences will be satisfied if it be true that

$$\frac{(\nu-1)\nu}{\nu-g}=(\nu-1+g)+\frac{(g-1)g}{\nu-g},$$

which is obviously the case, since $\nu^2-\nu-g^2-g=(\nu-g)(\nu+g-1)$.

Since, then, the assumed value of $[\mu, \nu, g]$ is correctly determined when $\nu=1$, it is obvious, from the form of the equation, that it holds good for all other values of ν, as was to be shown.

(26) From the equation

$$\frac{[\mu, \nu, g+1]}{[\mu, \nu, g]}=\frac{(\mu-g)(\nu-g)}{g(g+1)}$$

making $(\mu-g)(\nu-g)=g(g+1)$ or $g=\frac{\mu\nu}{\mu+\nu+1}$, we may readily infer that the value of g for which the probability $[\mu, \nu, g]$ is greatest is the integer part of $\frac{\mu\nu}{\mu+\nu+1}$, if that quantity is non-integer, or the quantity itself and the number next below it (indifferently) if it is an integer.

(27) If we apply a similar method to $[\mu, \nu, g+\frac{1}{2}]$, we obtain by aid of the formula above given,

$$\frac{[\mu, \nu, g+\frac{1}{2}]}{[\mu, \nu, g-\frac{1}{2}]}=\frac{2\mu\nu-(\mu+\nu)\gamma}{2\mu\nu+\mu+\nu-(\mu+\nu)\gamma}\cdot\frac{(\mu+1)-\nu(\nu+1-\gamma)}{\gamma^2};$$

and equating this ratio to unity, we obtain

$$\frac{2\mu\nu-(\mu+\nu)\gamma}{2\mu\nu+\mu+\nu-(\mu+\nu)\gamma}=\frac{\gamma^2}{(\mu+1)(\nu+1)-(\mu+\nu+2)\gamma};$$

or writing $\mu+\nu=p$, $\mu\nu=q$,

$$(p^2+p)\gamma^2-(3pq+4q+p^2+p)\gamma+2q(q+p+1)=0.$$

The roots of this equation will be both of them real, for its *determinant* is

$$p^2q^2+16pq^2+16q^2+(p^2+p^3)(\mu^2+\nu^2),$$

which is necessarily positive. Hence it follows that there are two positive roots of the equation. Whether there will exist values of g which give actual maxima or minima values, or one and the other to $[\mu, \nu, g+\frac{1}{2}]$, depends on the further condition being satisfied that the values of g in the above equation shall come out, one or both of them, not greater than either of the two numbers μ, ν. The inquiry connected with the satisfaction of this condition may be conducted by means of repeated applications of the

processes of STURM's theorem; but I shall not enter upon it, as it appears to lead to calculations of complexity disproportionate to the interest of the result.

(28) It may be noticed that the *average* value of $[\mu, \nu, g]$ can be calculated without any difficulty. This will be $\Sigma(g[\mu, \nu, g])$, or

$$\frac{\pi\mu\pi\nu}{\pi(\mu+\nu-1)}\left[1+\frac{(\mu-1)(\nu-1)}{1}+\frac{(\mu-1)(\mu-2)(\nu-1)(\nu-2)}{1.2^2}+\ldots\right]$$

$$=\frac{\pi\mu\pi\nu}{\pi(\mu+\nu-1)}\cdot\frac{\pi(\mu+\nu-2)}{\pi(\mu-1)\pi(\nu-1)}=\frac{\mu\nu}{(\mu+\nu-1)};$$

so that the average number of variations of sign in a per-rotatory pencil with μ positive and ν negative signs is $\frac{2\mu\nu}{\mu+\nu-1}$, or a little more than the harmonic mean between μ, ν.

In like manner, for a trans-rotatory pencil this number will be

$$\Sigma(2g+1)[\mu, \nu, g+\tfrac{1}{2}]=[\mu, \nu, \tfrac{1}{2}]+\Sigma\left((2g+1)\left(\frac{2\mu\nu}{g(\mu+\nu)}-1\right)[\mu, \nu, g]\right),$$

which, observing that $\Sigma[\mu, \nu, g]=1$, and $(\mu, \nu, \frac{1}{2})+\frac{2\mu\nu}{\mu+\nu}\Sigma\frac{\mu, \nu, g}{g}=2$, gives as the average number of variations of sign $\frac{4\mu\nu}{\mu+\nu}-\frac{2\mu\nu}{\mu+\nu-1}+1$.

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(29) The simplest mode of calculating the value of $[\mu, \nu, g]$ is the following:

Let $[\mu, \nu, g)$, $\left[\mu, \nu, (g-\frac{1}{2})\right)$ denote the probability that an arrangement in open line (in which, as is the case in applying DES CARTES's rule of signs, no account is taken of the relation of the extreme signs to each other) shall contain respectively $2g$ and $2g-1$ variations. Conceive a circular arrangement of γ groups of positive and γ groups of negative signs. If this circle be opened out into a line at an interval between a positive and negative sign (of which there are 2γ), one variation will be lost; but if at any of the remaining $\mu+\nu-\gamma$ intervals, the number of variations remains unaltered. Hence we derive immediately

$$[\mu, \nu, g)=\frac{\mu+\nu-2g}{\mu+\nu}[\mu, \nu, g] \text{ and } [\mu, \nu, g-\tfrac{1}{2})=\frac{2g}{\mu+\nu}[\mu, \nu, g].$$

But we may find $\left[\mu, \nu, g-\frac{1}{2}\right)$ by counting the arrangements which give μ, ν, $2g-1$ variations of sign. These may be all obtained, and without repetition, by intercalating every distribution of μ into g groups with every distribution of ν into the same; and the intercalation may be performed in *two* ways, according as the parcels of the μ signs, or those of the ν signs, are taken first in order. Hence we have

$$[\mu, \nu, g-\tfrac{1}{2})=\frac{2(\mu-1)(\mu-2)\ldots(\mu-g+1)}{1.2\quad\ldots\quad(g-1)}\;\frac{(\nu-1)(\nu-2)\ldots(\nu-g+1)}{1.2\quad\ldots\quad(g-1)}\;\frac{\pi\mu\pi\nu}{\pi(\mu+\nu)}$$

$$=\frac{2\pi\mu\pi(\mu-1)\pi\nu\pi(\nu-1)}{\pi(\mu+\nu)\pi(g-1)\pi(g-1)\pi(\mu-g)\pi(\nu-g)};$$

and thus

$$[\mu, \nu, g]=\frac{\mu+\nu}{2g}[\mu, \nu, g-\tfrac{1}{2}]=\frac{\pi\mu\pi(\mu-1)\pi\nu\pi(\nu-1)}{\pi(\mu+\nu+1)\pi g\pi(g-1)\pi(\mu-g)\pi(\nu-g)},$$

as previously found; also

$$[\mu, \nu, g)=\frac{(\mu+\nu-2g)\pi\mu\pi(\mu-1)\pi\nu\pi(\nu-1)}{\pi(\mu+\nu)\pi g\pi(g-1)\pi(\mu-g)\pi(\nu-g)}.$$

(30) Moreover, we thus see that the average number of variations in an open line with μ positive and ν negative signs, which is

$$\Sigma(2g-1)[\mu,\nu,g-\tfrac{1}{2}]+\Sigma 2g[\mu,\nu,g],$$

or

$$\Sigma 2g\left([\mu,\nu,g-\tfrac{1}{2}]+[\mu,\nu,g]\right)-\Sigma[\mu,\nu.g-\tfrac{1}{2}])$$

will be equal to

$$\Sigma 2g[\mu,\nu,g]-\Sigma\frac{2g}{\mu+\nu}[\mu,\nu,g]=\frac{\mu+\nu-1}{\mu+\nu}\Sigma 2g[\mu,\nu,g]=\frac{\mu+\nu-1}{\mu+\nu}\cdot\frac{2\mu\nu}{\mu+\nu-1}=\frac{2\mu\nu}{\mu+\nu}.$$

The total number of variations and continuations together is $\mu+\nu-1$. Hence the difference between the two is $\frac{4\mu\nu}{\mu+\nu}-(\mu+\nu-1)$, or $\frac{(\mu+\nu)-(\mu-\nu)^2}{\mu+\nu}$; so that the average number of variations is greater than, equal to, or less than that of the continuations, according as the difference between the numbers of the two sets is less than, equal to, or greater than the square root of the entire number of signs. Obviously the average should be the same for the variations as for the continuations if the number of signs, say $n+1$, is given, and each is supposed equally likely to be positive or negative. This is easily verified; for multiplying the probable value of each distribution of signs by the probable value of the number of variations corresponding thereto, we obtain the series

$$\frac{1}{(n+1)2^n}\left\{1.n.(n+1)+2(n-1)(n+1)\frac{n}{2}+3(n-2)\frac{(n+1)n.(n-1)}{1.2.3}+\dots\right\}=\frac{n(n+1)2^{n-1}}{(n+1)2^n}=\frac{n}{2}.$$

This is the final average of the number of variations of sign, and will be equal to that of the continuations, since the entire number of the two together is n.

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PART III.—ON THE NATURE OF THE ROOTS OF THE GENERAL EQUATION OF THE FIFTH DEGREE.

(31) In a foot-note, Part II. of this memoir, I have shown that when the discriminant of the canonizant (constituting an invariant of the twelfth order) of an equation of the fifth degree bears a particular sign, the character of the roots becomes completely determined by the sign of the discriminant of that equation.

This has naturally led me to investigate *de novo* the whole question of the character of the roots of an equation of that degree; and I have succeeded in obtaining under a form of striking and unexpected simplicity the invariantive criteria which serve to ascertain in all cases the nature of the equation as regards the number of real and imaginary roots which it contains; then passing to the expression for these criteria in terms of the roots themselves, I obtain expressions which exhibit the intimate connexion between this subject and a former theory of my own relative to the construction of the conditions for the existence of a given number and grouping of equal roots, which can hardly fail to lead eventually to the extension of the results herein obtained to equations of any odd degree whatever. It is the more needful that these results in a question of so high moment to the advancement of algebraical science should be made public, inasmuch as they do not seem to accord with those obtained by my eminent friend M. HERMITE, who has preceded me in this inquiry in a classic memoir, published in the year 1854 in

the ninth volume of the Cambridge and Dublin Mathematical Journal, since which time I am not aware that the subject has been resumed by any other writer. The discrepancy between our conclusions may be only apparent; but there can be no doubt of the superiority of the form in which they are herein presented, inasmuch as only three functions of the coefficients are required by my method, and five by M. HERMITE's. The solution offered by M. HERMITE is confessedly incomplete, but to this great analyst none the less will always belong the honour, not only of having initiated the inquiry, but of having emitted the fundamental conceptions through which it would seem best to admit of successful treatment. The arrow from my hand may have been the first to hit the mark, but it was his hand which had previously shaped, bent, and strung the bow.

Our methods of procedure, however, are widely dissimilar, and by employing my well-known canonical form for odd-degreed binary quantics, long since given to the world, I have succeeded in evading all necessity for the colossal labours of computation required in M. HERMITE's method, and am able to impart to my conclusions the clearness and certainty of any elementary proposition in geometry, not scrupling to avail myself for such purpose of that copious and inexhaustible well-spring of notions of continuity which is contained in our conception of space, and which renders it so valuable an auxiliary to Mathematic, whose sole proper business seems to me to be the development of the three germinal ideas—of which continuity is one and order and number the other two*.

SECTION I.—*Preparation of the General Binary Quantic of the Fifth Degree.*

(32) Let $$(a, b, c, d, e, i \between x, y)^5 = \mathrm{F}(x, y),$$
a cubic covariant of F is the canonizant C, where C represents the determinant

$$\begin{matrix} a & b & c & d \\ b & c & d & e \\ c & d & e & i \\ y^3 & -y^2x & yx^2 & -x^3. \end{matrix}$$

Let us first suppose that this form does not vanish identically, and has at least two distinct factors ξ, η linear functions of x, y, where of course ξ, η are each of them determinate to a constant factor *près*; giving any value to the constant factor for either of them, we may write $\mathrm{F}(x, y) = \Phi(\xi, \eta) = (\alpha, \beta, \gamma, \delta, \varepsilon, \iota \between \xi, \eta)^5$, and the canonizant of Φ with respect to ξ, η becomes the determinant T, where T represents

$$\begin{matrix} \alpha & \beta & \gamma & \delta \\ \beta & \gamma & \delta & \varepsilon \\ \gamma & \delta & \varepsilon & \\ \eta^3 & -\eta^2\xi & \eta\xi^2 & -\xi^3. \end{matrix}$$

* Herein I think one clearly discerns the internal grounds of the coincidence or parallelism, which observation has long made familiar, between the mathematical and musical ἦθος. May not Music be described as the Mathematic of sense, Mathematic as Music of the reason? the soul of each the same! Thus the musician *feels* Mathematic, the mathematician *thinks* Music,—Music the dream, Mathematic the working life—each to receive its consummation from the other when the human intelligence, elevated to its perfect type, shall shine forth glorified in some future MOZART-DIRICHLET or BEETHOVEN-GAUSS—a union already not indistinctly foreshadowed in the genius and labours of a HELMHOLTZ!

Hence since T to a constant factor *près* is identical with C, the coefficients of η^3 and ξ^3 in the above determinant must vanish in order that $\xi\eta$ may be contained in T.

Hence the two determinants

$$\begin{matrix} \alpha & \beta & \gamma \\ \beta & \gamma & \delta \\ \gamma & \delta & \varepsilon \end{matrix} \quad \text{and} \quad \begin{matrix} \beta & \gamma & \delta \\ \gamma & \delta & \varepsilon \\ \delta & \varepsilon & \iota \end{matrix}$$

both vanish.

Hence either α, β, γ, or otherwise γ, δ, ε, or else the first minors of

$$\begin{vmatrix} \beta & \gamma \\ \gamma & \delta \\ \delta & \varepsilon \end{vmatrix}$$

are each zero.

The first two suppositions must be excluded, since either of them would lead to the conclusion of T, and therefore C, being a perfect cube, contrary to hypothesis. The last supposition implies either that β, γ, δ, or otherwise that γ, δ, ε, or else that $\beta\delta-\gamma^2$ and $\gamma\varepsilon-\delta^2$ are each zero.

If β, γ, δ are each zero, T becomes a multiple of $\eta^2\xi$; if γ, δ, ε are each zero, T becomes a multiple of $\eta\xi^2$; that is to say, T, and consequently C, contains a square factor; and obviously the converse is true, so that when C contains a square factor F is reducible to the form $au^5+5euv^4+fv^5$. When this is not the case $\delta=\frac{\gamma^2}{\beta}$, $\varepsilon=\frac{\delta^2}{\gamma}=\frac{\gamma^3}{\beta^2}$. Hence

$$\mathrm{F}=\left(\alpha-\frac{\beta^2}{\gamma}\right)\xi^5+\frac{\beta}{\gamma}\left(\xi+\frac{\gamma}{\beta}\eta\right)^5+\left(\iota-\frac{\varepsilon^2}{\delta}\right)\eta^5,$$

which is of the form $\omega^5+\phi^5+\psi^5$, ω, ϕ, ψ being linear functions of x, y.

(33) We have supposed C not to be a perfect cube. When it is a perfect cube, say ξ^3, we may assume η any second linear function of x, y; and expressing F in the same manner as before in terms of ξ, η, it is clear that all the first minors of

$$\begin{matrix} \alpha & \beta & \gamma & \delta \\ \beta & \gamma & \delta & \varepsilon \\ \gamma & \delta & \varepsilon & \iota, \end{matrix}$$

except the one obtained by cancelling the last column in the above matrix, must vanish, consequently δ, ε, i must all vanish, so that Φ, and consequently F, must contain a cube factor identical with the canonizant itself.

Lastly, if the canonizant vanish entirely, every first minor in the above matrix, when we write again a, b, c, d, e, i in lieu of α, β, γ, δ, ε, ι, will be zero. Hence either a, b, c, d, or b, c, d, e, or c, d, e, i must each vanish, or else that must be the case with the first minors of

$$\begin{matrix} a & b & c & d \\ b & c & d & e, \end{matrix}$$

or of

$$\begin{matrix} b & c & d & e \\ c & d & e & i, \end{matrix}$$

or of

$$\begin{matrix} a & b & c & d \\ c & d & e & i. \end{matrix}$$

Under the first or third supposition F must contain four equal factors; under the second Φ becomes $a\xi^5+i\eta^5$; under the fourth or fifth it is readily seen that the form becomes

$$a\left(\xi+\frac{b}{a}\eta\right)^5+\left(i-\frac{e^2}{d}\right)\eta^5,\text{ or }\left(a-\frac{b^2}{c}\right)\xi^5+i\left(\eta+\frac{e}{i}\xi\right)^5$$

respectively, so that the second, fourth, and fifth suppositions conduct alike to the form $\omega^5+\varphi^5$, a particular case of the preceding one.

It remains only to consider the sixth supposition, viz. that the first minors of

$$\begin{matrix} a & b & c & d \\ c & d & e & i \end{matrix}$$

are all zero.

In this case if we write

$$\sqrt{a}x+\sqrt{c}y=u,$$

$$\sqrt{a}x-\sqrt{c}y=v,$$

$$A+B=\frac{1}{a^{\frac{4}{}}},$$

$$A-B=\frac{b}{a^2c^{\frac{1}{2}}}$$

and if neither a nor c is zero, it will readily be seen that $F(x, y)$ becomes Au^5+Bv^5 by virtue of the relations

$$d=\frac{c}{a}b,\quad e=\left(\frac{c}{a}\right)^2a,\quad i=\left(\frac{c}{a}\right)^2b\text{[29]}.$$

If $a=0$ or $c=0$, the preceding transformation fails.

But unless also $i=0$ or $e=0$ at the same time as $a=0$ or $c=0$, a legitimate transformation similar to the above may be performed by interchanging a, c, x, y with i, a, y, x.

If now

$a=0$, it will easily be seen that a, b, c, d or else a, c, e are each zero.

Similarly, if

$i=0$, it will easily be seen that i, e, d, c or else i, d, b are each zero.

Again, if

$c=0$, it will easily be seen that a, b, c, d or else c, e are each zero;

and if

$d=0$, it will easily be seen that c, d, e, i or else d, b are each zero.

[29] Thus we see that the equation $ax^5+5bx^4+10acx^3+10bcx^2+5ac^2x+bc^2=0$ belongs to the class of soluble forms.

Thus, then, if $a=0$ and $i=0$, all the coefficients, or else all except one, viz. b or e, are zero;

if $a=0$ and $d=0$, all the coefficients, or else only not e and i or only not b or only not i are zero;

so if $i=0$ and $c=0$, all must be zero except b and a or e or a;

if $c=0$ and $d=0$, only e and i or else a and b or else a and i will differ from zero.

Hence, then, in any case there will be at least four equal roots, or else F is of the form ax^5+iy^5.

Thus, then, for the first time has been here rigorously demonstrated, free from all doubt and subject to no exceptions, the following important proposition:

Every binary quantic function *not containing three or more equal roots* is reducible to one or the other of the two following forms,

$$u^5+v^5+w^5, \text{ or } au^5+5euv^4+fv^5.$$

The former is the case when the discriminant of the canonizant is different from zero, the latter when it is equal to zero; for it will be observed that, whether the canonizant has equal roots or totally disappears, its discriminant in both cases alike is zero.

(34) It has been seen that when the quintic has three equal roots the canonizant becomes a perfect cube; and it may not be out of place here to point out what the conditions (necessary and sufficient) are to ensure the quintic having four equal roots. These are all comprised in that of the quadratic covariant vanishing. To prove this, let η be a factor of $F(x, y)$, so that

$$F(x, y)=\Phi(x, \eta)=(\alpha, \beta, \gamma, \delta, \varepsilon, 0\between x, \eta)^5.$$

Then, since the similar covariant *quoad* x, y must also vanish, we have

$$\alpha\varepsilon-4\beta\delta+\gamma^2=0, \quad -3\beta\varepsilon+2\gamma\delta=0, \quad -4\gamma\varepsilon+3\delta^2=0.$$

If $\varepsilon=0$, then $\delta=0$, $\gamma=0$ by virtue of the two extreme equations, and Φ, and therefore F, contains four equal factors. If ε is not zero,

$$\gamma=\frac{3\delta^2}{4\varepsilon}, \; \beta=\frac{\delta^3}{2\varepsilon^2}, \; \alpha=\frac{5\delta^4}{16\varepsilon^3}, \text{ and } \Phi \text{ becomes } \frac{5\varepsilon}{16}x\left(\frac{\delta}{\varepsilon}x+2\eta\right)^4;$$

so that, as before, there are four equal factors. Conversely, it is obvious that if there are four equal factors u, so that $\Phi=au^5+5bu^4v$, the quadratic covariant of Φ disappears.

(35) The quadratic covariant also it was which led me to perceive the transformation applied in the antecedent article. For when the first minors of

$$\begin{matrix} a & b & c & d \\ c & d & e & f \end{matrix}$$

are all zeros, the quadratic covariant becomes

$$4(c^2-bd)x^2+4(d^2-ce)y^2.$$

Supposing neither of those coefficients to vanish, and calling its two factors u and v, and making

$$F(x, y)\Phi(u, v)=(\alpha, \beta, \gamma, \delta, \varepsilon, i\between u, v),$$

it is clear that the minors of

$$\begin{matrix} \alpha & \beta & \gamma & \delta \\ \gamma & \delta & \varepsilon & i \end{matrix}$$

can no longer all be zero, since in that case we should have

$$4(\gamma^2-\beta\delta)u^2+4(\delta^2-\gamma\varepsilon)v^2$$

containing u, v as factors. Consequently the canonizant of Φ must vanish under one or the other of those remaining suppositions which had been previously shown to conduct to the form au^5+bv^5, or else to the case of three or more equal roots. When the quadratic covariant vanishes, we know that there must be four equal roots; and when it becomes a perfect square but does not vanish, it will be found on examination that the equation has three equal roots.

(36) Returning to the general case, where $\Phi=u^5+v^5+w^5$, and making $\frac{u}{r^{\frac{1}{5}}}+\frac{v}{s^{\frac{1}{5}}}+\frac{w}{t^{\frac{1}{5}}}$ identically zero, and writing u', v', w' for $\frac{u}{r^{\frac{1}{5}}}$, $\frac{v}{s^{\frac{1}{5}}}$, $\frac{w}{t^{\frac{1}{5}}}$ respectively, Φ becomes $ru'^5+sv'^5+tw'^5$, or, if we please, $ru^5+sv^5+tw^5$, with the condition $u+v+w=0$.

Moreover u, v, w will all three be factors of the canonizant of F. For taking the canonizant of F with respect to u, v, it becomes

$$\begin{matrix} r-t & -t & -t & -t \\ -t & -t & -t & -t \\ -t & -t & -t & s-t \\ v^3 & -v^2u & vu^2 & -u \end{matrix} \quad \text{or } r\times \quad \begin{matrix} 1 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 1 \\ v^3 & -v^2u & vu^2 & -u^3 \end{matrix}$$

or $rst(uv^2+vu^2)$, i. e. $-rst(uvw)$.

Hence if $x+ey$, $x+fy$, $x+gy$ are three distinct factors of the canonizant of F with respect to x, y, if we choose the ratios $\lambda:\mu:\nu$ so that $\lambda+\mu+\nu=0$, $e\lambda+f\mu+g\nu=0$, we may make $u=\lambda(x+ey)$; $v=\mu(x+fy)$; $w=\nu(x+gy)$; and shall then have

$$F(x, y)=ru^5+sv^5+tw^5, \text{ with the condition } u+v+w=0,$$

where r, s, t may be found from three equations obtained by identifying any three of the six terms in F with the corresponding terms $ru^5+sv^5+tw^5$ expressed as a function of x, y. These equations being linear, it follows that ru^5, sv^5, tw^5 form a *single and unique* system of functions of x, y.

So when the canonizant has two equal roots and is of the form $C(x+py)(x+qy)^2$; in which case the reduced form is $au^5+5euv^4+fv^5$. The canonizant in respect to u, v becomes

$$\begin{array}{cccc} a & 0 & 0 & 0 \\ 0 & 0 & 0 & e \\ 0 & 0 & e & f \\ v^3 & -v^2u & vu^2 & -u^3, \end{array}$$

i. e. ae^2uv^2. Hence, writing

$$u=x+py, \quad v=x+qy, \quad F=au^5+5euv^4+fv^5,$$

a, e, f may be obtained, as before, by means of three linear equations, and the terms au^5, $5euv^4$, fv^5 form a single and unique system.

Finally, when the canonizant vanishes entirely, so that the form becomes au^5+fv^5, the quadratic covariant will take the form $C(x+ey)(x+fy)$; and making $u=x+py$, $v=x+qy$, a, f become determined by means of two linear equations, so that au^5, fv^5 form a single and unique system, as in the preceding cases.

(37) When the canonizant has three distinct roots, they may be all real, or one real and the other two imaginary. In the former case, in the expression $ru^5+sv^5+tw^5$, u, v, w may be considered as all real functions of x, y, and r, s, t will then also all of them be real. In the latter case w may be taken as a real function of x, y, u, v as conjugate imaginary functions; and consequently it is easy to see that, except when r, s are equal to each other, they will constitute a pair of conjugate imaginary quantities: in this case we may take for our canonizant form

$$r\left(\frac{-u+iv}{2}\right)^5+s\left(\frac{-u-iv}{2}\right)^5+tu^5;$$

or, if we please,

$$ru_i^5+sv_i^5+tw^5$$

understanding by u_i, v_i $\frac{-u+iv}{2}$, $\frac{-u-iv}{2}$ respectively. And it should be noticed that the determinant of u_i, v_i in respect to u, v will be

$$\begin{array}{cc} -\frac{1}{2} & \frac{i}{2} \\ -\frac{1}{2} & \frac{-i}{2} \end{array}$$

which is i.

(38) Let us proceed briefly to express the invariants of $ru^5+sv^5+tw^5$, which call Φ, with respect to u, v; the corresponding ones of $ru_i^5+sv_i^5+tw^5$, which call Φ_i, in respect to the same variables u, v will be found by attaching to these suitable powers of i.

$$\Phi=(r-t, -t, -t, -t, -t, s-t)(u, v)^5.$$

Hence its quadratic covariant is the quadratic invariant of

$$\left((r-t)u-tv,\ -tu-tv,\ -tu-tv,\ -tu-tv,\ -tu+(s-t)v\between u',\ v'\right)^4,$$

which is obviously

$$-rtu^2-stv^2+(rs-rt-st)uv.$$

Of this the quadratic invariant is

$$rt\,.\,st-\tfrac{1}{4}(rs-rt-st)^2;$$

or writing $\varrho=st$, $\sigma=tr$, $\tau=rs$, and calling this invariant (I),

$$(\mathrm{I})=-\tfrac{1}{4}(\varrho^2+\sigma^2+\tau^2-2\varrho\sigma-2\sigma\tau-2\tau\varrho).$$

Again, the cubic covariant or canonizant has been already shown to be $rst(u^2v+uv^2)$. Calling the discriminant of this (L), we have

$$(\mathrm{L})=-\tfrac{1}{27}r^4s^4t^4\,(^{30})=-\tfrac{1}{27}\varrho^2\sigma^2\tau^2.$$

Again, to find the discriminant (D) in respect to u, v.

When $ru^5+sv^5+tw^5=0$ has two equal roots, and $u+v+w=0$, it is easy to see that we have $ru^4+\lambda=0$, $sv^4+\lambda=0$, $tw^4+\lambda=0$.

Hence to a constant factor *près* (D) will be the *Norm* of

$$(st)^{\frac{1}{4}}+(tr)^{\frac{1}{4}}+(rs)^{\frac{1}{4}},\text{ i. e. of }\varrho^{\frac{1}{4}}+\sigma^{\frac{1}{4}}+\tau^{\frac{1}{4}}\,(^{31}).$$

To find the value of this norm, suppose $\varrho^{\frac{1}{4}}+\sigma^{\frac{1}{4}}+\tau^{\frac{1}{4}}=0$, then

$$\varrho+\sigma+\tau=2(\varrho^{\frac{1}{2}}\sigma^{\frac{1}{2}}+\sigma^{\frac{1}{2}}\tau^{\frac{1}{2}}+\tau^{\frac{1}{2}}\varrho^{\frac{1}{2}}),$$

and

$$\varrho^2+\sigma^2+\tau^2-2\varrho\sigma-2\varrho\tau-2\sigma\tau=8\varrho^{\frac{1}{4}}\sigma^{\frac{1}{4}}\tau^{\frac{1}{4}}(\varrho^{\frac{1}{4}}+\sigma^{\frac{1}{4}}+\tau^{\frac{1}{4}}).$$

Hence

$$(\varrho^2+\sigma^2+\tau^2-2\varrho\sigma-2\varrho\tau-2\sigma\tau)^2=64\varrho\sigma\tau\{(\varrho+\sigma+\tau)+2(\varrho^{\frac{1}{2}}\sigma^{\frac{1}{2}}+\sigma^{\frac{1}{2}}\tau^{\frac{1}{2}}+\tau^{\frac{1}{2}}\varrho^{\frac{1}{2}})\}=128\varrho\sigma\tau(\varrho+\sigma+\tau).$$

Hence (D) must contain $(\mathrm{J})^2-128\varrho\sigma\tau(\varrho+\sigma+\tau)$ as a factor; and since when $t=0$, $\varrho=0$, $\sigma=0$, and $(\mathrm{D})=\tau^4=(\mathrm{J})^2$, it is clear that $(\mathrm{D})=(\mathrm{J})^2-128(\mathrm{K})$, where

$$(\mathrm{K})=\varrho\sigma\tau(\varrho+\sigma+\tau).$$

(39) Although in the investigation in view (K) will only figure as an abbreviation of $\frac{(\mathrm{D})-(\mathrm{J})^2}{128}$, it may not be amiss to indicate a direct process for finding it. Let us for this purpose act upon the Hessian of Φ, treated as a function of u, v twice with the canonizant of Φ converted into an operator by substituting $\frac{d}{dv}$, $-\frac{d}{du}$ in place of u and v.

(30) For this is $\left(0,\frac{rst}{3},\frac{rst}{3},0\between u,v\right)^3$, and the discriminant of $\left(a,b,c,d\between u,v\right)^3$ is

$$a^2d^2+4ac^3+4db^3-3b^2c^2-6abcd.$$

(31) It is worthy of observation that (J) is also a Norm, viz. of $\varrho^{\frac{1}{2}}+\sigma^{\frac{1}{2}}+\tau^{\frac{1}{2}}$, so that (J) is the discriminant of $ru^2+sv^2+tw^2$. I have not been able to perceive the morphological significancy of this relation.

The Hessian of Φ may be obtained without difficulty under the form

$$rsu^3v^3+stv^3w^3+trw^3u^3 \text{ or } \tau u^3v^3+\varrho v^3w^3+\sigma w^3u^3\ (^{32}).$$

Operating upon this with

$$r^2s^2t^2\left(\frac{d}{dv}\cdot\frac{d}{du}\left(\frac{d}{du}-\frac{d}{dv}\right)\right)^2,$$

we obtain $\varrho\sigma\tau(A\tau+B\varrho+C\sigma)$, where

$$A=-2\left(\frac{d}{du}\right)^3\left(\frac{d}{dv}\right)^3u^3v^3=-72;$$

and as we know that this quantity must be of the form $\lambda(K)+\mu(J)^2$, we have $\mu=0$, $\lambda=-72$; so that, denoting the *operator* corresponding to the canonizant by T, and the Hessian by H, we have $(K)=-\frac{1}{72}T^2H\Phi$ (33). This gives a ready practical method for finding the discriminant of a general quintic F by means of the identity $D=J^2+\frac{16}{9}T^2H$, where D is the discriminant, H the Hessian, T the canonizantive operator, and J the quadratic invariant of F in respect to its own variables.

(40) If now we suppose the determinant of u, v in respect to x, y to be μ, where μ is by hypothesis a real quantity, and if we call the

Quadratic invariant in respect to x, y . .	$-\frac{1}{4}J$,
Discriminant of primitive „ „ . .	D,
Discriminant of the canonizant „ . .	$-\frac{1}{27}L$,

we have obviously

$$\left.\begin{aligned}J&=\mu^{10}(\varrho^2+\sigma^2+\tau^2-2\varrho\sigma-2\varrho\tau-2\sigma\tau),\\ K&=\mu^{20}\varrho\sigma\tau(\varrho+\sigma+\tau),\quad D=J^2-128K,\\ L&=\mu^{30}\varrho^2\sigma^2\tau^2,\end{aligned}\right\}\text{invariants of }\Phi.$$

This applies to the case where the reduced form is Φ, *i. e.* where the roots of the canonizant are all real, and consequently where $-L$ is negative, *i. e.* L positive.

When L is negative and the reduced form is $\Phi_{,}$, then, since the determinant of $u_{,}$, $v_{,}$ in respect to $u_{,}$ v is ι, we have

$$\left.\begin{aligned}J&=-\mu^{10}(\varrho^2+\sigma^2+\tau^2-2\varrho\sigma-2\varrho\tau-2\sigma\tau),\\ K&=\ \ \mu^{20}\varrho\sigma\tau(\varrho+\sigma+\tau),\quad D=J^2-128K,\\ L&=-\mu^{30}\varrho^2\sigma^2\tau^2,\end{aligned}\right\}\text{invariants of }\Phi_{,}$$

By means of the ratios $\frac{L}{J^3}$, $\frac{K}{J^2}$, it is obvious that in either case alike the ratios of ϱ, σ, τ

(32) It will be the quadratic invariant of $ru^3\xi^2+sv^3\eta^2+tw^3\zeta^2$ with respect to ξ, η, $\zeta+\eta+\zeta$ being zero; just as the quadratic covariant of Φ is the quadratic invariant of $ru\xi^4+sv\eta^4+tw\zeta^4$ with regard to the same variables. This latter is in fact $rsuv+stvw+trwu$.

(33) The intervening covariantic form of degree 8 in the variables and 5 in the coefficients, viz. $TH\Phi$, will easily be seen to be

$$rst^2(u^2v-uv^2)+str^2(v^2w-vw^2)+trs^2(w^2u-wu^2).$$

become determinable by means of the same cubic equations, viz.

$$\theta^3 - K\theta^2 + \frac{K^2-JL}{4}\theta - L^2 = 0;$$

ρ, σ, τ will be to each other as the roots of this equation([34]).

(41) Since $ru^5+sv^5+tw^5$ represents a function in x, y with real coefficients, it follows that when L is positive, u, v as well as w being real, $\alpha:\beta:\gamma$ are ratios of real quantities, and the roots of the preceding cubic will be real; when L is negative, u, v becoming conjugate imaginary functions of x, y, whilst w remains real, r, s, unless they are equal, must become conjugate imaginary constants. When r, s, t are all real, ρ, σ, τ will be so too; and when r, s are imaginary and t real, ρ, σ will be imaginary and τ real. Thus according as L is positive or negative the roots of θ are or are not all real. Hence understanding by Δ the discriminant of the preceding equation with respect to θ and 1, $\frac{\Delta}{L}$ must be always either zero or negative. We see *à priori* that $\frac{\Delta}{L}$ must be integer, because when $L=0$ the cubic has two equal roots, $\frac{L}{2}$. To compute its value more conveniently, write $K=6k$, $J=12j$. Then the equation becomes

$$(1,\ 2k,\ 3k^2-jL,\ L^2 \between \theta,\ -1)^3,$$

of which the discriminant is

$$L^4+4(3k^2-jL)^3+32k^3L^2-12k^2(3k^2-jL)^2-12kL^2(3k^2-jL).$$

Hence

$$\begin{aligned}\frac{\Delta}{L} &= L^3-108k^4j+36k^2j^2L-4j^3L^2+32k^3L\\ &\quad+\ 72k^4j-12k^2j^2L \qquad -36k^3L+12jkL^2\\ &= L^3-\ 36k^4j+24k^2j^2L-4j^3L^2-\ 4k^3L+12jkL^2.\end{aligned}$$

Accordingly, multiplying the above equation by $-3\cdot12^2$ in order to avoid fractions, replacing k, j by their values in terms of K, J, and naming G the quantity $-432\,\frac{\Delta}{L}$,

([34]) For since the absolute values of ρ, σ, τ are not in question, we may consider ρ, σ, τ as the roots of $\theta^3-K\theta^2+q\theta-r$, so that $\rho+\sigma+\tau=K$. We have then

$$\frac{\rho^4\sigma^4\tau^4}{(\rho\sigma\tau)^3(\rho+\sigma+\tau)^3}=\frac{L^2}{K^3}, \quad\text{or}\quad \frac{r}{K^3}=\frac{L^2}{K^3},$$

which gives $r=L^2$. Again,

$$\frac{\rho\sigma\tau K^2}{(K^2-4q)^2}=\frac{K^2}{J^2}, \quad\text{or}\quad \frac{(K^2-4q)^2}{r}=J^2, \quad\text{or}\quad (K^2-4q)^2=L^2J^2, \quad\text{or}\quad q=\frac{K^2\mp JL}{4}.$$

As regards the sign to be given to JL in q, since

$$\frac{J^3}{L}=\frac{(K^2-4q)^3}{r^2}=\frac{(K^2-4q)^3}{L^4},$$

we have $(K^2-4q)^3=J^3L^3$. Hence

$$q=\frac{K^2-1^{\frac{1}{3}}JL}{4}.$$

Consequently

$$q=\frac{K^2-JL}{4}, \text{ and } \textit{not } \frac{K^2+JL}{4}.$$

positive, or to speak more strictly non-negative, we have

$$G=JK^4+8LK^3-2J^2LK^2-72JL^2K-432L^3+J^3L^2\,(^{35}).$$

It is evident that G must be identical to a positive numerical factor *près* with the function which M. HERMITE denotes by I^2 (36).

(35) It will be observed that when J=0 and L=0, G vanishes. This is easily verifiable *à priori*; for when J=0 and L=0, the reduced form has been seen to be ax^5+5exy^4, of which the canonizant is

$$\begin{vmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & e \\ 0 & 0 & e & 0 \\ y^3 & -y^2x & yx^2 & -x^3 \end{vmatrix}$$

which equals $aexy^2$.

Hence the form and its canonizant have a common factor x, and consequently their resultant vanishes; hence I=0 and $G=I^2=0$. G also vanishes when K=0 and L=0, which is also easily verifiable; for then the reduced form becomes u^5+v^5, of which the canonizant vanishes, and consequently the resultant of the form and its canonizant becomes intensely zero; which accounts for the high power of K in (JK^4), the sole term of G in which L does not appear.

(36) (*) Compare expression for $16I^2$, Cambridge and Dublin Journal, p. 203. This will be found to contain nine terms, and to rise as high as the fifth power in Δ (which to a constant factor *près* is identical with my J); whereas in $\frac{-\Delta}{L}$ there are only six terms, and no power of J beyond the third. This seems to indicate that the K and L are more fortunately chosen than M. HERMITE's J_2, J_3, which are invariants of the like degrees 8 and 12. It is of course evident that the following relations exist between M. HERMITE's Δ_1, J_2, J_3 and the J, K, L of this paper,

$$\Delta=lJ,$$
$$J_2=mJ^2+nK,$$
$$J_3=pJ^3+qJK+rL,$$

where l, m, n, p, q, r are certain numerical quantities. Until these are ascertained, it is impossible to confront M. HERMITE's results with my own, to ascertain whether or not they are identical in substance, and, if not, wherein the difference consists. I therefore subjoin the necessary calculations for effecting this important object.

Let us first take the form $x^5+5exy^4+y^5$. The quadratic covariant of this is $x(ex+y)$.

Accordingly, to obtain M. HERMITE's A, B, C, C′, B′, A′ (Cambridge and Dublin Journal, vol. ix. p. 179), we must make

$$x=X; \quad ex+y=Y,$$

which gives (vide C. and D. J. p. 180)

$$F=X^5+5eX(Y-eX)^4+(Y-eX)^5$$
$$=(A, B, C, C', B', A' \between X, Y)^5,$$

where

$$A=1+4e^5,\ B=-3e^4,\ C=2e^3,\ C'=-e^2,\ B'=0,\ A'=1.$$

Accordingly (vide C. and D. J. p. 184),

$$AA'-3BB'+\ 2CC'=1+4e^5-\ 4e^5=1 \qquad =\sqrt{\Delta},$$
$$AA'+\ BB'-\ 2CC'=1+4e^5+\ 4e^5=1+\ 8e^5=\frac{I_1}{2\sqrt{\Delta^3}},$$
$$AA'+5BB'+10CC'=1+4e^5-20e^5=1-16e^5=\frac{\quad}{2\sqrt{\Delta^5}}.$$

Hence

$$\Delta=1,\quad I_1=2+16e^5,\quad I_2=2-32e^5.$$

Again (vide C. and D. J. p. 186. § vii.),

$$8J_1=I_1-\Delta^2=1+16e^5,\quad 24J_2=I_2-2I_1\Delta+\Delta^3=-1-64e^5;$$

(42) In fact M. HERMITE's octodecimal invariant is most simply obtained as the resultant of the primitive quartic and its canonizant. Using the reduced forms for these two

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but J_1, J_2 are subsequently *without warning* (compare expressions for AA′, BB′, CC′, pp. 186, 192) renamed J_2, J_3; so that

$$8J_2=1+16e^5,\quad 24J_3=-1-64e^5.$$

The corresponding values of J, K, L have been already calculated, and we have found

$$J=1,\quad K=-2e^5,\quad L=0.$$

Hence

$$A=1,\quad \frac{1}{8}+2e^5=B-2Ce^5,\quad \frac{-1}{24}-\frac{64}{24}e^5=D-2Ee^5.$$

Thus

$$A=1,\quad B=\frac{1}{8},\quad C=-1,\quad D=-\frac{1}{24},\quad E=\frac{4}{3}.$$

To find F, take another form convenient for the purpose, as $x^5+10dx^2y^3+y^5$.

Taking the emanant of this $(x, 0, dy, dx, y \between x', y')^4$, the quadratic covariant is obviously $xy+3d^2y^2$, so that $J=1$.

Also its discriminant is

$$\begin{matrix} 1 & 0 & 0 & d \\ 0 & 0 & d & 0 \\ 0 & d & 0 & 1 \\ y^3 & -y^2x & yx^2 & -x^3 \end{matrix}$$

$$\text{viz. } d^3y^3-d(-dx^3+y^2x)=d^3y^3-dy^2x+d^2x^3,$$

of which the discriminant is

$$d^{10}+4d^2\left(\frac{-d}{3}\right)^3=d^{10}-\tfrac{4}{27}d^5.$$

Hence by definition $\qquad L=e-\tfrac{27}{4}d^{10}+d^5.$

Again, to find A, B, C, C′, B′, A′, we must write

$$x+3d^2y=X,$$
$$y=Y,$$

and we have then

$$(X-3d^2Y)^5+10d(X-3d^2Y)^2Y^3+Y^5=(A, B, C, C', B', A' \between X, Y)^5.$$

Since $J=1$ and K is of the eighth order only in the coefficients, it is obvious that neither J^3 nor JK can contain a term involving d^{10}. In order therefore to find F, it will be sufficient to compare the coefficient of d^{10} in J_3 and in L.

Now $\qquad A=1,\quad B=-3d^2,\quad C=9d^4,\quad C'=27d^5+d,\quad B'=81d^8-12d^3,\quad A'=243d^{10}+90d^5+1.$

Also $\Delta=J=1$. Hence neglecting all but the terms which bring in d^{10}, $24J_3$ (p. 186, Memoir) is tantamount to I_2, and I_2 (p. 186) is tantamount to

$$2(243d^{10}-5.3.81d^{10}+10.9.27d^{10}),$$

which is

$$12\times 243d^{10}.$$

Hence in J_3 the term containing d^{10} is $\frac{243}{2}$.

Hence $-\frac{27}{4}F=\frac{243}{2}$, or $F=-18$.

Hence we have, finally,

$$\Delta=J,$$
$$J_2=-K+\tfrac{1}{8}J^2,$$
$$J_3=-18L+\tfrac{4}{3}JK-\tfrac{1}{24}J^3;$$

functions,

$$ru^5+sv^5-t(u+v)^5;\quad rstuv(u+v),$$

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and conversely,

$$J=\Delta,$$
$$K=-\frac{J}{2}+\frac{1}{8}\Delta^2$$
$$L=-\frac{J_3}{18}-\frac{2}{27}\Delta J_2+\frac{1}{8}\Delta^3.$$

Unhappily a further step is wanting to bring M. HERMITE's results to the final test of comparison; for the value of AA′ (p. 192) does not agree with that given for AA′ (p. 186) by simply changing J_1, J_2 into J_2, J_3 respectively; a further change of Δ into 2Δ becomes necessary to make the ratios of AA′, BB′, CC′ (p. 192) accord with the ratios of the same quantities at p. 186. Finally, even after making this change the expression for $16I^2$ (p. 203) does not accord (even to a constant coefficient *près*) with that with which it is meant to be identical, viz. $16I_3^2$ (p. 187); so that after great labour I am still baffled in my attempt to ascertain the agreement or discrepancy of my conclusions with those of my precursor in the inquiry. As will appear hereafter, the two sets of conclusions are undoubtedly discrepant in form; but whether they are so in substance or not, or rather whether they are or not in contradiction to each other, requires a close examination to discover, the more especially because, as will hereafter be shown, there is a certain necessary element of indeterminateness in the scheme of invariantive conditions which serve to fix the character of the roots. It is greatly to be lamented that so valuable a paper as M. HERMITE's should be to some extent marred, in respect of the important end it would serve as a term of comparison, by the existence of these numerical and notational inaccuracies. I have spent hours upon hours in endeavouring to reconcile these several texts of the same memoir, and, after all my labour, the work is left unperformed without which the truth as between the two methods cannot be elicited. I feel, however, as confident of the correctness of my own conclusions as of the truth of any proposition in Euclid.

([b]) It is worthy of notice that there is a failing case in M. HERMITE's process for finding I^2 in terms of Δ, J_2, J_3, just as there is one in mine for finding G in terms of J, K, L,—the failure of the process, however, in neither case entailing any corresponding defect in the results obtained. The process employed in this memoir fails when $L=0$: for then the general form $ru^5+sv^5+tw^5$ is superseded by the supplementary one, $au^5+5euv^4+fv^5$. M. HERMITE's fails when J (the J of *this* memoir) $=0$; for then the quadratic invariant becomes a perfect square, and the substitution of its factors in place of the original variables becomes inadmissible, since the two former coincide.

([c]) It may be as well here to notice the form which M. HERMITE's two linear covariants assume when referred to the canonical form above written. The quadratic covariant being $rsuv+stvw+trwu$, if we operate with the correlative of this obtained by writing in it $\frac{d}{dv}$, $-\frac{d}{du}$, $\frac{d}{du}-\frac{d}{dv}$ in lieu of u, v, w, viz.

$$-rs\,\frac{d}{du}\frac{d}{dv}-st\frac{d}{du}\left(\frac{d}{du}-\frac{d}{dv}\right)+tr\frac{d}{dv}\left(\frac{d}{du}-\frac{d}{dv}\right)$$

upon the primitive, we obtain to a factor *près* the canonizant $rstuvw$, which has been already obtained; repeating the process, it is easy to see that the first linear covariant of the fifth degree in the coefficient assumes the simple form $rst(stu+trv+rsw)$, or $rst(\varrho u+\sigma v+\tau w)$. Taking again the correlative of this, viz.

$$rst\left(\varrho\frac{d}{dv}-\sigma\frac{d}{du}+\tau\left(\frac{d}{du}-\frac{d}{dv}\right)\right),$$

and operating with it upon $rsuv+stvw+trwu$, it will be found without difficulty that the second linear covariant of the seventh degree in the coefficients becomes

$$rst\{(\sigma-\tau)(\sigma+\tau-\varrho)u+(\tau-\varrho)(\tau+\varrho-\sigma)v+(\varrho-\sigma)(\varrho+\sigma-\tau)w\},$$

which is distinguishable in species from the former one by its symmetry being only of the hemihedral kind.

([d]) It may not be out of place to notice here that the Hessian of the canonical form will be found to be

$$\varrho v^3w^3+\sigma w^3u^3+\tau u^3v^3.$$

their resultant in respect to u, v is obviously

$$(rst)^5(r-s)(s-t)(t-r)\,(^{37}),$$

(*) Again, if we write

$$\begin{aligned} rst(\rho u+\sigma v+\tau w)&=\xi \\ rst(w-\tau)(\sigma+\tau-\rho)u+(\tau-\rho)(\tau+\rho-\sigma)v+(\rho-\sigma)(\rho+\sigma-\tau)w)&=\eta, \\ u+v+w&=0, \end{aligned}$$

and from these equations deduce the values of u, v, w, and substitute them in $ru^5+sv^5+tw^5$, we shall obtain M. HERMITE's "forme-type" expressed in terms of the parameters of the reduced form, and every coefficient therein will be invariantive.

The resultant of the equations above written (on making $\xi=0$, $\zeta=0$) will appear in the denominator of each such coefficient. Hence it appears, from M. HERMITE's expressions (Camb. and Dubl. Math. Journal, vol. ix. p. 193), where J_3 will be seen to enter into the denominator of A, B, C, C′, B′, A′, that this resultant to a factor *près* is his J_3. Its value may easily be calculated, and will be found to be

$$\rho\sigma\tau(\rho+\sigma+\tau)^3-4(\rho+\sigma+\tau)(\rho\sigma+\rho\tau+\sigma\tau)+9\rho\sigma\tau=\mathrm{JK}+9\mathrm{L}.$$

Accordingly as L (to use Dr. SALMON's convenient elliptical expression) is the condition of the failure of my *general* reduced form, so is $9\mathrm{L}+\mathrm{JK}$ the condition of the failure of M. HERMITE's "forme-type." As particular cases of this last failure, we may suppose $\mathrm{J}=0$, $\mathrm{L}=0$, or $\mathrm{K}=0$, $\mathrm{L}=0$. In the former case the reduced form is ax^5+5ex^4y, of which the simplest quadratic and cubic covariants are respectively aex^2; ae^2y^2x. Thus to find L, the first linear covariant, we have to operate upon ae^2y^2x with $ae\left(\frac{d}{dy}\right)^2$, which gives a^2e^3x; and to find L_2, we have to operate on $(aex^2)^2$ with $ae^2\left(\frac{d}{dx}\right)^2\frac{d}{dy}$, or, if we please (according to M. HERMITE's method), with $\left(a^2e^3\frac{d}{dy}\right)$ on aex^2, showing that L_2 vanishes, but L_1 continues to subsist. When, secondly, $\mathrm{K}=0$, $\mathrm{L}=0$, the reduced form is ax^5+ey^5, and the canonizant disappears entirely, so that the first, and consequently also the second, linear covariants, each of them becomes a *null*.

(^{37}) By aid of the reduced forms of the invariants J, K, L, I given in the text, it is easy to prove that every other invariant, say Ω of a quintic, is a rational integral function of these four. In what follows, let a parenthesis enclosing the symbol of any invariant signify its value when any two of the quantities u, v, w in the reduced form $ru^5+sv^5+tw^5$; $[u+v+w=0]$ are taken as the independent variables. We have then

$$(\mathrm{J})=\rho^2+\sigma^2+\tau^2-2\rho\sigma-2\rho\tau-2\sigma\tau,\quad (\mathrm{K})=\rho\sigma\tau(\rho+\sigma+\tau),\quad (\mathrm{L})=\rho^2\sigma^2\tau^2,\quad (\mathrm{I})=\rho^2\sigma^2\tau^2(\rho-\sigma)(\sigma-\tau)(\tau-\rho),$$

ρ, σ, τ meaning st, tr, st.

The degree of Ω must be of the degree $4m$ or $4m+2$. 1. Let it be of the form $4m$. Then, since the interchange of any two of the variables u, v, w must leave (Ω) unaltered, (Ω) will be unaltered by the interchange of any two of the letters r, s, t, and is consequently a symmetric function of ρ, σ, τ, the roots of the equation

$$\theta^3-\frac{(\mathrm{K})}{(\mathrm{L})^{\frac{1}{2}}}\theta^2+\frac{(\mathrm{K})^2-(\mathrm{J})(\mathrm{L})}{(\mathrm{L})}-(\mathrm{L}^{\frac{1}{2}})=0.$$

Hence

$$(\Omega)=\frac{\mathrm{F}((\mathrm{J}),(\mathrm{K}),(\mathrm{L}))}{(\mathrm{L})^{2m}},$$

F denoting a rational integral function-form of the quantities it affects. Consequently

$$\Omega=\frac{\mathrm{F}(\mathrm{J},\mathrm{K},\mathrm{L})}{\mathrm{L}^{2m}}.$$

Hence since Ω cannot become infinite when $\mathrm{L}=0$, which merely implies that the general form reduces to

$$(a, 0, 0, 0, e, i\between x, y)^5,$$

$\Omega=\Phi(\mathrm{J},\mathrm{K},\mathrm{L})$, a rational integral function of J, K, L.

2. If the degree Ω is of the form $4m+2$, (Ω) will be a function of r, s, t, which changes its sign when u and v

and consequently, if we call I the resultant in respect to x, y, we have

$$\pm \mathrm{I} = \mu^{45}\varrho^2\sigma^2\tau^2(\sigma-\varrho)(\tau-\sigma)(\varrho-\tau),$$

and

$$\mathrm{I}^2 = \mu^{90}\varrho^4\sigma^4\tau^4(\sigma-\varrho)^2(\tau-\sigma)^2(\varrho-\tau)^2$$
$$= \mu^{30}(\sigma-\varrho)^2(\tau-\sigma)^2(\varrho-\tau)^2\mathrm{L}^2.$$

(43) Thus we see that the two quantities G, I^2, which are both rational integral functions of the degree 36 in the coefficients of $\mathrm{F}(x, y)$, cannot one vanish without the other, at all events when L is not equal to zero. This is sufficient to show that they are identical to a numerical factor *près*, whatever L may be, zero or not zero([38]), and consequently that the quantity called G, proved to be positive upon the supposition of L not being zero, must also remain positive when L is zero, because it is in fact the square of a rational function of the coefficients. But we may also prove this independently by virtue of the supplementary reduced form $au^5+5euv^4+fv^5$ applicable to the case of L zero.

For when L=0, G becomes JK^4; so that the condition "G not negative" implies simply that J is positive unless K vanishes.

Now the canonizant, when it does not vanish, i. e. when e is not zero, contains v^2u as a factor, and, its coefficients being real, u, v are both of them necessarily real functions of x, y. Consequently J, which by definition is $-4\times$discriminant of quadratic covariant, becomes $-4\mu^{10}\times$discriminant of $au(eu+fv)$ in respect to u, v, which $=\mu^{10}a^2f^2$, μ being real. Consequently J is positive, since the reality of u, v implies that of a, e, f, when e is not zero. When e *is* zero u, v may be either real or imaginary; for u^5+v^5 may be real whether u, v be real or conjugate imaginary functions of x, y; but in that case K, which is found by operating twice upon the Hessian with a canonizant turned into an operator, vanishes, since then all the coefficients of the canonizant vanish([39]). Hence the rule that G cannot be negative is seen to be true, whatever L may be.

or any two of its quantities u, v, w, are interchanged, such interchange having the effect of introducing as a multiplier the $5(2m+1)$th power of the determinant of substitution (-1). Hence (Ω) is of the form

$$(\varrho-\sigma)(\sigma-\tau)(\tau-\varrho)\mathrm{F}(\varrho, \sigma, \tau), \text{ i. e. } \frac{(\mathrm{I}).\mathrm{F}(\varrho, \sigma, \tau)}{(\mathrm{L})^{\frac{3}{2}}},$$

which again is of the form

$$\frac{(\mathrm{I}).\mathrm{F}\big((\mathrm{J}), (\mathrm{K}), (\mathrm{L})\big)}{(\mathrm{L})^{2m-8}},$$

so that Ω is of the form

$$\frac{\mathrm{I}.\mathrm{F}(\mathrm{J}, \mathrm{K}, \mathrm{L})}{\mathrm{L}^{2m-8}}.$$

Hence since, as before, Ω cannot become infinite when L=0, and since, furthermore, I does not vanish (for if so then G, which is I^2, would vanish) when L=0, Ω must be of the form $\mathrm{I}\Phi(\mathrm{J}, \mathrm{K}, \mathrm{L})$. Q. E. D.

([38]) For if $\mathrm{Q}^2=\mathrm{KI}^2$ for an indefinite number of systems of values of a, b, c, d, e, f, of which Q, I are rational integral functions, Q^2 and KI^2 must be *absolutely* identical; this of course is the case when Q^2 and KI^2, as proved in the text, are known to be identical for all values of a, b, c, d, e, f which do not make L zero.

([39]) (*) In the more general form $au^5+5euv^4+fv^5$, taking $\mu=1$. The canonizant is ae^2uv^2; this squared and

It may be said that the case of three or more equal roots existing in F (x, y) has been

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turned into an operator becomes $a^2e^4\left(\frac{d}{dv}\right)^2\left(\frac{d}{du}\right)^4$, which, applied to the Hessian, viz. $3aeu^4v^2+afu^3v^3-e^2v^6$, after multiplying by $-\frac{1}{72}$, gives $K=-2a^3e^5$, so that $D=J^2-128K=a^4f^4+256a^3e^5$, which is capable of easy verification. In fact D becomes the resultant of au^4+ev^4 and $v^3(4eu+fv)$; v^3 introduces the factor a^3 into D; and further, making $u:v::-f:4e$ and substituting in au^4+ev^4, we obtain the other factor af^4+256e^5.

If we adopt $u^5+5euv^5+v^4$ as the reduced form for the failing case (a form analogous to the well-known one, $u^4+6cu^2v^2+v^4$, for the general quartic), to find e we have $J=\mu^{10}$, $K=-2\mu^{20}e^5$. Hence $e^5=-\frac{K}{2J^2}$; thus when $K=0$, $e=0$.

(^{b}) By a linear transformation we may always take away any two (except the two first or last) coefficients of a given quintic, but the vanishing of more than two coefficients always corresponds to some invariantive condition. Thus, *ex. gr.*, in the form

$ax^5+5exy^4+fy^5$	$L=0$		
ax^5+fy^5	$L=0$	$K=0$	
ax^5+5exy^4	$L=0$	$J=0$	
$ax^5+10dx^2y^3$	$J=0$	$K=0$	
$ax^5+5bx^4y+10cx^3y^2$	$L=0$	$J=0$	$K=0$.

(^{c}) The condition for the existence of four equal roots in a quintic is the vanishing of the quadratic covariant; that is to say, we must have

$$ae-4bd+3c^2=0, \quad af-3be+2cd=0, \quad bf-4ce+3d^2=0.$$

The three quantities equated to zero are not separately invariants, but constitute in their *ensemble* an invariantive plexus.

(^{d}) [It may here be noticed incidentally that the conditions for equal roots in the biquadratic form are as follows. For two equal roots, of course, the discriminant is zero, for three equal roots the two lowest invariants are each zero, and for two pairs of equal roots the Hessian $(A, B, C, D, E \between x, y)^4$ becomes to a factor *près* identical with the primitive $(a, b, c, d, e \between x, y)^4$, so that all the first minors of the matrix

$$\begin{matrix} a, & b, & c, & d, & e, & f \\ A, & B, & C, & D, & E, & F \end{matrix}$$

vanish. *Quære*, whether the character of the five-rayed pencil (centre at origin), in which a, A; b, B; c, C; d, D; e, E mark points, may not serve to distinguish between the case of four real and four imaginary roots.]

(^{e}) When $J=0$ and $K\neq 0$, but *not* $L=0$, it is obvious that $\rho:\sigma:\tau::1:\iota:\iota^2$, ι being any imaginary cube root of unity, and the reduced form is $u^5+\iota v^5+\iota^2w^5$, with the relation $u+v+w=0$.

J and K being zero, D will be so too, and accordingly the equation $u^5+\iota v^5+\iota^2w^5=0$ will have two equal roots. It will easily be found that these equal roots correspond to the system of ratios $u=1$, $v=\iota^2$, $w=\iota$. In fact, if we write $u=1+\rho$, $v=\iota^2+\iota\rho$, $w=\iota+\iota^2\rho$, the equation becomes $u^5+\iota v^5+\iota^2w^5=\rho^2(30\rho+3\rho^3)=0$.

Hence, understanding by ϵ either of the two prime sixth roots of unity, the complete system of ratios of u, v, w may be expressed as follows:—

$u=1$	$v=\iota^2$	$w=\iota$
$u=1$	$v=\iota^2$	$w=\iota$
$u=1-\sqrt[3]{10}$	$v=\iota^2-\sqrt[3]{10}$	$w=\iota-\iota^2\sqrt[3]{10}$
$u=1+\sqrt[3]{10}\epsilon$	$v=\epsilon^4-\sqrt[3]{10}$	$w=\epsilon^2+\sqrt[3]{10}\epsilon^5$
$w=1+\sqrt[3]{10}\epsilon^5$	$v=\epsilon^4+\sqrt[3]{10}\epsilon$	$w=\epsilon^2-\sqrt[3]{10}$.

Thus, when $J=0$ and $K=0$, u, v, w (with the relation $u+v+w=0$) may first be found, in terms of x, y, by

lost sight of; but we know, and it is capable of immediate verification by taking as the

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solving the cubic equation, obtained by equating to zero the canonizant of $(a, b, c, d, e, f \between x, y)$, and then x, y will be known from the above system of values for any two of the quantities u, v, w.

(f) It is obvious that the form $ax^5+dx^2y^3$ gives J=0 and K=0; but it seems desirable to prove the converse, viz. that when J=0 and K=0, but not L=0, the form is always reducible to $au^5+10\delta u^2v^3$, which may be done as follows. Since J=0 and K=0 the discriminant is zero, and we may assume

$$F=ax^5+5bx^4y+10cx^3y^2+10dx^2y^3,$$

and we have J= discriminant of

$$(-4bd+3c^2)\xi^2+2cd\xi\eta+3d^2\eta^2.$$

Hence

$$3d^2(3c^2-4bd)-c^2d^2=0;$$

d cannot be zero, for then we should have J=0, K=0, L=0, contrary to hypothesis. Hence $8c^2-12bd=0$.

If $b=0$ and $c=0$, F is already reduced to the desired form; but if not, $d=\frac{2c^2}{3b}$, and F becomes

$$ax^5+\frac{5b}{6}x^2\left(6x^2y+\frac{12c}{b}xy^2+\frac{8c^2}{b^2}y^3\right);$$

or, making

$$a-\frac{5b}{6}=\alpha,\quad \frac{b}{6}=2\delta,\quad x+\frac{2cy}{b}=v,$$

$F=\alpha x^5+10\ \delta x^2v^3$, as was to be shown.

The corresponding converses for the case of J=0, L=0, and of K=0, L=0 have been already established.

(g) It will be observed that under a certain point of view L for binary quintics is the analogue of Δ the *discriminant* for binary quartics, the condition of failure in the *general* reduced form in the two cases being L=0 and $\Delta=0$ respectively. The mere vanishing of the discriminant in the case of the quintic function, unattended by any other condition, does not affect the nature of the reduced form.

(h) It has been shown previously in the text that when L=0 the primitive is reducible to the form

$$(a, 0, 0, 0, e, f \between x, y)^5.$$

Hence if I_{12} is any duodecimal invariant which vanishes when $b=0$, $c=0$, $d=0$, I_{12} must vanish whenever L vanishes, and consequently, since L is of as high a degree as I_{12}, I_{12} must be a numerical multiple of L. In Mr. CAYLEY's Third Memoir on Quintics, "No. 29" represents a duodecimal invariant calculated by M. FAÀ DE BRUNO, and characterized morphologically by Mr. CAYLEY as being that duodecimal invariant in which "the leading coefficient a does not rise above the fourth degree." On examining No. 29 it will be found to contain no term in which b, c, d are all simultaneously absent. Hence it is, by virtue of the above observation, a multiple of my L: to determine the numerical factor, let all the coefficients in the primitive except a, d be supposed zero; then the canonizant becomes

$$\begin{vmatrix} a & 0 & 0 & d \\ 0 & 0 & d & 0 \\ 0 & d & 0 & 0 \\ y^3 & -y^2x & yx^2 & -x^3 \end{vmatrix} = d^3y^3+ad^2x^3.$$

Hence L becomes $-27a^2d^{10}$, but "No. 29" becomes $27a^2d^{10}$. Hence we have the important relation "No. 29" $=-$L, so that No. 29 is a discriminant, an *intrinsic* property of the calculated invariant, which, I believe, was not suspected.

(i) It will at once be recognized that "No. 19" given in Mr. CAYLEY's Second Memoir upon Quantics is identical with the J of this memoir, whence it follows from Mr. CAYLEY's equation (No. 26)=(No. 19)$^2-1152$ No. 26, that K=9 (No. 25). Thus abstraction made of a mere numerical factor, Mr. CAYLEY and myself agree upon perfectly distinct grounds in recognizing K and L as the true simplest invariants of their respective degrees, an accordance as satisfactory as it was unexpected, and which must be considered as setting at rest the question of what should be deemed the, so to say, *staple* invariants of the Binary Quintic.

reduced form $au^5+5bu^4v+10cu^3v^2$, that on such hypothesis all the invariants J, K, L must vanish, so that JK^4 is still non-negative[40].

(44) It is most important to notice that G can only become zero by virtue of two of the quantities ϱ, σ, τ, and therefore of r, s, t becoming equal. When u, v are imaginary, it is the coefficients r, s which must become equal, as otherwise the reduced form would not be a real function of x, y. By equating r to s, and using as an auxiliary variable the ratio $\frac{r}{t}$ or $\frac{s}{t}$, we shall be able to study the composition and inward nature of G with the utmost clearness and facility.

SECTION II.—*On the Criteria which decide the Number of Real and Imaginary Roots.*

(45) Since in the preceding section we have supposed that u, v are always real linear functions of x, y, it is obvious that the character of the roots of the given quintic in x, y is completely identical with that of the roots in the reduced form, and it has been shown that only one reduced form corresponds to a given system of values of J, D, L[41].

Let us suppose J, D, L to be taken as coordinates of a point in space; when J, D, L are so related that the condition G non-negative is satisfied, the point will correspond to an equation with real coefficients, and may be termed a *facultative* point. But when G is negative it will correspond to an equation of the kind alluded to in the recent section of this paper, and there called conjugate: such a point may be termed non-facultative. Thus the whole of space will be divided into two parts, separated by the surface $G=0$, which may be termed respectively facultative and non-facultative (as being made up of facultative or non-facultative points[42]). It is clear that these two portions will be exactly equal, similar, and symmetrical with regard to the axis of D; by which I mean that, if two points be taken in any line perpendicular to the axis of D at equal distances from that axis, one will be facultative and the other non-facultative, as is evident from the fact that when J, L become $-$J, $-$L (K, and therefore D or J^2-128K, remaining unaltered), G is converted into $-$G. Thus by a semirevolution

(40) When the form is $au^5+5euv^4+fv^5$ so that $L=0$, the canonizant, as has been seen before, is ae^2v^2u; the resultant of these two is $a^6e^{10}a^2f=a^7e^{10}f$. Again, $J=a^2f^2$, $K=-2a^3e^5$; thus the square of the resultant $=\frac{1}{16}JK^4$; so that if we call this resultant, which we may take as the definition of the Octodecimal Invariant I, we have $G=16I^2$.

(41) It should be well noticed that the mere ratios $\frac{D}{J^2}$, $\frac{L}{J^3}$ do not suffice to determine the character of the roots. When these ratios are given, it is true that the ratios r, s, t in the reduced form are given, but according as L is positive or negative, the arguments u, v in $ru^5+sv^5+tw^5$ (supposing w to be the real linear function of x, y) will be real or imaginary. When J, L, D are all given *absolutely*, then the character of the roots is completely determined. The *indelible* marks of a quintic function are three in number, viz. the ratios $\frac{K}{J^2}$, $\frac{L}{J^3}$, and the sign of L or J, as for a quartic function they are two in number, viz. $\frac{s^3}{t^2}$ and the sign of s.

(42) It will also be convenient to call the coordinates J, D, L corresponding to any facultative point a facultative system of invariants, and $\frac{D}{J^2}$, $\frac{L}{J^3}$ corresponding to the same (for a *given sign* of J) a facultative system of invariantive ratios.

round the axis of D the facultative and non-facultative portions may be made to exchange places.

(46) The axis of D itself lies on the surface of G, and like every other portion of this surface is facultative, for there is no reason for disallowing G to become zero. Conversely, if, instead of a real equation, we take one of the conjugate class (described in the second section), the whole of the facultative portion of space (except the separating surface G) becomes non-facultative, and the non-facultative part becomes facultative, but G itself remains facultative. When the invariants, or any of them, become imaginary, we are put out of space altogether, and the system can belong neither to a real nor to a conjugate family, but to one with coefficients at the same time imaginary and non-conjugate. $G=0$ (43), it may be remarked, will in all cases be the condition of an equation capable of linear transformation into one of recurrent (44) form; for the reduced form then in general becomes $ru^5+rv^5-t(u+v)^5$. The case when G becomes zero by virtue of $J=0$ and $L=0$, that is to say when the function is reducible by real or imaginary linear substitutions (see footnote (39) (f)) to the form $u(u^4\pm v^4)$, is the one which might for a moment be supposed to offer an exception to the rule; but only the exception is only apparent, since $u(u^4-v^4)$, on writing $u=p+q$, $v=p-q$, becomes $16(p+q)pq(p^2+q^2)$.

(47) To every point in space, it has been remarked, will correspond one particular family of equations all of the same character as regards the number they contain of real or imaginary roots, because capable of being derived from one another by real linear substitutions, such family consisting of an infinite number of ordinary or conjugate equations according as the point is facultative or non-facultative; but it may be well to notice that, conversely, every point does not correspond to a distinct family. In fact every point in the curves $D=pJ^2$, $L=qJ^3$ (p, q being constants) will denote a curve divided into two branches by the origin of coordinates, one of which will be facultative and the other non-facultative; but in each separate branch every point will represent the very same family. Any such separate branch may be termed an isomorphic line; and we see that the whole of space may be conceived as permeated by and made up of such lines radiating out from the origin in all directions.

(48) The origin at which $J=0$, $D=0$, $L=0$, as already noticed, corresponds to the case of three equal roots. The theorem that, when more than half as many roots are equal to each other as there are units in the degree of any binary form, all the invariants vanish, was remarked by myself originally in the very infancy of the subject, before Mr. CAYLEY's paper, alluded to by M. HERMITE, appeared in Crelle. The method of proof which then occurred to me is the simplest that can be given. For instance, in

(43) I shall hereafter allude to the surface denoted by $G=0$ under the name of the Amphigenous Surface, as being the locus of the points which give birth to real and conjugate forms indifferently.

(44) The roots of recurring equations, geometrically represented, in general go in quadruplets, A, A′; B, B′, where A and B, as also A′, B′, are mutual optical images of each other in respect to a fixed line, and A, A′, as also B, B′, are electrical images of each other in respect to a circle of which the fixed line is a diameter—with liberty, of course, for the images taken in either mode of combination to coalesce so as to reduce the quadruplet to a simple pair.

the case before us, if the quintic have three equal roots, we may reduce it to the form

$$ax^5+5bx^4y+10cx^3y^2.$$

Suppose now, if possible, an invariant of the degree m; the *weight* of each term therein, say $a^rb^sc^t$, in respect to x or y would be the same $\left(\text{viz. } \frac{5m}{2}\right)$, so that we should have

$$5r+4s+3t=\frac{5m}{2}=s+2t, \text{ or } 5s+3s+t=0,$$

and therefore $r=0$, $s=0$, $t=0$, $m=0$. So for a sextic with three equal roots reduced to the form $(a, b, c, 0, 0, 0)(x, y)^6$. Supposing any term in one of its invariants to be $a^rb^sc^t$, we should have

$$6r+5s+4t=\frac{6m}{2}=s+2t, \text{ or } 6r+4s+2t=0,$$

which is absurd, unless $r=0$, $s=0$, $t=0$, $m=0$, and so in general for a binary form of any degree. If in the above example for the degree m only three roots were equal *inter se* (the form assumed being $(a, b, c, d, 0, 0, 0)(x, y)^6$, any term in a supposed invariant being $a^rb^sc^td^u$, where $r+s+t+u=m$, we should have

$$6r+5s+4t+3u=3m=s+2t+3u,$$

and, as before,

$$6r+4s+2t=0, \ r=0, \ s=0, \ t=0;$$

no longer, however, $m=0$, but $m=u$, which is left undetermined.

(49) Before proceeding further it will be proper to consider under what circumstances a variation (in the coefficients of any equation) arbitrary, except that the coefficients are to remain real, can affect the character of the roots.

Let $F(x)=0$ be any algebraical equation with real coefficients, and let $\delta(Fx)$ be the variation of F due to the variation of the coefficients, $dF(x)$ the variation due to the change of x into $x+dx$. If, now, r be a root of $Fx=0$, and $r+dr$ the corresponding root of $F(x)+\delta F(x)=0$, we have

$$Fr=0, \quad F(r+dr)+\delta F(r)=0, \quad \text{or } \delta F(r)+\frac{d}{dr}F(r)dr+\frac{1}{1.2}\left(\frac{d}{dr}\right)^2 Fr(dr)^2+\&c.=0.$$

Hence, unless $\frac{dF}{dr}=0$, i. e. unless there are two equal roots r, we shall have $dr=-\frac{\delta F(r)}{\frac{d}{dr}F(r)}=$ a real quantity; so that the character of the root $r+dr$ will be the same as that of r.

But if

$$\frac{dF}{dr}=0, \ \frac{d^2F}{dr^2}=0, \ \ldots \left(\frac{d}{dr}\right)^{i-1}F=0,$$

so that there are i roots r, i being any integer greater than zero, then to find dr we have the equation

$$(dr)^i+\frac{\Pi(i)\delta Fr}{\left(\frac{d}{dr}\right)^i F(r)}=0.$$

Thus dr will have i distinct values; of these, if i is odd, all but one will be imaginary, but if i is even they will be all imaginary, or only all but two imaginary and the remain-

ing two real, according as the sign of $\delta F(r)$ is the same as or the contrary to that of $\left(\frac{d}{dr}\right)^i F(r)$. Accordingly, if r is real [45] and i even, the nature of the *ensemble* of the i roots $r+dr$ will not be the same when $\delta F(r)$ is positive as when $\delta F(r)$ is negative.

(50) So, further, if $Fx=0$ have $2m$ equal roots r, $2n$ equal roots s, and so on, the deduced corresponding groups of roots in $F(x)+\delta F(x)=0$ will, or may at least each of them, undergo a change of character to the extent of one pair of the r group changing their nature with the sign of $\delta F(r)$, one pair of the s group changing their nature with the sign of $\delta F(s)$, and so on; but in no case, except $F(x)$ possess some equal roots (*i. e.* unless its discriminant be zero), can an infinitesimal variation in the constants affect the character of the roots [46].

(51) To every facultative point corresponds a certain set of values of J, D, L; and when these are given, it has been shown that the equation $(a, b, c, d, e, f \between x, y)^5$ is reducible to the form $ru^5+sv^5+tw^5$, where $u+v+w=0$, or to the form $ru_i^5+sv_i^5+tw^5$, where

$$u_i+v_i+w=0, \text{ and } u_i=\frac{-w+iv}{2}, \; v_i=\frac{-w-iv}{2},$$

or to the form $au^5+5euv^4+fv^5$, u, v, w being always real linear functions of x, y, with the sole exception that when $J=0$, $K=0$, $L=0$, the reduced form is

$$au^5+5bu^4v+10cu^3v^2.$$

When these three invariants are not all zero, the coefficients in the reduced form r, s, t or a, e, f are known functions of J, D, L, and the character of the roots is perfectly determinate; so that to every facultative point corresponds an infinite family of equations with real linear coefficients all deducible from each other by real linear substitutions. Thus then, with the sole exception of the origin, every facultative point corresponds to a determinate character of equation, viz. to an equation with four, or two, or no imaginary roots; so that by a bold figure of speech we may be permitted to speak of every point but one in facultative space having a determinate quality, as masculine, feminine, or neuter. The origin alone is exempt from this law, and may be considered to be of epicene gender, since the factor $au^2+5buv+10v^2$ may have its roots real or imaginary. As we travel continuously from point to point in the facultative portion of space we pass from family to family, or, if we please, from an individual of one family to an individual of another family, differing from the former individual by an infinitesimal variation of the constants.

[45] r, although supposed to be one of a group of equal roots, is not *necessarily* real, for it may belong to a factor $(x^2+2e\cos\theta+e^2)^2$.

[46] Compare this statement with the corresponding one given by M. HERMITE, Camb. and Dub. Journal, vol. ix. p. 204, where only one parameter is supposed to undergo a change. I think that greater breadth and at the same time greater precision and clearness are gained by the mode of exposition employed in the text above. It will be observed that for a change of character to be possible when the function passes through a phase of equal roots, it is not enough that there shall exist a group of equal roots r, but there must be an even number of such roots in the group, and, furthermore, the equal roots must be *real*; when this last supposition is not satisfied, no change in the character of dr will affect the character of $r+dr$: an instructive exemplification of this remark will occur in the sequel.

(52) If, then, we insulate any portion of facultative space, and in the block so insulated it is possible to pass from one point to any other—that is to say, if we can draw a *continuous* curve of any sort from one point to another without passing out of the block, and without cutting or touching the plane $D=0$, then by virtue of the principle just laid down, we see that all the points in such block have the same character, and the nature of the roots will be the same in the infinite number of families, each containing an infinite number of individuals which the points in that block severally represent. Now imagine a block taken so extensive as to admit of no further augmentation, except accompanied with a violation of the condition of the capability of free communication between point and point without cutting or touching the surface D; such a block may be termed a *region*, and the whole of facultative space will be capable of subdivision into a certain number of these regions. This being supposed effected, the character of each region will be known when we know the character of a single point in it; that is to say, every region will have a determinate character of positive, negative, or neuter. It will presently be shown that the number of such regions is only three([47]) (the least number it could be to meet the three cases of four, two, or no imaginary roots), one masculine, one feminine, one neuter; and consequently there will be but three cases to consider when the invariantive coordinates J, D, L are given; according as J, D, L belong to one or the other of these three regions, the equation to which they belong will have all its roots real, or only one real, or three real and two imaginary. The origin, it need hardly be added, constitutes a region *per se*, in which, so to say, the characters of masculine and feminine are blended.

(53) Let it be observed that we can see *à priori* that, were it not for the distinction between facultative and non-facultative portions of space, it would be impossible for each point corresponding to a given system of invariants to possess an unequivocal character; for in such case there would necessarily be free continuous communication possible between all the points on each side of D *inter se*, and consequently we should be landed in the absurdity of conceiving the general equation of the fifth degree not to admit of division into cases of four, two, or no imaginary roots; D being negative, we know, would imply two roots, and not more than two, being imaginary; and accordingly D positive would imply either that four roots are imaginary or none—not sometimes one and sometimes the other, but in all cases alike four imaginary, to the exclusion of the supposition of the roots being all real, or else of all the roots being real and never four imaginary. Thus we see that the mere fact of a given system of invariants communicating a definite character to the roots, implies the necessity of the invariants exercising a restraining action over each other's limits, and that where this restraint does not exist it is impossible that the character of the roots can be determined by the values of the invariants.

([47]) It is clear from the definition, that a *region* can only be bounded by G the amphigenous surface, and D the plane of the discriminant: and granted (as will be shown hereafter) that G and D *touch* each other in only one continuous line, it becomes obvious *à priori* that there can be but two regions on one side of D and a single region on the other.

(54) This is precisely what happens in biquadratic equations. In such we know the fundamental invariants t, s, or, if we please, t, Δ (where $\Delta=s^3+27t^2$), are perfectly independent and subject to no equation of condition; so that if we consider t, Δ as the coordinates of points in a plane, the whole of the plane will be made up of facultative points. When Δ is negative, *i.e.* for representative points lying on one side of the line Δ, it is true we know that there is just one pair of imaginary roots constituting what may be termed the neuter case; but when the representative points lie on the other side of this plane, they cannot be said to be either masculine or feminine, but will every one of them possess that epicene character which is peculiar to the origin alone in the case of quintic forms. A single example will make this clear.

Take the two reduced forms

$$u^4+6(1+\varepsilon)u^2v^2+v^4,$$
$$\omega^4+6(1-\varepsilon)\omega^2\theta^2+\theta^4,$$

where u, v are real linear functions of x, y, and ω, θ conjugate imaginary ones of the same; and suppose s, the quadrinvariant in respect to x, y, to be the same for both forms. For greater convenience of computation consider ε to be infinitesimal.

Then in the one case the t is of the same sign as

$$(1+\varepsilon)\left(1-(1+\varepsilon)^2\right),\text{ i. e. } -2\varepsilon,$$

and in the other the t is of the contrary sign to

$$(1-\varepsilon)\left(1-(1-\varepsilon)^2\right),\text{ i. e. } 2\varepsilon,$$

so that t is of the same *sign* (viz. negative) in each case.

Again, in the two cases respectively

$$\frac{t^2}{s^3}=\frac{4\varepsilon^2}{1+3(1\pm\varepsilon)^2}=4\varepsilon^2.$$

Hence t as well as s, and consequently t and Δ are alike for both forms.

But in the one first written the roots are of the same nature as those of $u^4+6u^2v^2+v^4$, i.e. are all impossible, and in the other of the same nature as in

$$\left(\frac{u+iv}{2}\right)^4+6\left(\frac{u+iv}{2}\right)^2\left(\frac{u-iv}{2}\right)^2+\left(\frac{u-iv}{2}\right)^4=0,$$

where u, v are real linear functions of x, y and $i=\sqrt{-1}$, in which case the roots are all possible. Thus we see that the very same values of t, Δ may correspond either to the case of four real or four imaginary roots, showing that the point t, Δ is what we have termed *epicene*. If we choose to take s, t as the coordinates, the same remarks would apply, except that Δ instead of a straight line would become a semicubical parabola. All the points on one side of this curve would have a definite neuter character, but those on the opposite side would be neither masculine nor feminine, but epicene.

(55) With a view to its subsequent distribution into regions, I now proceed to ascertain the form of that moiety of space which I have termed facultative.

Let $J^2=qK$, $J^3=\nu L$. Then

$$\frac{G}{J^9}=\frac{1}{q^4}+\frac{8}{\nu q^3}-\frac{2}{\nu q^2}-\frac{72}{\nu^2 q}-\frac{432}{\nu^3}+\frac{1}{\nu^3}, \text{ and } \frac{D}{J^2}=1-\frac{128}{q}.$$

We may for the moment make abstraction of the section of G made by the plane of L; that being done, and J, K, L being referred to the form $ru^5+sv^5+tw^5$ or $ru_{\prime}^5+sv_{\prime}^5+tw^5$, calling μ^{10}, M, and, as before, using ϱ, σ, τ to denote st, tr, rs, we have

$$\begin{aligned}\pm J &=M(\varrho^2+\sigma^2+\tau^2-2\varrho\sigma-2\varrho\tau-2\sigma\tau),\\ K&=M^2\varrho\sigma\tau(\varrho+\sigma+\tau),\\ \pm L&=M^3\varrho^2\sigma^2\tau^2.\end{aligned}$$

Now when $G=0$, we may suppose $\varrho=\sigma$, $\frac{\tau}{\varrho}=\frac{\tau}{\sigma}=\theta+4$, θ being a new auxiliary variable. We have then

$$\begin{aligned}\pm J &=M(\tau^2-4\varrho\tau), =M\varrho\tau\theta,\\ K&=M^2\varrho^2\tau(2\varrho+\tau)=M^2\varrho^2\tau^2\left(1+\frac{2}{\theta+4}\right),\\ \pm L&=M^3\varrho^4\tau^2 \qquad =M^3\varrho^3\tau^3\frac{1}{\theta+4},\end{aligned}$$

and consequently

$$\begin{aligned}\nu&=\frac{J^3}{L}=\theta^4+4\theta^3,\\ q&=\frac{J^2}{K}=\frac{\theta^2(\theta+4)}{\theta+6}.\end{aligned}$$

(56) In general we have $\theta^4+4\theta^3-\nu=0$.

By a well-known corollary to DESCARTES's rule this equation can never have more than two real roots; when ν is positive there will always be two real roots of opposite signs; but when ν is negative and inferior to a certain negative limit, *all the roots become imaginary*. When ν lies between zero and that limit, two roots of θ will be real and both negative. To find that limit we may make $4\theta^3+12\theta^2=0$, or $\theta=-3$, which gives $\nu=81-108=-27$.

(57) When $D=0$, $q=\frac{J^2}{K}=128$, i. e. $\theta^3+4\theta^2-128\theta-768=0$, or $(\theta+8)^2(\theta-12)=0$; so that the roots of θ, when $D=0$, are -8, -8, 12, and the corresponding values of ν are 2^{11}, 2^{11}, $2^{10}27$.

If now we make $\theta^4+4\theta^3=2^{11}$, one of the real values of θ we know is -8, and the other will be the real root of the cubic equation $\theta^3-4\theta^2+32\theta-256=0$.

When $\theta=5$, the left-hand side of the equation $=125+160-100-256=-71$.

When $\theta=6$, the left-hand side of the equation $=216+192-144-256=\ \ 8$.

Hence the real root lies between 5 and 6, and q lies between $\frac{225}{11}$ and $\frac{360}{12}$. Thus $q<30$ and $\frac{D}{J^2}=1-\frac{128}{q}$ is negative.

Again, if we take $\theta^4+4\theta^3=27\cdot 2^{10}$, and take out the root $\theta=12$, the resulting cubic becomes

$$\theta^3+16\theta^2+192\theta+2304=0,$$

where it will easily be seen the real root lies between -12 and -16.

When $\theta=-12$,

$$q=\theta^2\frac{\theta+4}{\theta+6}=144\times\frac{8}{6}=192\,;$$

and when $\theta=-16$,

$$q=256\times\frac{12}{10}=307\tfrac{1}{5}.$$

Moreover, when q is a maximum or minimum, it will readily be found that $\theta^2+11\theta+24=0$; so that $\theta=-3$, or $\theta=-8$. Hence for the value of θ found from the above cubic $q<192$ and $\frac{D}{J^2}=1-\frac{128}{q}$ is *positive.*

(58) When $J=0$, $\nu=0$; and when $L=0$, $\nu=\infty$.
For these two cases it will be more simple to dispense with the auxiliary variable θ, and to revert to the original equation between J, K, L.

Accordingly, when $J=0$, we find $8LK^3-432L^3=0$. Hence

$$L=0,\ \text{or}\ K^3=54L^2,\ \text{i. e.}\ \left(\frac{-D}{128}\right)^3=54L^2;$$

so that the complete section of G made by the coordinate plane J becomes a straight line, viz. the axis of D, and a semicubical parabola whose axis is the negative part of D. When J is very nearly zero, ν becomes a positive or negative infinitesimal in the equation $\theta^4+4\theta^3=\nu$.

One real root of this equation is $\theta=\left(\frac{\nu}{4}\right)^{\frac{1}{3}}$.

The other is $-4+\delta$, where $\left(4(-4)^3+12(-4)^2\right)\delta=\nu$,

or

$$\delta=-\frac{\nu}{64}.$$

Now

$$\frac{K^3}{L^2}=\left(\frac{\theta+6}{\theta+4}\right)^3(\theta+4)^2=\frac{(\theta+6)^3}{(\theta+4)}.$$

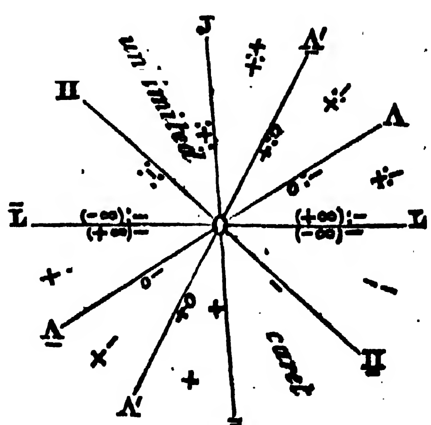
The first value of θ gives $K^3=54L^3$ to an infinitesimal *près*; the other value gives

$$K^3=-\frac{512}{\nu}L^2,$$

or, to an infinitesimal *près*,

$$\left(\frac{D}{128}\right)^3=\frac{512}{\nu}L^2;$$

so that D passes from $+\infty$ to $-\infty$, *i. e.* $\frac{J^3}{L}$ passes through zero.



(59) In the annexed figure[48], the plane of the paper represents the plane of D, *i. e.* the plane for which $D=0$; $JO\bar{J}$ is the axis of J, OJ being the positive and $O\bar{J}$ the negative direction; $LO\bar{L}$ is the axis of L, OL being the positive and $O\bar{L}$ the negative direction. In order to avoid any appearance of an attempt at a practicably impossible accuracy of drawing, I use straight lines to

[48] I shall refer, when I have occasion to do so, to this figure, which contains a synopsis of the whole theory, under the name of the Dial figure.

denote cubical parabolas, and pay no attention whatever to relative magnitudes, but only to the order or progression of magnitudes, using the lines which are drawn in the figure not as *copies* but as *symbols* of the actual curves which are to be mentally imagined.

Thus the line $JO\bar{J}$ is used to represent the straight line $L=0$; $\Lambda'O\underline{\Lambda}'$ the cubical parabola $J^3=27\cdot2^{10}L$; $\Lambda O\underline{\Lambda}$ the cubical parabola $J^3=2^{11}L$; $\Pi O\underline{\Pi}$ the cubical parabola $J^3=-27L$([49]).

It will be observed that certain combinations of *plus*, *zero*, *minus*, positive and negative *infinity* are placed along the lines and inside the sectorial spaces. The meaning of these will be sufficiently obvious from what has preceded. They refer to the signs of the two values of D in the surface G for each point in the line or sector along or within which they are placed. At every point along the line $O\bar{J}$, $\frac{D}{J^2}$ has only one value, and that positive; along $\Lambda'O\underline{\Lambda}'$, $\frac{D}{J^2}$ has two values, one positive and the other zero. Along $\Lambda O\Lambda$, $\frac{D}{J^2}$ has two values, one positive the other negative. Immediately below $\bar{L}OL$ two values, one $+\infty$, the other finite and negative. Immediately above $\bar{L}OL$ two values, one $-\infty$, the other finite and negative. Along $\Pi O\underline{\Pi}$ one value, finite and negative.

Moreover D has been shown to be never zero, except along $\Lambda'O\underline{\Lambda}'$, $\Lambda O\underline{\Lambda}$. Hence it is obvious that *inside* $\Lambda'O\bar{J}$ and the opposite sector D has two values, both *plus*; inside the next pairs of opposite sectors two values, one *plus*, the other *minus*; inside the next pair of sectors also two values, one plus, the other minus; inside the next pair of sectors two values both *minus*, and in the pair of sectors left vacant, for which $\nu<-27$, it has been shown that D becomes impossible.

([49]) It has been shown in the preceding articles that corresponding to the line $\bar{J}OJ$ and to the line $\Pi O\underline{\Pi}$, the vertical ordinate D of the amphigenous surface ($G=0$) has only one value positive for the former, negative for the latter; along the line $\Lambda'O\underline{\Lambda}'$ two values, one positive the other negative; for the space between $\Lambda O\Lambda'$, $\bar{L}OL$ indefinitely near to the latter two values, one positively infinite, the other negative; and for the space indefinitely near to the same on the opposite of it, two values, one negatively infinite, the other negative. These results are collected and represented symbolically in the Table annexed.

$\bar{J}$	Λ'	Λ	$\bar{L}$	Π
	+	0	$(+\infty)$ −	
+				−
	0	−	− $(-\infty)$	

Thus, corresponding to the upper sheet of G, we have the succession

+	+	0	$(+\infty)$	−	−

and to the lower sheet

+	0	−	−	$(-\infty)$	−

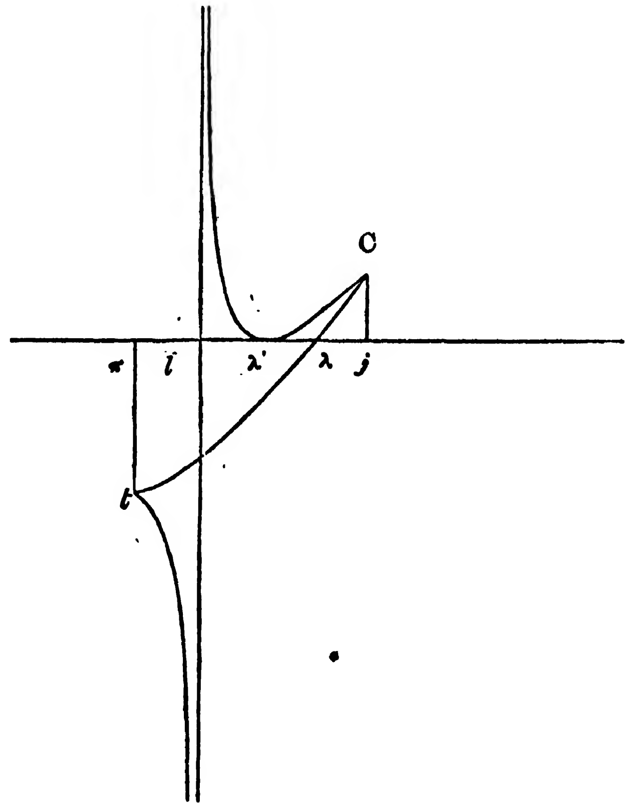
the two sheets coming together at a cuspidal edge above $\bar{J}OJ$ and below $\Pi O\bar{\Pi}$.

Moreover these are the only positions of the line revolving in the plane of D corresponding to which a change in the nature of D can take place, and thus we can without further examination fill up the Table, giving the nature of D for the intervening spaces, and may thus obtain the Table embodied in the *dial-figure* above, viz.,

$\bar{J}$		Λ'		Λ		$\bar{L}$		Π
	+	+	+	0	+	$(+\infty)$ −	−	
+								−
	+	0	−	−	−	− $(-\infty)$	−	

(60) Thus it will be seen that the surface G consists of two opposite portions precisely similar and symmetrical in respect to the axis of D.

Let us trace that one of these whose ground-plan is comprised within the sector $\Pi O\bar{J}$. It will consist of two sheets coming to a cuspidal edge (a common parabola) in the superior part of the plane of L. The upper sheet will touch the plane of D in $O\Lambda$[50], and, remaining above the plane of D, approach continually to the plane of J as an asymptotic plane. The lower sheet will cut the plane of D in $O\Lambda'$, pass under the plane of D, cut the plane of J, progress to a maximum distance from it, and then approach indefinitely to J as its asymptotic plane. This will become apparent by taking a vertical section of this portion, cutting the lines $O\bar{L}$, $O\bar{J}$; for the nature of the flow of the two branches of the section will evidently be as figured below, where j, λ, λ', l, π represent the points in which the lines $O\bar{J}$, $O\Lambda'$, $O\Lambda$, $O\bar{L}$, $O\Pi$ are cut by the secant plane. [It should be particularly noticed that this figure is only intended to exhibit, under its most general aspect, the nature of the flow of the two branches of the curve; it is drawn in other respects almost at random, and makes no pretension whatever to giving a representation of the actual form of the curve.]



No part of the surface G lies under or above the sector ΠOJ, except the axis of D. The cusp C, where the two branches meet, is the intersection of the cutting plane with the parabola $J=D^2$ lying in the plane of L, and there will be another cusp at t, the point of maximum recession from the plane of J.

(61) I now proceed to discriminate, by aid of this surface, the facultative from the non-facultative portion of space.

If in the expression for G as a function of J, K, L we substitute for K its value $-\frac{D}{128}+\frac{J^2}{128}$, we obtain $G=\frac{J}{(128)^4}D^4+$ terms involving only lower powers of D; so that, calling D_1, D_2 the two real values of D in the upper and lower sheets of G respectively corresponding to any point J, L,

$$G=J(D-D_1)(D-D_2)Q,$$

Q being a quantity essentially positive.

Hence when J is negative the *facultative* points in any line parallel to D will be those for which D lies between D_1, D_2, but when J is positive, the facultative points must be exterior to the segment D_1D_2; I denote this difference in the figure by placing a colon between the signs in each sector for which J is positive, indicating thereby that the facultative points lie between $+\infty$ and D_1, and between D_2 and $-\infty$; but where no

[50] For the value of D for this sheet is zero all along $O\Lambda$, and positive on either side of it.

colon is interposed, then it is to be understood that the facultative points lie between D_1 and D_2. Thus, if we turn back for a moment to the section of G last drawn, the whole of the space included between the two branches and the asymptote is facultative, because up to the asymptote J is negative, and beyond the asymptote the whole of the space not included between the asymptote and the lower branch is facultative, because beyond the asymptote J becomes positive. Thus, then, we see that the whole of that portion of the plane which lies on the left-hand side of the entire curve is facultative, and the portion on the right-hand side of the same non-facultative; the curve separating facultative from non-facultative space as a coast-line, indefinitely extended, separates land from water; so that there is, as of course we might have anticipated, no break of continuity in passing through the plane J.

If we take a corresponding section of the opposite portion of space corresponding to the ground-plan $JL\underline{\Pi}$, it is obvious that precisely the contrary takes place, because the sign of J is opposite in the opposite sectors; so that what was facultative becomes non-facultative, and *vice versâ*.

(62) It is now clear that the whole of the facultative part of space is divided into three, and only three of the *regions* previously defined. One region will consist of that portion of it which is entirely under the plane of D: the second region will be so much of the upper portion as stands upon the acute sector $\bar{J}O\Lambda$; and the third of so much of the remainder of this portion as stands on the sector $\Lambda OJJO\underline{\Pi}$ [51]. Again, as regards the second region, the line $O\Lambda'$ is quite inoperative against its unity, because we have vertical ordinates above $O\Lambda'$ through which free communication can take place between the blocks over $JO\Lambda'$ and $\Lambda'O\Lambda$; but when we come to $O\Lambda$, where G touches the plane of D, there we have an effective line of demarcation between the adjoining blocks *above* the plane of D; for it is impossible to pass from one into the other without going under D and coming up again through that plane, or else descending to the line $O\Lambda$ and so meeting the plane of D [52].

[51] It will be borne in mind that the whole of the infinite prism, both above and below, standing on ΠOJ belongs to *facultative* space: the prism standing on the opposite section $\bar{J}O\bar{\Pi}$, or, to speak more strictly, on the *inside* of this last-named sector, is wholly unfacultative. The facultative line D which passes through O is completely isolated from the facultative portion which stands over ΛOJ, except at the point O (which we are forbidden to pass through if we would remain in the same region), and is of course a rectilinear edge to the facultative prism above referred to.

[52] Two superior regions we know *à priori* must exist to correspond respectively to the two cases of five and of one real root. Moreover we know *à priori* that two regions can only meet on the plane of D, and an inspection of the *dial-figure* shows that only $O\Lambda$ can be such line. Thus without completely making out the geometry of the question as regards the remarkable line ($J=0$, $L=0$) (the axis of D) which lies on the surface G, we may feel assured that the upper part of this line (which is easily found to belong to the 1-real-root region) cannot have any point except the origin in common with the 5-real-roots region, since otherwise these two regions would communicate along this line and merge into one. When it is considered that G is a surface of the ninth order in J, D, L, it will not appear surprising that some difficulty arises in forming a mental conception of certain of its local properties; on the contrary, the subject of wonder rather is that enough can be ascertained about it in a very brief compass to shed all the needful light upon the analytical problem which it illustrates.

(63) It remains only to fix the characters of the several regions; but this requires no calculation to effect, for we know that when D is negative there is one and only one pair of imaginary roots. This disposes of the first of the regions above enumerated. Again, we know that when L is positive so that the reduced form is the superlinear equation $ru^5+sv^5+tw^5=0$, u, v, w being *real* functions, D being also positive, there must be four imaginary roots, as follows from the theory of the second section. Hence the third region has for its character two pairs of imaginary roots; and consequently the only remaining region, the second described, must correspond to the case of no imaginary roots, since otherwise we should be absurdly assuming the impossibility in any case of a quintic equation having all its roots real.

(64) It may, however, be an additional satisfaction to see how the change of character comes to pass at the critical line OΛ from one to five real roots.

Along the line OΛ we have found that, calling the reduced form $ru_i^5+sv_i^5+tw^5$,

$$r=s \quad \frac{\tau}{\varrho}=\frac{rs}{st}=\frac{r}{t}=\theta+4=-4.$$

Hence the equation becomes

$$4u_i^5+4v_i^5+(u_i+v_i)^5=0,$$

u_i, v_i being of the form $\frac{-u+iv}{2}$, $\frac{-u-iv}{2}$, because L is negative.

Hence $u_i+v_i=0$, or

$$4(u_i^4-u_i^3v_i+u_i^2v_i^2-u_iv_i^3+v_i^4)+(u_i+v_i)^4=0,$$

$$\text{i. e. } 5u_i^4+10u_i^2v_i^2+v_i^4=0,$$

$$\text{i. e. } (u_i^2+v_i^2)=0;$$

so that there are two pairs of equal roots of $\frac{u_i}{v_i}$, viz. $\pm\iota$; to these values of $\frac{u_i}{v_i}$ correspond

$$\frac{u-iv}{u+iv}=\iota, \quad \frac{u-\iota v}{u+\iota v}=-\iota.$$

Hence

$$(1-\iota)u=(\iota-1)v, \text{ or } (1+\iota)u=(\iota+1)v;$$

so that the two pairs of equal roots of $\frac{u}{v}$ are ±1, the outstanding root corresponding to $u_i+v_i=0$ being $\frac{u}{v}=0$.

Now, *still keeping upon the surface* G, which we know is facultative, let θ become $-8+4\epsilon$, where ϵ is an infinitesimal, then

$$\delta\left(\frac{J^3}{L}\right)=\delta\nu=(4\theta^3+12\theta^2)\delta\theta=-5120\epsilon;$$

also the supposed equation becomes

$$(4-4\epsilon)(u_i^5+v_i^5)+(u_i+v_i)^5=0,$$

or

$$(\iota v-u)^5-(\iota v+u)^5+8(1+\epsilon)u^5=0;$$

or, calling $\frac{v}{u}=\varrho$,

$$(\iota\varrho-1)^5-(\iota\varrho+1)^5+8(1+\epsilon)=0.$$

Let $\varrho=\pm1+\sigma$, where σ is an infinitesimal. Hence

$$\left(-10(\pm\iota-1)^5+10(\pm\iota+1)^5\right)\sigma^2-8\epsilon=0,$$

or

$$20(1-10+5)\sigma^2-8\epsilon=0,$$

or

$$\sigma^2=\frac{-\epsilon}{10}=+\frac{1}{51200}\delta\left(\frac{J^3}{L}\right).$$

Hence calling σ_1, σ_2 the two values of σ, the four roots that at OΛ were 1, 1, −1, −1 become $1+\sigma_1$, $1+\sigma_2$, $-1+\sigma_1$, $-1+\sigma_2$, when $\frac{J^3}{L}$ becomes varied by $\delta\left(\frac{J^3}{L}\right)$, and consequently become all real if $\frac{J^3}{L}$ is increased, and all imaginary if $\frac{J^3}{L}$ is decreased, *i. e.* become real or imaginary according as the line OΛ sways towards or away from O$\bar{\text{J}}$, conformably with what has been shown on other grounds.

It will be noticed that in the line OΛ produced in the opposite direction, *i. e.* along the line O$\underline{\Lambda}$, L being positive, the reduced form is

$$4(u^5+v^5)+(u+v)^5=0,$$

and the roots of $\frac{u}{v}$ become $\frac{u}{v}=-1$, $\frac{u}{v}=\pm\iota$, $\frac{u}{v}=\pm\iota$; so that, according to the canon laid down at the commencement of this discussion (see foot-note (46)), no change in the character of the roots can possibly take place along OΛ, and accordingly we have seen that this curved line does not correspond to any demarcation of regions.

(65) It is easy to express the conditions to be satisfied by the coordinates of a point according as it lies in one or another of the three regions which have now been mapped out, and it is clear that we have the following rule:

When D is negative the equation has two imaginary roots.

When D is positive the equation has *no* imaginary roots, provided the two criteria J and $2^{11}L-J^3$ are both negative([53]); but if either of these is zero or positive, there are two pairs of imaginary roots([54]).

The duodecimal criterion-invariant, $2^{11}L-J^3$, and the invariants of the like order, $27\cdot2^{16}L-J^3$, $-27L-J^3$, I shall henceforth call Λ, Λ′, Π respectively. It has been just above shown that the three invariants J, D, Λ of the 4th, 8th, and 12th orders respectively are sufficient for ascertaining the character of the roots of the quintic to which they appertain.

([53]) Observe that this implies L also being negative; so that $2^{11}-\frac{J^3}{L}$ is positive and $\frac{J^3}{L}<2^{11}$.

([54]) ([a]) Observe that in general when $2^{11}L-J$ is zero there are no facultative points above the plane of D, but when J and $2^{11}L-J$, and consequently L and J are both simultaneously zero, a facultative right line springs into existence, viz. the axis of D extending both above and below the plane of D. The reduced form of equation (as previously demonstrated) corresponding to this singular line is $u^5\pm uv^4=0$.

([b]) It may further be noticed that on each side of the line O$\underline{\Lambda}$ the limits of D are between positive infinity and a positive quantity, and between negative infinity and a negative quantity; so that as we pass from O$\underline{\Lambda}$ to either side of it no facultative point can be found lying in the plane of D, showing that we cannot pass by a real infinitesimal variation of coefficients from an equation with two pairs of equal imaginary roots to an equation with a single pair of equal roots, as is apparent also on purely analytical grounds.

(66) The assertion that the *whole* of facultative space is divisible into three regions, in strictness requires a slight modification. It is obvious that the plane of D itself cannot be said to belong to any of the regions; and in order to make our theory quite complete, so as to furnish criteria applicable to equations having equal roots, and to enable us to distinguish between the case of the unequal roots being all three real, or two imaginary and one real, we must examine what takes place in this plane, and under what circumstances a passage from one point of it to another will or may be accompanied with a change of character in the roots.

If the roots of $f(x)=0$ are supposed to be a, a, c, d, e, where c, d, e are unequal, on varying the constants of fx in such a manner that the variation of the discriminant D is zero, the two equal roots a, a will remain equal. Now *in general* we have $\delta f(a)+f''(a)\frac{(da)^2}{2}=0$; if this, under the particular supposition made, continued to obtain, da would have two distinct values, and the two equal roots would cease to continue to be equal, contrary to hypothesis. Hence we see that $D=0$, $\delta D=0$ necessarily implies $\delta f(a)=0$ (55), and consequently $\delta f(a+da)$ is no longer δfa, but $\delta f'ada$; so that we obtain $da=0$, or $da=-\frac{2\delta f'a}{f''a}$, and no change of character in the five roots results. If, however, the original roots are a, a, c, c, e, then, as shown in the general case, δc will have two distinct values, which will be both real or both imaginary. Accordingly we see that in

(55)(a) This is a somewhat curious theorem (whether new or otherwise I know not) thus incidentally established in the text, viz. that if $D(f)$ represent the discriminant of f, and if $D(f)=0$ and $\delta D(f)=0$, then when $f=0$ we must have $\delta(f)=0$. The very simplest example that can be chosen will serve to illustrate this proposition. Let

$$f=ax^2+2bxy+cy^2.$$

Suppose

$$D(f)=ac-b^2=0,$$

and also

$$\delta D(f)=a\delta c+c\delta a-2b\delta b=0,$$

we have

$$\delta(f)=x^2\delta a+2xy\delta b+y^2\delta c.$$

Now if $f=0$ we may write $x=b$, $y=-a$, and δf becomes

$$\begin{aligned}&b^2\delta a-2ab\delta b+a^2\delta c\\ &=b^2\delta a-2ab\delta b+2ab\delta b-ac\delta a\\ &=(b^2-ac)\delta a=0,\end{aligned}$$

according to the theorem.

If we make $f=(x, 1)^n$, D we know becomes a syzygetic function of f and f' $\left(\text{meaning by the latter } \frac{df}{dx}\right)$. Hence since δD vanishes when $fx=0$, $D=0$, and $\delta f(x)=0$, we learn that $\delta(D)$ is a syzygetic function of $(f, f', \delta f)$.

The theorem thus stated easily admits of extension to the higher variations of D, and so extended takes I believe the following form:

$$\delta^i(D)=\text{ a syzygetic function of } (f, f', f'', \ldots f^i, \delta f).$$

(b) Professor CAYLEY has since informed me that the theorem in (55) (a), about whose originality I was in doubt, will be found in SCHLÄFLI's 'De Eliminatione.' This is not the first unconscious plagiarism I have been guilty of towards this eminent man, whose friendship I am proud to claim. A much more glaring case occurs in a note by me in the 'Comptes Rendus,' on the twenty-seven straight lines of cubic surfaces, where I believe I have followed (like one walking in his sleep), down to the very nomenclature and notation, the substance of a portion of a paper inserted by SCHLÄFLI in the 'Mathematical Journal,' which bears my name as one of the editors upon its face!

the plane of D no change can possibly take place except in crossing the line which corresponds to a family of *two pairs* of equal roots.

(67) It has already been pointed out, in a foot-note, that we cannot pass facultatively from $O\underline{\Lambda}$ to either side of this curve line. Hence the separation of the plane of D into subregions can only take place along the line $O\Lambda$, and it remains but to ascertain the character of the points on either side of this line, which we know, therefore, *à priori*, must possess opposite characters, since otherwise we should be admitting the absurd proposition of its being impossible to construct an equation of the fifth degree having two equal roots without the remaining three being always of *one character*, either all real or all not real. Let us, then, ascertain the character of the points in OJ for which $D=0$, $L=0$, and J is positive[56].

Since $L=0$, the reduced form is $u^5+5euv^4+v^5$.

This equation, by DESCARTES's rule, must contain imaginary roots. Hence in the sector $\Lambda O\bar{J}$ the roots are all real, and in the remainder of the facultative portion of the plane (from which it may be noticed the sector $\underline{\Lambda}O\bar{J}$ is excluded) two of the roots are imaginary.

Along $O\Lambda$ itself there are, as already observed, two pairs of real equal roots, and along $O\underline{\Lambda}$ two pairs of imaginary equal roots. Thus, finally, we have the *complete rule*.

If D is negative,	2 roots imaginary.	
If D is positive.		
When J, Λ are both negative,	0 roots imaginary.	
„ J, Λ are *not* both negative,	4 roots imaginary.	
If D is zero.		
When J, Λ are both negative,	0 roots imaginary	1 pair of equal roots.
„ J, Λ are *not* both negative,	2 roots imaginary	1 pair of equal roots.
„ J is negative, Λ zero,	0 roots imaginary	2 pairs of equal roots.
„ J is positive, Λ zero,	4 roots imaginary	2 pairs of equal roots.
„ J is zero, Λ zero,		3 equal roots[56 bis].

Thus we see that our space referred to an arbitrary origin, and with the invariants J, D, Λ for the coordinates, has been first divided into facultative and non-facultative space. The former has then been resolved prismatically into two regions above and one below the plane of D. The plane of D itself, or the facultative part of it, into two

[56] We could not take J negative, for the facultative points of D in $\bar{J}$ are two positive quantities. See dial figure.

[56 bis] When $D=0$, $\Lambda=0$, there are two pairs of equal roots. If J is negative these pairs are both real. If J is positive they are both imaginary. When J is zero there are no longer two pairs, but a single triad of equal roots. This perfectly explains what at first sight has the air of a paradox, viz. that the discrimination between the two kinds of double equality of an apparently equal order of generality that may subsist between the roots of an equation, depends on the fulfilment or failure of an algebraical equality. The fact is, as shown above, that there are not, as commonly supposed, two, but three kinds of double equality, according as there are two pairs of real, two pairs of imaginary, or one triad of equal roots; and the last is a sort of transition case between the other two.

planar regions on opposite sides of the line $\Lambda O \underline{\Lambda}$; and again this line into two linear regions on either side of the origin O, which last corresponds to the case of three equal roots, and constitutes a region or microcosm in itself.

(68) It may as well be noticed here that the ambiguity of character in the points representing the different families of biquadratic forms when t and D are taken as the coordinates (and the same would be true if s and D were employed), which prevails when these points lie above the line $D=0$, equally obtains along this line itself. For the reduced form, when $D=0$, is $ax^4+4bx^3y+6cx^2y^2$. In that case, calling the determinant of transformation μ, we have $s=3\mu^{12}c^2$, $D=-\mu^{24}c^3$; and thus, whatever s and D may be, the character of the unequal roots is left undecided.

It may also be noticed that the blending of characters at the *origin* for the quintic form is not precisely of the same nature as that for the points above the line D in the biquadratic form; for at these points it is the cases of 4 and 0 imaginaries which become undistinguishable invariantively; whereas at the origin for quintics the reduced form becomes $ax^5+5bx^4y+10x^3y^2$, and the characters left undistinguished are those of 4 and of 2 imaginary roots—unless, indeed, we consider equal real roots as belonging indifferently to the class of real and imaginary; on which supposition all the three genders (so to say), masculine, feminine, and neuter, become blended together at that point. But if we consider equal real roots as exclusively of the real class, then the *origin* for quartics ceases to be epicene; for when there are three equal roots all of them must be real. Thus the origin in quintics is the only epicene point, and in quartics the only non-epicene point—understanding by epicene the blending of the masculine (4 *imaginary roots*) and feminine (*no imaginary roots*) characters.

(69) We may draw some further important inferences from an inspection of the "dial figure," or the section of facultative space which follows it.

Within the prism $JO\Lambda'$ [57] it will be observed D is always positive [58]. Hence, when J is negative and Λ' is negative, all the roots *must* be real, and the necessity for using the criterion D is done away with.

Again, when J and L are both negative, D is always negative, so that just two of the roots must be imaginary; and in this case also it becomes unnecessary to apply the criterion D.

Again, since there is no facultative prism corresponding to $\underline{\Pi}OJ$, the combination of L and D, both negative, can never occur unless $\underline{\Pi}$ is negative.

When L is negative, but J not negative, there may be two or four imaginary roots, according to the sign of D; but all the roots cannot be real.

(70). M. HERMITE's rule is as follows. For remarks on the relation between his Δ, J_2, J_3 and the J, K, L of this paper, see foot-note [36]. D is still the discriminant.

If D is negative (of course) two roots are imaginary.

If D is positive.

[57] By which I mean within the facultative prism of which $JO\Lambda'$ is the section made by the plane of D.

[58] The vertical section of facultative space in this supposition (see figure) is the area $\lambda C \lambda'$, which lies wholly *above* the plane of D.

When Δ is negative, $25\Delta^2-3.2^{10}J_3$ negative and J_3 positive, no roots are imaginary.

Δ is negative, $25\Delta^2-3.2^{10}J_3$ positive, $25\Delta^2-2^{11}J_3$ negative, no roots are imaginary.

Δ is positive, ... four roots are imaginary.

Δ is negative, $25\Delta^2-3.2^{10}J_3$ positive, $25\Delta^2-2^{11}J_3$ positive, four roots are imaginary(59).

(71) What is the effect of the condition "Λ *positive* or *negative*," as the case may be? or rather, how does this condition arise? The ground of it is simply this, that $\Lambda=0$ represents a cylindrical surface passing through the curve $O\Lambda$ (see dial figure), which curve is the *edge* of separation between two regions of opposite characters above the plane of D; the cylinder in question cuts the facultative position of space below the plane of D, but above this plane (except along the vertical line $J=0$, $L=0$, *i. e.* the axis of D) it passes exclusively through non-facultative space, never again cutting or meeting the surface G (the facultative boundary). Now it is clear that any surface whatever which passes through $O\Lambda$ and never meets the surface G above the plane $D=0$, except along the axis of D (*i. e.* the line $J=0$, $L=0$), may be substituted for Λ(60) and will serve equally well with Λ to distinguish between the masculine and feminine regions of space. $\Lambda-\rho JD$ will fulfil the condition of passing through the line $O\Lambda$,

(59) (a) The last four conditions ought to tally (and be in effect coextensive) with the two given by me for the case of D positive. The third of them, viz. the case of D positive Δ positive, I have already noticed, as inferences from the dial figure; for M. Hermite's Δ, if not identical with my J, is at all events a positive multiple of it. I do not see how the case of Δ negative, $25\Delta^3-3.2^{10}J_3$ negative with D positive, is met by this system of criteria, since J_3, as well as Δ, may be negative consistently with the second condition. I have not been able to ascertain whether in the memoir such a combination is shown to be impossible. M. Hermite admits, and indeed has been always aware of, the existence of a *lacuna* in the conditions above stated, which, I understand from him, it is his intention at some future time to fill up, and thus to complete his original solution. In the meanwhile he has been led to study the question from a different point of view, and has succeeded in obtaining a new set of criteria adequate to a complete solution of the question without calling in the aid of the principle of continuity. In this new system my Λ criterion is replaced by an invariant of the twenty-fourth degree, which is of course an objection as far as it goes, but in no wise diminishes the extraordinary interest that attaches to this altered mode of approaching the question, which bears to his original method and my own the same relation as the proof of Sturm's theorem by the law of inertia for quadratic forms bears to that given by Sturm himself.

(b) It is apparent from the fact that when $D=0$, G (M. Hermite's I^3) becomes $(25\Delta^3-3.2^{10}J_3)(25\Delta^3-2^{11}J_3)^2$ (Camb. and Dub. Journal, vol. ix. p. 206), that the factors of this product are respectively of the form $a\Lambda'+bJD$, $c\Lambda+eJD$, a, b, c, e being certain numerical quantities. This gives rise to a singular reflection, *to wit*, that my own criteria for the case of D positive may be varied by the addition of a term λDJ to Λ (λ being a numerical coefficient), provided λ lies within certain limits, the form of the criteria in all other respects remaining unchanged. This proposition, fraught with the most important consequences, and not unlikely to lead to an entire revolution in the mode of attacking the general problem of criteria, I proceed to establish in the text.

(60) The surface to be employed will be $\Lambda-\rho JD$, which call M. Λ and M (or at least their upper portions above the plane of D) may then be regarded as the two sides of a sack, of infinite dimensions, open at the top, and seamed together at the bottom, along the curved line $D=0$, $\Lambda=0$, and in the vertical direction along the straight line $J=0$, $L=0$. The surface Λ serving as a screen of separation between the two upper regions, it is clear that M will serve equally well as such screen, provided no superior facultative points lie in the interior of the sack.

whose equation is $\Lambda=0$, $D=0$, and obviously is the only invariant not exceeding the twelfth order capable of so doing; it only remains to ascertain within what limits the numerical coefficient ϱ must be taken so as to fulfil the condition that the combined equations $\Lambda-\varrho JD=0$, $G=0$ shall be incapable of being satisfied by any positive value of D.

(72) Substituting for Λ and D their values, the equation to be combined with $G=0$ becomes

$$J^3-2^{11}L+\varrho J(J^2-128K)=0.$$

Returning to the notation of art. (55), and dividing by JK, this equation, when $G=0$, becomes

$$q-2^{11}\frac{q}{\nu}+\varrho(q-128)=0,$$

or

$$(1+\varrho)q\nu-2^{11}q=128\varrho\nu,$$

which, substituting for q, ν in terms of θ, gives

$$\frac{(1+\varrho)\theta^5(\theta+4)^2}{\theta+6}=2^{11}\frac{\theta^3+4\theta^2}{\theta+6}-128\varrho\theta^3(\theta+4),$$

or

$$(\theta+4)\theta^2(\theta+8)\big((\theta^3-4\theta^2+32\theta-256)+(\theta^3-4\theta^2-96\theta)\varrho\big)=0.$$

When $\theta+8=0$, $D=0$, see art. (57); neglecting, then, this factor, the condition to be satisfied is that when from the equation

$$(\theta+4)\theta^2\big((\theta^3-4\theta^2+32\theta-256)\varrho+(\theta^3-4\theta^2-96\theta)\big)=0$$

a value of θ has been deduced, the values of D corresponding thereto shall not be a positive finite quantity.

(73) Now

$$\frac{D}{J^2}=1-\frac{128(\theta+6)}{\theta^2(\theta+4)}=\frac{\theta^3+4\theta^2-128(\theta+6)}{\theta^2(\theta+4)}=\frac{(\theta+8)^2(\theta-12)}{\theta^2(\theta+4)}.$$

If $\theta=0$, or $\theta+4=0$, since D cannot be infinite, we have $J=0$, so that $\Lambda-\varrho JD$ becomes identical with the original criterion Λ. Hence the factor $(\theta+4)\theta^2$ in the quantity just above equated to zero may be neglected, and the condition to be fulfilled by ϱ is that if θ be any root of the equation

$$\frac{-\theta^3+4\theta^2-32\theta+256}{\theta^3-4\theta^2-96\theta}=\varrho,$$

θ shall be between -4 and 12; this equation on making $\theta=-4\phi$, so that $1>\phi>-3$, becomes

$$-\varrho=\frac{\phi^3+\phi^2+2\phi+4}{\phi^3+\phi^2-6\phi},$$

or, writing $\sigma=\frac{-1-\varrho}{4}$,

$$\sigma=\frac{2\phi+1}{\phi^3+\phi^2-6\phi}=\frac{2\phi+1}{(\phi-2)\phi(\phi+3)}.$$

(74) We wish to ascertain what values of σ will be incompatible with the violation of the limits just assigned to ϕ, and accordingly we must inquire what is the range of values assumed by σ when $\phi>1$ or $\phi<-3$; any values of σ *not* included within this range will be admissible for the purpose in view.

When $\phi < -3$, σ is always positive, and proceeds continuously from ∞ to 0 as ϕ passes from $-3-\epsilon$ (ϵ being infinitesimal) to $-\infty$. Consequently σ must not be allowed to have any positive value. When $\phi=\infty$, $\sigma=0$, and when $\phi=1$, $\sigma=-\frac{3}{4}$.

Hence, if no minimum value of σ (i. e. no maximum value of $-\sigma$) occurs between $\phi=1$, $\phi=\infty$, σ may have any value between 0 and $-\frac{3}{4}$; but if such a minimum value, $-\mathrm{M}$, where $\mathrm{M}>\frac{3}{4}$, should exist, the admissible values of σ would become more enlarged, and might be taken between 0 and $-\mathrm{M}$.

Making then $\delta\sigma=0$, we have

$$\frac{2}{2\phi+1}=\frac{3\phi^2+2\phi-6}{\phi^3+\phi^2-6\phi},$$

or

$$4\phi^3+5\phi^2+2\phi-6=0;$$

which, substituting $1+\psi$ for ϕ, becomes

$$4\psi^3+17\psi^2+24\psi+5=0;$$

so that there can be no real root of the equation in ϕ greater than unity.

Hence the admissible values of σ are defined by the inequalities $0>\sigma>-\frac{3}{4}$,

$$\text{i. e.}\quad 0>-\frac{1+\varrho}{4}>-\frac{3}{4},\qquad \text{or } 0>-(1+\varrho)>-3,\qquad \text{or } 2>\varrho>-1.$$

(75) We have thus obtained the complete solution of the problem of assigning invariantive criteria, such that their signs (positive, negative, or zero) shall serve to fix the nature of the roots. These criteria we now see are

$$\mathrm{J},\ \mathrm{D},\ \Lambda+\mu\mathrm{JD},$$

where μ (the negative, it must be noticed, of ϱ) is any numerical quantity intermediate between 1 and -2 (61).

(76) This important modification of the original criteria J, D, Λ I proceed to apply to the problem of obtaining the *simplest* and *most symmetrical* expression for the criteria in terms of the roots of the equation. Let a, b, c, d, e be the roots, and write

$$\mathrm{Z}=\Sigma\{(a-b)^2(a-c)^2(b-c)^2(a-d)^4(a-e)^4(b-d)^4(b-e)^4(c-d)^4(c-e)^4\},$$

or say

$$\mathrm{Z}=\Sigma\left\{\zeta(a,b,c)\begin{pmatrix} a & b & c \\ & de & \end{pmatrix}\right\}\ (62).$$

(61) Strictly it has only been proved that the surface $\Lambda+\mu\mathrm{JD}$, which passes through the line Λ, D, contains no superior facultative points except those comprised in the line L=0, J=0. It is, I think, not difficult to see from this, that, if in the "*sack*" formed between Λ and $\Lambda+\mu\mathrm{JD}$ any such points were contained, L=0, J=0, *i. e.* the axis of D would be a double or multiple line on the surface G, which is easily disproved by examining the algebraical form of G in art. 41, where K represents $\frac{-\mathrm{D}+\mathrm{J}^2}{128}$; any obscurity, however, which may be supposed to cling to this view is immaterial, as a demonstration capable of being followed *in plano* and leaving nothing to be desired in point of perspicuity, will be found in the Note appended to this Part.

(62) Agreeable to the meaning assigned to ζ and to a couple of rows of letters in my memoir on Syzygetic Relations, in the Philosophical Transactions.

Then, since each letter occurs the same number of times (12) in each term, Z will be an invariant.

(77) Again, suppose any two roots to become equal, say that e becomes d, then Z reduces to the single term $\zeta\,(a, b, c)\begin{pmatrix} a & b & c \\ d & d & \end{pmatrix}$; for any such factor as $\zeta(a, b, d)$ will be accompanied with the factor $\begin{pmatrix} a & b & d \\ c & d & \end{pmatrix}$ which vanishes.

If, further, we suppose any two of the letters a, b, c to become equal, then Z disappears entirely, since on that supposition $\zeta(a, b, c)$ vanishes. Hence Z is an invariant of the twelfth order, possessing the property of vanishing when the equation to which it belongs has two pairs of equal roots. Hence Z is of the form $p\Lambda+q\text{JD}$, and it becomes of importance to ascertain the value of the ratio $\frac{q}{p}$.

To do this let us suppose $e=0$, $a=-b$, $c=-d$.

The ten terms in Z correspond to the following ten partitions:—

(1)	(2)	(3)	(4)
abc	abd	acd	bcd
de	ce	be	ae
	(5)	(6)	
	abe	cde	
	cd	ab	
(7)	(8)	(9)	(10)
ace	bde	ade	bce
bd	ac	bc	ad

(78) The corresponding values of the terms will be

$$4a^2(a^2-c^2)^2.16(a^2c^2)8^2(a^2-c^2)^4;\ 4a^2(a^2-c^2)^2 16a^2c^2(a^2-c^2)^4;\ 4c^2(a^2-c^2)^2.16a^2c^2(a^2-c^2)^4;$$
$$4c^2(a^2-c^2)^2 16a^2c^2(a^2-c^2)^4;\ 4a^6c^8(a^2-c^2)^2;\ 4c^6a^8(a^2-c^2)^2;\ (a-c)^2 256a^4c^4(a+c)^8;$$
$$a^2c^2(a-c)^2 256a^4c^4.a^4c^4(a+c)^8;\ (a+c)^2 256a^4c^4(a-c)^8;\ (a+c)^2 256a^4c^4(a-c)^8.$$

Collecting and simplifying these terms, and observing that

$$(a-c)^2(a+c)^8+(a+c)^2(a-c)^8=(a^2-c^2)\big((a+c)^6+(a-c)^6\big)=4(a^4-c^4)(a^4+14a^2c^2+c^4),$$

we find

$$\begin{aligned}Z=128(a^2+c^2)a^8c^8(a^2-c^2)^6+4(a^2+c^2)a^6c^6(a^2-c^2)^8\\+1024(a^2+c^2)(a^4+14a^2c^2+c^4)(a^2-c^2)^2a^{10}c^{10}.\end{aligned}$$

Let $(a^2-c^2)^2=p$, $a^2c^2=q$, and let $Z_{\prime}=\frac{Z}{(a^2+c^2)q^3}$. Then

$$\begin{aligned}Z_{\prime}&=16384pq^3+1024p^2q^2+128p^3q+4p^4\\&=2^{14}pq^3+2^{10}p^2q^2+2^7p^3q+2^2p^4.\end{aligned}$$

(79) We must now calculate J, D, L:

$$D=\frac{1}{5^5}\zeta(a, -a, c, -c, 0)$$

$$=\frac{1}{5^5}4a^6c^6(a^2-c^2)^4;$$

or writing

$$D=\frac{D}{q^3},$$

$$D_{,}=\frac{4}{5^5}p^2.$$

Again, for J. The form to which it belongs is

$$x^5-(a^2+c^2)x^3y^2+a^2c^2xy^4,$$

or

$$(1, 0, -\frac{a^2+c^2}{10}, 0, \frac{a^2c^2}{5}, 0)(x, y)^5;$$

so that the coefficients of the biquadratic Emanant are

$$x; \quad -\frac{a^2+c^2}{10}y; \quad -\frac{a^2+c^2}{10}x; \quad \frac{a^2c^2}{5}y; \quad \frac{a^2c^2}{5}x.$$

Hence the quadratic covariant becomes

$$\frac{a^2c^2}{5}x^2+\frac{2}{25}(a^2+c^2)a^2c^2y^2+\frac{3}{100}(a^2+c^2)^2x^2$$

$$=\frac{20a^2c^2+3(a^2+c^2)^2}{100}x^2+\frac{2}{25}(a^2+c^2)(a^2c^2)y^2.$$

Hence, by definition, J (which $=-4\times$ Discriminant of the Quadratic Covariant)

$$=-\frac{4}{1250}(a^2c^2)(a^2+c^2)\big(3(a^2-c^2)^2+32a^2c^2\big);$$

and making

$$J_{,}=\frac{J}{(a^2+c^2)q},$$

$$J_{,}=-\frac{6}{625}p-\frac{64}{625}q=-\frac{6}{5^4}p-\frac{2^8}{5^5}q.$$

Finally, to calculate L. The canonizant of the form

$$\begin{matrix} 1 & 0 & A & 0 \\ 0 & A & 0 & B \\ A & 0 & B & 0 \\ y^3; & -xy^2; & x^2y; & -x^3 \end{matrix}$$

is

$$(A^3-AB)x^3+(B^2-A^2B)xy^2,$$

of which the discriminant is

$$-4\frac{AB^3}{27}(A^2-B)^4,$$

where

$$A=-\frac{a^2+c^2}{10}, \qquad B=\frac{a^2c^2}{5}.$$

Hence, by definition,

$$L=AB^3(A^2-B)^4=-\frac{1}{125.10^9}(a^2+b^2)(a^6b^6)\left((a^2-b^2)^2-16a^2b^2\right);$$

and making

$$L_{,}=-\frac{L}{(a^2+c^2)q^3},$$

$$L_{,}=\frac{1}{125.10^9}(p-16q)^4=-\frac{1}{5^{12}.2^7}(p^2-16q)^4.$$

(80) Now let us write

$$\frac{1}{5^{12}}Z=\eta L+eJD(^{63})+\varepsilon J^3.$$

This gives

$$\frac{1}{5^{12}}Z_{,}=eqJ_{,}D_{,}+\varepsilon(p+4q)J_{,}^3+\eta L_{,}$$

or

$$4p^4+128q^3p^3+1024q^2p^2+16384pq^3$$
$$=125(256p^2q^2+24p^3q)e+(p+4q)(6p+64q)^3\varepsilon+\frac{1}{2^7}(p-16q)^4\eta),$$

by means of which identity we can obtain linear equations for finding the values of e, ε, η.

Thus, equating the coefficients of p^4, q^4, p^3q respectively, we obtain

$$4=216\varepsilon+\frac{1}{2^7}\eta,$$

$$4.64^3\varepsilon+\frac{16^4}{2^7}\eta=0,$$

which gives $\eta=-2^{11}\varepsilon$ (as it ought to do),

$$128=(24\times125)e+(4\times216+108\times64)\varepsilon+64\ 2^{11}\varepsilon$$
$$=3000e+8800\varepsilon.$$

Hence

$$200\varepsilon=4, \quad \varepsilon=\frac{1}{50}, \quad \eta=-\frac{2^{10}}{25},$$

$$3000e=128-176=-48, \quad e=-\frac{2}{125} \text{ and } \frac{e}{\varepsilon}=-\frac{4}{5}.$$

In order to verify the value of e, let $p=-4$, $q=1$; then, assuming the correctness of the above determinations, we ought to find

$$4^5-128.4^3+1024.16+16384=125(256.16-24.64).\frac{-2}{125}+\frac{1}{128}.160000.-2^{11}.\frac{1}{50},$$

or

$$2^{10}(1-8+16-64)=(-32.256+48.64)-\frac{8}{25}\times160000,$$

or

$$2^{10}(-55)=-5120-25.2048=2^{10}(-5-50),$$

which is right.

(63) Since Z has been proved to be of the form $p\Lambda+qJD$, we know *à priori* the value of $\frac{\varepsilon}{\eta}$; but I have thought it safer to determine ε, η independently, as an additional check upon the accuracy of the computations.

(81) Thus

$$-Z=\frac{5^{10}}{2}\left(2^{11}L-J^{3}+\frac{4}{5}JD\right)$$

$$=\frac{5^{10}}{2}\left(\Lambda+\frac{4}{5}JD\right);$$

and accordingly we have proved that $-Z$ is of the form $(\Lambda+\frac{4}{5}JD)$; and consequently, since $\frac{4}{5}$ lies within the allowed limits 1 and -2, $-Z$ may be used to replace Λ in the system of criteria.

(82) On examining the composition of Z, it will be found to have a remarkable relation to the lower criterion J.

J we know is, to a numerical factor *près*, of the form

$$\Sigma\{(d-e)^4\zeta(a, b, c)\},$$

ζ denoting, as usual, the squared product of the differences of the quantities which it affects; and Z, it will readily be seen, is of the form

$$\left(\zeta(a, b, c, d, e)\right)^2\Sigma\frac{1}{\zeta(a, b, c)(d-e)^4};$$

and the squared factor is always positive whatever the roots may be, for ζ is always real.

Hence the essential part of our rule thus transformed comes to this, that if

$$\Sigma\left(\zeta(a, b, c)\times(d-e)^4\right) \text{ and } \Sigma\left\{\left(\zeta(a, b, c)\right)^{-1}(d-e)^{-4}\right\}$$

are both of them positive, then when the discriminant is positive, so that the case of two of the five quantities a, b, c, d, e being conjugate and the other three real is excluded, and the choice lies between supposing all or only one of them real, we are able to affirm that they will all be real. Nothing could be easier than to multiply tests expressed by simple symmetric functions of differences of the roots, any infringement of which would contradict the hypothesis of all the five letters denoting real quantities; the difficulty consists in discovering a system of the least number that will suffice of decisive tests, such that not only their infringement shall contradict the hypothesis of imaginary roots, but whose fulfilment shall ensure the roots being all real. This is what has been proved to be effected by means of the invariants J, D, $\Lambda+\frac{4}{5}JD$.

In the case before us it is clear that when the roots are all real, each of the sums above written must be positive and greater than zero. That their being both positive and greater than zero is inconsistent with four of the letters a, b, c, d, e being imaginary would probably not admit of an easy direct demonstration.

Z we have seen is only a particular value of the general invariant $\Lambda+\mu JD$, which may be called M, where μ is an arbitrary constant limited to lie between 1 and -2.

(83) It may be well to notice the effect of using *as a criterion*, in conjunction with J and D, the value of M corresponding to either extreme value of μ. In such case, supposing M to become zero, it might for a moment appear doubtful to which region

that point representing the family of forms is to be referred. But since the doubt can only arise when J is negative and D positive, and since by hypothesis we have $\Lambda=-\mu JD$, we see that Λ takes the sign of μ; and consequently the sign of M, when it becomes zero, is to be understood as following the sign of μ, i. e. as positive when μ is 1 and negative when μ is -2.

(84) The method above given for ascertaining the nature of the roots of a quintic involves the use of only three criteria. It may be inquired how many would become needful in applying STURM's method. In the case of a cubic equation only the last of the two Sturmian criteria comes into use; and it seems therefore desirable to ascertain whether all four of the Sturmian criteria applicable to that case are required, or whether a smaller number are sufficient. I speak of four criteria, inasmuch as the leading terms fx and $f'x$ cannot be considered as such, their signs being fixed; so that we are at liberty to consider them both positive. Suppose all six Sturmian functions to be written down, including fx (a function of x of the fifth degree) and $f'x$, and let us characterize by the index (r, s) any succession of signs of the leading coefficients which contain r continuations and s variations, and which therefore will correspond to the case of $(r-s)$ roots.

The total number of cases to be considered are the sixteen following:

(5, 0)	+	+	+	+	+	+
(4, 1)	+	+	+	+	+	−
	+	+	+	+	−	−
	+	+	+	−	−	−
	+	+	−	−	−	−
(3, 2)	+	+	+	+	−	+
	+	+	+	−	+	+
	+	+	+	−	−	+
	+	+	−	+	+	+
	+	+	−	−	+	+
	+	+	−	−	−	+
(2, 3)	+	+	+	−	+	−
	+	+	−	+	+	−
	+	+	−	+	−	−
	+	+	−	−	+	−
(1, 4)	+	+	−	+	−	+

the successions corresponding to the indices (2, 3), (1, 4) will become impossible, as corresponding to a *negative* number of real roots. An inspection of the eleven cases corresponding to the indices (5, 0), (4, 1), (3, 2) will show that no *ternary* combination of signs in the third, fourth, and sixth columns belongs to any of the three characters (5, 0), (4, 1), (3, 2) exclusively, and consequently all four signs must be used; and therefore, if the method of STURM is employed, four criteria are indispensable for determining

effectually the character of the roots in an equation of the fifth degree[64]; whereas in the symmetrical and invariantive method which I have employed three have been seen to suffice.

In an equation of the seventh degree the case of 0 or 4 will be distinguishable from that of 2 or 6 imaginary roots by the sign of the discriminant, and then again the case of 0 from that of 4, and of 2 from that of 6, by other invariantive criterion-systems. So for an equation of the ninth degree, the first separation will be that of the 0, 4, or 8 case from that of 2 or 6; then it may be conjectured the 2 case will be invariantively separated from the 6, and the 0 or 8 from that of 4, and, finally, 0 and 8 from each other—the reduction of cases apparently depending upon the relation of the index of the equation to the powers of the number 2. This much we know (from art. 49) as matter of certainty, that no single criterion other than the discriminant can ever serve to distinguish one form of roots from another so that all other criteria must accompany each other in groups; and accordingly the scheme of criteria established in the foregoing investigation is in kind the very simplest *à priori* conceivable.

[64] [a] For an equation of the nth degree there are $n-1$ variable criteria, each capable of being + or −, and thus giving rise to 2^{n-1} conceivable diversities of combination. The actual number possible, however, is considerably less than this; and I find by an easy method that this number, when n is odd, is $2^{n-2}+\dfrac{\Pi(n-1)}{2\left(\Pi\frac{n-1}{2}\right)^2}$, and when n is even, is $2^{n-2}+\dfrac{\Pi(n-1)}{\Pi\frac{n}{2}\Pi\left(\frac{n}{2}-1\right)}$.

[b] Not quite foreign to this subject is the inquiry as to the comparative probability of each different succession or each different family of successions possessing equivalent characters; and, as connected therewith, the comparative probability of a certain specified number of the roots of an equation of a given degree being real and the remainder imaginary. In the simplest case of a quadratic equation of which the coefficients are real but otherwise arbitrary, I find that upon the particular hypothesis of the squares of the three coefficients being limited by one and the same quantity, the probability of the roots being imaginary is $\frac{31}{72}-\frac{\log 2}{12}$, or ·3727932, a little less than $\frac{3}{8}$, this being the value of the integral $\int_0^{\frac{1}{2}}db\int_{b^2}^{1}da\left(1-\frac{b^2}{a}\right)$; but we are not at liberty to infer from this the value of the probability in question when the coefficients are left absolutely unlimited. A case in point, as illustrating the effect of imposing a limit in questions of this kind, occurs in the problem (which I raised in my lectures on Partitions) of finding the probability that four points placed at hazard in a plane will form the angles of a reentrant quadrilateral, which Professor CAYLEY has shown is exactly $\frac{1}{4}$ in the absence of any limit. For if ABCD be the four points, and ABC the greatest of the four triangles of which they may be regarded as the angular points, and if through A, B, C be drawn lines parallel to BC, CA, AB respectively, the triangle $\alpha\beta\gamma$ so formed will be four times as great as ABC, and the point D must be somewhere within $\alpha\beta\gamma$, otherwise ABC would not be less than each of the three other triangles ABD, BCD, CAD; and consequently, since D must lie within ABC when the quadrilateral is reentrant, the probability in question is $\frac{ABC}{\alpha\beta\gamma}$, or $\frac{1}{4}$. Now it is easy to see, by using the very same construction, that if any contour whatever be imposed as a limit upon the positions of the four points, the probability referred to will exceed $\frac{1}{4}$ by a finite quantity—a result somewhat paradoxical, since *à priori* one would have supposed that the value of it for the case of *no limit* would be the *mean* of the values corresponding to the respective suppositions of every possible form of limit.

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Note on the arbitrary constant which appears in one of the criteria for distinguishing the case of four from that of no imaginary roots, and on the curve whose coordinates express the limiting relations of all the octodecimal invariants of a binary quintic, &c.

(85) The appearance of an arbitrary constant in a criterion is a circumstance so unexampled and remarkable that I have thought it desirable to give a more complete, or at least a more palpable proof of the validity of the substitution of $\Lambda+\mu\text{JD}$ for Λ than that furnished in the foregoing text, where some indistinctness arises from the difficulty of raising up in the mind a clear conception of the form of the amphigenous surface, and the two portions of space which it separates. That difficulty is entirely obviated, and the theory rendered palpable to the senses by the following investigation, where the problem is so handled as to involve the contemplation of two dimensions only of space. We have in general

$$\text{D}=\text{J}^2-128\text{K},\quad \Lambda=2048\text{L}-\text{J}^3,$$

and at the amphigenous surface (see art. 57)

$$\frac{\text{K}}{\text{J}^2}=\frac{\theta+6}{(\theta+4)\theta^2},\quad \frac{\text{L}}{\text{J}^3}=\frac{1}{(\theta+4)\theta^3}.$$

Let

$$\theta=4\phi,\quad y=\frac{\text{D}}{\text{J}^2},\quad x=\frac{\Lambda}{\text{J}^3}.$$

Then

$$y=1-128\frac{\theta+6}{(\theta+4)\theta^2}=1-\frac{8\phi+12}{\phi^2(\phi+1)}=\frac{(\phi+2)^2(\phi-3)}{\phi^2(\phi+1)},$$

$$x=-1+\frac{2048}{(\theta+4)\theta^3}=-1+\frac{8}{\phi^4+\phi^3}=\frac{-(\phi+2)(\phi^3-\phi^2+2\phi-4)}{\phi^3(\phi+1)};$$

and consequently

$$\delta y=\frac{4(\phi+2)(4\phi+3)}{\phi^3(\phi+1)^2}\delta\phi,\quad \delta x=-\frac{8(4\phi+3)}{\phi^4(\phi+1)^2}\delta\phi,\quad \frac{\delta y}{\delta x}=-\frac{\phi^2+2\phi}{2}:$$

x, y may be considered as the coordinates (inclined to each other at any angle) of a curve of the fourth order, whose form, so far as is essential to the object in view, I proceed to determine. It is obvious, furthermore, that this curve will be a section of the amphigenous surface made by the plane $\text{J}=1$.

(86) This curve will be seen to consist of four branches, coming together in pairs or two cusps, so as to form two distinct horns([65]). For when $\phi=\infty$, or $\phi=-\frac{3}{4}$, δy, δx will

([65]) ([a]) Since $$\phi^4+\phi^3-\frac{\text{J}^3}{256\text{L}}=0,$$
we see at once, from DESCARTES's rule, that ϕ can never have more than two real values to one of $\frac{\text{L}}{\text{J}^3}$, or consequently of x, and consequently there can only be two values of y to each of x.

([b]) When $\text{J}=0$, the cusp of the left-hand horn and the two points of intersection of the dexter horn with the axis of L coincide at the origin; the upper branch of the latter and the linear of the former become the lower and upper parts of the axis of D, whilst the lower and upper branches of the same respectively become the left and right-hand branches of the semicubical parabola $27.2^{22}\text{L}^2=-\text{D}^5$.

each of them be zero. Hence there is a cusp at the point where $x=-1$, $y=1$([66]), and again at the point where

$$x=-1+\frac{8\times 256}{81-108}=-76\tfrac{23}{27},\quad y=\frac{(\frac{5}{4})^2(-\frac{9}{4})}{(\frac{3}{4})^2\frac{1}{4}}=-25.$$

(87) When $\varphi=0$, and also when $\varphi=-1$, x and y each become infinite; when $\varphi=\pm\infty$, x and y each become unity.

As φ passes from $+\infty$ to 0, δy is always negative, and x always positive; so that there will be one branch of the curve (CMP in Plate XXV.) extending from $x=-1$ to $x=+\infty$, for which y commences at $y=1$, which cuts the axis of x when $\varphi=3$, i. e. $x=-\frac{25}{27}$([67]), and which, for the remaining part of its course, lies completely under the axis of x, becoming infinite when x becomes indefinitely great.

Again, as φ passes from $-\infty$ to -1, δx remains always positive, but δy is negative so long as $\varphi<-2$ vanishes when $\varphi=2$, and ever afterwards continues positive. Thus there is a second branch, COQ, which starts from the cusp C, touches the axis of x at the origin, ever afterwards remaining positive, and increasing up to positive infinity.

Since when $\varphi=\infty$, $\frac{\delta y}{\delta x}=\infty$, the tangent at C is parallel to the axis of y, and consequently the two branches which start from C lie on the same side of the tangent, so that the cusp at this point is of the second or ramphoidal kind; in Professor CAYLEY's nomenclature a cusp-node, and equivalent to the union of a double point and a cusp of the first kind.

There remains to account for the values of φ in the interval between 0 and -1. Throughout this interval y and x remain both of them negative, and $\frac{\delta y}{\delta x}=-\frac{\varphi(\varphi+2)}{2}$([68, 69]) is always positive.

There will thus be two branches, in each of which x and y increase simultaneously in the negative direction, coming to a cusp necessarily of the first kind at the point $x=-76\frac{23}{27}$, $y=-25$, one branch corresponding to the values of φ from $-\frac{3}{4}$ to 0, the other to the values of φ from $-\frac{3}{4}$ to -1, both of them lying completely under the axis of x, and becoming respectively infinite at the extreme values of φ (0 and -1).

([66]) Where this branch cuts the axis of y we have $\varphi^3-\varphi^2+2\varphi-4=0$, of which the real root will be a trifle less than $\frac{3}{2}$.

([67]) From this it is easily seen that, whatever may be supposed to be the inclination of the axes x, y, the curve in question is rectifiable by means of elliptic functions; for $\frac{ds}{d\varphi}$ will be expressible as a rational function of φ and the square root of a quartic function of φ. The same conclusion will hold for the curve obtained by making J constant when J, together with any invariant of the eighth and any of the twelfth order, are taken as the coordinates of the amphigenous surface.

([68]) To ascertain which range of φ gives the superior and which the inferior outline of the sinister horn, let $\varphi=\varepsilon$, an infinitesimal; then $\varphi^4+\varphi^3=\varepsilon^3$, and the other value of φ is $-1-\eta$, where $\eta=\varepsilon^3$. Hence the two values of y corresponding to φ nearly zero and φ nearly -1 respectively will be

$$y_1=-\frac{12\varepsilon}{\varepsilon^3}=-\frac{12}{\varepsilon^2}\quad\text{and}\quad y_2=\frac{-4(-1-\eta)}{\varepsilon^3}=\frac{4}{\varepsilon^3}.$$

Thus y_1 is negative for ε positive or negative, but y_2 is positive in the one case and negative in the other, as

Again,

$$2y-x+5=\frac{\phi+2}{\phi^4+\phi^3}\left((2\phi^3-2\phi^2+2\phi)+(\phi^3-\phi^2+2\phi-4)\right)+5$$

$$=\frac{\phi+2}{\phi^3}(3\phi^2-6\phi-4)+5=\frac{8\phi^3-16\phi-8}{\phi^3}.$$

Hence when $\phi=-1$, for which value of ϕ x and y both become infinite, $2y-x+5=0$; hence the straight line $2y-x+5=0$, represented by AN in the diagram, will be an asymptote to the curve (70).

If now we draw the straight line $2y-x=0$, represented by OB in the figure and join OC, the curvilinear triangle OCM will be completely under OC, and the curvilinear infinite sector XOP completely under OB.

(88) What we have to prove is, that so long as μ lies between 2 and 1, so long may $\Lambda+\mu JD$ be substituted as a criterion in lieu of Λ, it being remembered that Λ only plays the part of a criterion when D is positive and J is not positive. Hence, since when $J=0$ $\Lambda+\mu JD$ and Λ coincide, we have only to show that, so long as D is positive and J is negative, $\Lambda+\mu JD$ and Λ will bear the same sign for all such values of J, D, L as constitute a facultative system, *i. e.* coordinates to a facultative point in space.

Now at any facultative point G (the function of the amphigenous surface), or say rather $G(J, K, L)>0$, or $\frac{1}{J^3}G\left(1, \frac{D}{J^2}, \frac{L}{J^3}\right)>0$, and consequently considering $\frac{D}{J^2}$, $\frac{L}{J^2}$ as the coordinates of a plane curve, the line $G\left(1, \frac{D}{J^2}, \frac{L}{J^3}\right)=0$ (the sign of J being fixed) will separate those points for which J, K, L constitute a facultative system from those

already seen for the dexter horn. We see also that y_2 becomes indefinitely greater than y_1, so that it is the value of ϕ near to -1 which gives the inferior branch; and consequently the superior branch of the sinister horn belongs to the range from $-\frac{3}{4}$ to 0, and the inferior to the range from $-\frac{3}{4}$ to -1.

(69) It may further be noticed that each horn so called is a true horn, being destitute of any point of contrary flexure, except at infinity; for otherwise we should have

$$\frac{d^2y}{dx^2}=\frac{d\phi}{dx}\cdot\frac{d\frac{dy}{dx}}{d\phi}=-\frac{d\phi}{dx}(\phi+1)=\frac{(\phi+1)^3\phi^4}{8(4\phi+3)}=0,$$

which implies $\phi=0$ or $\phi=-1$, for each of which values of ϕ x and y become infinite. It will be seen hereafter that it is only for the value corresponding to $\phi=0$ that there does exist at infinity a point of inflexion.

(70) The two points where the asymptote cuts the curve will be found by writing

$$\frac{\phi^3-2\phi-1}{\phi+1}=\phi^2-\phi-1=0,$$

which gives

The superior sign corresponds to a point x, y in the inferior branch of the dexter horn, and the lower sign, for which $\phi>-\frac{3}{4}$, to the superior branch of the sinister horn. It is easy to see that there can be no other asymptote; for x, y only become infinite when $\phi=-1$, or $\phi=0$; so that if $\lambda x+\mu y+\nu$ is an asymptote, it must contain $(\phi+1)^2$, or ϕ^2 as a factor. The first condition is only satisfied when $\lambda:\mu:\nu::-1:2:5$; and the latter cannot be satisfied at all.

in which J, K, L constitute a non-facultative one. But the curve above traced is obviously a homographic derivative of that line $\left(\text{for G is the resultant of } \frac{K}{J^2}=\frac{\theta+6}{(\theta+4)\theta^2},\ \frac{L}{J^3}=\frac{I}{(\theta+4)\theta^3}\right)$.

Hence this latter curve will also separate systems of values of J, D; Λ corresponding to facultative from those corresponding to non-facultative points. Moreover when J is negative and D positive, it has been shown (see dial figure) that the values of D (in facultative systems) corresponding to finite values of J are *limited* in magnitude; hence, upon the same suppositions, facultative systems of J, D, Λ will correspond to the interior and contour of the curve we have been considering.

(89) Accordingly, since D is supposed positive, our sole concern will be with the curvilinear triangle CMO and the infinite sector QOX, and we have to show that for all points not exterior to those areas Λ and $\Lambda+\mu JD$ have the same sign; that is to say, $1+\mu\frac{JD}{\Lambda}$, or $1+\mu\frac{y}{x}$ is *positive.*

When y and x have opposite signs (as is the case in the triangle CMO), all negative values of μ, and when y and x have the same signs (as is the case in the sector XOQ), all positive values of μ obviously make $1+\mu\frac{y}{x}$ positive. But furthermore $\frac{y}{x}$, which is -1 for the line OC, is greater than -1 for all points in the triangle just named; and again, $\frac{y}{x}$, which is $\frac{1}{2}$ for OB (the parallel to the asymptote through O), will be less than $\frac{1}{2}$ for all points in the sector QOX. Thus, then, as regards points either in the triangle or in the sector, $\frac{y}{x}$ is always intermediate between -1 and $\frac{1}{2}$; so that when μ lies between 1 and -2, $1+\mu\frac{y}{x}$ will be always positive, and Λ and $\Lambda+\mu JD$ will bear the same sign O, so that $\Lambda+\mu JD$ may be used to replace Λ as a criterion. Q.E.D.

(90). It is apparent from the nature of the preceding demonstration that Λ may be replaced by an invariant containing not one merely, but an infinite number of arbitrary constants (limited), provided we are indifferent to the degree which the substitute for Λ may assume. To this end we have only to draw any algebraical curve $F(x, y)=0$ passing through the origin, and with its parameter subject to such conditions of inequality as will ensure the mixtilinear triangle and sector COM, XOQ lying on opposite sides of the curve. If its degree be n, the number of parameters in F left arbitrary within limits will be $\frac{n^2+3n-2}{2}$, and $\epsilon F(\Lambda, JD)$, where ϵ means one of the two quantities $+1$ or -1, may be used as a criterion in lieu of Λ. For instance, a common parabola with its axis coincident with that of x and passing through O will obviously serve as a screen between these figures; its equation will be $y^2-x=0$, and the invariant $D^2-J\Lambda$, which is of the sixteenth degree in the coefficients, will serve together with J and D to fix the nature of the roots; so in general we may obtain invariants of any degree of the form $4i$ from twelve

upwards. Thus M. HERMITE, by a method not introducing the notion of continuity, has found one of the twenty-fourth degree, which he has been so obliging as to communicate to me, viz. $(D_1-5\Delta J_2)^2+(9D-25\Delta^2)J_3^2$, where $D_1=16J_3+25\Delta J_2$; and D is his discriminant, which I cannot safely attempt to express in terms of x, y for want of a certain knowledge of the arithmetical relations between his Δ, J_2, J_3 D, and my own J, K, L; but were this transformation effected, the curve so represented must, *ex necessitate*, pass clear between the triangle and sector above referred to, or else the invariant in question could not be a criterion. I have ascertained without difficulty that it passes through the origin and represents one of the principal species of NEWTON's diverging parabolas.

(91) The curve which we have been discussing will, on reference to PLÜCKER's 'Algebraischen Curven,' p. 193, be seen to belong to his sixteenth species of curves of the fourth order having two double points; but as in reality one of these is tantamount to the union of two, it may be considered as having three, the maximum possible number of such points, and consequently comes under the operation of CLEBSCH's rule, given in the last Number of CRELLE's Journal, and accordingly its coordinates have been seen to be rational functions of a single variable. The equation connecting x, y may of course be obtained by means of a simple and obvious substitution operated upon the G of art. 41, or it may be found directly by writing

$$\frac{x+1}{8}=\xi=\frac{1}{\varphi^4+\varphi^3}\quad\frac{y-1}{4}=\eta=-\frac{2\varphi+3}{\varphi^3+\varphi^2},$$

whence we obtain

$$\varphi^4+\varphi^3-\frac{1}{\xi}=0, \quad \ldots \quad (1)$$

$$2\varphi^2+3\varphi+\frac{\eta}{\xi}=0. \quad \ldots \quad (2)$$

Calling $\varphi_1\varphi_2$ the two roots of equation (2), making

$$\left(\varphi_1^4+\varphi_1^3-\frac{1}{\xi}\right)\left(\varphi_2^4+\varphi_2^3-\frac{1}{\xi}\right)=0,$$

and substituting the values of the symmetric functions of φ_1, φ_2 found from the same equation, we obtain without difficulty

$$\eta^4-\xi\eta^3-8\xi\eta^2+36\xi^2\eta+16\xi^2-27\xi^3=0$$

for the equation in question. The curve thus denoted I propose to call the Bicorn. Its figure is given in Plate XXV., in which ξ, η are taken at right angles, but they may of course be supposed to be inclined at any angle whatever. If we now assume at pleasure any two new axes U, V in the place of the Bicorn, the coordinates u, v will be always respectively proportional to two invariants of the twelfth order of the given quintic, whose particular forms will depend upon the positions of the two new axes so taken. If one of these axes, say that of u, be made coincident with the axis of ξ, v will be proportional to JD, and u to some other invariant of the twelfth degree. When

this is the case, then in general v, as u travels from one end of infinity to the other, will sometimes have four, and sometimes two, or else sometimes two and sometimes no real values, as will be obvious by inspection of the figure. There is, however, one direction of the axis of v which will cause v in all cases to have two, and only two real values. This direction is that of the line joining the two cusps. At the node-cusp, for which $\phi=\infty$, $\xi=0$, $\eta=0$; at the other cusp, for which $\phi=-\frac{3}{4}$, $\xi=-\frac{256}{27}$, $\eta=-\frac{32}{3}$. Hence the equation of the joining line is $9\xi-8\eta=0$. Now $\frac{\mathrm{K}}{\mathrm{J}^2}=-\frac{\eta}{32}$, $\frac{\mathrm{L}}{\mathrm{J}^3}=\frac{\xi}{256}$. Hence along this line $9\mathrm{L}+\mathrm{JK}=0$; and consequently, if the axis of v be taken parallel to this line and passing through the origin, whilst u is proportional to $9\mathrm{L}+\mathrm{JK}$, v will be proportional to JD; and thus we see that for every value of $9\mathrm{L}+\mathrm{JK}$, which is HERMITE's J_3 (see foot-note (34)(e)), D at the amphigenous surface (*i. e.* when $\mathrm{G}=0$, and therefore when HERMITE's $\mathrm{I}=0$) will always have two, and only two real values. This perfectly agrees with M. HERMITE's conclusion (71), and in an unexpected manner confirms the correctness of the concordance established, in the foot-note cited, between his J_3 and my J, K, L. Had M. HERMITE employed any duodecimal invariant whatever other than J_3, a mere inspection of the Bicorn shows that a similar conclusion could not have obtained.

(92) The intersections of the curve whose equation is written in the preceding article with infinity evidently lie in the lines $\eta^3=0$, $\eta-\xi=0$. This latter is the equation to a line parallel to the asymptote which touches the highest and lowest of the four branches of the curve, and corresponds to the value -1 of ϕ. Thus we see that there is a point of inflexion corresponding to the point at infinity at which the second and third branches of the Bicorn may be conceived to unite. It is easy to show that the Bicorn has no double tangent; for we have seen that

$$\frac{dy}{dx}=-\frac{\phi^2+2\phi}{2},$$

and consequently the values of ϕ corresponding to the two supposed points of contact may be regarded as the two roots ϕ_1, ϕ_2 of the equation $\phi^2+2\phi+2\lambda=0$, and we shall have

$$-\frac{2\phi_1+3}{\phi_1^3+\phi_1^2}+\frac{2\phi_2+3}{\phi_2^3+\phi_2^2}=\lambda\left(\frac{2}{\phi_1^4+\phi_1^3}-\frac{2}{\phi_2^4+\phi_2^3}\right),$$

$$i.\ e.-(2\phi_1+3)(\phi_2^3+\phi_2^2)+(2\phi_2+3)(\phi_1^3+\phi_1^2)=(\phi_2^4+\phi_2^3)-(\phi_1^4+\phi_1)^3,$$

or

$$4\lambda\,.\,(-2)+4\lambda+3(4-2\lambda)+6\big(-2(4-4\lambda)+(4-2\lambda)\big)=0,$$

or

$$(-8+4-6+8-2)\lambda+12-6-8+4=0,$$

$$i.\ e.\ -4\lambda+2=0,\ \lambda=\tfrac{1}{2},\ \phi^2+2\phi+1=0,$$

and the two values of ϕ coincide, contrary to hypothesis.

It is also easy to find its class; for when $\frac{d\eta}{d\xi}$ corresponds to any point in which the

(71) Lemma 3, p. 202, Cambridge and Dublin Journal, vol. ix.

curve is met by a tangent drawn from the point whose ξ, η coordinates are a, b, we have

$$\left(\frac{2\varphi+3}{\varphi^3+\varphi^2}+b\right)+\frac{d\eta}{d\xi}\left(\frac{1}{\varphi^4+\varphi^3}-a\right)=0;$$

but

$$\frac{d\eta}{d\xi}=2\frac{dy}{dx}=-(\varphi^2+2\varphi);$$

hence

$$\frac{(2\varphi+3)-(\varphi+2)}{\varphi^3+\varphi^2}+(\varphi^2+2\varphi)a+b=0;$$

hence

$$a\varphi^4+2a\varphi^3+b\varphi^2+1=0;$$

and φ having four values, four tangents (real or imaginary) can be drawn to the Bicorn from every point in its plane. It is thus of the fourth order, fourth class, possesses a common cusp and a cusp-node, no double tangent, and one point of inflexion at infinity. These results accord with those given by PLÜCKER (Algebraischen Curven, p. 222).

(93) The canonical form of the equation to the Bicorn is $(pr+q^2)^2+pq^3=0$, as seen in PLÜCKER, p. 193, where $p=0$, $r=0$, $q=0$ will obviously be the equations to the tangent at the node-cusp, to the tangent at the common cusp, and to the line of junction of the two cusps. This leads to a remarkable transformation of the invariant G of art. (41). Thus we may write $p=\xi$, $q=\mu(9\xi-8\eta)$; and to find r, we must draw the tangent to the lower cusp, for which $\varphi=-\frac{3}{4}$, which gives

$$\xi=-\frac{256}{27},\quad \eta=-\frac{32}{3},\quad \frac{d\eta}{d\xi}=-\frac{15}{16}\text{(72)};$$

consequently we may write $r=\lambda(144\eta-135\xi+256)$, and then proceed to satisfy, by assigning suitable values to λ, μ, ν, the identity

$$\left(\lambda(144\eta\xi-135\xi^2+256\xi)+\mu^2(8\eta-9\xi)^2\right)^2+\mu^3\xi(8\eta-9\xi)^3$$
$$=\nu(\eta^4-\eta^3\xi-8\eta^2\xi+36\eta\xi^2+16\xi^2-27\xi^3)=\nu\,.\,2^{20}\text{G}.$$

On performing the necessary calculations it will be found that

$$\lambda=-\frac{1}{2^{12}},\quad \mu=\frac{1}{2^6},\quad \nu=\frac{1}{2^{12}}.$$

Hence we see that J^3G may be expressed under the form $(\text{LL}_1+c\text{J}_3^2)^2+e\text{LJ}_3^3$, where L_1 is a new duodecimal invariant, and c, e are two known numbers; in fact

$$\text{J}^3\text{G}=\left(\text{L}(18\text{JK}+135\text{L}^2-\text{J}^3\text{L})+(\text{JK}+9\text{L})^2\right)^2+64\text{L}(\text{JK}+9\text{L})^3.$$

I am indebted to my friend Dr. HIRST for these references to the immortal work of PLÜCKER.

(94) The existence has been demonstrated of a linear asymptote which is a tangent

(72) I find, by a calculation which offers no difficulty, that the value of φ at the point where this tangent cuts the curve will be given by the equation

$$-256\varphi^4-256\varphi^3+288\varphi^2+432\varphi+135=0;$$

and taking away the factor $(4\varphi+3)^3$ which belongs to the cusp, there remains $\varphi=\frac{5}{4}$, which corresponds to a point in the lower branch of the superior horn.

at infinity to the first and fourth branch. A cubic asymptote touches the intermediate branches in the point at infinity corresponding to $\varphi=0$. For we have

$$\xi=\frac{1}{\varphi^3(1+\varphi)}=\frac{1}{\varphi^3}(1-\varphi+\varphi^2-\varphi^3\ldots.);$$

and writing v for $-\eta$,

$$v=\frac{3+2\varphi}{\varphi^2(1+\varphi)}=\frac{1}{\varphi^2}(3-\varphi+\varphi^2-\varphi^3\ldots.),$$

$$v^{\frac{1}{2}}=\frac{3^{\frac{1}{2}}}{\varphi^3}\left(\varphi^2-\frac{1}{6}\varphi^3+\ldots.\right),\quad v^{\frac{3}{2}}=\frac{3^{\frac{1}{2}}}{\varphi^3}\left(3-\frac{3}{2}\varphi+\frac{13}{8}\varphi^2-\frac{27}{16}\varphi^3\ldots.\right).$$

Hence we may determine

A, B, C, D so that $Av^{\frac{3}{2}}+Bv+Cv^{\frac{1}{2}}+D-\xi$ shall $=\lambda\omega^n+\mu\omega^x+\ldots.$,

and I find

$$A=\frac{1}{3^{\frac{1}{2}}},\qquad B=-\frac{1}{6},\qquad C=\frac{7}{72},\qquad D=-\frac{2}{9}.$$

Thus the cubic asymptote will have for its equation

$$\left(\xi+\frac{1}{6}v+\frac{2}{9}\right)^2=3v\left(\frac{v}{9}+\frac{7}{72}\right)^2,$$

which is a divergent cubic parabola with a conjugate point, viz. the point for which

$$v=-\frac{7}{8},\qquad \xi+\frac{1}{6}v+\frac{2}{9}=0,\qquad \text{or } \eta=\frac{7}{8},\ \ \xi=-\frac{9}{128}.$$

(95) It is obvious from the preceding article, that we may expand ξ in terms of v by the descending series

$$\xi=Av^{\frac{3}{2}}+Bv+Cv^{\frac{1}{2}}+D+\frac{E}{v^{\frac{1}{2}}}+\ldots.$$

But we may also obtain an ascending series for ξ in terms of v which will exhibit the nature of the curve of the cusp-node at which point $\varphi=\infty$. Let $\varphi=\frac{1}{\omega}$, then

$$\xi=\frac{1}{\varphi^3(\varphi+1)}=\frac{\omega^4}{1+\omega}=\omega^4(1-\omega+\omega^2-\omega^3\ldots.),$$

$$v=\frac{2\varphi+3}{\varphi^2(\varphi+1)}=\omega^2\left(\frac{2+3\omega}{1+\omega}\right)=\omega^2(2+\omega-\omega^2+\omega^3\ldots).$$

Hence

$$v^2=\omega^4(4+4\omega-3\omega^2+2\omega^3\ldots),$$

$$v^{\frac{5}{2}}=\omega^4(4\sqrt{2}\,\omega+5\sqrt{2}\,\omega^2-\frac{25}{8}\sqrt{2}\,\omega^3\ldots),$$

$$v^3=\omega^4(8\omega^2+12\omega^3\ldots),$$

$$v^{\frac{7}{2}}=\omega^4(\sqrt{2}\,\omega^3\ldots),$$

&c.=&c.

from which we may easily deduce

$$\xi=2\left(\frac{v}{2}\right)^{2}-\left(\frac{v}{2}\right)^{\frac{5}{2}}+\frac{7}{4}\left(\frac{v}{2}\right)^{3}-\frac{109}{32}\left(\frac{v}{2}\right)^{\frac{7}{2}},\ \&c.,$$

in which it will be observed that the indices of the powers of v are precisely complementary to those in the preceding expansion, the two series of indices together comprising all multiples of $\frac{1}{2}$ from positive to negative infinity.

(96) We now see how, supposing the curve to be given with ξ and η at any angle, the axes corresponding to $\frac{K}{J^2}$, $\frac{L}{J^3}$ may be defined: viz., the origin of coordinates will be at the cusp-node; η, along which $\frac{K}{J^2}$ is reckoned, will be in the direction of the tangent at that point; and ξ, along which $\frac{L}{J^3}$ is reckoned, will be the axis of that common parabola which at the same point has the closest contact with the given curve.

It seems desirable, with a view to a more complete comprehension of the form of the amphigenous surface, i. e. the *limiting surface* of invariantive parameters, to ascertain the nature of the systems of plane sections of it, parallel to each of the three coordinate planes. The sections parallel to J, which are curves of the fourth order, have been already satisfactorily elucidated. It remains to consider briefly the sections parallel to J and D, which will be curves of the ninth order.

(97) When L is constant, writing $J=z$, $D=y$, where for facility of reference we may conceive y horizontal and z vertical, and making $L=\frac{k^3}{256}$, we have

$$z^3=k^3\phi^3(\phi+1),\quad y=z^2\frac{(\phi+2)^2(\phi-3)}{\phi^2(1+\phi)}=k^2\frac{(\phi-3)(\phi+2)^2}{(1+\phi)^{\frac{1}{3}}},$$

$$\frac{\delta y}{y}=\frac{2}{3}\frac{(\phi-1)(4\phi+3)}{(\phi+2)(\phi-3)(\phi+1)}\delta\phi,\quad \frac{\delta z}{z}=\frac{1}{3}\frac{4\phi+3}{\phi(\phi+1)}\delta\phi,\quad \frac{\delta z}{\delta y}=\frac{1}{2k}\frac{(\phi+1)^{\frac{2}{3}}}{(\phi-1)(\phi+2)}\delta\phi,$$

when	$\phi=-1$,	$z=0$,	$y=\infty$,
,,	$\phi=-\frac{3}{4}$,	$\delta y=0$,	$\delta z=0$,
,,	$\phi=0$,	$z=0$,	$y=-12k^2$,
,,	$\phi=1$,	$\frac{\delta y}{\delta z}=0$,	
,,	$\phi=+\infty$,	$z=+\infty$,	$y=+\infty$,
,,	$\phi=-2$,	$y=0$,	$\frac{\delta z}{\delta y}=\infty$
,,	$\phi=-\infty$,	$z=+\infty$,	$y=+\infty$.

Hence it appears that the curve consists of three branches, two coming together at an ordinary cusp at the point corresponding to $\phi=-\frac{3}{4}$, and the third completely separate. The nature of the sign of k does not affect the nature of the curve. If, for greater clearness, k be supposed positive, the first branch, having the negative part of

the axis of y for its asymptote, lies entirely in the $-y$, $-z$ quadrant, and is always convex to the axis of y; the second branch, joining the first at a cusp corresponding to $\varphi=-\frac{8}{4}$, is concave to the origin, cuts the axis of y negatively and of z positively, and goes off to infinity; the third branch, having the positive part of the axis of y for its asymptote, lies in the $+y$, $+z$ quadrant, is always convex to the axis of z, which it touches at a point below that where it is cut by the second branch, and also goes off to infinity, lying entirely under the second branch. A straight line, according to the direction in which it is drawn, may cut the curve in one, three, or five real points.

(98) When D is constant, writing $J=z$, $L=x$, we have

$$z^2=D\frac{\varphi^2(\varphi+1)}{(\varphi+2)^2(\varphi-3)},\quad x=\frac{Dz}{(\varphi-3)\varphi(\varphi+2)^2}.$$

The form of the curve changes with the sign of D. For sections parallel to and above the plane of D, we may make

$$D=c^2,\quad \tau^2=\frac{\varphi+1}{\varphi-3},\quad\text{or}\quad \varphi=\frac{3\tau^2+1}{\tau^2-1};$$

then the complete equation-system to the curve will be

$$z=c\tau\frac{3\tau^2+1}{5\tau^2-1},\quad x=c^3\tau\frac{(\tau^2-1)^4}{4(5\tau^2-1)^3},$$

it being unnecessary to affect c with a double sign, since z and x change their signs with that of τ.

Also

$$\frac{\delta x}{x}=\frac{(\tau^2+1)(15\tau^2+1)\delta\tau}{\tau(\tau^2-1)(5\tau^2-1)},\qquad \frac{\delta z}{z}=\frac{(\tau^2-1)(15\tau^2+1)\delta\tau}{\tau(3\tau^2+1)(5\tau^2-1)},$$

$$\delta x=\frac{c^3}{4}\frac{(\tau^2+1)(15\tau^2+1)(\tau^2-1)^3}{(5\tau^2-1)^4}\delta\tau,\quad \delta z=c\frac{(15\tau^2+1)(\tau^2-1)}{(5\tau^2-1)^2}\delta\tau,$$

$$\frac{dx}{\delta z}=\frac{c^2}{4}\frac{(\tau^2+1)(\tau^2-1)^2}{(5\tau^2-1)^2}.$$

To the values of τ included between $+\sqrt{\frac{1}{5}}$ and $-\sqrt{\frac{1}{5}}$ will correspond one branch of the curve passing through the origin, where it has a point of contrary flexure, and extending to infinity in both directions.

When $(5\tau^2-1)$ is positive $\frac{x}{\tau}$ is always positive; and when $\tau^2=1$,

$$\delta x=0,\quad \delta z=0,\quad \frac{\delta x}{\delta z}=0.$$

Hence there will be a cusp of the second kind when $x=0$, $z=\pm c$, the axis of z being a tangent to the curve at each cusp. One pair of branches has its cusp at the point $x=0$, $z=c$, and the values of x, z increase indefinitely in the respective branches as τ passes from 1 to $+\infty$ and from 1 to $\sqrt{\frac{1}{5}}$. This pair lies in the $+x$, $+z$ quadrant, and there will be a precisely similar and similarly situated pair in the $-x$, $-z$ quadrant. Thus there will be in all one infinite $\int$-formed branch passing through the origin, and

two detached pairs of infinite branches lying in opposite quadrants(73). The value $\frac{1}{5}$ for τ^2, it will of course be seen, corresponds to -2 for φ, and gives, as it ought to do, the position of the cusp.

(99) Finally, for sections parallel to the plane of the discriminant and lying below it, making $D=-k^2$, $t^2=\frac{1+\varphi}{3-\varphi}$, we obtain in like manner

$$z=kt\frac{3t^2-1}{5t^2+1},\quad x=k^3t\frac{(t^2+1)^4}{4(5t^2+1)^3},\quad \frac{\delta x}{x}=\frac{(t^2-1)(15t^2-1)}{t(t^2+1)(5t^2+1)}\delta t,\quad \frac{\delta z}{z}=\frac{(t^2+1)(15t^2-1)}{t(3t^2-1)(5t^2+1)},$$

$$\delta x=\frac{k^3}{4}\frac{(t^2-1)(15t^2-1)(t^2+1)^3}{(5t^2+1)^4},\quad \delta z=k\frac{(15t^2-1)(t^2+1)}{(5t^2+1)^2},\quad \frac{\delta x}{\delta z}=\frac{k^2}{4}\frac{(t^2-1)(t^2+1)^2}{(5t^2+1)^2}.$$

When $t^2=\frac{1}{15}$ there will be an ordinary cusp, and when $t^2=1$, $\frac{\delta x}{\delta z}=0$.

There will therefore be three branches,—one corresponding to the values of t between $-\sqrt{\frac{1}{15}}$ and $+\sqrt{\frac{1}{15}}$, the other two to values of t between these limits and $-$ and $+$ infinity respectively. The middle branch passes through the origin, where it undergoes an inflexion, and comes to a cusp at a finite distance from the origin in two opposite quadrants. The connected branch at each cusp crosses the axis of x, sweeps convexly towards the axis of z, arrives at a minimum distance from it, and then goes off to infinity.

The value $\frac{1}{15}$ for t^2 corresponds to $-\frac{8}{4}$ for φ, and gives, as it ought to do, the cusp-node. In fact the values $\varphi=-\frac{3}{4}$, $\varphi=-2$ correspond respectively to a cuspidal and to a cusp-nodal line in the limiting surface whose sections we have been considering.

When the cutting plane is that of D itself, the section becomes a double cubic parabola and a single cubical parabola crossing each other at the origin.

(73) Let ϵ be an infinitesimal, and $\theta^2=1+\epsilon$; then

$$\delta z=\frac{4}{c^2}\frac{(4+5\epsilon)^2}{(2+\epsilon)\epsilon^2}\delta x=\frac{32}{c^2}(1+2\epsilon)\frac{\delta x}{\epsilon^2}.$$

Hence at either cusp the branch the further removed from the axis of x corresponds to the values of θ^2 between 1 and ∞, and the inferior branch to its values between 1 and $\frac{1}{5}$; so that the order of continuity of the five branches of the curve may be read as follows:—from the infinite point in the higher branch of the upper pair to its cusp; thence to the infinite point in the connected branch, which is contiguous to the infinite point in the opposite extremity of the middle branch; thence along this branch to its contrary infinite extremity; thence to the infinite point in the upper branch of the inferior pair; along that branch to its cusp; and thence, finally, along the lower branch to infinity.

Description of the Plates.

PLATE XXIV.

The (ϵ, η) equation is $(1, \epsilon, \epsilon^2, \eta^2, \eta, 1 \between x, y)^5=0$, of which two roots are always imaginary; its extreme criteria are 0, 0; its middle criteria $\epsilon^4-\epsilon\eta^2$, $\eta^4-\eta\epsilon^2$,

$$p=\epsilon\eta-1, \quad \sigma=(\epsilon^3-\eta^3)(\epsilon^2-\eta^2).$$

Points (p, σ) above the discriminatrix indicate 2 pairs of associated roots in the (ϵ, η) equation.

Points (p, σ) on the discriminatrix indicate 2 equal roots in the (ϵ, η) equation.

Points (p, σ) under the discriminatrix indicate 3 solitary roots in the (ϵ, η) equation.

Points (p, σ) above the equatrix indicate ϵ, η real and unequal.

Points (p, σ) on the equatrix indicate ϵ, η equal.

Points (p, σ) under the equatrix indicate ϵ, η imaginary and conjugate.

Points (p, σ) above the loop of the indicatrix indicate middle criteria not *both* positive.

Points (p, σ) on the loop of the indicatrix indicate middle criteria of opposite signs.

Points (p, σ) under the loop of the indicatrix indicate middle criteria not *both* negative.

The discriminatrix is a closed curve, the *whole* of which is figured on the Plate, and is shaped somewhat like a harp: it has a cusp of the fourth order at the origin.

The equatrix consists of two branches coming together at a cusp at the distance 1 from the origin; the upper branch touches the horizontal axis at the origin; the lower branch, after touching the discriminant at a single point, sweeps out from and round it, cutting the vertical axis at the distance 4 below the origin. Both branches go off to infinity to the right, and lie completely under the horizontal axis. Where the lower branch touches the discriminatrix, the discriminant of the (ϵ, η) equation passes through zero without changing its sign.

The indicatrix consists of a single branch extending indefinitely in both directions. It passes from infinity below and to the left until, at the distance 1 from the origin, it touches the axis, which at the origin it crosses at an angle of 45°, after which it goes off to infinity in the positive direction. Its *loop* extends from $p=0$ to $p=-1$. The two portions of it figured in the Plate join on together, coming to a maximum at a great distance below the horizontal axis. The narrow tract marked "Region of Real parameters" is that portion of the harp-shaped space for which alone, ϵ, η being *real*, the (ϵ, η) equation can have more than one real root. The areas of each of the three regions into which the discriminatrix is divided by the equatrix and indicatrix may readily be expressed numerically in terms of algebraic and inverse circular functions only.

I am indebted to Gentleman Cadet S. L. Jacob, of the Royal Military Academy, for the tracing of the curves of which the above Plate is a somewhat imperfect reproduction.

PLATE XXV.

Described in text, p. 658.

CONTENTS.

SUPPLEMENTAL REFERENCES.

XVIII. *On a New Series of Bodies in which Nitrogen is substituted for Hydrogen.*

By PETER GRIESS, *Esq.* *Communicated by* A. W. HOFMANN.

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IN some former papers* I have had occasion to describe a peculiar class of nitrogen-compounds, obtained by the action of nitrous acid upon amido-compounds, by the exchange of some of the hydrogen of the latter for the nitrogen of the nitrous acid. This substitution may be effected in two different ways; accordingly every amido-compound may give rise to two distinctly different series of bodies. By viewing the amido-compounds as constructed according to the general formula $\mathrm{C}_x \mathrm{H}_{y-2} (\mathrm{N H}_3)'' \mathrm{O}_z$†, the changes may be expressed as follows:—

$$\text{I.}\quad 2[\mathrm{C}_x \mathrm{H}_{y-2}(\mathrm{N H}_3)'' \mathrm{O}_z]+\mathrm{NH\,O}_2=\mathrm{C}_{2x}\mathrm{H}_{2(y-2)}(\mathrm{N H}_3)''(\mathrm{N}_2)''\mathrm{O}_{2z}+2\mathrm{H}_2\mathrm{O}.$$

$$\text{II.}\quad \mathrm{C}_x \mathrm{H}_{(y-2)} (\mathrm{N H}_3)'' \mathrm{O}_z+\mathrm{NH\,O}_2=\mathrm{C}_x\mathrm{H}_{(y-2)}(\mathrm{N}_2)\mathrm{O}''_z+2\mathrm{H}_2\mathrm{O}.$$

In the first equation one atom of nitrogen is substituted for three atoms of hydrogen contained in *two* atoms of the amido-compound, but in the second the substitution affects only *one* atom of the latter. I have hitherto directed my attention more particularly to the members of the first group, whether derived from amido-acids (such as diazo-amido-benzoic acid), or whether corresponding to amido-bases (diazo-amidobenzol). The bodies which I now shall have to describe in this communication are derived according to the second of the above general equations, and I have restricted myself almost entirely to the study of those which can thus be obtained from aniline and similar bases.

With regard to the chemical nature of these bodies, I may mention generally that they are capable of combining with acids and bases, but that their basic character preponderates. They are remarkable for the great variety of compounds which they

* Ann. der Chem. und Pharm. Bd. cxiii. p. 201; Bd. cxvii. p. 1; Bd. cxxi. p. 257; Supplement I. 1861, p. 100. Proceedings of the Royal Society, vol. ix. p. 594; vol. x. p. 309.

† Chemists are not agreed upon the rational constitution of amido-compounds. They are frequently referred to the ammonia-type, and almost as frequently to the same type to which the nitro-compounds from which they are derived belong. In the latter case the group NH_2 is considered as replacing one atom, or NH_3 as taking the place of two atoms of hydrogen. Aniline can thus be written in three different ways, and expressed by the three formulæ,

$$\underbrace{\left.\begin{matrix}\mathrm{C}_6\mathrm{H}_5\\ \mathrm{H}_2\end{matrix}\right\}\mathrm{N},}_{\text{Phenylamine.}}\qquad \underbrace{\mathrm{C}_6\mathrm{H}_5(\mathrm{H}_2\mathrm{N})',}_{\text{Amidobenzol.}}\qquad \underbrace{\mathrm{C}_6\mathrm{H}_4(\mathrm{H}_3\mathrm{N})''.}_{\text{Ammoniabenzol.}}$$

The two latter formulæ appear to me to be capable of explaining in the most natural manner the formation of bodies in which nitrogen is substituted for hydrogen.

produce, such as is not met with in any other portion of the field of organic chemistry. When in the free state they are remarkable for their instability; their compounds, however, are somewhat more stable; and it is for that reason that the latter have chiefly engaged my attention, and have been employed in the experiments to be described hereafter. The very striking physical properties of these bodies, as well as the large number of products of decomposition to which they give rise, likewise deserve to be specially noticed. Altogether they may be looked upon as one of the most interesting groups of organic compounds. I have avoided, as much as possible, discussing their rational composition, and have abstained from theoretical speculation. I have, however, come to the conclusion that the two atoms (or the molecule) of nitrogen, N_2, they contain must be considered as equivalent to two atoms of hydrogen, and it is in accordance with this view that the names of the new compounds have been framed.

PART I.—COMPOUNDS OF DIAZOBENZOL.

Nitrate of Diabenzol, $C_6H_4N_2, NHO_3$.

This substance can be prepared in various ways; most readily, however, by acting with nitrous acid upon a solution of nitrate of aniline. This salt of aniline is not very soluble in cold water; therefore, in order to obtain a concentrated solution of the new compound, it is best to make a thick paste by grinding up a portion of the nitrate with water and submitting it to the action of the gas, when the undissolved portion of the aniline-salt rapidly disappears, and the whole is converted into the new body. The reaction being accompanied by considerable increase of temperature, it is necessary to keep the solution cool, and to guard against passing a too-rapid current of the gas. The temperature of the solution should not rise much above 30° C. The operation is interrupted as soon as the whole of the aniline has disappeared. This can be ascertained by adding a little strong solution of potassa to a portion of the liquid on a watch-glass, when, if no more aniline is liberated, it may safely be inferred that the reaction is complete. The nitrate of diazobenzol is almost insoluble in ether, and even in ether mixed with much dilute alcohol, in consequence of which it may readily be obtained in a crystalline state thus:—the solution is first filtered, to remove traces of a brown resin, then mixed with about three times its volume of strong alcohol, and ether added until the precipitation is complete. The crystals are allowed to subside, and then filtered from the mother-liquor. The small quantity of the new body which remains in the mother-liquor may be neglected altogether, as its recovery is accompanied with great difficulty. To remove the last traces of colouring matter, the crystals are taken up with cold dilute alcohol and reprecipitated by the addition of ether, when they are obtained as long white needles.

Analysis, as will be shown further, proves that the new compound has the formula $C_6H_4N_2, NHO_3$. Its formation may be explained by the following equation:—

$$\underbrace{C_6H_7N, NHO_3}_{\text{Nitrate of aniline.}} + \underbrace{NHO_2}_{\text{Nitrous acid.}} = \underbrace{C_6H_4N_2, NHO_3}_{\text{Nitrate of diazobenzol.}} + \underbrace{2H_2O}_{\text{Water.}}$$

Another method of preparing nitrate of diazobenzol is based upon the action of nitrous acid upon diazo-amidobenzol, a body described in a previous communication to the Royal Society. On dissolving this latter compound in cold ether, and passing a current of nitrous acid gas through the solution, long acicular crystals of the new substance soon appear. The action should be continued as long as crystals form. The compound so obtained is collected on a filter and washed with ether. The reaction is expressed by the following equation:—

$$\underbrace{C_{12}H_{11}N_3}_{\text{Diazo-amidobenzol.}}+NHO_2+2NHO_3=2\underbrace{(C_6H_4N_2,\ NHO_3)}_{\text{Nitrate of diazobenzol.}}+2H_2O.$$

However elegant and simple this method of preparation of the nitrate of diazobenzol may appear, it will scarcely ever be employed, since it involves the previous preparation of diazo-amidobenzol, a body which it is rather troublesome to obtain in large quantities. Nitrate of diazobenzol can also be procured by the direct action of nitrous acid upon a mixture of aniline with about four times its volume of alcohol. The gas is passed into the alcoholic solution till, on the addition of ether to a small portion of it, a copious precipitation of white acicular crystals is produced. When this point is reached, the whole of the reddish-brown solution is mixed with ether, and the precipitate purified as already described.

This method of preparing the nitrate, however, is not suitable when large quantities have to be prepared. It is similar in principle to the preparation by means of diazo-amidobenzol, since to all appearance the aniline is first converted into this compound.

The methods just described, although simple, will only give favourable results when strict attention is paid to the directions given above; for it happens sometimes, and especially when the temperature of the solution is allowed to rise too high, that a copious evolution of nitrogen gas ensues, which cannot be stopped by any means before the whole of the substance has been destroyed.

By employing the first of the above processes, it is sometimes found that, on the addition of ether to the weak alcoholic solution of the crude compound, no crystals separate, but that an aqueous layer is deposited at the bottom of the vessel. This, however, only happens when an insufficient amount of alcohol has been originally added to the solution, because in this case the ether, not being able to mix with the liquor, causes no separation of crystals to take place. If this occurs, it is best to remove the ethereal liquor, and to dilute the residuary aqueous solution with strong alcohol, when, on the addition of ether, crystallization invariably ensues.

The nitrate of diazobenzol, by whatever method it may have been prepared, crystallizes in long white needles, which have been obtained several inches in length and are very soluble in water, less so in alcohol, and almost insoluble in ether and benzol. They can be dried over sulphuric acid without undergoing any change. Heated even below 100° C., they explode with unparalleled violence, far surpassing that of fulminating mer-

cury or iodide of nitrogen. About a gramme of this substance causes by its explosion a concussion like that produced by firing a pistol.

The destructive action of such an explosion is likewise extreme. Iron slabs of several lines in thickness were found smashed to atoms when a somewhat larger quantity was exploded upon them. Friction, pressure, and concussion also cause it to explode. The smallest particles of this substance, accidentally dropped upon the floor of a room, when trodden upon when dry, gave rise to a series of explosions attended with flashes of light. The properties of the nitrate of diazobenzol render it absolutely necessary that the greatest precaution should be observed when manipulating it. The portion of the substance required for analysis having been well washed with ether*, was placed in a platinum crucible and dried over sulphuric acid. Concussion or pressure had to be carefully avoided, especially with the dry substance, on account of its great explosiveness.

For the above-mentioned reasons I abstained from analyzing this compound in the usual manner, especially as I had opportunities of ascertaining the composition of analogous but less dangerous compounds by the ordinary analytical method. I have, however, been enabled to arrive at a knowledge of the composition of nitrate of diazobenzol by the very interesting change which an aqueous solution undergoes by the action of heat. By ebullition the compound is transformed under the influence of the water into carbolic acid, nitrogen, and nitric acid; and by estimating the quantities of the two latter products I have arrived at the true composition of the explosive substance.

$$\underbrace{C_6 H_4 N_2,\ NHO_3}_{\text{Nitrate of diazobenzol.}} + H_2 O = \underbrace{C_6 H_6 O}_{\text{Carbolic acid.}} + NHO_3 + N_2.$$

The method employed for the determination of the quantity of nitrogen evolved by the ebullition of the aqueous solution of the explosive compound is as follows:—The solution was introduced into a flask and a stream of carbonic acid gas passed through it. When the air was expelled, the delivery-tube was placed under a graduated cylinder containing solution of potassa, and the contents of the flask heated to ebullition†.

* When it is intended to recover the ether employed in the preparation of nitrate of diazobenzol, it is advisable, in order to avoid explosions, to shake the ethereal mother-liquor first with a little water, so as to dissolve any traces of the diazo-compound suspended in it. I have had a most dangerous explosion by neglecting this simple precaution. A large quantity of ether, which had been employed for the precipitation of the new compound from its alcoholic solution, had accumulated. From this liquid a few crystals had been observed to have separated. Their number seemed, however, to be so small, that it was deemed unnecessary to remove them from the vessel containing the ethereal liquid for distillation. As soon as the vessel became warm in the water-bath, and before the boiling-point of the ether had been reached, a fearful explosion took place, shattering the whole of the distilling apparatus to pieces and setting fire to the ether, the flame of which spread most alarmingly through the laboratory. The gas-flames, which were burning at the time in the room, were suddenly extinguished by the violent pressure upon the atmosphere, and all those working in the laboratory (who fortunately escaped unhurt) were for a moment deprived of their breath. The explosive properties of the nitrate of diazobenzol may perhaps at a future period find practical application.

† In all analyses mentioned herein the analyzed substances are understood to be desiccated by sulphuric acid previously, unless the contrary is distinctly stated.

I. 0·605 grm. of the compound, treated as above, gave 80·4 cub. centims. of nitrogen at 0°, and 760 millims. bar. pressure=0·101 grm. of nitrogen, or 16·70 per cent.

Theory.	Experiment.
N=16·77	16·70

II. ·492 grm. of substance, dissolved in water and boiled, required for neutralization 29·5 cub. centims. of alkali solution, corresponding to 0·185 grm. of nitric acid, or 37·77 per cent.

	Calculated.		Found.
$C_6H_4N_2$	190	62·27	——
NHO_3	63	37·73	37·77
	253	100·00	

Sulphate of Diazobenzol, $C_6H_4N_2$, SH_2O_4.

Acid sulphate of aniline, when treated with nitrous acid, is converted into sulphate of diazobenzol according to the equation

$$*\underbrace{C_6H_7N, SH_2O_4}_{\text{Acid sulphate of aniline.}}+NHO_2=\underbrace{C_6H_4N_2, SH_2O_4}_{\text{Sulphate of diazobenzol.}}+2H_2O.$$

On account of the slight solubility of sulphate of aniline, and in order to avoid employing a large quantity of liquid, it becomes necessary to suspend the salt in water, and to expose it in this state to the action of nitrous acid. The reaction, however, is very slow, and the salt is only gradually converted into the diazo-compound. I found it therefore more convenient to prepare the sulphate from the nitrate of diazobenzol, by treating a concentrated aqueous solution† of the latter with a sufficient quantity of sulphuric acid, previously diluted to avoid rise of temperature upon addition.

The solution thus obtained is then mixed with three times its volume of absolute alcohol, and lastly with a sufficient amount of ether, which causes the sulphate of diazobenzol, together with some water, to separate in a layer at the bottom of the vessel, the liberated nitric acid, together with the excess of sulphuric acid, remaining mixed with the supernatant alcohol and ether. This latter is decanted, and the solution of the sulphate (with a view to the removal of a further quantity of water) is once more treated with absolute alcohol and reprecipitated by means of ether. The precipitated liquid is then placed in flat dishes over sulphuric acid, when it soon solidifies to a magma of white crystals. In order to purify these crystals completely, they are washed on a filter with a mixture of alcohol and ether, which removes any trace of sulphuric and carbolic acids which may have been produced by a partial decomposition of the sulphate of diazobenzol, then dissolved in weak cold alcohol, and precipitated by the addition of

* Ordinary sulphate of aniline containing excess of free sulphuric acid is understood by this formula.

† The crude solution obtained by the action of nitrous acid upon nitrate of aniline may be employed.

ether. The crystals are separated without delay from the mother-liquor and dried over sulphuric acid.

0·4848 grm. of substance gave 0·6278 grm. of carbonic acid gas and 0·1397 grm. of water, corresponding to 35·32 per cent. of carbon and 3·20 per cent. of hydrogen.

0·4185 grm. gave, on boiling with water, 44·7 cub. centims. of nitrogen at 14° C. and 763 millims. bar. pressure = 44·9 cub. centims., at 0° and 760 millims. bar. pressure = 0·05642 grm., or 13·49 per cent.

	Calculated.		Found.
C_6	72	35·65	35·32
H_6	6	2·97	3·20
N_2	28	13·86	13·49
S	32	15·83	——
O_4	64	31·69	——
	202	100·00	

For the estimation of the sulphuric acid 0·558 grm. of substance, precipitated by chloride of barium, gave 0·647 grm. of sulphate of barium, corresponding to 48·76 per cent. of sulphuric acid.

	Calculated.		Found.
$C_6H_4N_2$	104	51·49	——
SH_2O_4	98	48·51	48·76
	202	100·00	

Sulphate of diazobenzol crystallizes in prisms, which readily dissolve in water, but are soluble with difficulty in absolute alcohol and insoluble in ether. The aqueous as well as the alcoholic solution is decomposed on boiling with evolution of gas. Exposed to the air, this compound attracts moisture very rapidly and becomes liquid, and gradually decomposes. Heated alone, it deflagrates feebly at about 100° C.

Hydrobromate of Diazobenzol, $C_6H_4N_2$, HBr.

This compound is prepared by the action of bromine upon diazo-amidobenzol. When an ethereal solution of bromine is gradually added to a rather concentrated solution of the diazo-amido-compound, each drop of the bromine-solution is seen to produce a precipitation of the hydrobromate of diazobenzol. When no more crystals are formed the precipitate is separated from the mother-liquor, washed with ether till quite white, and then dried over sulphuric acid. All these operations must be performed as speedily as possible, since the new compound is of a very unstable nature and rapidly decomposes.

Should it be found necessary to recrystallize the precipitate, it must be dissolved in the least possible quantity of cold alcohol, and ether added to the solution till no more precipitation takes place.

0·3025 grm. of the substance gave, on precipitation with nitrate of silver, 0·304 grm. of bromide of silver, corresponding to 43·3 per cent. of hydrobromic acid.

	Calculated.		Found.
$C_6H_4N_2$	104	56·22	—
HBr	81	43·78	43·30
	185	100·00	

Hydrobromate of diazobenzol crystallizes in white nacreous scales, which, like the compound previously described, are readily soluble in water, less so in alcohol, and insoluble in ether. The solution of this compound is distinguished by a strong acid reaction. Heat, friction, and pressure cause the crystals of the hydrobromate to explode with the same violence as the nitrate of diazobenzol. Even when in a perfectly dry state this compound can only be kept for a short time without undergoing decomposition (which is accompanied by the production of a peculiar aromatic odour), the decomposition being complete in a few days. It is formed according to the equation

$$\underbrace{C_{12}H_{11}N_3}_{\text{Diazo-amido-benzol.}}+6Br=\underbrace{C_6H_4N_2,\ HBr}_{\text{Hydrobromate of diazobenzol.}}+\underbrace{C_6H_4Br_3N}_{\text{Tribromaniline.}}+2HBr.$$

Tribromaniline remains in the ethereal mother-liquor, from which it crystallizes on evaporation.

It is most likely that, by the action of chlorine upon diazo-amidobenzol, hydrochlorate of diazobenzol may be obtained. An aqueous solution of this latter compound, however, may also be prepared by treating a solution of the hydrobromate with moist chloride of silver.

Perbromide of Diazobenzol, $C_6H_4N_2$, HBr, Br_2.

On adding a small quantity of bromine-water to an aqueous solution of the nitrate of diazobenzol, a white crystalline precipitate of tribromophenylic acid is usually obtained, owing to the presence of a small quantity of phenylic acid formed by the spontaneous decomposition of the nitrate by water. On removing the tribromophenylic acid as quickly as possible by filtration, and on the addition of a large excess of bromine-water containing free hydrobromic acid to the filtrate, the new bromine-compound separates as a brownish-red oil, which solidifies to a crystalline mass soon after the supernatant liquor has been removed. The crystals are obtained pure for analysis by washing with a little ether.

I. 0·7154 grm. gave 0·5504 grm. CO_2, corresponding to 20·98 per cent. of carbon and 1·60 of hydrogen.

II. 0·4805 grm. of this substance, digested with excess of aqueous ammonia *, gave, on expulsion of the excess of ammonia and precipitation with nitrate of silver, 0·8005 grm. of bromide of silver, corresponding to 70·68 bromine per cent.

* The transformation which this substance thus undergoes, and which renders it possible to determine the bromine in this manner, will be fully explained further on.

	Calculated.		Found.
C_6	72	20·87	20·98
H_5	5	1·45	1·60
N_2	28	8·12	——
Br_3	240	69·56	70·89
	345	100·00	

Perbromide of diazobenzol, which crystallizes in yellow plates, is insoluble in water, rather difficultly soluble in alcohol, and insoluble in ether. It is comparatively stable when in a dry state. Its alcoholic solution, on the other hand, rapidly decomposes, even in the cold, with evolution of gas. For this reason it is impossible to recrystallize it without loss. In order to purify this unstable perbromide, it is most advantageous to dissolve it in cold alcohol, and to allow the solvent to evaporate spontaneously in shallow vessels in the open air. Very fine crystals are usually obtained in this manner, but contaminated with trifling quantities of an oily product of decomposition, which, however, may be removed by washing with a little cold ether. On application of heat this compound deflagrates at a comparatively low temperature. The constitution of the perbromide of diazobenzol seems the same as the periodide of tetrethylammonium and similar compounds of other bases.

Platinum-salt of Hydrochlorate of Diazobenzol, $C_6 H_4 N_2$, H Cl, Pt Cl_2,

is obtained by the addition of bichloride of platinum to a rather concentrated aqueous solution of the nitrate of diazobenzol. The fine yellow prisms which are precipitated are almost insoluble in alcohol and ether. They are rather stable; when kept for some time, however, they acquire a brownish colour, and are gradually but completely decomposed. On heating they deflagrate, hence it is impossible to estimate the platinum by simple ignition.

0·760 grm., ignited with carbonate of sodium, gave 0·241 grm. of platinum, corresponding to 31·71 per cent.

$C_6 H_4 N_2$, HCl, Pt Cl_2.

Theory.	Experiment.
Pt=31·82	31·71

Gold-salt of Hydrochlorate of Diazobenzol, $C_6 H_4 N_2$, HCl, Au Cl_3.

By the addition of terchloride of gold to a dilute aqueous solution of the nitrate of diazobenzol, this compound is obtained in the form of a light-yellow crystalline precipitate, insoluble in water, but soluble in alcohol, especially when warm, from which, on cooling, it is deposited in the form of small golden-yellow plates. This salt cannot, however, be recrystallized without some loss, especially when the alcoholic solution is heated to boiling. The decomposition is indicated invariably by an evolution of gas; continued boiling with alcohol destroys it completely. 0·6965 grm. of this salt

gave, on precipitation with sulphuretted hydrogen and ignition of the tersulphide of gold, 0·309 grm. of gold, corresponding to 44·36 per cent. of that metal.

$C_6H_4N_2$, HCl, AuCl_3.

	Theory.	Experiment.
Gold	44·37	44·36

It is well known that when bromine, chlorine, or hyponitric acid is substituted in organic compounds for certain hydrogen atoms, the product formed is distinguished from the original compound, if the latter be an acid, by stronger acid properties, or if a base, by less pronounced basic characters; and also that the contrary is observed in the case of substitution of potassium, NH_2, &c. for the same hydrogen atoms.

Nitrogen, however, exhibits in this respect a peculiar double nature, since it is found only slightly (or not at all) to affect the basic properties of a compound, whilst at the same time it exerts so decided an acidulating action as to impart to strong bases an acid nature. I have already clearly shown that diazobenzol has all the characters of an organic base, capable, like aniline, of forming with acids saline compounds. It possesses, at the same time, the property of combining with metallic hydrates, playing, to a certain extent, the part of an acid. These metallic derivatives are, as a rule, distinguished by the same instability as the compounds of diazobenzol with acids. They are less affected, however, by heat. Their aqueous or alcoholic solutions can be heated to boiling for some time without suffering complete decomposition. Heated alone they explode, although at a much higher temperature, and not with so great violence as the bodies previously described. The compounds which are soluble in water are mostly well crystallized, whilst the insoluble ones, such as those formed with silver and lead, are obtained as amorphous precipitates.

Compound of Hydrate of Potassium with Diazobenzol, $C_6H_4N_2$, KHO.

By introducing a very concentrated solution of nitrate of diazobenzol into an excess of an equally concentrated solution of caustic potash, drop by drop, a yellowish liquid is obtained possessing a peculiar aromatic odour, and solidifying, by evaporation in the water-bath, to a crystalline mass. This is a mixture of the compound of hydrate of potassium with diazobenzol and nitre, together with a brownish-red amorphous body—the result of a secondary reaction, which a portion of the original substance undergoes, and which is always indicated by the evolution of gas. In order to separate these bodies it is necessary to remove the excess of caustic potash. This is best done by putting the crystalline mass into a strong linen cloth and squeezing it powerfully between porous stones. The dry cake is then treated with absolute alcohol, which readily dissolves the compound of hydrate of potassium with diazobenzol, leaving the nitrate of potassium insoluble; it is then separated by filtration. The alcoholic filtrate (which, on account

of the above-mentioned secondary product of decomposition, possesses an intense reddish-brown colour) is then evaporated on the water-bath. The residue is once more pressed and washed with a mixture of alcohol and ether to remove the reddish-brown substance, when the compound is obtained nearly white. By again pressing and dissolving the dry cake in a small quantity of absolute alcohol, filtering, and adding a sufficient amount of ether, the substance is obtained in white plates, which must be dried at once over sulphuric acid.

0·735 grm. gave 0·3992 grm. of sulphate of potassium, corresponding to 35·00 per cent. of KHO.

	Calculated.		Found.
$C_6H_4N_2$	104	64·92	——
KHO	56·2	35·08	35·00
	160·2	100·00	

The compound of hydrate of potassium with diazobenzol crystallizes in small, white, soft plates, which become reddish by exposure to the air; they are very readily soluble in water and alcohol, but insoluble in ether. The solution has a strong alkaline reaction. A freshly prepared aqueous solution is but slightly coloured; but by keeping for a short time it rapidly acquires a yellow colour, and ultimately a reddish-yellow substance is precipitated. Ebullition does not seem to accelerate this decomposition materially. The dry substance is very stable, and can be kept a long time unchanged. Heated alone it explodes at a little above 130°; the explosion is accompanied by a slight report.

Compound of Hydrate of Silver with Diazobenzol, $C_6H_4N_2$, Ag HO*.

This is obtained as a white or slightly chocolate-coloured precipitate, by treating a freshly prepared solution of the previous compound with a solution of nitrate of silver. After removing the mother-liquor, the precipitate is thoroughly washed with water, dried by pressing between filter-paper, and finally over sulphuric acid.

0·5830 grm. gave 0·6772 grm. of CO and 0·126 grm. of water, corresponding to 31·68 per cent. of carbon and 2·40 per cent. of hydrogen.

0·9645 grm. gave 0·5955 grm. of chloride of silver, corresponding to 46·46 per cent. of silver.

* It deserves notice that diazo-amidobenzol and the analogous double compounds likewise combine with bases. When speaking of these bodies (Ann. der Chem. und Pharm. Bd. cxxi, p. 362) I have pointed out that they all possess the property of forming insoluble precipitates with nitrate of silver. Without examining these precipitates more closely, I then came to the conclusion that (judging from the manner in which they were obtained, and taking a few silver-determinations into consideration), they were simply combinations of the respective diazo-amido-compound with nitrate of silver. This view has proved erroneous; for, on closer examination, it was found that these bodies contain no nitric acid, but must be regarded as combinations of silver with diazo-amido-compounds, viz. $C_{12}H_{11}N_3$, Ag. I intend subsequently to refer to this subject.

	Theory.		Experiment.
C_6	72	31·44	31·68
H_5	5	2·19	2·40
N_2	28	12·23	——
Ag	108	47·16	46·46
O	16	6·98	——
	229	100·00	

The compound of hydrate of silver with diazobenzol is insoluble in all the ordinary neutral solvents. Nitric acid even when cold dissolves it with great facility. It is distinguished by great stability; for even after having been kept for weeks, not the least sign of decomposition could be discovered. Exposed to a higher temperature it explodes with some violence. Since the constitution of the two compounds just described affords sufficiently fixed data for establishing the composition of other combinations of metallic hydrates with diazobenzol, I have not thought it necessary to extend my analyses to the other salts.

Their preparation and constitution, moreover, present no important characteristics, and I will therefore describe them very briefly.

Compound of Hydrate of Barium with Diazobenzol

is obtained when a solution of a very soluble barium-salt is added to a rather concentrated solution of the potassium-compound. It is precipitated in the form of white, microscopic, indistinct needles or plates, which become yellowish in consequence of a gradual decomposition. It is difficultly soluble in water.

The compound of diazobenzol with hydrate of zinc is a white amorphous powder insoluble in water. The compound with hydrate of lead is also a white powder, but acquiring rapidly a yellow colour. With sulphate of copper a brown precipitate changing to green is obtained. Mercurial chloride gives no precipitate.

Diazobenzol, $C_6H_4N_2$.

This remarkable substance is obtained when an aqueous solution of the compound of hydrate of potassium with diazobenzol is treated with a sufficient quantity of acetic acid. A thick yellow oil is liberated, which possesses a peculiar odour, and is remarkable for its extraordinary instability. Its existence is very ephemeral, and after a short time nitrogen gas begins to be evolved, and the oil is rapidly converted into a brownish-red substance. The heat which is produced when larger quantities of the oil undergo this spontaneous decomposition is sufficient to give rise to dangerous explosions.

The addition of ether to the oil dissolves it instantaneously, producing a red solution, a tumultuous evolution of gas taking place. It combines with nitric and sulphuric acids, and with hydrate of potassa, terchloride of gold, &c., forming the compounds previously described.

COMPOUNDS OF DIAZOBENZOL WITH ORGANIC BASES.

Diazo-amidobenzol, as is well known, is formed by the action of nitrous acid upon an alcoholic solution of aniline, according to the equation

$$\underbrace{2C_6H_7N}_{\text{Aniline.}}+NHO_2=\underbrace{C_{12}H_{11}N_3}_{\text{Diazo-amidobenzol.}}+2H_2O.$$

By viewing this body as a double compound of diazobenzol and aniline,

$$C_{12}H_{11}N_3=\begin{cases}C_6H_4N_2\\C_6H_7N\end{cases},$$

I was led to prepare it by the direct action of aniline upon compounds of diazobenzol. This reaction proceeds readily on mixing an aqueous solution of nitrate of diazobenzol with aniline, when a viscid yellow mass is speedily produced, which becomes crystalline after a short time, and which can be obtained in a perfectly pure state by several recrystallizations * from alcohol. The formation of the diazo-amidobenzol may be expressed as follows:—

$$\underbrace{C_6H_4N_2, NHO_3}_{\text{Nitrate of diazobenzol.}}+\underbrace{2C_6H_7N}_{\text{Aniline.}}=\underbrace{C_{12}H_{11}N_3}_{\text{Diazo-amidobenzol.}}+\underbrace{C_6H_7N, NHO_3}_{\text{Nitrate of aniline.}}.$$

On referring to the equation given on page 669 for the formation of nitrate of diazobenzol from diazo-amidobenzol,

$$C_{12}H_{11}N_3+NHO_2+2NHO_3=2(C_6H_4N_2, NHO_3)+2H_2O,$$

it is seen that these two bodies may be readily transformed into one another.

Many other bases deport themselves with nitrate of diazobenzol exactly like aniline, and we may therefore look forward to the discovery of a large number of double compounds analogous to diazo-amidobenzol.

It is not my intention to give a full history of these compounds, as a few short statements will show clearly how closely they are allied to their prototype diazo-amidobenzol.

Diazobenzol-amidobromobenzol, $\begin{Bmatrix}C_6H_4N_2\\C_6H_6BrN\end{Bmatrix}$.

This compound is obtained by the action of bromaniline upon nitrate of diazobenzol. It crystallizes in very fine small yellow plates or needles, which are rather difficultly soluble in alcohol, but readily soluble in ether. Its platinum-salt,

$$C_{12}H_{10}BrN_3, H_2Cl_2, 2PtCl_2,$$

is obtained as a buff-coloured precipitate consisting of fine hair-like crystals. Nitrate of silver produces, in an alcoholic solution, a yellow precipitate similar to that of the compound previously described.

* Any excess of aniline must be removed by means of acetic acid before crystallization.

In the formation of the above bodies two atoms of base enter into chemical action with one atom of nitrate of diazobenzol, forming, together with the diazo-amido-compound, the nitrate of the base employed. Naphtalidine (amidonaphtol), however, combines directly in equal numbers of atoms with nitrate of diazobenzol, giving rise to a compound which has the formula

$$C_{16}H_{14}N_4O_3,$$

and which, as will be shown, must be viewed as nitrate of diazobenzol-amidonaphtol. This compound is obtained in an impure state as a violet crystalline precipitate on adding an aqueous solution of nitrate of diazobenzol to an alcoholic solution of naphtalidine. It is purified by repeated washings with cold alcohol and recrystallization from the same, forming beautiful green prisms.

0·2303 grm. gave 0·5235 grm. of carbonic acid and 0·0975 grm. of water, corresponding to 61·99 per cent. of carbon and 4·70 per cent. of hydrogen.

These numbers lead to the formula

$$C_{16}H_{14}N_4O_3 = \left.\begin{matrix} C_6H_4N_2 \\ C_{10}H_9N \end{matrix}\right\} NHO_3.$$

	Calculated.		Found.
C_{16}	192	61·94	61·99
H_{14}	14	4·52	4·70
N_4	56	18·06	——
O_3	48	15·48	——
	310	100·00	

The new compound is formed according to the equation

$$\underbrace{C_6H_4N_2, NHO_3}_{\text{Nitrate of diazobenzol.}} + \underbrace{C_{10}H_9N}_{\text{Naphtalidine.}} = \underbrace{C_{16}H_{14}N_4O_3}_{\text{Nitrate of diazobenzol-amidonaphtol.}}$$

It is one of the finest bodies of which chemistry can boast. It crystallizes in well-defined prisms, which by reflected light are of a magnificent grass-green colour, but ruby-red by transmitted light. The crystals are almost insoluble in water and ether. Hot alcohol dissolves them freely, and redeposits them almost entirely on cooling.

Diazobenzol-amidonaphtol, $\left.\begin{matrix} C_6H_4N_2 \\ C_{10}H_9N \end{matrix}\right\}.$

This compound is obtained from the previous substance by removing the nitric acid by means of ammonia or potassa. It crystallizes in very brilliant ruby-red prisms, readily soluble in alcohol and ether, forming yellow liquids. Acids impart a beautiful violet colour to these solutions. Bichloride of platinum produces a purple-blue crystalline precipitate; nitrate of silver a yellow precipitate, which consists of small fine needles.

It is worth mentioning that these diazobenzol-amido-compounds can also be obtained by acting with an aqueous solution of the salts of the respective bases upon the com-

pound of hydrate of potassium with diazobenzol. The reaction which takes place may be expressed by the following equation:—

$$\underbrace{C_6H_4N_2,\ KHO}_{\text{Hydrate of potassium with diazobenzol.}}+\underbrace{C_6H_7N,\ HCl}_{\text{Hydrochlorate of aniline.}}=\underbrace{C_{12}H_{11}N_3}_{\text{Diazoamido-benzol.}}+KCl+H_2O.$$

COMPOUNDS OF DIAZOBENZOL WITH AMIDO-ACIDS.

Amido-acids are likewise capable of entering into combination with diazobenzol. One would have expected that the compounds to which this reaction gives rise would deport themselves in an analogous manner to the sulphate or nitrate of diazobenzol, which, like the salts of organic bases, are capable of double decomposition. This, however, is not the case. The compounds of diazobenzol with amido-acids behave more like simple bodies, exhibiting much similarity to the diazo-amidobenzol, being capable, like the latter, of forming, with bichloride of platinum, double compounds of the nature of the potassio-bichloride of platinum. They possess, moreover, the property of combining with metals, giving rise to bodies which correspond entirely to the salts of simple acids.

Diazobenzol-amidobenzoic acid, $\begin{cases} C_6H_4N_2 \\ C_7H_7NO_2 \end{cases}$.

This compound is obtained by simply mixing an aqueous solution of the nitrate of diazobenzol (1 molecule) with a solution of amidobenzoic acid (2 molecules). It separates as a yellow crystalline precipitate which is readily freed from the mother-liquor, and when dry dissolved in ether and filtered. The ethereal solution deposits on evaporation yellow crystals, which are obtained pure for analysis by washing with cold alcohol.

0·3333 grm. of substance gave 0·785 grm. at CO_2 and 0·1458 grm. of water, corresponding to 64·23 per cent. of carbon and 4·86 per cent. of hydrogen.

0·49 grm. gave 68·8 cub. centims. of nitrogen at 0° C. and 760 millims. pressure, =17·4 per cent.

	Theory.		Experiment.
C_{13}	156	64·73	64·23
H_{11}	11	4·56	4·86
N_3	42	17·43	17·65
O_2	32	13·28	——
	241	100·00	

This body is formed according to the equation

$$\underbrace{C_6H_4N_2,\ NHO_3}_{\text{Nitrate of diazobenzol.}}+\underbrace{2C_7H_7NO_2}_{\text{Amidobenzoic acid.}}=\underbrace{C_{13}H_{11}N_3O_2}_{\text{Diazobenzol-amido-benzoic acid.}}+\underbrace{C_7H_7NO_2,\ NHO_3}_{\text{Nitrate of amidobenzoic acid.}}.$$

Diazobenzol-amidobenzoic acid forms small indistinct plates or crystalline grains. It

is almost insoluble in water, very difficultly soluble in alcohol, and easily soluble in ether. Solutions of ammonia, potash, or carbonate of potassium dissolve the acid readily, the solution acquiring a yellow colour. On heating a little of the substance on platinum-foil, it fuses and is rapidly decomposed, the decomposition being accompanied by a violent evolution of gas.

Cold dilute mineral acids act upon it but slowly; on heating, however, speedy decomposition ensues. Acetic acid, even when highly concentrated, has no action in the cold, but on the application of heat destroys it.

Platinum-salt of the Hydrochlorate of Diazobenzol-amidobenzoic Acid,
$C_{13}H_{11}N_3O_2$, 2HCl, $2PtCl_2$.

This compound forms small yellowish-white indistinct plates, and is obtained by adding an alcoholic solution of bichloride of platinum to the ethereal solution of the diazobenzol-amidobenzoic acid.

0·7455 grm. of the substance gave, on ignition with carbonate of sodium, 0·234 grm. of platinum.

$$C_{13}H_{11}N_3,\ H_2Cl_2,\ 2PtCl_2.$$

	Calculated.	Found.
Pt=	30·80	31·39

Diazobenzol-amidobenzoic acid in its behaviour with bases is similar to a bibasic acid, since it combines with metallic bodies in two ways. All the salts formed are comparatively stable; those, *e. g.*, which are soluble in water, as the potassium-salt, will even bear recrystallization. With the oxide of silver and barium it forms insoluble precipitates. A more minute description of these compounds I must defer to a future time.

Compounds analogous to the diazobenzol-amidobenzoic acid are formed when amidodracylic acid (isoamidobenzoic acid), amidoanisic acid, &c. are made to react in the manner previously described upon nitrate of diazobenzol. It is my intention to investigate these compounds more specially, and I therefore abstain from entering further upon their description, only noticing that they possess a remarkable resemblance to diazobenzol-amidobenzoic acid.

IMIDOGEN COMPOUNDS OF DIAZOBENZOL.

By this name are designated a peculiar class of diazobenzol-compounds obtained by the action of ammonia, ethylamine, and analogous organic bases upon perbromide of diazobenzol ($C_4H_4N_2$, HBr, Br_2). All the compounds hitherto described exist in a solid state, and none of them can be volatilized without decomposition; the new compounds to be described, on the other hand, are liquors which can be distilled, and which possess the narcotic odour of some vegetable bases, such as conidine and nicotine. Beyond this odour, however, they have nothing in common with these natural bases. They are indifferent bodies, combining with neither acids nor bases.

Diazobenzolimide, $C_6H_5N_3 = \left.\begin{matrix}(C_6H_4N_2)''\\H\end{matrix}\right\}N.$

On treating perbromide of diazobenzol with aqueous ammonia, a speedy decomposition ensues with evolution of much heat. The products of the reaction are, first, bromide of ammonium, this passes into solution; secondly, diazobenzolimide, which separates as a heavy oil, rather highly coloured by a brown substance simultaneously produced in small quantity. By repeatedly distilling the oil with water it is obtained perfectly pure, and of a slightly yellowish colour; the substance which imparts to it the intense brownish colour, not being volatile, is left behind in the retort. The purified oil is separated from the water by means of a separating funnel, placed over chloride of calcium, and then distilled once more from a water-bath *in vacuo*.

0·1148 grm. gave 0·253 grm. carbonic acid and 0·0505 grm. of water.

	Calculated.		Found.
C_6	72	60·50	60·10
H_5	5	4·20	4·88
N_3	42	35·30	——
	119	100·00	

Its formation is explained by the following equation:—

$$\underbrace{C_6H_4N_2,\ HBr,\ Br_2}_{\text{Perbromide.}} + 4NH_3 = \underbrace{\left.\begin{matrix}C_6H_4N_2\\H\end{matrix}\right\}N}_{\text{Diazobenzolimide.}} + 3NH_4Br^{*}.$$

Diazobenzolimide is remarkable for its narcotic, aromatic-ammoniacal odour. It is volatilized by distillation with water, and also when heated *in vacuo*, as shown above. Distilled at the ordinary atmospheric pressure, it is decomposed with explosive violence. Alcohol and ether dissolve it rather difficultly. I could not succeed in solidifying the oil by using a frigorific mixture of nitre and sal-ammoniac. Hydrochloric acid, even when concentrated, and aqueous potassa have no effect upon it. Strong nitric and sulphuric acids dissolve it with decomposition.

Ethyldiazobenzolimide $= \left.\begin{matrix}(C_6H_4N_2)''\\C_2H_5\end{matrix}\right\}N.$

This body is obtained in a manner exactly similar to the one employed for the preparation of diazobenzolimide, viz. by acting with ethylamine upon the perbromide of diazobenzol. It is likewise a yellowish-coloured oil, having a deceptive resemblance to the previously described compound.

* This decomposition of the perbromide explains the method of analysis adopted for the estimation of the bromine described in page 673.

PRODUCTS OF DECOMPOSITION OF THE DIAZOBENZOL COMPOUNDS.

The transformations which the molecule of diazobenzol undergoes through the influence of various reagents are numerous, and there is probably no other body to be met with in the large field of organic chemistry that surpasses it in this respect. The products of decomposition to which it gives rise frequently possess new and very distinctive features; more frequently, however, they belong to the benzol or phenyl group, in which latter case their formation depends upon the great inclination which the two nitrogen atoms of the diazobenzol exhibit to escape and to cede their place to other atomic groups of the same value (HH).

Deportment of Diazobenzol Compounds when boiled in an aqueous solution.

The transformation of nitrate of diazobenzol, under the influence of boiling water, has already been noticed. A similar change is observed with regard to its sulphate and hydrobromate, as will be seen from the following equations:—

$$\underbrace{C_6H_4N_2, SH_2O_4}_{\substack{\text{Sulphate}\\\text{of diazobenzol.}}}+H_2O=\underbrace{C_6H_6O}_{\substack{\text{Phenylic}\\\text{acid.}}}+SH_2O_4+N_2$$

$$\underbrace{C_6H_4N_2, HBr}_{\substack{\text{Hydrobromate}\\\text{of diazobenzol.}}}+H_2O=\underbrace{C_6H_6O}_{\substack{\text{Phenylic}\\\text{acid.}}}+HBr+N_2.$$

On the other hand, the compounds of diazobenzol with metallic hydrates exhibit a very different deportment with boiling water. By neutralizing the metal, however, with a mineral acid, the decomposition takes place in accordance with the above equations.

Action of Alcohol upon Nitrate of Diazobenzol.

By gradually and cautiously introducing the nitrate of diazobenzol into a moderate quantity of alcohol (previously warmed to about 50° C. in order to accelerate the solution), and submitting the whole to distillation in a water-bath, a yellow residue remains, which solidifies to a crystalline mass on cooling, and dissolves readily in alkaline solutions, whence it was inferred that it had the character of an acid. There was, in fact, no difficulty in ascertaining that this body was nothing else than dinitrophenylic acid, possessing all the well-pronounced properties of this compound; and to prove this most conclusively, it was converted into the characteristic amidonitrophenylic acid. Dinitrophenylic acid, however, is not the only product to which this reaction gives rise; for on mixing the distillate with water an oily body separates, which collects upon the fluid, especially when its specific gravity has been increased by the addition of a solution of chloride of sodium. This oil may be removed by means of a separating funnel and dried over chloride of calcium. It possesses, when rectified by distillation, all the properties of ordinary benzol. I have converted it into dinitrobenzol by the action of fuming nitric acid, and have found this latter identical with that prepared from coal-tar

benzol. The fusing-point of the dinitrobenzol was in both instances 89° C. The production of dinitrocarbolic acid and of benzol can be expressed by the following equation:—

$$\underbrace{2(\mathrm{C_6\,H_4\,N_2,\ NHO_3})}_{\text{Nitrate of diazobenzol.}}+\underbrace{\mathrm{C_2\,H_6\,O}}_{\text{Alcohol.}}=\underbrace{\mathrm{C_6\,H_4(NO_2)_2\,O}}_{\text{Dinitrophenylic acid.}}+\underbrace{\mathrm{C_6\,H_6}}_{\text{Benzol.}}+\underbrace{\mathrm{C_2\,H_4\,O}}_{\text{Aldehyde.}}+\mathrm{N_4}+\mathrm{H_2O}.$$

Sulphate of diazobenzol, when distilled with alcohol, is acted upon in a similar manner. The alcoholic distillate contains the benzol, whilst the residue in the flask consists of sulphuric acid and a small quantity of an organic acid.

Deportment of Nitric Acid and Nitrate of Diazobenzol.

Ordinary nitric acid, as well as the fuming acid, has no action upon this compound in the cold. The diazo-compound was dissolved in the strongest fuming nitric acid, allowed to stand for one hour, and cautiously diluted with water so as to prevent any rise of temperature; solution of terchloride of gold was then added, which gave an immediate precipitate. Analysis showed this precipitate to be identical with the gold-salt of hydrochlorate of diazobenzol.

0·8295 grm. gave, on precipitation with sulphuretted hydrogen and ignition of the tersulphide of gold, 0·3685 grm. of gold, corresponding to 44·42 per cent.

$$\mathrm{C_6\,H_4\,N_2,\,HCl,\,Au\,Cl_3}.$$

	Calculated.	Found.
Gold	44·37	44·42

On boiling the solution of nitrate of diazobenzol in fuming nitric acid, trinitrophenylic acid is produced. By employing a somewhat weaker acid, an admixture of tri- and di-nitrophenylic acid is obtained.

Action of Sulphuric Acid upon Sulphate of Diazobenzol.

On dissolving the sulphate in a small quantity of concentrated sulphuric acid, and heating in a water-bath, a copious evolution of nitrogen gas ensues, and a brownish-coloured liquid remains, consisting of a mixture of the excess of acid employed and a new sulpho-acid. The separation of the two acids is readily accomplished by preparing their barium-salts. After diluting the brownish liquid with a sufficient quantity of water and adding carbonate of barium as long as the solution exhibits an acid reaction, the insoluble sulphate of baryta may be separated by filtration from the barium-salt of the new sulpho-acid. The filtrate is evaporated till a pellicle forms, when, on cooling, a large quantity of crystals of the barium-salt of the new sulpho-acid appears. After separating the crystals by filtration, and then evaporating the mother-liquor further and cooling, a new quantity of crystals is obtained. A single recrystallization from water renders them perfectly pure. This salt forms fine, white, well-developed prisms, attaining frequently to the length of half an inch, when the solution is allowed to cool very slowly. The crystals are readily soluble in hot, rather difficultly in cold water, and

almost insoluble in alcohol and ether. The solutions have a neutral reaction. The substance was dried at 160° C.; and on analysis

0·4587 grm. gave 0·3067 grm. of carbonic acid and 0·0649 grm. of water.

0·4505 grm., precipitated with sulphuric acid, gave 0·2495 grm. of sulphate of barium.

0·321 grm., analyzed by CARIUS's method, gave 0·379 grm. of sulphate of barium*.

These numbers lead to the formula

$$C_6 H_6 Ba_2 S_2 O_8.$$

	Theory.		Experiment.
C_6	72	17·70	18·23
H_6	6	1·47	1·55
Ba_2	137	33·66	33·59
S_2	64	15·72	16·21
O_8	128	31·45	——
	407	100·00	

The salt, dried over sulphuric acid, contained $3\frac{1}{2}$ atoms of water of crystallization, which escaped entirely at 130° C. No further loss of water was observed on heating to a higher temperature.

I. 0·5435 grm. of substance, dried over sulphuric acid, and lastly at 130° C., lost 0·0755 grm. of water.

II. 0·5398 grm. was dried at 190° C. and lost 0·073 grm. of water.

$$C_6 H_6 Ba_2 S_2 O_8 + 3\tfrac{1}{2} H_2 O.$$

Calculated.	Found.	
	I.	II.
13·44	13·89	13·52

In accordance with the analysis of the barium-salt, the free acid must be expressed by the formula $C_6 H_8 S_2 O_8$, and I propose to call it disulphophenylenic acid, since it may be viewed as a compound of two molecules of sulphuric acid, with the hypothetical hydrocarbon $C_6 H_4$ (phenylene), viz. $C_6 H_4 S_2 H_4 O_8$. Its formation may be thus expressed:—

$$\underbrace{C_6 H_4 N_2 S H_2 O_4}_{\text{Sulphate of diazobenzol.}} + S H_2 O_4 = \underbrace{C_6 H_4 S_2 H_4 O_8}_{\text{Disulphophenylenic acid.}} + N_2$$

The free acid is easily prepared by dissolving the barium-salt in water, carefully precipitating the barium with sulphuric acid, and concentrating the filtrate on a water-bath till it acquires a syrupy consistency; when it is placed over sulphuric acid, it crystallizes out in the form of warty crystals, which are exceedingly soluble in water and alcohol, and deliquesce in a moist atmosphere.

* It is not even requisite to heat the substance with nitric acid in sealed glass tubes; for the sulphur in the substance is entirely converted into sulphuric acid by the action of concentrated nitric acid at the ordinary pressure. The acid is converted into a crystallized nitro-acid, resembling picric acid.

Besides the compound just described, there exists yet another barium-compound of disulphophenylenic acid of the composition $C_6 H_4 S_2 Ba_3 H O_8$, formed by the exchange of a third equivalent of the hydrogen for barium. It is obtained by digesting either the barium-compound with two atoms of barium, or the free disulphophenylenic acid with baryta-water for some time. The excess of baryta is neutralized by carbonic acid, and the filtrate evaporated till it begins to crystallize. The new salt forms very thin white plates, which, on being left for some time in contact with the mother-liquor, are likewise converted into well-formed prisms.

This salt differs, moreover, from the salt containing only two equivalents of barium, by its greater solubility in water*, and the strong alkalinity of its solutions, which is not destroyed by the carbonic acid. When freshly prepared, the crystals of this compound are clear and transparent; they soon, however, lose a portion of their water of crystallization, and are reduced to a white powder. The substance employed for the following analyses was dried at 160° C., at which temperature the water of crystallization is rapidly given off.

I. 0·4518 grm. of substance gave 0·261 grm. of carbonic acid and 0·0396 grm. of water.
II. 0·3025 grm. gave 0·2225 grm. of sulphate of barium.

	Calculated.		Found.	
			I.	II.
C_6	72	15·18	15·75	——
H_5	5	1·05	0·97	——
S_2	64	13·48	——	——
Ba_3	205·5	43·31	——	43·24
O_8	128	26·98		
	474·5	100·00		

It is possible that two more barium-salts of disulphophenylenic acid exist, of the respective formulæ $C_6 H_4 S_2 Ba H_3 O_8$ and $C_6 H_4 S_2 Ba_4 O_8$.

Disulphophenylenic acid is likewise capable of combining in two proportions with other metals; with lead it seems to combine even more freely, forming apparently no less than five distinct salts, viz. $C_6 H_4 S_2 H_3 Pb O_8$, $C_6 H_4 S_2 H_2 Pb_2 O_8$; $C_6 H_4 S_2 H Pb_3 O_8$, $C_6 H_4 S_2 Pb_4 O_8$, and $C_6 H_4 S_2 Pb_4 O_8 + Pb_2 O$.

The description of the preparation and properties of these bodies will be reserved for a future communication. The silver-salt of disulphophenylenic acid, however, may find a place here, since it exhibits the peculiar chemical deportment of the new acid in a striking manner.

Disulphophenylenate of Silver.—This salt is obtained by treating an aqueous solution of the free acid with carbonate of silver, evaporating the filtrate first on the water-bath, and lastly over sulphuric acid. It crystallizes either in warty masses or in small plates.

* On preparing the salt $C_6 H_4 S_2 H_2 Ba_2 O_8$, as described above, with excess of carbonate of barium, a certain amount of the second barium-compound is formed, which remains in the mother-liquor from which the first salt has crystallized.

It is easily soluble in water, difficultly so in alcohol, and almost insoluble in ether. It does not contain any water of crystallization. After drying over sulphuric acid it does not lose weight, even when heated to upwards of 160° C.

I. 0·5535 grm. of the salt, dried at 160°, gave 0·3112 grm. of carbonic acid and 0·0443 grm. of water.

II. 0·9153 grm. gave 0·5133 grm. of carbonic acid and 0·0726 grm. of water.

III. 0·502 grm. gave 0·3082 grm. of chloride of silver.

IV. 0·4975 grm. gave 0·302 grm. of chloride of silver.

These numbers lead to the formula $C_6 H_4 S_2 Ag_2 O_7$.

	Theory.		Experiment.			
			I.	II.	III.	IV.
C_6	72	15·39	15·33	15·29	——	——
H_4	4	0·85	0·89	0·88	——	——
S_2	64	13·68	——	——	——	——
Ag_2	216	46·15	——	——	46·20	45·69
O_7	112	23·93	——	——	——	——
	468	100·00				

To judge from the composition of the silver-salt of disulphophenylenic acid, it appears to be bibasic ($C_6 H_4 S_2 H_2 O_7$), whilst the formula deduced from its barium and lead salts establishes its tetrabasic character ($C_6 H_4 S_2 H_4 O_8$). The new acid exhibits therefore the rare property of varying basicity, such as is possessed by phosphoric acid in the inorganic, and by terebinic acid in organic chemistry, as shown by EKMAN*.

Action of Sulphuretted Hydrogen upon the Gold-salt of Hydrochlorate of Diazobenzol.

By passing a current of sulphuretted hydrogen gas through cold water in which the gold-salt has been suspended, all the gold is converted into the trisulphide, whilst the diazobenzol is transformed into a volatile product. When the reaction is complete, the liquid is submitted to distillation, when a very nauseous, heavy, yellowish oil is found to pass over with the aqueous vapour. I have not pursued its examination further than to convince myself that it is not the phenylmercaptan ($C_6 H_6 S$) described by VOGT, which at first sight it appeared to be. Its mercaptanic odour and its derivation from the diazobenzol in the gold-compound (according to the equation

$$C_6 H_4 N_2 + SH_2 = C_6 H_6 S + N_2)$$

rendered this view very probable. Neither basic acetate of lead, however, nor nitrate of silver, even in the presence of ammonia, gave precipitates with the oil, which proved convincingly that it differed entirely from phenylmercaptan.

On examining the residue in the retort after the oil had been distilled off, it was found to consist of tersulphide of gold and an aqueous liquid; the latter is found to contain chloride of ammonium, together with a little free hydrochloric acid, and a small

* LIMPRICHT, Lehrbuch der Chemie, p. 1016.

quantity of the hydrochlorate of a base which is no other than aniline. The occurrence of these bodies shows that a small portion of the diazobenzol is decomposed as follows:—

$$C_6H_4N_2+3SH_2=C_6H_7N+NH_3+3S.$$

The same products of decomposition are met with when an alcoholic solution of the gold-salt is treated with sulphuretted hydrogen.

On passing the latter, however, over the dry gold-compound, it speedily causes an explosion. By employing only a very small quantity of the substance, and by spreading it in a thin layer in a glass tube, it is possible to avoid explosion and secure a quiet decomposition. The reaction is over when no more hydrochloric acid fumes escape with the current of sulphuretted hydrogen. The black residue which is left behind in the glass tube appears to contain, besides tersulphide of gold, free diazobenzol. I was not able to isolate this latter. The explosive nature of the residue and its deportment with ether, which speedily produces a rapid evolution of gas, leave, however, little doubt of its presence. Hydrochloric acid, tersulphide of gold, and diazobenzol are therefore the products of decomposition of the gold-salt by means of sulphuretted hydrogen.

Action of Nascent Hydrogen upon Diazobenzolimide.

If hydrogen is generated by means of zinc and sulphuric acid in an alcoholic solution of this body, a point is reached in a comparatively short time when, on the addition of water, no more turbidity occurs, indicating that the diazobenzolimide has completely disappeared.

By removing the excess of zinc and evaporating the alcohol on a water-bath, the residue, when treated with potassa, evolves much ammonia, and an oily base simultaneously separates; this is purified by distillation, and is found identical with ordinary aniline. The decomposition may be thus expressed:—

$$\underbrace{C_6H_5N_3}_{\text{Diazobenzolimide.}}+8H=\underbrace{C_6H_7N}_{\text{Aniline.}}+2NH_3.$$

Action of Carbonate of Barium upon Nitrate of Diazobenzol.

By treating a cold aqueous solution of this salt with levigated carbonate of barium, a feeble evolution of gas is observed, which lasts for several days. A reddish-brown mass is produced insoluble in water, which remains with the excess of carbonate of barium when the reaction is over. The residue is a mixture of two distinct bodies. By filtering off the solution containing nitrate of barium and removing the excess of carbonate by means of dilute hydrochloric acid, these two products can be readily separated by treatment with cold alcohol, in which they are very unequally soluble. In order to obtain the more soluble one in a pure state, the alcohol is evaporated and the residue treated with ammonia. An intensely yellow-coloured solution is produced, which must be filtered to remove a small quantity of a resinous substance, and decomposed with hydrochloric acid, when the new compound is precipitated in crystals. These are obtained perfectly pure for analysis by repeated crystallization from weak alcohol.

0·250 grm. of substance gave 0·6679 of carbonic acid and 0·1135 grm. of water, corresponding to 72·52 per cent. of carbon and 5·04 of hydrogen.

0·307 grm. gave 35·7 cub. centims. of nitrogen at 0° C. and 760 millims. bar. pressure, =0·04461 grm. of nitrogen, or 14·52 per cent.

The following formula is deduced from these numbers:—

$$C_{12}H_{10}N_2O.$$

	Calculated.		Found.
C_{12}	144	72·73	72·52
H_{10}	10	5·05	5·04
N_2	28	14·14	14·52
O	16	8·08	——
	198	100·00	

The formation of this compound may be expressed by the following equation:—

$$\underbrace{2(C_6H_4N_2, NHO_3)}_{\text{Nitrate of diazobenzol.}}+H_2O=\underbrace{C_{12}H_{10}N_2O}_{\text{New compound.}}+2NHO_3+2N.$$

It will be readily seen that this formula contains the elements of phenylic acid and diazobenzol *

$$C_6H_4N_2+C_6H_6O=C_{12}H_{10}N_2O,$$

and I will therefore call it phenol-diazobenzol without prejudging its rational constitution. This body usually crystallizes from alcohol and ether (in which it is very easily soluble) in brittle brownish-yellow warts. It is almost insoluble in cold water, slightly so in boiling water, from which, on cooling, it crystallizes in small yet well-formed rhombic prisms of a fine yellow colour, with a tinge of violet. These crystals fuse at 148° C. to a brownish-yellow oil, which cannot be volatilized without decomposition, and is destroyed at a higher temperature with formation of yellow vapour. Although phenol-diazobenzol has the properties of an acid, forming with certain metals saline compounds, its acid character is so slightly pronounced, that it is not even capable of decomposing carbonates. On evaporating a solution of phenol-diazobenzol with aqueous carbonate of potassium to complete dryness, the former will be left behind unaltered. Evaporated with aqueous ammonia, the whole of the ammonia is driven off. Treated with nitrate of silver, a scarlet-red silver compound precipitates, which appears specially suited to serve for the determination of the atomic weight of this compound. It deserves mention that phenol-diazobenzol is isomeric with azoxybenzide. The properties just described show conclusively, however, that it has nothing in common with the latter compound beyond the formula.

The second product of the above-mentioned reaction may usually be obtained per-

* Nitrosoethyline, recently described by Geuther and Kreutzhage, will probably come under the same class of compounds, since its composition may be expressed by the addition of the hypothetical diazohydride of ethyl and alcohol, viz.,

$$C_2H_4N_2+C_2H_6O=C_4H_{10}N_2O.$$

fectly pure by recrystallizing it once or twice from strong alcohol, afterwards dissolving it in ether, and allowing the solution to evaporate spontaneously. Sometimes, however, the crystals are slightly contaminated with traces of a body which has to be removed by means of caustic potassa*, in which the new compound is completely soluble, whilst the foreign substance remains behind as a brown resin. Hydrochloric acid precipitates it from the alkaline solution, and it can now be completely purified by crystallization from alcohol or ether.

I. 0·2155 grm. gave 0·564 grm. of carbonic acid and 0·096 grm. of water, corresponding to 71·31 per cent. of carbon and 4·95 of hydrogen.

II. 0·33 grm. gave 47·6 cub. centims. of nitrogen at 0° C. and 760 millims. bar. pressure, =0·059813 grm. of nitrogen, equal to 18·12 per cent.

III. 0·4605 grm. gave 66·5 cub. centims. of nitrogen at 0° C. and 760 millims. bar. pressure, =0·08356 grm. of nitrogen, equal to 18·14 per cent.

These numbers led to the formula

$$C_{18} H_{14} N_4 O.$$

	Calculated.		Found.		
			I.	II.	III.
C_{18}	216	71·52	71·38	——	——
H_{14}	14	4·63	4·95	——	——
N_4	56	18·55	——	18·12	18·14
O	16	5·30	——	——	——
	302	100·00			

Its formation is explained by the equation

$$\underbrace{3(C_6 H_4 N_2, NHO_3)}_{\text{Nitrate of diazobenzol.}}+H_2O=\underbrace{C_{18} H_{14} N_4 O}_{\text{New compound.}}+N_2+3NHO_3.$$

It may likewise be viewed as composed of phenol and diazobenzol, viz.

$$2C_6 H_4 N_2+C_6 H_6 O=C_{18} H_{14} N_4 O,$$

and I would therefore propose the name phenol-bidiazobenzol.

This new compound crystallizes in brownish-red needles or plates; it is readily soluble in ether, difficultly soluble in cold alcohol. Hot water dissolves it but very slightly. On heating, it deports itself like the compound previously described. It fuses at 113° C. Phenol-bidiazobenzol is an almost perfectly neutral body. Its deportment with potassa (in which it dissolves readily, forming a ruby-red solution) reminds one, however, of the properties of an acid. Ammonia-water dissolves it with difficulty, and aqueous carbonate of potassa not at all. Dilute acids exert likewise no solvent action; concentrated acids, however, dissolve it with a blood-red colour. Decomposition ensues when the latter solutions are heated.

* Instead of treating the mixture of the crude products of decomposition and carbonate of barium with hydrochloric acid in order to remove the latter, the new compounds may be extracted with potassa, then precipitated with hydrochloric acid, and lastly separated, as already described, by means of alcohol.

Action of Potassa upon Nitrate of Diazobenzol.

On mixing *diluted* aqueous solutions of these two bodies*, a yellow liquid is obtained, which possesses a peculiar aromatic odour and soon begins to evolve nitrogen gas, a reddish-brown neutral substance being simultaneously formed. At the common temperature this reaction is very slow, and requires several weeks to be completed. If, however, heat is employed, the decomposition proceeds rapidly, and the reddish-brown substance is separated as a resinous, semifluid mass, quite insoluble in water, and only very slightly soluble even in boiling alcohol. Ether dissolves it readily. I have not been able to obtain this substance in crystals. By allowing its ethereal solution to evaporate spontaneously it is left behind in a resinous state. As a powder it is very electric. Boiling with nitric acid produces a new yellowish crystalline body.

In order to purify the amorphous product of decomposition, I have first washed it thoroughly with water, then boiled with alcohol, and finally dissolved it in ether; the substance remaining after the evaporation of the ether was submitted to analysis.

I. 0·2958 grm., dried *in vacuo*, gave 0·8888 grm. of carbonic acid and 0·1447 grm. of water.

II. 0·220 grm. gave 0·6667 grm. of carbonic acid and 0·1082 grm. of water.

III. 0·3926 grm. gave 29·8 cub. centims. of nitrogen at 0° C. and 760 millims. pressure.

IV. 0·5054 grm. gave 32·0 cub. centims. of nitrogen at 0° C. and 760 millims. pressure.

These numbers agree best with the formula

$$C_{24} H_{18} N_2 O.$$

	Calculated.		Found.			
			I.	II.	III.	IV.
C_{24}	288	82·29	81·95	82·68	——	——
H_{18}	18	5·14	5·44	5·45	——	——
N_2	28	8·00	——	——	9·53	7·66
O	16	4·57	——	——	——	——
	350	100·00				

The formation of this substance can be expressed by the following equation:—

$$4(C_6 H_4 N_2, NHO_3)+H_2 O=C_{24} H_{18} N_2 O+4NHO_3+6N.$$

If instead of an aqueous, an alcoholic solution of potassa be added to the nitrate of diazobenzol, dissolved in water, the reaction which takes place is much more complicated. In this case, in addition to the reddish-brown body, two volatile substances are formed, viz. benzol and phenyl, $C_{12} H_{10}$, the hydrocarbon recently discovered by FITTIG†. When the reaction is conducted in a retort, on the application of heat the benzol passes over with the alcohol, and may be separated from it by the addition of water. The

* The compound of hydrate of potassium and diazobenzol (page 675) does not seem to be formed in this case, as I was only able to separate this substance when very *concentrated* solutions of the nitrate of diazobenzol and potassa had been employed in its preparation.

† Ann. der Chem. und Pharm. Bd. cxxi. p. 363.

phenyl being less volatile, is obtained after the whole of the alcohol has distilled over; it condenses into a crystalline mass in the receiver. Repeated recrystallizations from alcohol render it quite pure, in white plates, resembling naphtaline, and fusible at 70° C. These properties leave no doubt as to its identity with the phenyl of Dr. FITTIG, to whom I am indebted for a small portion of his substance, which resembles in every respect the body obtained as mentioned above.

The third product of the decomposition in question, viz. the amorphous brown substance, is left behind in the retort as a resinous mass.

From the foregoing observations it will be seen that potassa, in the presence of alcohol, causes simultaneously three different decompositions of the diazobenzol-molecule, which may be expressed as follows:—

$$\text{I.}\quad \underbrace{C_6H_4N_2}_{\text{Diazobenzol.}} + \underbrace{C_2H_6O}_{\text{Alcohol.}} = \underbrace{C_6H_6}_{\text{Benzol.}} + \underbrace{C_2H_4O}_{\text{Aldehyd.}} + N_2.$$

$$\text{II.}\quad 2C_6H_4N_2 + C_2H_6O = \underbrace{C_{12}H_{10}}_{\text{Phenyl.}} + C_2H_4O + N_4.$$

$$\text{III.}\quad 4C_6H_4N_2 + H_2O = \underbrace{C_{24}H_{18}N_2O}_{\text{Reddish-brown substance.}} + N_6.$$

Action of Ammonia upon aqueous Nitrate of Diazobenzol.

On adding diluted ammonia to an aqueous solution of this diazo-compound, a similar reaction takes place as when the potassa is employed. On treating the brown mass, however, with alcohol, it becomes evident that it consists of two bodies. The difficultly soluble portion is absolutely identical with the nitrogenous body previously described, as was proved by the following analysis:—

0·245 grm. gave 0·744 grm. of carbonic acid and 0·1237 grm. of water. Carbon = 82·75, and hydrogen 5·60 per cent.

$C_{24}H_{18}N_2O$.

	Calculated.	Found.
C	82·29	82·75
H	5·14	5·60

The body accompanying it, and which is readily soluble in alcohol, is obtained, by repeated crystallization, in the form of small, light-yellow plates which detonate on heating, and whose alcoholic solution gives precipitates with nitrate of silver and bichloride of platinum. This compound is, in fact, no other than the diazo-amidobenzol previously mentioned. The coincidence with this latter was so complete, that it appeared to me loss of time to analyze it. With regard to the first of the two products of decomposition, it is clear that its formation must be expressed by the same equation which illustrated the reaction with caustic potassa upon nitrate of diazobenzol, whilst the formation of the latter (the diazo-amidobenzol) is explained by assuming that a portion

of the original compound is decomposed, in the following manner:—

$$\underbrace{2(C_6H_4N_2, NHO_3)}_{\text{Nitrate of diazobenzol.}}+3NH_3=\underbrace{C_{12}H_{11}N_3}_{\text{Diazo-amidobenzol.}}+N_2+2(NH_4, NO_3).$$

Two atoms of nitrogen in two equivalents of diazobenzol have, therefore, been simply replaced by one of ammonia.

Decomposition of the Platinum-salt of Diazobenzol, and of the Perbromide of Diazobenzol, by the action of heat.

When speaking of the platinum-salt, I had occasion to mention that it detonates when heated. By mixing it, however, with a large excess of perfectly dry carbonate of sodium *, and heating the whole in a retort on a sand-bath, it is quietly decomposed. The decomposition begins at a moderate heat, and is marked at first by the evolution of gas, and subsequently by the distillation of an oily body. The residue in the retort consists of carbonate of sodium, metallic platinum, and chloride of sodium. The oily distillate contains chlorine. It is obtained perfectly pure by distilling once with chloride of calcium, and forms an almost colourless oil, which is heavier than water and has the odour of benzol. These properties, as well as the chlorine determination, which was made by igniting the substance with caustic lime, prove that it is chloro-benzol, C_6H_5Cl †.

0·205 grm. gave 0·268 grm. of chloride of silver.

	C_6H_5Cl. Calculated.	Found.
Chlorine . .	31·56	32·34

The formation of chlorobenzol may be expressed by the following equation:—

$$\underbrace{C_6H_4N_2, HCl, PtCl_2}_{\text{Platinum-salt.}} = \underbrace{C_6H_5Cl}_{\text{Chlorobenzol.}}+Pt+Cl_2+N_2.$$

The platinum-salt of the bromide of diabenzol ‡ is decomposed in an exactly similar manner. The resulting bromobenzol differs in no way from the bromobenzol obtained by COUPER by the action of bromine upon benzol, as may easily be shown by converting it into nitrobromobenzol, which possesses all the properties of nitrobromobenzol prepared from coal-tar oil.

Bromobenzol can also be produced by the decomposition of perbromide of diazobenzol by heat, according to the equation

$$\underbrace{C_6H_4N_2, HBr, Br_2}_{\text{Perbromide of diazobenzol.}}=\underbrace{C_6H_5Br}_{\text{Bromobenzol.}}+N_2+Br_2.$$

* It is self-evident that other carbonates, such as carbonate of barium, calcium, &c., may be used instead of carbonate of sodium.

† In all probability it is identical with chlorobenzol obtained from benzol or phenol.

‡ This compound is obtained as an insoluble reddish-yellow precipitate, on mixing bibromide of platinum with an aqueous solution of nitrate of diazobenzol.

In order to decompose larger quantities of the perbromide of diazobenzol in this manner, it is likewise requisite to mix it first with a sufficient quantity of carbonate of sodium, to avoid a violent explosion.

By heating the mixture in a retort bromobenzol is obtained almost perfectly pure. The perbromide, when heated with alcohol, likewise gives rise to a decomposition in accordance with the previous equations. Bromobenzol separates as a heavy oil on the addition of water to the alcoholic solution.

It deserves to be mentioned that all these reactions are very well defined, and that the amount of the products of decomposition corresponds almost theoretically with the quantities employed.

Appendix.

It may be of some interest to mention an experiment I made to obtain ethylated diazobenzol compounds. For this purpose I submitted nitrate of ethylaniline to the same reaction which produced from nitrate of aniline the nitrate of diazobenzol. I obtained a body crystallizing, like the latter, in long needles. If this body had really been nitrate of ethyldiazobenzol, I expected to obtain, by boiling with water, a reaction according to the equation

$$\underbrace{C_6H_3(C_2H_5)N_2,\ NHO_3}_{\text{Nitrate of ethyldiazobenzol.}}+H_2O=\underbrace{C_6H_5(C_2H_5)O}_{\text{Ethylphenylic acid.}}+NHO_3+N_2.$$

It soon became evident, however, that the oily body produced was nothing else than ordinary phenylic acid; and since no other organic product of decomposition could be traced, I had to come to the conclusion that the above-mentioned crystals were nothing else than ordinary nitrate of diazobenzol. In order to decide this question the gold-salt was prepared, and, after being purified by recrystallization from alcohol, the well-known golden, brilliant crystals were obtained, which gave, on analysis—

0·759 grm., decomposed by sulphuretted hydrogen and ignition of the tersulphide of gold, left 0·337 grm. of gold.

$$C_6H_4N_2,\ HCl,\ AuCl_3.$$

	Calculated.	Found.
Gold . . .	44·37	44·40

The compound $C_6H_3(C_2H_5)N_2,\ HCl,\ AuCl_3$, requires 41·74 per cent. of gold.

The action of nitrous acid upon nitrate of ethylaniline may therefore be expressed by the equation

$$\underbrace{C_6H_6(C_2H_5)N,\ NHO_3}_{\text{Nitrate of ethylaniline.}}+NHO_2=\underbrace{C_6H_4N_2,\ NHO_3}_{\text{Nitrate of diazobenzol.}}+\underbrace{C_2H_6O}_{\text{Alcohol.}}+H_2O.$$

PART II.

The peculiar and somewhat remarkable properties of the compounds derived from aniline in the manner previously described, have induced me to try whether bromaniline, nitraniline, &c., when similarly treated, would be converted into the corresponding substituted diazobenzols.

This I have succeeded in doing, and the new substances thus obtained exhibited all the properties which I found so characteristic of the normal diazobenzol-compounds. They are, if anything, more stable, owing, if I may so express myself, to the comparatively larger amount of more solid and stable materials which are more firmly combined with the extremely volatile and easily disturbed nitrogen. This property renders them more fit for many experiments in which the non-substituted diazobenzol-compounds are liable to give rise to dangerous accidents. They are likewise remarkable for great beauty, a property which certainly encourages a closer acquaintance.

Nitrate of Diazobromobenzol, $C_6 H_3 Br N_2, NHO_3$.

This compound may be prepared either by the action of nitrous acid gas upon nitrate of bromaniline or upon diazo-amidobromobenzol, and in an exactly similar manner to the nitrate of diazobenzol. If an aqueous solution of nitrate of bromaniline be employed, nitrous acid gas must be passed through very rapidly at first, or else diazo-amidobromobenzol (even in the presence of much free nitric acid) begins to separate. This it is very difficult to convert into the desired compound in an aqueous solution. Nitrate of diazobromobenzol remaining comparatively constant in aqueous solutions, it is possible to concentrate them, without any great loss, by spontaneous evaporation in the open air. In this manner solutions which are too dilute may be concentrated before precipitating by means of alcohol and ether. It can be obtained perfectly pure by repeatedly dissolving in alcohol and precipitating with ether, when it is obtained in the form of purely white scales which, when crystallizing out rapidly, present themselves in the form of regular rhombic plates. These crystals, like those of nitrate of diazobenzol, are exceedingly soluble in water, difficultly so in strong alcohol, and almost insoluble in ether. They explode when they are heated, struck, or compressed, though not so readily nor with the same violence as the nitrate of diazobenzol, and they can therefore be mixed without danger with oxide of copper and burnt in the usual mode of organic analysis.

0·4692 grm. of the substance gave 0·503 grm. of carbonic acid and 0·0745 grm. of water. These numbers correspond with the formula

$$C_6 H_3 Br N_2, NHO_3.$$

	Calculated.		Found.
C_6	72	29·27	29·24
H_4	4	1·63	1·76
Br	80	32·53	——
N_3	42	17·07	——
O_3	48	19·50	——
	246	100·00	

The formation of nitrate of diazobromobenzol may be represented by the following equations:—

$$\text{I.}\quad \underbrace{C_6H_6BrN,\ NHO_3}_{\text{Nitrate of bromaniline.}} + HNO_2 = \underbrace{C_6H_3BrN_2,\ NHO_3}_{\text{Nitrate of diazobromobenzol.}} + 2H_2O.$$

$$\text{II.}\quad \underbrace{C_{12}H_9Br_2N_3}_{\text{Diazoamidobromobenzol.}} + HNO_2 + 2NHO_3 = 2(\underbrace{C_6H_3BrN_2,\ NHO_3}_{\text{Nitrate of diazobromobenzol.}}) + 2H_2$$

Sulphate of Diazobromobenzol, $C_6H_3BrN_2$, SH_2O_4.

The preparation of this compound from nitrate of diazobromobenzol and sulphuric acid corresponds so closely with that of the non-substituted (abromous) sulphate, that it may suffice simply to refer to the description already given of the preparation of the latter, and I will therefore restrict myself to a few remarks on its properties.

Sulphate of diazobromobenzol crystallizes in very fine colourless prisms, which are very soluble in water, very difficultly soluble in alcohol, and almost insoluble in ether. The new body is comparatively stable, and can be crystallized from water without suffering the least decomposition, by allowing its solution to evaporate over sulphuric acid. Boiling water decomposes the compound, and heat causes it to explode. Its formula has been established by a determination of the sulphuric acid only.

0·377 grm. gave 0·315 grm. of sulphate of barium.

	Calculated.		Found.
$C_6H_3BrN_2$	183	65·12	——
SH_2O_4	98	34·88	35·15
	281	100·00	

Hydrobromate of Diazobromobenzol, $C_6H_3BrN_2$, HBr.

This compound is prepared either by decomposing an aqueous solution of the previous salt by means of a sufficient quantity of bromide of barium and spontaneous evaporation of the filtrate, or by the action of an ethereal solution of bromine upon an ethereal solution of diazo-amidobromobenzol. If prepared by this latter method the new compound speedily separates in crystals, on account of its insolubility in ether, and is obtained pure by filtering off from the mother-liquor and washing the crystals with ether. Its formation may be expressed by the following equation:—

$$\underbrace{C_{12}H_9Br_2N_3}_{\text{Diazoamidobromo-benzol.}} + 4Br = \underbrace{C_6H_3BrN_2HBr}_{\text{Hydrobromate of diazo-bromobenzol.}} + \underbrace{C_6H_4Br_3N}_{\text{Tribromaniline.}} + HBr.$$

The hydrobromate of diazobromobenzol forms pearly white shining scales which dissolve very readily in water; like the hydrobromate of diazobenzol, they are more difficulty soluble in alcohol and quite insoluble in ether. In the dry state it can be preserved a long time without undergoing decomposition. On heating, it explodes

almost as violently as the corresponding nitrate. By treating an aqueous solution with freshly precipitated chloride of silver, the hydrobromate is converted into the hydrochlorate of diazobenzol.

0·5315 grm. of substance, precipitated with nitrate of silver, gave 0·379 grm. of bromide of silver.

	Calculated.	Found.
$C_6H_3BrN_2$	69·32	——
HBr	30·68	30·72
	100·00	

Perbromide of Diazobromobenzol, $C_6H_3BrN_2$, HBr, Br_2.

By treating an aqueous solution* of any one of the previously-described diazobromobenzol-compounds with excess of bromine-water, a crystalline orange precipitate speedily falls, increasing rapidly till all the diazobromobenzol has been precipitated. If too much bromine has been added the precipitate becomes generally of an oily consistency, solidifying, however, to a yellow crystalline mass as soon as the mother-liquor has been removed and the excess of bromine allowed to evaporate spontaneously. In order to obtain the perbromide, thus prepared, in fine crystals, it is dissolved in the smallest possible quantity of warm, not boiling, alcohol, from which it separates on cooling in yellow monoclinic prisms. A small portion only remains in the alcoholic mother-liquor, from which evaporation rarely recovers it, since it usually undergoes decomposition. It will, however, be seldom necessary to run this risk of losing part of the substance by recrystallization, for the compound is almost perfectly pure from the very first, and at all events quite fit to be employed for the experiments to be described further on.

Under certain circumstances this perbromide is formed during the preparation of hydrobromate of diazobromobenzol from diazo-amidobromobenzol, in which case the two compounds are readily separated from one another by washing with cold water. Perbromide of diazobromobenzol crystallizes, as has been already remarked, in orange monoclinic prisms, insoluble in water, freely soluble in warm, difficultly so in cold alcohol, and very difficultly soluble in cold ether.

On boiling an alcoholic solution decomposition ensues, and on heating the crystals alone they explode feebly, with disengagement of bromine vapour and nitrogen gas.

Analysis gave the following results:—

0·558 grm. of substance gave 0·304 grm. of carbonic acid and 0·054 grm. of water.

0·4551 grm., ignited with caustic lime, gave with nitrate of silver 0·8135 grm. of bromide of silver.

These numbers lead to the formula

$$C_6H_3BrN_2HBr_3.$$

* The crude aqueous solution obtained by the action of nitrous acid upon the nitrate of bromaniline may conveniently be employed.

	Theory.		Experiment.
C_6	72	16·98	17·11
H_4	4	0·94	1·08
N_2	28	6·60	——
Br_4	320	75·48	76·06
	424	100·00	

Platinum-salt of the Hydrochlorate of Diazobromobenzol, $C_6H_3BrN_2$, HCl, $PtCl_2$.

Bichloride of platinum, even from a very dilute solution of the nitrate or sulphate of diazobromobenzol, separates a mass of small yellow crystals, which appear under the microscope as fractured plates, and which are almost insoluble in every neutral solvent. They are stable at 100° C.

0·568 grm. gave, on ignition with carbonate of sodium, 0·1447 grm. of platinum.

$C_6H_3BrN_2$, HCl, $PtCl_2$.

	Calculated.	Found.
Platinum . .	25·36 per cent.	25·47

Gold-salt of the Hydrochlorate of Diazobromobenzol, $C_6H_3BrN_2$, HCl, $AuCl_3$.

This compound is precipitated from an aqueous solution of nitrate of diazobromobenzol on the addition of terchloride of gold, at first as a yellow oil, which, however, rapidly solidifies to a crystalline mass. The crystals are insoluble in water, but can be recrystallized from warm alcohol without much loss, and are thus obtained in the form of splendid small golden-yellow shining plates.

0·469 grm. of substance, dissolved in alcohol and decomposed with sulphuretted hydrogen, gave, after ignition of the tersulphide of gold, 0·1763 grm. of gold.

$C_6H_3BrN_2$, HCl, $AuCl_3$.

	Calculated.	Found.
Gold . . .	37·67 per cent.	37·59

Compound of Hydrate of Potassium with Diazobromobenzol, $C_6H_3BrN_2$, KHO.

On the addition of caustic potash to a concentrated solution of nitrate of diazobromobenzol a lemon-yellowish precipitate of diazobromobenzol separates at first, but is soluble in excess of potash. On evaporation by means of a water-bath, the solution solidifies, when sufficiently concentrated, to a reddish-coloured crystalline paste of nitre and the new compound of hydrate of potassium and diazo-bromobenzol.

The latter is separated and purified exactly in a similar manner to the analogous compound of diazobenzol. It differs from this compound in being precipitated from its alcoholic solution by means of ether as a white gelatinous mass, and not in the form of crystals. White plates are obtained by allowing its aqueous solution to evaporate on a

watch-glass over sulphuric acid, which, however, turn red on keeping, owing to a partial decomposition.

Compound of Hydrate of Silver with Diazobromobenzol, $C_6 H_3 Br N_2, Ag HO$.

This body is obtained as an almost white insoluble precipitate, very similar to the hydrate of silver with diazobenzol. I abstain from describing any more of the compounds of diazobromobenzol with metallic hydrates, since they entirely resemble the corresponding diazobenzol-compounds in every respect.

Diazobromobenzol, $C_6 H_3 Br N_2$.

This compound is obtained in slender, bright-yellow needles by adding weak acetic acid to the compound of hydrate of potassium with diazobromobenzol, or as a bright yellow amorphous precipitate on the addition of dilute potassa to the aqueous solution of the nitrate of diazobromobenzol. In either case it is necessary to remove it speedily from the mother-liquor, and to dry it rapidly over sulphuric acid.

Diazobromobenzol is an exceedingly dangerous compound; for the slightest pressure, or even touch with a rough object, causes it to go off in a fiery explosion almost exceeding in violence that of the nitrate of diazobenzol. Although much more stable than diazobenzol, it can, however, be kept only for a short time in a perfectly unchanged condition. After being kept for some time, a reddish-brown residue is left, which no longer explodes even on heating, and it seems that the nitrogen has been gradually eliminated. Ether dissolves diazobromobenzol and causes a violent evolution of gas, frequently of such intensity as to give rise to explosions. When freshly prepared, this body is soluble in caustic potassa as well as in mineral acids, with formation of the previously-described saline bodies.

COMPOUNDS OF DIAZOBROMOBENZOL WITH AMIDO-COMPOUNDS.

Diazobromobenzol, like diazobenzol, can enter into combination with amido-bases and amido-acids. Since, however, these new bodies possess absolutely no fresh chemical interest, but resemble in every respect the amido-compounds of diazobenzol, it may suffice to mention a few only, as briefly as possible.

Diazo-amidobromobenzol, $\left\{\begin{matrix} C_6 H_3 Br N_2 \\ C_6 H_6 Br N \end{matrix}\right\}$,

separates in yellow needles or small plates when a concentrated aqueous solution of nitrate of diazobromobenzol is treated with an alcoholic solution of bromaniline. This compound was obtained by me on a former occasion* by the action of nitrous acid upon alcoholic aniline.

It needs no further proof that both methods furnish a product of identical properties.

* Annalen der Chem. und Pharm. Bd. cxxi. p. 273.

The action of aniline upon nitrate of diazobromobenzol gives rise to diazobromo-amidobenzol, which not only has the same empirical composition as the diazobenzol-amidobromobenzol ($C_{12}H_{10}BrN_3$) (page 678), but resembles it in every other respect. It would, no doubt, be of interest to determine whether both compounds are identical or only isomeric. I must, however, defer answering this question till a future opportunity.

Diazobromobenzol-amidobenzoic acid, $\left\{\begin{matrix} C_6H_3BrN_2 \\ C_7H_7NO_2 \end{matrix}\right\}$.

This body is obtained as a yellow crystalline precipitate on mixing an aqueous solution of nitrate of diazobromobenzol and amidobenzoic acid, according to the equation

$$\underbrace{C_6H_3BrN_2NHO_3}_{\text{Nitrate of diazobromo-benzol.}}+\underbrace{2C_7H_7NO_2}_{\text{Amidobenzoic acid.}}=\underbrace{\left\{\begin{matrix} C_6H_3BrN_2 \\ C_7H_7NO_2 \end{matrix}\right\}}_{\text{Diazobromobenzol-amidobenzol.}}+\underbrace{C_7H_7NO_2,\ NHO_3}_{\text{Nitrate of amidobenzoic acid.}}.$$

Recrystallized from ether it forms roundish lumps of small needles or plates. In every other respect it is identical with diazobenzol-amidobenzoic acid.

IMIDOGEN COMPOUNDS OF DIAZOBROMOBENZOL.

Diazobromobenzolimide, $\left.\begin{matrix} (C_6H_3BrN_2)'' \\ H \end{matrix}\right\}N$.

On mixing the yellow crystals of perbromide of diazobromobenzol with solution of ammonia, they are speedily converted into a yellowish oil, which, after a single distillation with water, is obtained in an almost colourless condition. This oil is diazobromobenzolimide in a perfectly pure state.

If the temperature of the atmosphere be not too high, the oil usually solidifies after a short time. If this does not take place, artificial cold must be resorted to. The new compound is obtained for analysis by removing the water and drying over sulphuric acid.

0·432 grm. of substance gave 0·5803 grm. of carbonic acid and 0·0958 grm. of water.

0·3213 grm. gave 53·8 cub. centims. of nitrogen at 0° C. and 760 millims. barom. pressure = 0·06704 grm. of this gas.

	Calculated.		Found.
C_6	72	36·36	36·63
H_4	4	2·02	2·45
Br	80	40·40	——
N_3	42	21·22	21·04
	198	100·00	

The formation of this compound is expressed by the following equation:—

$$\underbrace{C_6H_3BrN_2HBr_3}_{\text{Perbromide.}}+4NH_3=\underbrace{\left.\begin{matrix}C_6H_3BrN_2\\H\end{matrix}\right\}N}_{\text{Diazobromobenzolimide.}}+3NH_4Br.$$

The quantity of diazobromobenzolimide and bromide of ammonium which is obtained from a given quantity of perbromide corresponds accurately with the theoretical proportions.

0·8305 grm. of perbromide of diazobromobenzol, after decomposition with ammonia, gave with nitrate of silver 1·0945 grm. of bromide of silver, corresponding to 56·1 per cent. of bromine.

According to the above equation, one equivalent of the perbromide gives rise to the production of three equivalents of bromide of ammonium; the quantity of bromine capable of precipitation after the action of ammonia will therefore be 56·6 per cent.

Diazobromobenzolimide presents itself generally as a white or slightly yellowish mass of small crystalline plates, which melt at about 20° C. to a heavy oil. It is insoluble in water, rather difficultly soluble in alcohol, and easily soluble in ether and benzol. It distils readily in the presence of water; heated alone it explodes feebly. Left exposed to the open air it appears gradually to volatilize, giving off the same aromatic ammoniacal odour which characterizes diazobenzolimide. It resembles the latter, moreover, in its behaviour with various reagents; caustic potassa, hydrochloric acid, and bromine have no action upon it, strong sulphuric and nitric acids decompose it readily.

By the action of ethylamine, aniline, &c. upon perbromide of diazobromobenzol, corresponding substitution-compounds of diazobromobenzolimide are obtained.

Ethyldiazobromobenzolimide, $\left.\begin{matrix}(C_6H_3BrN_2)''\\C_2H_5\end{matrix}\right\}N$, forms a yellowish oil which does not solidify even when cooled much below 0° C.

Phenyldiazobromobenzolimide, $\left.\begin{matrix}(C_6H_3BrN_2)''\\C_6H_5\end{matrix}\right\}N$, is obtained in the form of orange crystals.

REMARKS ON THE PRODUCTS OF DECOMPOSITION OF THE COMPOUNDS OF DIAZOBROMOBENZOL.

The great similarity existing between the properties of the diazobromobenzol-compounds and those of the corresponding abromous bodies, is likewise encountered in their products of decomposition obtained under similar circumstances; and I have therefore generally abstained from verifying the latter by analysis, having restricted myself to bringing forward analytical numbers in a few cases only.

On heating an aqueous solution of nitrate or hydrobromate of diazobromobenzol,

an evolution of nitrogen gas takes place, bromophenylic acid (which separates as a brownish oil of the odour of creosote) being simultaneously produced.

$$\underbrace{C_6 H_3 Br N_2, H Br}_{\text{Hydrobromate of diazo-bromobenzol.}} + H_2 O = \underbrace{C_6 H_5 Br O}_{\text{Bromophenylic acid.}} + HBr + N_2.$$

Ebullition of nitrate of diazobromobenzol with alcohol gives rise to the formation of bromobenzol, which distils over with the vapour of alcohol, a yellow acid (probably bromodinitrophenylic) being left behind. Ammonia forms a yellow body with the simultaneous production of diazo-amidobromobenzol, which, owing to its greater solubility in alcohol, can be readily separated from the former. The formation of diazo-amidobromobenzol takes place according to the formula

$$\underbrace{2 C_6 H_3 Br N_2}_{\text{Diazobromobenzol.}} + NH_3 = \underbrace{C_{12} H_9 Br_2 N_3}_{\text{Diazo-amidobromobenzol.}} + N_2.$$

Sulphate of diazobromobenzol, when heated with sulphuric acid, is converted into a sulpho-acid, which is most probably disulphobromophenylenic acid, $C_6 H_3 Br, S_2 H_4 O_8$.

Nascent hydrogen, generated by the action of zinc upon sulphuric acid, in the presence of an alcoholic solution of bromobenzolimide converts the latter into bromaniline and ammonia.

$$\underbrace{C_6 H_4 Br N_3}_{\text{Diazobromoben-zolimide.}} + 8H = \underbrace{C_6 H_6 Br N}_{\text{Bromaniline.}} + 2H_3 N.$$

When the addition of water no longer gives rise to a precipitate, indicating that the whole of the imide has been decomposed, the bromaniline is most easily isolated by evaporating the alcoholic solution on a water-bath, and distilling with caustic potassa.

Bromaniline passes over in oily drops, which quickly solidify, and may be crystallized from alcohol in octahedra. These crystals, as well as the properties of the platinum-salt, prove it to be identical with the ordinary bromaniline.

0·2584 grm. of the latter left on ignition 0·0661 grm. of platinum, corresponding to 26·5 per cent.

$C_6 H_6 Br N, HCl, Pt Cl_2$ requires 26·1 per cent.

By heating the platinum-salt of diazobenzol with carbonate of sodium, chlorobenzol was obtained; so in like manner by the action of heat on the platinum-salt of hydrochlorate of diazobromobenzol, chlorobromobenzol is produced, according to the following equation:—

$$\underbrace{C_6 H_3 Br N_2, HCl, Pt Cl_2}_{\text{Platinum-salt of diazobromo-benzol.}} = \underbrace{C_6 H_4 Br Cl}_{\text{Chlorobro-mobenzol.}} + Pt + Cl_2 + N_2.$$

By pressing the crystals which condense in the neck of the retort between filter paper, and after a second distillation and crystallization from alcohol, they are obtained sufficiently pure for analysis.

0·441 grm. of substance gave 0·6072 grm. of carbonic acid and 0·0832 grm. of water.

These numbers lead to the formula C_6H_4BrCl.

	Calculated.		Found.
C_6	72	37·60	37·52
H_4	4	2·09	2·10
Br	80	41·77	——
Cl	35·5	18·54	——
	191·5	100·00	

Chlorobromobenzol is rather difficultly soluble in alcohol, readily so in ether, and crystallizes in white needles or plates from a hot saturated alcoholic solution on cooling, or by evaporation of the ethereal solution. The crystals possess a peculiar odour, resembling that of benzol.

The double salt of dibromide of platinum and hydrobromate of diazobromobenzol is obtained in the form of ruby-red crystals, by mixing a tolerably concentrated aqueous solution of the diazo-salt with dibromide of platinum. This compound shows, as might be expected, a similar deportment. Like the previous platinum-salt, it breaks up according to the equation

$$\underbrace{C_6H_3BrN_2HBr, PtBr_2}_{\text{Platinum-salt of the hydrobromate of diazobromobenzol.}}=\underbrace{C_6H_4Br_2}_{\text{Dibromobenzol.}}+Br_2+N_2+Pt.$$

Dibromobenzol is readily prepared also from the perbromide of diazobromobenzol by heating the latter alone*, or with carbonate of sodium, when it is decomposed according to the equation

$$\underbrace{C_6H_3BrN_2HBr_3}_{\text{Perbromide of diazobromobenzol.}}=\underbrace{C_6H_4Br_2}_{\text{Dibromobenzol.}}+Br_2+N_2.$$

A still more convenient method of preparing dibromobenzol consists in the decomposition of the perbromide with alcohol, which is completed after a few minutes' boiling; and if too much alcohol has not been employed, a large portion of the dibromobenzol separates at once in the form of crystals. The remaining portion is precipitated on the addition of water, in the form of a thick oil, which soon solidifies to a crystalline mass. After washing with a little alcohol and pressing between bibulous paper, this mass, together with the crystals first precipitated, is subjected to distillation. Dibromobenzol distils over as an almost colourless oil, which speedily solidifies.

Dibromobenzol resembles very much chlorobromobenzol in its various physical properties. It crystallizes from ether in the form of rectangular prisms or small plates, which are frequently very regular and well formed, sometimes, however, agglomerated in various ways. It fuses at 89° C.

* Only small quantities must be employed, in order to prevent violent explosions. By heating the perbromide in a long-necked flask the dibromobenzol condenses in the cold part of the vessel.

The properties of the dibromobenzol prepared in the manner just described can leave no doubt that it is identical with the dibromobenzol described by COUPER, obtained by the action of bromine upon benzol. Although large quantities of dibromobenzol can be prepared by COUPER'S method with perhaps greater facility, it is always difficult to obtain the dibromo- quite free from tribromo-benzol, which is simultaneously formed.

The above method might be preferred in all cases when absolutely pure dibromobenzol is required, as, for instance, for certain physical purposes—the more so, since it invariably yields theoretical quantities. The same may be said of several other derivatives of benzol and its homologues, which are obtained from diazo-compounds to be described hereafter.

DIAZODIBROMOBENZOL COMPOUNDS.

Nitrate of Diazodibromobenzol, $C_6 H_2 Br_2 N_2$, NHO_3.

This compound is readily obtained by the action of a rapid current of nitrous acid upon an aqueous solution of nitrate of bromaniline containing free nitric acid. The liquid is allowed to evaporate spontaneously, the residue taken up with weak alcohol, and the new compound precipitated by means of ether. It can be recrystallized without loss from water or alcohol by evaporating the respective solutions below their boiling-points. Its aqueous solution remains remarkably constant. Continued boiling even for hours frequently leaves some of the substance undecomposed.

Nitrate of diazodibromobenzol crystallizes in fine white prisms, or elongated hexagonal plates. It does not detonate with the same violence as the corresponding bodies previously described.

Perbromide of Diazodibromobenzol, $C_6 H_2 Br_2 N_2$, HBr_3.

This compound is formed by the addition of bromine-water to an aqueous solution of the nitrate of diazodibromobenzol, when it is precipitated in the form of fine long needles. On boiling with alcohol, and the subsequent addition of water, an oily substance is thrown down which soon solidifies, and can be obtained perfectly pure by pressing the precipitate between bibulous paper, then distilling, and finally crystallizing from alcohol. Thus purified it presents itself in the form of fine long silky needles. This substance is evidently tribromobenzol, presenting all the properties of that compound obtained by the distillation of $C_6 H_6 Br_6$ with alkalies by LASSAIGNE*.

Platinum-salt of the Hydrochlorate of Diazodibromobenzol, $C_6 H_2 Br_2 N_2$, HCl, Pt Cl_2.

It separates in small orange oval plates, which are difficultly soluble in water, on adding dichloride of platinum to the hydrochlorate.

0·722 grm. gave 0·154 grm. of platinum:

$C_6 H_2 Br_2 N_2$, HCl, Pt Cl_2.

	Calculated.	Found.
Platinum	21·08 per cent.	21·33

* Rev. Scient. vol. v. p. 360.

Diazodibromobenzolimide, $\left.\begin{matrix}(C_6H_2Br_2N_2)'' \\ H\end{matrix}\right\}N.$

This compound is easily obtained in crystals by mixing the perbromide,

$$C_6H_2Br_2N_2, HBr_3,$$

with ammonia. Repeated recrystallization from hot alcohol yields it in the form of white needles, which fuse at 62° C., and which detonate slightly at a higher temperature. It is very little soluble in water, more soluble in hot alcohol, and very readily so in ether.

DIAZOCHLOROBENZOL COMPOUNDS.

Perfect analogy exists between these bodies, as well as between the diazodichlorobenzol compounds, and the compounds I have just described, with regard not only to the modes of preparation from chlorine and dichloraniline, but also in respect to their various physical properties. I therefore abstain from entering into a minute description, and will simply enumerate some few experiments, which I hope will satisfactorily prove their great similarity.

Nitrate of Diazochlorobenzol, $C_6H_3ClN_2, NHO_3$.

This substance crystallizes in small white plates, which yield on boiling with water chlorophenylic acid in the form of a brownish oil possessing the odour of creosote.

Perbromide of Diazochlorobenzol, $C_6H_3ClN_2, HBr_3$,

forms yellow columns which are decomposed in boiling alcohol with formation of bromochlorobenzol, the composition of which is thus expressed, C_6H_4ClBr*.

0·2005 grm. of the perbromide was decomposed with ammonia, and when precipitated with nitrate of silver gave 0·2952 grm. of bromide of silver.

$$C_6H_3ClN_2, HBr_3.$$

	Calculated.	Found.
Bromine	63·24	62·64

Platinum-salt of the Hydrochlorate of Diazochlorobenzol, $C_6H_3ClN_2, HCl, PtCl_2$, forms fine yellow needles. On heating with carbonate of sodium it yields dichlorobenzol, which is obtained, according as it crystallizes slowly or rapidly, in long fine needles, or in elongated four-sided plates, possessing the same peculiar aromatic odour as the dibromobenzol.

Diazochlorobenzolimide, $\left.\begin{matrix}(C_6H_3ClN_2)'' \\ H\end{matrix}\right\}N$, forms readily fusible crystals.

* I leave it undecided whether the bromochlorobenzol thus obtained is identical with the body described on page 702 as derived from $C_6H_3BrN_2$, HCl, $PtCl_2$, and possessing the same elementary composition. Some observations, which shall be fully described when treating of the products of decomposition of diazonitrobenzol, do not favour the view of their identity.

Diazochlorobenzol, $C_6 H_3 Cl N_2$, is obtained as a highly explosive lemon-yellow precipitate.

DIAZODICHLOROBENZOL COMPOUNDS.

Nitrate of diazodichlorobenzol presents itself in the form of white plates. The perbromide, $C_6 H_2 Cl_2 N_2$, HBr_3, which is formed by the action of bromine-water, crystallizes in yellow prisms; the platinum-salt in small, beautiful, yellow, very brilliant plates.

0·3975 grm. of the latter compound gave 0·1015 grm. of platinum.

$$(C_6 H_2 Cl_2 N_2, HCl, PtCl_2).$$

	Calculated.	Found.
Platinum	26·01 per cent.	25·54.

DIAZOIODOBENZOL COMPOUNDS.

It will suffice if I give likewise only a short outline of these bodies, owing to the great resemblance which usually exists between them and the compounds of diazobromobenzol, both in regard to their preparation and chemical deportment, and also with respect to their physical properties.

Nitrate of Diazoiodobenzol, $C_6 H_3 IN_2$, NHO_3,

is prepared from nitrate of iodaniline, precisely like the corresponding bromo-compound, and crystallizes in white needles or prisms, which are exceedingly soluble in water.

Sulphate of Diazoiodobenzol, $C_6 H_3 IN_2$, $SH_2 O_4$,

crystallizes in small plates, which are easily soluble in water, difficultly so in alcohol.

0·5665 grm. of substance gave 0·4025 grm. of sulphate of barium.

		Calculated.	Found.
$C_6 H_3 IN_2$	230	70·12	——
$SH_2 O_4$	98	29·88	29·92
	328	100·00	

Perbromide of Diazoiodobenzol, $C_6 H_3 IN_2$, HBr_3,

forms small lemon-yellow slender plates.

0·4395 grm. of substance, decomposed with solution of ammonia, gave 0·525 grm. of bromide of silver.

$$C_6 H_3 IN_2, HBr_3.$$

	Calculated.	Found.
Bromine	50·95 per cent.	50·83

On boiling this compound with alcohol it yields bromoiodobenzol, $C_6 H_4 Br I$, which crystallizes from ether or alcohol in large white plates, which are volatile without decomposition.

0·453 grm. of bromoiodobenzol gave 0·4082 grm. carbonic acid and 0·0612 grm. water.

	Calculated.		Found.
C_6	72	25·44	24·58
H_4	4	1·41	1·50
Br	80	28·27	
I	127	14·88	
	283	100·00	

Platinum-salt of the Hydrochlorate of Diazoiodobenzol, $C_6H_3IN_2$, HCl, $PtCl_2$, forms small bright yellow clusters of needles.

$$\textit{Diazoiodobenzolimide,}\ \left.\begin{matrix}(C_6H_3IN_2)''\\ H\end{matrix}\right\}N,$$

forms yellowish-white crystals, which are readily fusible, and pass over on distilling with water as a yellowish oil, soon solidifying. This likewise is of a peculiar aromatic ammoniacal odour.

$$\textit{Diazoiodobenzol,}\ C_6H_3IN_2,$$

is obtained as a yellow explosive precipitate.

DIAZONITROBENZOL COMPOUNDS.

The remarkable isomerism observed between α and β nitraniline, and hitherto left unexplained by any chemical theory, likewise extends, as I have shown upon a former occasion, to the double compounds which I have described as α and β Diazo-amidonitrobenzol. This isomerism, however, is somewhat less striking, since these bodies retain one-half the nitraniline required for their formation in the original condition, as will be clearly seen on examining the rational formula

$$\left\{\begin{matrix}C_6H_3(NO_2)N_2\\ C_6H_4(NO_2)H_2N\end{matrix}\right\}$$

which I assign to these bodies. The simple diazo-compounds derived from these isomeric nitranilines cannot, however, be viewed similarly, and it was impossible therefore to decide *à priori* the question of their isomerism. The experiments presently to be described nevertheless leave no doubt that a similar isomerism exists between the several members of both series, and that, although of identical composition, differences in their physical properties become manifest, which are as striking as those observed between the original bases.

*α Diazonitrobenzol compounds**.

Nitrate of Diazonitrobenzol, $C_6H_3(NO_2)N_2HNO_3$.

The preparation of this compound by the action of nitrous acid upon *α* nitraniline differs in no way from that of the analogous bodies already referred to. By gradually adding ether to its alcoholic solution till crystallization commences, it is obtained in fine long needles, which, however, do not exhibit any distinct form of crystallization.

This compound, like all the rest of the like bodies, is very readily soluble in water, more difficultly so in alcohol, and insoluble in ether, and explodes with the same violence on heating.

Perbromide of α Diazonitrobenzol, $C_6H_3(NO_2)N_2, HBr_3$.

On mixing an aqueous solution of the former compound with bromine-water, this new body is speedily thrown down in slender orange prisms, which are almost insoluble in water, and scarcely soluble in ether. They dissolve, however, readily in warm alcohol, from which they are deposited, on cooling, in well-defined crystals.

Platinum-salt of the Hydrochlorate of α Diazonitrobenzol, $C_6H_3(NO_2)N_2$, HCl, $PtCl_2$.

On the addition of chloride of platinum to a hot aqueous solution of the hydrochlorate of diazonitrobenzol, this double salt, which crystallizes in long yellow needles, is precipitated. On recrystallizing it from boiling water, in which it is soluble (although difficultly), the crystals are obtained as prisms, frequently very well formed.

α Diazonitrobenzolimide, $\left.\begin{matrix}(C_6H_3(NO_2)N_2)'' \\ H\end{matrix}\right\}N$.

This compound is prepared by the action of ammonia upon perbromide of *α* diazonitrobenzol. By repeated recrystallization from alcohol it may be obtained in a perfectly pure state in the form of very brilliant yellow, rounded plates, which are so soluble in hot alcohol that a magma of crystals separates on cooling from a saturated solution. The crystals dissolve as readily in ether. In boiling water they fuse, producing a yellow oil but slightly soluble in water, the portion dissolved yields very fine, almost white crystals on cooling. The fusing-point of the crystals obtained by recrystallization from alcohol was found to be 71° C. The substance explodes when heated a little above the fusing-point.

0·3392 grm. of substance gave 0·548 grm. of carbonic acid and 0·805 grm. of water.

* The *α* nitraniline employed for these experiments was obtained according to ARPPE's method, by the action of alkaline liquids upon nitrated anilides.

The *β* nitraniline was prepared according to HOFMANN and MUSPRATT's method, from dinitrobenzol by the reduction with sulphuretted hydrogen.

	Calculated.		Found.
C_6	72	43·90	44·06
H_4	4	2·44	2·64
N_4	56	34·15	——
O_2	32	19·51	——
	164	100·00	

β Diazonitrobenzol compounds.

Nitrate of β Diazonitrobenzol, $C_6H_3(NO_2)N_2, HNO_3$,

is obtained from the nitrate of β nitraniline. The β does not differ much from the corresponding α compound in solubility; they however exhibit perceptible differences in the forms of their crystals, which in the β compound are columns, frequently approaching the cubical form, whilst the crystals of the nitrate of α diazonitrobenzol are long needles of a somewhat unpronounced character.

Perbromide of β Diazonitrobenzol, $C_6H_3(NO_2)N_2, HBr_3$.

On treating an aqueous solution of the nitrate of β diazonitrobenzol with bromine-water the new compound is thrown down generally as an oil, which soon solidifies. It forms small plates or prisms of an orange colour, which seem to differ from the crystals of the α compound by their want of stability when treated with warm alcohol, and which I have scarcely ever been able to recrystallize. I have obtained the perbromide of β diazonitrobenzol also by the action of bromine upon β diazo-amidonitrobenzol by suspending the latter in water and adding bromine till the whole is converted into a heavy brownish-red oil. The supernatant aqueous mother-liquor is decanted and the excess of bromine allowed to evaporate spontaneously, when the oil solidifies to a crystalline mass.

Before, however, complete solidification occurs, thick yellowish-red prisms are frequently seen to shoot out, of about an inch in length, which consist of almost pure perbromide containing mere traces of tribromnitraniline, a body which forms in the reaction to nearly the same extent, as will be seen from the following equation:—

$$\underbrace{C_{12}H_9(NO_2)_2N_3}_{\beta\text{ diazoamidotribenzol.}}+8Br=\underbrace{C_6H_3(NO_2)N_2, HBr_3}_{\beta\text{ perbromide.}}+\underbrace{C_6H_3(NO_2)Br_3N}_{\text{Tribromnitraniline.}}+2HBr.$$

The adhering tribromnitraniline* can be removed by washing the finely pulverized crystals with ether, in which the perbromide is almost insoluble.

* In order to obtain the tribromnitraniline in a pure state the ethereal solution is evaporated to dryness, and the residue dissolved by warm alcohol. Water is then added to the alcoholic solution till it becomes milky and deposits crystals. These are thrown upon a filter, and then pressed between sheets of filter paper, in order to remove any adhering bromonitrobenzol, and purified further by recrystallization from weak alcohol. Tribromnitraniline crystallizes in small, slightly yellowish plates, which cannot be sublimed without decomposition. The analysis of this compound will be found in another place.

If no crystals are formed after the bromine has evaporated in the manner described, but only a crystalline mass of perbromide and tribromaniline, the latter is pressed between bibulous paper, and then washed with ether in order to separate the two compounds. This, however, cannot be done without incurring a considerable loss of perbromide, since by the action of the ether it is partly converted into the nitrobromobenzol. Thus prepared it gave on analysis the following results:—

0·548 grm. gave 0·3895 grm. of carbonic acid and 0·0565 grm. of water.

0·363 grm. gave, on ignition with lime, 0·5275 grm. of bromide of silver.

	Theory.		Experiment.
C_6	72	18·46	19·38
H_4	4	1·03	1·14
N_3	42	10·77	——
Br_3	240	61·54	61·85
O_2	32	8·20	——
	390	100·00	

Platinum-salt of the Hydrochlorate of β Diazonitrobenzol, $C_6H_3(NO_2)N_2$, HCl, $PtCl_2$.

This compound crystallizes likewise in needles or prisms resembling much the crystals of the α compound.

0·847 grm. left, on ignition with carbonate of sodium, 0·236 grm. of platinum =27·86 per cent.

$C_6H_3(NO_2)N_2$, HCl, $PtCl_2$.

	Calculated.	Found.
Platinum	27·79 per cent.	27·86

β Diazonitrobenzolimide, $\left.\begin{matrix}(C_6H_3(NO_2)N_2)'' \\ H\end{matrix}\right\}N$,

differs from the α compound by crystallizing invariably in orange-coloured needles which fuse at 52° C., and consequently much below the fusing-point of the α compound. It is likewise somewhat soluble in hot water, from which it crystallizes on cooling in whitish needles, which closely resemble those of the α diazonitrobenzolimide recrystallized also from hot water. It somewhat possesses the odour of nitrobenzol.

0·4317 grm. of substance gave 0·6927 grm. of carbonic acid and 0·1008 grm. of water.

	Calculated.		Found
C_6	72	43·90	43·76
H_4	4	2·44	2·59
N_4	56	34·15	——
O_2	32	19·51	——
	164	100·00	

ON THE PRODUCTS OF DECOMPOSITION OF THE DIAZONITROBENZOL COMPOUNDS.

These substances, as far as my observations go, are decomposed under the influence of reagents like the other diazobenzol compounds; and their transformations may be expressed by corresponding equations. The nature of their products of decomposition may in fact be safely predicted beforehand, as the same isomerism which characterizes the compounds already described is observable. It appeared of sufficient interest to trace them somewhat more fully. I could not have given the comparative history of the products of decomposition, however, without deviating too far from the main direction of this investigation, and I must therefore reserve for a future opportunity a more intimate acquaintance with these interesting bodies. A few illustrations will show how promising a harvest of interesting results may be expected from such an investigation. One peculiar fact deserves mention here. I had taken it for granted that the same relations would be exhibited between the isomeric diazonitrobenzol compounds and the isomeric acids of the formula $C_6H_5(NO_2)O$ (the nitrocarbolic acid of HOFMANN and the isonitrocarbolic acid of FRITSCHE) as that observed between the compounds of diazobenzol and carbolic acid, and that their deportment would be represented by the formula

$$\underbrace{C_6H_3(NO_2)N_2,\ NHO_3}_{\alpha\text{-}\beta \text{ Nitrate of diazonitrobenzol.}}+H_2O=\underbrace{C_6H_5(NO_2)O}_{\alpha\text{-}\beta \text{ Nitrocarbolic acid.}}+NHO_3+N_2.$$

The assumption has not been verified. On boiling these diazonitrobenzol compounds with water, they are certainly decomposed, after some time, with evolution of nitrogen gas, but neither the α nor β compound furnishes under these circumstances either of the well-known phenylic substitutes. In both cases a brownish and easily fusible substance is obtained, which readily dissolves in alcohol and ether, but which can in no manner be made to crystallize. I have not ascertained whether this substance deports itself differently according to its origin (from the α or β compound), nor have I analyzed it, but I presume that, judging from the progress of its formation, and also from the weak acid properties it exhibits (dissolving in potassa and being reprecipitated by hydrochloric acid), its composition is probably represented by the formula

$$C_6H_5(NO_2)O.$$

On heating an alcoholic solution of the perbromide of α diazonitrobenzol, it is readily decomposed according to the equation

$$\underbrace{C_6H_3(NO_2)N_2,\ HBr_3}_{\alpha \text{ perbromide.}}=\underbrace{C_6H_4(NO_2)Br}_{\text{Bromonitrobenzol.}}+N_2+Br_2.$$

A portion of the bromonitrobenzol formed in this manner separates in crystals on cooling, provided the amount of alcohol employed has not been too large; the rest is precipitated on the addition of water. The crystals are purified by pressure between filter paper and distillation, when they pass over as a slightly yellowish oil, which soon solidifies to a crystalline mass. The bromonitrobenzol thus prepared is difficultly soluble

in cold, readily in hot alcohol and ether. It crystallizes from these solvents in long almost white needles, which fuse at 126° C., and which possess the odour of nitrobenzol. A bromine-determination gave the following results:—

0·3355 grm. was ignited with lime and gave 0·317 grm. of bromide of silver.

$$\mathrm{C_6H_4(NO_2)Br}.$$

	Calculated.	Found.
Bromine	39·61 per cent.	40·19

If the perbromide of β diazonitrobenzol is decomposed in a similar manner with boiling alcohol, and the solid product of decomposition which forms purified as described before, a compound is obtained to which the formula $\mathrm{C_6H_4(NO_2)Br}$ must likewise be assigned, which, however, differs greatly in its physical appearance and properties from the former bromonitrobenzol. It crystallizes from alcohol and ether, in which it seems to be more easily soluble, in the form of well-made, slightly yellowish, or almost white rhombic prisms, sometimes also in plates, and fuses at 56° C., or 70° below the fusing-point of the bromonitrobenzol derived from the α compound. The analysis of this body gave the following results:—

0·2745 grm. of substance gave 0·361 grm. of carbonic acid and 0·0535 grm. of water.

0·3365 grm. of substance gave, on ignition with caustic lime, 0·3145 grm. of bromide of silver.

	Calculated.		Found.
$\mathrm{C_6}$	72	35·64	35·87
$\mathrm{H_4}$	4	1·98	2·16
Br	80	39·61	39·77
N	14	6·93	
$\mathrm{O_2}$	32	15·84	
	202	100·00	

The existence of these bromonitrobenzols suggests the question whether one of them be not identical with the bromonitrobenzol prepared by COUPER by the action of fuming nitric acid upon bromobenzol. COUPER's description of this body appeared to me scarcely conclusive of its nature; and in order to decide this question I have treated bromobenzol, prepared from coal-tar naphtha, with fuming nitric acid, and have compared the perfectly pure product, after repeated recrystallizations from alcohol, with the bromonitrobenzols prepared by my method. The striking coincidence between the crystalline form of COUPER's compound and of the bromonitrobenzol corresponding to the α nitraniline, which I will now call α bromonitrobenzol, became at once perceptible; and since both compounds crystallize in white needles that could by no means be distinguished from one another, I felt justified in coming to the conclusion that both are identical. In order, however, to make quite sure I have also determined the fusing-point of the respective compounds, and have found it uniformly at 120° C.*

* COUPER states (Ann. de Chim. et de Phys. [3] vol. lii. p. 309) that his compound fused below 90° C. This is evidently erroneous.

They are acted upon in a like manner by various reagents. When reduced in alcoholic solutions by sulphide of ammonium, they are both converted into bromaniline, which crystallizes in octahedra, and proves itself to be completely identical with that obtained from bromisatine or bromacetanilid by distillation with potash. Very different from this is the deportment of bromonitrobenzol (β bromonitrobenzol) prepared from perbromide of β diazonitrobenzol. On treating this compound with sulphide of ammonium, it is certainly also converted into bromaniline, but this base differs greatly from the above-described bromaniline. Ordinary bromaniline (which I would now designate as α bromaniline) crystallizes, as is well known, in octahedra which fuse at 57° C. The new base, however (the β bromaniline), forms an oil which does not solidify even in winter. The chemical deportment of both compounds is the same; they form, under like conditions, a series of derivatives which are of the same composition, and differ only in their physical properties.

In order to establish the composition of the β bromaniline experimentally, I have analyzed the hydrochlorate and its platinum-salt. *The hydrochlorate of β bromaniline* forms white, nacreous, brilliant plates, which are readily soluble in water and alcohol, and which are rapidly coloured red when exposed to the air.

0·2605 grm. of substance gave 0·176 grm. of chloride of silver.

$$C_6 H_6 Br N, HCl.$$

	Calculated.	Found.
Chlorine	17·02 per cent.	16·71

The Platinum-salt of the Hydrochlorate of β Bromaniline, $C_6 H_6 Br N, HCl, Pt Cl_2$, crystallizes in yellow, often well-formed prisms, which are far more soluble in water than the slender highly lustrous plates of the platinum-salt of α bromaniline.

0·3735 grm. gave 0·097 grm. of platinum.

$$C_6 H_6 Br N, HCl, Pt Cl_2.$$

	Calculated.	Found.
Platinum	26·12 per cent.	25·97

I need scarcely state that *nitrochlorobenzol* compounds corresponding to α and β nitrobromobenzol can readily be obtained by submitting the platinum-salts of α and β diazonitrobenzol to distillation with carbonate of sodium. These two bodies differ likewise most characteristically; α nitrochlorobenzol crystallizes invariably in long, almost white needles which fuse at 83° C., whilst β nitrochlorobenzol crystallizes from its ethereal solution in thick columns which fuse at 46° C. The former is converted by sulphide of ammonium into the ordinary (α) chloraniline, the latter into a new base (β chloraniline), which is distinguished by its remaining an oil at the common temperature. The platinum-salt presents itself in the form of yellow crystals, which differ likewise considerably in form and solubility from the small slender plates of the platinum-salt of α bromaniline.

It may, I think, be safely deduced from these facts that two distinctly different series of compounds are obtained by the substitution of two atoms of hydrogen in benzol by two different elements, or groups of atoms, according as this substitution is accomplished, and that these two series differ most distinctly in their physical properties, although their chemical composition is the same. The great differences between the fusing-points form one of the most important means of distinction between the members of the two series. The fusing-points of series α lie considerably higher than those of the β series; sometimes a difference of 70° C. is observable, as will be seen by the accompanying Table.

	α Series.	β Series.
Bromonitrobenzol . . .	126° C.	56° C.
Chloronitrobenzol . . .	83° C.	46° C.
Nitraniline	141° C.	108° C.
Bromaniline	57° C.	Liquid at the ordinary temperature.
Chloraniline		Liquid at the ordinary temperature.
Diazo-amidonitrobenzol .	245° C.	195° C.
Diazonitrobenzolimide . .	71° C.	53° C.

The compounds to which the experiments just described gave rise belonged exclusively to the aniline group. Now there could be little doubt that the homologues of aniline, and the similarly constituted bases, migh tbe also converted into diazo-compounds. This I have confirmed by experiment, the more readily since it was to be foreseen that the decomposition of some of these compounds would give rise to the formation of certain bodies which could not have been obtained by the ordinary methods, *e. g.* naphtylic alcohol, the existence of which, however, could not be doubted for a moment.

DIAZOTOLUOL COMPOUNDS.

These compounds exhibit considerable analogy with regard to preparation, solubility, and many other properties to the corresponding aniline derivatives. They seem, however, to be somewhat more constant, and to crystallize more readily.

Nitrate of Diazotoluol, $\mathrm{C_7 H_6 N_2, NHO_3}$.

This substance is best prepared by the action of nitrous acid upon an aqueous solution of nitrate of toluidine, but it can also be obtained from diazo-amidotoluol. It forms long white needles, which are decomposed on boiling with water, according to the following equation:—

$$\underbrace{\mathrm{C_7 H_6 N_2, NHO_3}}_{\text{Nitrate of diazotoluol.}}+\mathrm{H_2 O}=\underbrace{\mathrm{C_7 H_8 O}}_{\text{Cresyl-alcohol.}}+\mathrm{NHO_3}+\mathrm{N_2}.$$

The other compounds are readily obtained from the nitrate in the ordinary manner.

Sulphate of Diazotoluol, $C_7 H_6 N_2, SH_2 O_4$,

is obtained in brilliant needles, plates, or prisms, according to the circumstances under which it crystallizes. On heating with sulphuric acid in the manner described when speaking of the sulphate of diazobenzol, it is converted into a sulpho-acid.

The barium-salt of this acid crystallizes in long white needles, and is disulpho-toluylenate of barium, which, according to the subjoined analysis, has the formula $C_7 H_6 S_2 Ba_2 O_7$.

0·530 grm. dried at 140° gave 0·276 grm. of sulphate of barium.

$C_7 H_6 S_2 Ba_2 O_7$.

	Calculated.	Found.
Barium	33·99 per cent.	33·74

Perbromide of Diazotoluol, $C_7 H_6 N_2$, HBr, Br_2,

is thrown down as a yellow oil, which, after evaporation of the excess of bromine, solidifies to a crystalline mass.

Platinum-salt of the Hydrochlorate of Diazotoluol, $C_7 H_6 N_2$, HCl Pt Cl_2,

is precipitated from a dilute solution of the hydrochlorate of diazotoluol by means of bichloride of platinum, and forms fine yellow prisms. When ignited with soda it yields an aromatic oil, chlorotoluol ($C_7 H_7 Cl$). It remains to be seen whether this oil be identical, or only isomeric with the chlorobenzyl discovered by CANIZZARO.

0·360 grm. of substance gave 0·5765 grm. of carbonic acid and 0·1315 grm. of water.

0·6305 grm. gave, on ignition with carbonate of sodium, 0·1925 grm. of platinum.

	Calculated.		Found.
C_7	84	25·92	26·20
H_7	7	2·16	2·43
N_2	28	8·63	——
Cl_3	106·5	32·85	——
Pt	98·7	30·44	30·53
	324·2	100·00	

Diazotoluol-amidobenzol, $\begin{cases} C_7 H_6 N_2 \\ C_6 H_5 (H_2 N). \end{cases}$

This body is obtained by the action of aniline upon nitrate of diazotoluol. It crystallizes in beautiful long yellow needles.

DIAZONITRANISOL COMPOUNDS.

These compounds are nearly related to the diazonitrobenzol-compounds, both with regard to physical properties and their deportment with reagents. They are prepared also in a similar manner, and it is only necessary therefore to refer respecting their preparation to these analogous bodies.

Nitrate of Diazonitranisol, $C_7H_5(NO_2)N_2O, NHO_3$,

is obtained from nitrate of nitranisol. It separates from an alcoholic solution on the addition of ether, forming small white plates, which are difficultly decomposed on heating with water, giving rise to the formation of a brownish-red substance.

Perbromide of Diazonitranisol, $C_7H_5(NO_2)N_2O, HBr_3$,

forms small yellow plates, which, on boiling with alcohol, furnish bromonitranisol ($C_7H_6(NO_2)Br$), crystallizing in light-yellow opake needles, which may be sublimed, and possess the odour of nitrobenzol.

Diazonitranisolimide, $\left.\begin{matrix}(C_7H_5(NO_2)N_2O)'' \\ H\end{matrix}\right\}N$,

crystallizes in light-yellow needles, and possesses the odour of bitter almonds.

Platinum-salt of the Hydrochlorate of Diazonitranisol, $C_7H_5(NO_2)N_2O, HCl, PtCl_2$.

When precipitated from a moderately concentrated solution, it forms a yellow powder, which under the microscope is found to consist of fine needles. Recrystallized from boiling water, it is obtained in orange-red, well-formed prisms. On heating with carbonate of sodium chloronitranisol ($C_7H_6(NO_2)ClO$) distils over, which crystallizes in fine, almost white needles.

0·8795 grm. of the platinum-salt gave 0·222 grm. of platinum.

$C_7H_5(NO_2)N_2O, HCl, PtCl_2$.

	Calculated.	Found.
Platinum	25·62 per cent.	25·24

DIAZONAPHTOL COMPOUNDS.

Nitrate of Diazonaphtol, $C_{10}H_6N_2, NHO_3$,

is prepared by the action of nitrous acid on moist nitrate of napthalidine (amidonaphtol); an amorphous reddish-brown substance* is formed at the same time, which must be separated by filtration when the reaction is over. As nitrate of diazonaphtol is not precipitated from its aqueous solution by alcohol and ether, it is not so easily obtained in a solid state as the corresponding bodies previously described; if, however, its aqueous solution be allowed to evaporate spontaneously in a shallow vessel, long white needles are formed, which are very soluble in water and alcohol, and likewise very explosive.

Perbromide of Diazonaphtol, $C_{10}H_6N_2, HBr_3$,

is obtained in the form of orange crystals by the action of bromine-water upon the crude solution of the nitrate.

Platinum-salt of the Hydrochlorate of Diazonaphtol, $C_{10}H_6N_2, HCl, PtCl_2$,

forms truncated, yellowish crystals, which are almost insoluble in water, alcohol, and ether.

* Probably impure nitrate of diazonaphtol-amidonaphtol.

1·0565 grm. of substance gave 0·2887 grm. of platinum.

$$C_{10}H_6N_2, HCl, PtCl_2.$$

	Calculated.	Found.
Platinum . . .	27·40 per cent.	27·33

Diazonaphtolimide, $\left.\begin{matrix}(C_{10}H_6N_2)'' \\ H\end{matrix}\right\}N$,

passes over as a yellowish-coloured oil (which becomes brown when exposed to the air) by distilling with water the substance obtained by the action of ammonia upon the perbromide. It possesses somewhat the odour of naphtaline.

OBSERVATIONS ON THE PRODUCTS OF DECOMPOSITION OF THE DIAZONAPHTOL COMPOUNDS.

After being convinced that the products of decomposition of the diazonaphtol compounds so much resembled those of the other diazo-bodies, I could not entertain for a moment the intention of pursuing their study in all directions, especially since known bodies would frequently have formed the subject of such study. The decomposition of the perbromide, for instance, by means of alcohol gives rise to the formation of bromonaphtaline, that of the platinum-salt when ignited with carbonate of sodium to the formation of chlornaphtaline. I thought it, however, of sufficient importance to ascertain whether nitrate of diazonaphtol would split up according to the equation

$$C_{10}H_6N_2, NHO_3+H_2O=C_{10}H_8O+N_2+NHO_3,$$

since the possibility of obtaining the long-sought-for naphtyl-alcohol presented itself. On boiling the solution of the nitrate of diazonaphtol, an immediate evolution of gas takes place, and a viscid violet-brown mass separates which remains on the filter when the solution is filtered hot.

The filtrate deposits generally small white plates, retaining, however, a portion of the substance dissolved, which can be recovered by shaking the aqueous solution with ether. On evaporating the ether it remains behind as a violet-coloured oil which quickly solidifies. This latter, as well as the plates first deposited, is in fact almost pure naphtyl-alcohol, whilst the violet-brown mass on the filter contains, besides naphtyl-alcohol, a considerable quantity of a reddish-brown body. In order to purify the naphtyl-alcohol, the united portions are treated with a cold solution of potassa, the residuary brown-red body* is filtered off, and the filtrate treated with acetic acid as long as precipitation ensues. Naphtyl-alcohol separates first as an oil, soon solidifying to a network of small plates, which are still somewhat violet-coloured. They are thrown on a filter, washed with cold water (which removes the mother-liquor), and then submitted to distillation.

* This body is soluble in alcohol, to which it imparts a blood-red colour. On concentrating the alcoholic solution, reddish-brown indistinct crystals are deposited, which, on rubbing, acquire a green metallic lustre.

This process must be repeated until the mass of crystals, which is invariably found deposited in the condensing-tube, is quite white, and fuses, on heating, to a limpid oil. This body gave on analysis the following results:—

0·2003 grm. of substance gave 0·6138 grm. of carbonic acid and 0·1047 grm. of water.

	Calculated.		Found,
C_{10}	120	83·33	83·57
H_8	8	5·56	5·81
O	16	11·11	——
	144	100·00	

Naphtyl-alcohol crystallizes in small white shining plates, fusing at 91° C. to a colourless, highly refractive oil which may be volatilized without decomposition. It is but slightly soluble in water, readily soluble in alcohol, ether, and benzol. When inflamed it burns with a thick smoky flame. Its formation and its physical properties prove its relationship with phenylic alcohol; it possesses a similar burning taste, and a creosote-like odour somewhat resembling that of naphtaline. The relation between it and phenylic alcohol is likewise indicated by its chemical deportment. It stands on the same narrow boundary line between acid and alcohol; and naphtyl-alcohol may, in like manner with phenol, be classified with alcohols or with acids. The strong bases form with the new alcohol (acid) a series of salts which are as unstable as those of phenylic acid, and are mostly decomposed even by the carbonic acid of the air. The potassium- or sodium-salt may also be obtained by the action of the respective metals upon the fused naphtylic acid with evolution of hydrogen. In both cases a crystalline saline mass is obtained which dissolves in water and alcohol. Basic acetate of lead produces in a solution of naphtyl-alcohol a white voluminous precipitate. An ammoniacal silver solution is decomposed, metallic silver being deposited. Nitric acid of sp. gr. 1·4 dissolves naphtyl-alcohol in the cold with evolution of red fumes. If the solution be boiled for some time and water added, a difficultly-soluble yellow acid precipitates, whilst a second readily-soluble acid is deposited in yellow crystals on evaporating the mother-liquor. The latter appears to be the picric acid of the naphtyl group. By treating the aqueous solution of naphtyl-alcohol with bromine-water, it deposits bromonaphtylic acid in the form of an oil which speedily crystallizes.

Many of the experiments described here and in the previous part of this investigation, have been made in the laboratories of the Royal College of Chemistry, London, and the University of Marburg; and it gives me great pleasure to express my sincere thanks to Professors HOFMANN and KOLBE for the kind manner in which the laboratories of these institutions have been placed at my disposal.

PART III.

All the diazo-compounds which have formed the subject of Parts I. and II. of this inquiry are derived from monoamido-compounds. If the composition of the latter be again expressed by the general formula $C_x H_{(y-2)} (NH_3)'' O_z$ (see page 667), it is at once perceived that the process of formation of the diazo-compounds is similar to the reaction which takes place by the action of nitrous acid on ammonia:

$$\text{I.} \quad \underbrace{C_x H_{(y-2)} (NH_3)'' O_z}_{\text{Monoamido-compound.}} + NHO_2 = \underbrace{C_x H_{(y-2)} (N_2)'' O_z}_{\text{Diazo-compound.}} + 2H_2O\,^{*}.$$

$$\text{II.} \quad \underbrace{(NH_3)''}_{\text{Ammonia.}} + NHO_2 = \underbrace{(N_2)''}_{\text{Nitrogen.}} + 2H_2O.$$

By viewing the diamido-compounds in like manner as represented by the general formula $C_x H_{(y-4)} (NH_3)_2''$, O_z, it is evident that the action of nitrous acid may give rise to two decompositions:

$$\text{I.} \quad C_x H_{(y-4)} (NH_3)_2^{\mathrm{iv}} O_z + NHO_2 = C_x H_{(y-4)} (NH_3)'' (N_2)'' O_z + 2H_2O.$$

$$\text{II.} \quad C_x H_{(y-4)} (NH_3)_2^{\mathrm{iv}} O_z + 2NHO_2 = C_x H_{(y-4)} (N_4)^{\mathrm{iv}} O_z + 4H_2O.$$

The hydrogen contained in one atom of ammonia can thus be replaced by nitrogen, or both atoms of ammonia present in the original compound may be replaced by nitrogen, giving rise to a tetrazo-compound.

Although this theory has not been fully confirmed by the action (presently to be described) that nitrous acid exerts on benzidine, since an intermediate compound according to equation I. could not be obtained, it is nevertheless highly probable that nitrogenous compounds in accordance with equation I. exist, and that further experiments with other diamido-compounds will confirm this view.

Action of Nitrous Acid upon Benzidine.

Benzidine, which Zinin, its original discoverer, expressed by the formula

$$C_6 H_6 N,$$

has been found, on further investigation by P. W. Hofmann, to have double the atomic weight first assigned to it, and to be a base capable of combining with two molecules of acid. Very recently Fittig has shown that it must be viewed as the diamido-compound of his newly discovered hydrocarbon-phenyl, and that it ought to be expressed by the rational formula $C_{12} H_8 (NH_2)_2$, or $C_{12} H_6 (NH_3)''_2$. I entirely abide by Fittig's view, but find it necessary to select for both compounds a somewhat modified nomenclature, in order to avoid the use of the same names for some derivatives of these bodies (which I shall have occasion to describe) already employed for several long-known derivatives

* It must be borne in mind that nitrous acid acts upon the nitrates of the amido-compounds; if otherwise, the reactions would frequently give rise to the formation of diazoamido-compounds formerly described by me.

of phenylic acid. I propose, therefore, to designate FITTIG's hydrocarbon, $\text{Ɵ}_{12}H_{10}$, as diphenyl, and to call benzidine diamidodiphenyl.

Nitrate of Tetrazodiphenyl, $\text{Ɵ}_{24}H_{6}N_{4}, 2NH\text{Ɵ}_{3}$.

This compound is most readily and copiously obtained by passing nitrous acid through a cold concentrated aqueous solution of nitrate of diamidobenzidol, when only traces of a brown amorphous body are formed, whilst from an alcoholic solution the latter is deposited in considerable quantities. When a sufficient current of gas has been passed through the solution, the brown body is separated by filtration, the filtrate mixed with twice its volume of strong alcohol, and ether added as long as any white crystals are deposited. By once more dissolving the crystals in a very small quantity of water, and reprecipitating with alcohol and ether, they are obtained perfectly pure: it will be readily perceived that this mode of preparation closely resembles that for the preparation of nitrate of diazobenzol, which body presents a striking analogy to the nitrate of tetrazodiphenyl. The latter crystallizes in white or slightly yellowish-tinged needles, which are readily soluble in water, more difficultly so in alcohol, and insoluble in ether. On heating they explode with the same violence as the analogous diazo-compound.

Sulphate of Tetrazodiphenyl, $\text{Ɵ}_{12}H_{6}N_{4}, 3SH_{2}O_{4}$.

After mixing a concentrated aqueous solution of the nitrate of tetrazodiphenyl with a sufficient quantity of cold sulphuric acid, diluted previously with its own bulk of water, this new body separates, on the addition of strong alcohol, either in the form of a white crystalline powder, or in white needles. If alcohol does not completely precipitate it, ether must be added to complete its separation. It is very soluble in water. On heating in a dry state an explosion ensues.

I. 0·820 grm. of substance gave, on direct precipitation with chloride of barium, 0·811 grm. of sulphate of barium.

II. 0·441 grm. gave 0·4355 grm. of sulphate of barium.

	Calculated.		Found. I.	Found. II.
$2\text{Ɵ}_{12}H_{6}N_{4}$	412	58·36	—	—
$3SH_{2}\text{Ɵ}_{4}$	294	41·64	41·60	41·54
	706	100·00		

This compound partakes of the nature of an acid salt.

Platinum-salt of the Hydrochlorate of Tetrazodiphenyl, $\text{Ɵ}_{12}H_{6}N_{4}, 2HCl, 2PtCl_{2}$.

This salt is precipitated from a moderately dilute solution of the nitrate or sulphate by means of bichloride of platinum. It forms light-yellow very small narrow plates. By employing a very dilute solution, it falls slowly, in small, elongated, well-formed

hexagonal plates. These crystals are almost insoluble in water, alcohol, and ether. Exposure to light during drying causes them to assume a slightly brown colour.

0·6125 grm. gave 0·5252 grm. of carbonic acid and 0·0915 grm. of water.

0·6065 grm. gave 44·1 cub. centims. of nitrogen at 0° C. and 760 millims. bar. pressure, =0·055415 grm.

0·739 grm. gave on ignition with carbonate of soda 0·232 grm. of platinum.

	Calculated.		Found.
C_{12}	144	23·29	23·38
H_8	8	1·30	1·66
N_4	56	9·05	9·44
Pt_2	197·4	31·92	31·40
Cl_4	213	34·44	——
	618·4	100·00	

Perbromide of Tetrazodiphenyl, $C_{12}H_6N_4$, 2HBr, Br_4.

This compound is formed on the addition of bromine-water to an aqueous solution of the nitrate of the tetrazodiphenyl, being precipitated in the form of round reddish crystals, which are collected on a filter, thoroughly washed with water, and dried without delay over sulphuric acid and caustic lime. They are thus obtained sufficiently pure for analysis. Further purification indeed is impossible, since this body is decomposed by dissolving in alcohol, with evolution of gas. Even at the ordinary temperature it undergoes a gradual decomposition and evolves bromine, which is recognizable by its odour. For this reason the bromine-determinations, which were made with portions of this body that had been left for several days under the desiccator, were found a little too low.

I. 0·583 grm., decomposed with alcoholic potash, gave 0·931 grm. of bromide of silver =67·96 per cent.

II. 0·742 grm. gave, on ignition with caustic lime, 1·192 grm. of bromide of silver =68·36 per cent.

$C_{12}H_6N_4$, 2HBr, Br_4.

	Calculated.	Found. I.	Found. II.
Bromine	69·77	67·96	68·36

Tetrazodiphenyl-amidobenzol, $\left\{\begin{array}{l} C_{12}H_6N_4 \\ C_6H_7N \\ C_6H_7N \end{array}\right.$

By adding aniline to an aqueous solution of nitrate of tetrazodiphenyl, this complex substance, corresponding to the diazo-amidobenzol in the diazobenzol series, is at once separated as a yellow crystalline mass; by repeatedly washing with alcohol it is easily obtained in a pure state for analysis.

0·2693 grm. gave 0·7233 grm. of carbonic acid and 0·1315 grm. of water. These numbers correspond to the above formula, $C_{24}H_{20}N_6$.

	Calculated.		Found.
C_{24}	288	73·47	73·24
H_{20}	20	5·10	5·42
N_6	84	21·43	——
	392	100·00	

The formation of this new substance can be expressed as follows:

$$\underbrace{C_{12}H_6N_4,\ 2NHO_3}_{\text{Nitrate of tetrazo-diphenyl.}}+\underbrace{4C_6H_7N}_{\text{Aniline.}}=\underbrace{C_{24}H_{20}N_6}_{\text{Tetrazodiphenyl-amidobenzol.}}+\underbrace{2(C_6H_7N,\ NHO_3)}_{\text{Nitrate of aniline.}}.$$

Tetrazodiphenyl-amidobenzol is insoluble in water, and, although very little soluble even in boiling alcohol and ether, can be recrystallized from them. By this means lance-shaped crystals are obtained, which are generally grouped together in a star-like form. When heated they explode; boiled with mineral acids they are decomposed with evolution of nitrogen gas.

Diphenylimide, $\left.\begin{matrix}(C_{12}H_6N_4)^{iv}\\ H_2\end{matrix}\right\}N_2$.

This body is obtained in crystals when the perbromide is acted upon by solution of ammonia. The crystals are purified by repeated recrystallization from strong alcohol, when they are obtained in small very brilliant white, or slightly yellowish plates, insoluble in water, very difficultly soluble in cold alcohol, and only moderately soluble in ether. Boiling alcohol, however, dissolves them very readily, and yields on cooling a mass of crystals. The compound fuses at 127° C. to a yellow oil, which is decomposed at a higher temperature, giving rise to slight explosions. It does not combine with acids or alkalies, and is perfectly neutral to test-paper. No change is produced even by boiling with concentrated hydrochloric acid, nor with aqueous or alcoholic potash. It is decomposed, however, by nitric or strong sulphuric acid.

I. 0·2503 grm. of substance gave 0·5577 grm. of carbonic acid and 0·0816 grm. of water.

II. 0·3118 grm. gave 0·6918 grm. of carbonic acid and 0·0983 grm of water.

	Calculated.		Found. I.	Found. II.
C_{12}	144	61·02	60·76	60·51
H_8	8	3·39	3·62	3·50
N_6	84	35·59	——	——
	236	100·00		

The formation of this compound may be expressed by the following equation:—

$$\underbrace{C_{12}H_6N_4,\ 2HBr,\ Br_4}_{\text{Perbromide.}}{}^{*}+8NH_3=\underbrace{C_{12}H_8N_6}_{\text{Tetrazodiphenylimide.}}+6NH_4Br.$$

It has been my endeavour to study somewhat more closely the compounds which tetrazodiphenyl forms with metallic hydrates, but all my attempts to obtain them more definitely have failed, and their preparation seems indeed to be beset with insurmountable difficulties. I omit a lengthened description of the unsuccessful attempts, and will merely show by one instance how tetrazodiphenyl can play the part of an acid. An aqueous solution of nitrate of tetrazodiphenyl, when mixed with caustic potash, yields a yellow liquid exhibiting such properties as might be looked for in a solution of the compound of hydrate of potassa with tetrazodiphenyl. On treating it with chloride of platinum, it gives rise to the formation of the platinum-compound above described, a proof that the tetrazodiphenyl remains unchanged in the alkaline solution. It is invariably decomposed on the application of heat, with evolution of gas and separation of a reddish-brown amorphous substance.

PRODUCTS OF DECOMPOSITION OF THE COMPOUNDS OF TETRAZODIPHENYL.

By applying the laws of classification just now accepted by chemists to the tetrazotized derivatives of benzidine, specially taking into account their manner of formation and combination, these bodies must be classified with the diatomic compounds, whilst the respective diazo-compounds belong to the monatomic bodies. It has been of late a favourite subject of chemical research to trace the analogies which monatomic bodies exhibit under the influence of certain agents with polyatomic bodies. In illustration of this I may refer to the results obtained in the comparative study of the products of decomposition of the ethyl- and ethylene-alcohols in order to show how simple are the laws which regulate these chemical transformations. The experimental results which I am able to adduce prove likewise that the decomposition of the tetrazo-compound gives rise to derivatives which differ in nothing from those obtained under similar circumstances from diazo-compounds, beyond the distinctive features imparted to them by the polyatomic nature of the compound from which they are derived.

Action of Water upon Nitrate of Tetrazodiphenyl.

An aqueous solution of this body, when left in a cold place, gradually undergoes a spontaneous decomposition. When heated it gives rise to a copious evolution of nitrogen gas with separation of two substances—one an amorphous brown, and the other a white crystalline body. An additional quantity of the latter is obtained when the liquid has

* Leaving aside the hydrobromic acid, this compound may be looked upon as the bromide of a tetratomic radical $(C_{12}H_6N_4)^{IV}Br_4$.

become quite cold. It is easy to separate these two bodies by filtering off the acid mother-liquor, pressing the residue between bibulous paper, and repeatedly extracting with dilute alcohol, which takes up the crystalline substance and leaves the brown body undissolved: this latter being very probably identical with the brown compound obtained as a by-product in the formation of nitrate of tetrazobenzidol, and being moreover of a very unpromising nature, I have not pursued its study any further. The crystalline product of decomposition dissolved by the alcohol is obtained on evaporation as a yellowish crystalline mass, from which the least traces of the brown body are removed by repeatedly dissolving in ether, and lastly recrystallizing from dilute alcohol.

0·3037 grm. of substance gave 0·8603 grm. of carbonic acid and 0·154 grm. of water.

Qualitative tests showed the absence of nitrogen; these numbers correspond therefore with the formula $\mathrm{C}_{12} \mathrm{H}_{10} \mathrm{O}_2$, as will be seen from the following calculations:—

	Theory.		Experiment.
C_{12}	144	77·42	77·26
H_{10}	10	5·38	5·63
O_2	32	17·20	——
	186	100·00	

The formation of this compound, for which I propose the name of diphenyl-alcohol or diphenylic acid, takes place according to the equation

$$\underbrace{C_{12} H_6 N_4, 2NHO_3}_{\text{Nitrate of tetrazodiphenyl.}} + 2H_2O = \underset{\text{Diphenyl-alcohol.}}{C_{12} H_{10} O_2} + 2NHO_3 + 4N.$$

Diphenyl-alcohol (diphenylic acid) crystallizes in small white, or slightly tinged plates or needles, which possess slight solubility in water, but are soluble to any extent in alcohol and ether. It fuses on heating, and can be sublimed in small quantities by heating it cautiously in a test-tube. It is thus obtained in the form of soft, white, very lustrous plates. The chemical deportment of diphenyl-alcohol shows it to be most closely allied to the class of compounds the best-known representative of which is phenylic alcohol. It may in fact be considered as the first biatomic representative of this peculiar group of chemical compounds. The new alcohol is readily soluble in potash, and can be reprecipitated by the addition of an acid. Concentrated ammonia also dissolves it. On heating an ammoniacal solution with basic acetate of lead, a white voluminous precipitate ensues. Heated with ordinary strong nitric acid it is changed into a nitro-acid, which crystallizes out in yellow roundish crystals; the ammonium-salt of this acid crystallizes in beautiful long needles.

Action of Alcohol upon Sulphate of Tetrazodiphenyl.

On heating an admixture of alcohol and sulphate of tetrazodiphenyl, violent decomposition takes place. The products formed are nitrogen, sulphuric acid, and a solid sub-

tance, which separates in small indistinct plates on mixing the alcoholic fluid (after the reaction is entirely over) with a large quantity of water. In order to free the precipitate from a trace of a brown substance, which likewise forms in the reaction, it is filtered off from the mother-liquor, dried, and then sublimed through paper, according to the method proposed by GORUP-BESANEZ. The substance is thus obtained in the form of perfectly white plates, which dissolve easily in ether and hot alcohol, and which crystallize from the latter very much like naphtaline. The new compound fuses at 70° C. to an oil, which distils at a higher temperature without decomposition. It possesses a peculiar aromatic odour like that of cinnamol and benzol combined.

0·1787 grm. of substance gave 0·607 grm. of carbonic acid and 0·107 grm. of water.

These numbers lead to the formula $C_{12}H_{10}$.

	Calculated.		Found.
C_{12}	144	93·50	92·70
H_{10}	10	6·50	6·65
	154	100·00	

Its chemical composition, combined with the before-mentioned physical properties, prove that the hydrocarbon is identical with the diphenyl described by FITTIG*. Its formation may be described by the following equation:—

$$\underbrace{2(C_{12}H_6N_4, 3SH_2O_4)}_{\text{Sulphate of tetrazodiphenyl.}} + \underbrace{4C_2H_6O}_{\text{Alcohol.}} = \underbrace{2C_{12}H_{10}}_{\text{Diphenyl.}} + \underbrace{4C_2H_4O}_{\text{Aldehyde.}} + N_8 + 6SH_2O_4.$$

Nitrate of tetrazodiphenyl is decomposed in a similar manner by boiling alcohol, but simultaneously the formation of a yellow nitro-acid takes place, which stands probably to the diphenylic alcohol in the same relation as the nitrophenol to phenylic alcohol (compare also the corresponding decomposition of nitrate of diazobenzol, p. 683).

Action of Sulphuric Acid upon Sulphate of Tetrazodiphenyl.

On heating sulphate of tetrazodiphenyl dissolved in a small quantity of oil of vitriol, a violent evolution of nitrogen gas is observed. The brown liquid which remains after the reaction has ceased contains, besides the excess of sulphuric acid employed, two new sulpho-acids, which can be separated by means of their barium-salts. For this purpose the brown liquid is diluted with at least thirty times its volume of water, boiled, and saturated with carbonate of barium. The precipitated sulphate of barium is then filtered off, the saline solution evaporated to dryness on the water-bath, and the residue several times extracted with hot water. The portion remaining undissolved is the

* If the diphenyl, prepared according to my method, has not been previously sublimed through paper as ....ribed, it crystallizes only in the form of small indistinct silvery white plates, which are so different from the large naphtaline-like plates described by FITTIG, that they appear at first sight to be crystals of quite a different compound.

barium-salt of a new acid which I will call tetrasulphodiphenylenic acid; the soluble portion contains another new acid, for which the name trisulphodiphenylenic acid may be adopted.

I will first endeavour to give a brief description of the former. The barium-salt of this acid, obtained as before mentioned, being only very slightly soluble in water, could not well be purified by recrystallization; I preferred therefore to convert it into the ammonium-salt, which served me as a starting-point for the preparation of all the saline compounds presently to be described. The ammonium-salt is readily obtained by decomposing the barium-salt with a solution of carbonate of ammonium. The mixture is heated for a short time, the insoluble carbonate of barium filtered off, and the filtrate concentrated on a water-bath till the ammonium-salt crystallizes out on cooling. One recrystallization from alcohol renders it in the form of perfectly pure white prisms.

Barium-salt of Tetrasulphodiphenylenic Acid, $\mathrm{C_{12} H_6, S_4 H_2 Ba_4 O_{15}}$.

To a rather concentrated boiling solution of the ammonium-salt chloride of barium is added, when this salt is precipitated as white needles or prisms; the crystals are allowed to subside, and, after the mother-liquor has been separated by filtration, repeatedly washed with cold water; in this manner they are obtained quite pure. The substance employed for the following analyses was dried between 150° and 160°.

I. 0·612 grm. gave 0·4055 grm. of carbonic acid and 0·070 grm. of water.

II. 0·365 grm., decomposed by nitric acid, gave 0·4327 grm. of sulphate of barium, corresponding to 16·28 per cent. of sulphur.

III. 0·2845 grm., treated with sulphuric acid, gave 0·177 grm. of sulphate of barium =34·14 per cent. of barium.

IV. 0·6185 grm., treated as before, gave 0·4327 grm. of sulphate of barium=34·77 per cent. of barium.

These numbers lead to the formula

$$\mathrm{C_{12} H_8 S_4 Ba_4 O_{15}}.$$

	Calculated.		Found. I.	II.	III.	IV.
$\mathrm{C_{12}}$	144	18·14	18·07	——		
$\mathrm{H_8}$	8	1·00	1·27	——		
$\mathrm{S_4}$	128	16·21	——	16·28		
$\mathrm{Ba_4}$	274	34·51	——	——	34·14	34·77
$\mathrm{O_{15}}$	240	31·35	——	——		
	794	100·00				

Dried over sulphuric acid, the salt retains four molecules of water of crystallization. 0·452 grm., dried as above, lost between 150° and 160° 0·037 grm. of water.

$$\mathrm{C_{12} H_8 Ba_4 S_4 O_{15} + 4 H_2 O}.$$

	Calculated.	Found.
$4H_2O$	8·03	8·00

If the ammonium-salt of tetrasulphophenylenic acid be, instead of chloride of barium, treated with baryta-water, another barium-salt is formed, which, according to the subjoined barium-determination, has the composition $C_{12}H_6, S_4Ba_6O_{15}$. It is precipitated as a white amorphous powder, which, on washing with water, is converted into small prisms.

0·6085 grm., dried at 150°, gave 0·4585 grm. of sulphate of barium.

$$C_{12}H_6, S_4Ba_6O_{15}.$$

	Calculated.	Found.
Ba	44·24	44·29

Silver-salt, $C_{12}H_6, S_4H_3Ag_3O_{15}$.

On mixing a concentrated solution of the ammonium-salt with an equally concentrated solution of nitrate of silver, separation of warty crystals of this salt takes place after some time. They may be purified by recrystallization from water, in which they are easily soluble. The concentration of their solution must be conducted *in vacuo*.

0·3955 grm., dried at 150°, gave 0·2025 grm. of chloride of silver.

$$C_{12}H_9S_4Ag_3O_{15}.$$

	Calculated.	Found.
Ag	38·34	38·50

0·4205 grm. of silver-salt, dried over sulphuric acid, lost at 150° 0·025 grm. of water.

$$C_{12}H_9S_4Ag_3O_{15}+3H_2O.$$

	Calculated.	Found.
$3H_2O$	6·00	5·95

When describing disulphophenylenic acid (page 687), it was shown that it is capable of forming two series of salts exhibiting an analogous relationship to each other, as metaphosphates to ordinary phosphates. Tetrasulphodiphenylenic acid behaves in a similar manner. Thus the salts previously described point most naturally to the conclusion that it is hexabasic, while the salts which will be presently mentioned indicate its capability of assuming also an octabasic character.

Lead-salt, $C_{12}H_6, S_4Pb_8O_{16}$.

When a boiling solution of tetrazodiphenylenate of ammonium is mixed with a solution of neutral acetate of lead, this salt is precipitated in the form of white needles. Repeated washings with water render it quite pure.

1·0195 grm., dried at 150°, gave 0·3797 grm. of carbonic acid and 0·054 grm. of water.

0·3135 grm.* gave 0·2755 grm. of sulphate of lead, corresponding to 60·04 per cent. of lead and 9·28 per cent. of sulphur.

$$C_{12}H_6S_4Pb_8O_{16}.$$

	Calculated.		Found.
C_{12}	144	10·57	10·16
H_6	6	0·45	0·59
Pb_8	828	60·79	60·04
S_4	128	9·39	9·28
O_{16}	256	18·80	——
	1362	100·00	

Basic Lead-salt, $C_{12}H_6, S_4Pb_8O_{16}+P_4O_2$.

This salt is obtained by treating the hot solution of the ammonium-salt with basic acetate of lead. It is a white amorphous precipitate.

0·5935 grm., dried at 150°, gave 0·592 grm. of sulphate of lead.

$$C_{12}H_6S_4Pb_{12}O_{18}.$$

	Calculated.	Found.
Pb	68·69	68·14

Tetrasulphodiphenylenic acid is easily to be obtained in a free state, either by decomposing the lead- or silver-salt with sulphuretted hydrogen, or by treating the barium-salts with an equivalent quantity of sulphuric acid. The filtered aqueous solution is evaporated on a water-bath to a syrupy consistence, and placed over sulphuric acid; after some time white needles or plates are obtained, which are very soluble in water and alcohol, but not deliquescent in the air. I have not yet analyzed this acid; but according to the above-mentioned salts it seems very probable that it can exist in two different states, as expressed by the formulæ $C_{12}H_6, S_8H_6O_{15}$ and $C_{12}H_6, S_8H_8O_{16}$.

The second compound (trisulphodiphenylenic acid) to which the reaction of sulphuric acid or sulphate of tetrazobenzidol gives rise deserves likewise a few passing remarks. The separation of its barium-salt from the barium-salt of tetrasulphodiphenylic acid by means of hot water has already been described. The aqueous extracts thus obtained, sufficiently evaporated and allowed to cool, will generally solidify to a gelatinous mass from which distinct crystals cannot be obtained, even by repeated solution and evaporation. Crystallization may, however, be effected thus: The gelatinous salt is converted into the ammonia-compound, by boiling with an aqueous solution of carbonate of ammonium.

* This substance was boiled with concentrated nitric acid till red fumes ceased to be evolved, and evaporated to dryness. The sulphate of lead was then collected on a filter and well washed with alcohol; the alcoholic filtrate contains neither a trace of lead nor sulphuric acid.

The excess of ammonia is removed by evaporation, and the crystalline residue dissolved in a little hot water and treated with a solution of chloride of barium. On cooling, warty crystals, or globular groups of small plates, of the barium-salt are obtained, which can be freed from the difficultly-soluble tetrasulphodiphenylenate of barium with which it may still be contaminated, and also from the mother-liquor, by repeated crystallization from water.

The analysis of this salt (dried at 130° C., at which temperature it turns to a dirty green colour owing to the loss of water of crystallization) gave the following numbers:

0·526 grm. of substance gave 0·4403 grm. of carbonic acid and 0·0766 grm. of water.

0·453 grm. gave 0·2535 grm. of sulphate of barium.

0·3605 grm. gave 0·388 grm. of sulphate of barium.

These amounts lead to the formula

$$C_{12}H_6S_3HBa_3O_{11}.$$

	Calculated.		Found.
C_{12}	144	22·91	22·80
H_7	7	1·11	1·60
Ba_3	205·5	32·70	32·80
S_3	96	15·28	14·78
O_{11}	176	28·00	——
	628·5	100·00	

Lead-salt, $C_{12}H_6, S_3Pb_6O_{12}$.

White amorphous precipitate obtained by treating a hot solution of the barium-salt with a solution of neutral acetate of lead.

0·711 grm., dried at 130°, gave 0·604 grm. of sulphate of lead.

$$C_{12}H_6, S_3Pb_6O_{12}.$$

	Calculated.	Found.
Pb	58·63	58·05

0·7375 grm., dried over sulphuric acid, lost at 130° 0·0265 grm. of water.

$$C_{12}H_6S_3Pb_6O_{12}+2H_2O.$$

	Calculated.	Found.
$4H_2O$	3·29	3·52

Basic Lead-salt, $C_{12}H_6, S_3Pb_6O_{12}, Pb_4O_2$,

is precipitated from a solution of the ammonium- or barium-salt with basic acetate of lead. It scarcely differs in its properties from the previous salt.

0·750 grm., dried at 130°, gave 0·2483 grm. of carbonic acid and 0·0557 grm. of water.

0·5395 grm., dried at the same temperature, gave 0·532 grm. of sulphate of lead.

	Calculated.		Found.
C_{12}	144	9·57	9·03
H_6	6	0·40	0·82
S_3	96	6·38	——
Pb_{10}	1035	68·77	68·59
O_{14}	224	14·88	——
	1505	100·00	

Trisulphodiphenylenic Acid

is obtained in a free state in exactly the same manner as tetrasulphodiphenylenic acid, which it resembles in every other respect. It is evident at a glance that its composition may also be expressed in two different ways, viz. $C_{12} H_6, S_3 H_4 O_{11}$, or $C_{12} H_6, S_3 H_6 O_{12}$.

In accordance with the experiment just described, the decomposition which the tetrazodiphenyl undergoes by the action of sulphuric acid may be expressed by the following equations:—

$$\text{I.}\quad \underbrace{C_{12} H_6 N_4}_{\text{Tetrazodiphenyl.}} + S_4 H_8 O_{16} = \underbrace{C_{12} H_6 S_4 H_8 O_{16}}_{\text{Tetrasulphodiphenylenic acid.}} + N_4.$$

$$\text{II.}\quad \underbrace{C_{12} H_6 N_4}_{\text{Tetrazodiphenyl.}} + S_3 H_6 O_{12} = \underbrace{C_{12} H_6, S_3 H_6 O_{12}}_{\text{Trisulphodiphenylenic acid.}} + N_4.$$

Decomposition of the Platinum-salt of Tetrazodiphenyl and of the Perbromide.

On mixing the platinum-salt of tetrazodiphenyl with from four to six times its weight of carbonate of sodium, and heating the mixture in a retort, a copious evolution of gas speedily ensues; and on increasing the heat an oily body distils, which solidifies in the neck of the retort to a white mass. By pressing this body between bibulous paper, and by repeated recrystallization from boiling alcohol, it is obtained perfectly pure. Analogy leads to the supposition that the formation of this body takes place according to the equation

$$\underbrace{C_{12} H_6 N_4, 2H Cl, 2Pt Cl_2}_{\text{Platinum-salt.}} = \underbrace{C_{12} H_8 Cl_2}_{\text{New body.}} + Cl_4 + Pt_2 + N_4.$$

The following chlorine-determination corroborates this transformation:—
0·235 grm. gave 0·2938 grm. of chloride of silver = 30·93 per cent. of chlorine.

Calculated.	Found.
Chlorine = 31·84 per cent.	30·93.

This new body, which I will call dichlorodiphenyl, crystallizes in white, usually well-formed prisms; it is difficult of solution even in boiling alcohol, but readily soluble in

ether, and quite insoluble in water. It fuses at 148° C. to a yellowish oil, which can be distilled without decomposition.

On heating in like manner the perbromide of tetrazodiphenyl with carbonate of sodium, a mixture of bromine and nitrogen gas is evolved, and on heating more strongly an oily substance distils over, which quickly solidifies. It is obtained pure for analysis by repeated recrystallization from ether.

0·322 grm. gave 0·5513 grm. of carbonic acid and 0·082 grm. of water.

These numbers correspond, as might be expected, with the formula

$$C_{12} H_8 Br_2.$$

	Theory.		Experiment.
C_{12}	144	46·15	46·69
H_8	8	2·57	2·84
Br_2	160	51·28	——
	312	100·00	

This compound, which may be called dibromodiphenyl, resembles the previously described chlorine-compound; it crystallizes likewise in well-formed prisms, which appear to be even more insoluble in alcohol and ether than the above, and which fuse at 164° C. This substance is obtained, moreover, by boiling the perbromide with alcohol, as in the following equation:—

$$\underbrace{C_{12} H_6 N_4 H_2 Br_2 Br_4}_{\text{Perbromide.}} = \underbrace{C_{12} H_8 Br_2}_{\text{Dibromodiphenyl.}} + N_4 + Br_4.$$

The substance separates from the alcoholic solution on cooling in crystals; it is best purified by distillation.

Dr. FITTIG informs me that he obtained a compound of like composition by acting with bromine upon diphenyl. The description of this new compound given by him applies so entirely to the dibromodiphenyl prepared by me, that no doubt remains of the identity of the bodies obtained by different methods.

XIX. *On the Differential Equations which determine the form of the Roots of Algebraic Equations.*

By GEORGE BOOLE, *F.R.S., Professor of Mathematics in Queen's College, Cork.*

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1. Mr. HARLEY* has shown that any root of the equation

$$y^n - ny + (n-1)x = 0$$

satisfies the differential equation

$$y - \frac{\left(D - \frac{2n-1}{n}\right)\left(D - \frac{3n-2}{n}\right)\ldots\left(D - \frac{n^2-n+1}{n}\right)}{D(D-1)\ldots(D-n+1)} e^{(n-1)\theta} y = 0, \quad . \quad . \quad . \quad . \quad (1)$$

in which $e^\theta = x$, and $D = \frac{d}{d\theta}$, provided that n be a positive integer greater than 2. This result, demonstrated for particular values of n, and raised by induction into a general theorem, was subsequently established rigorously by Mr. CAYLEY by means of LAGRANGE's theorem.

For the case of $n=2$, the differential equation was found by Mr. HARLEY to be

$$y - \frac{D - \frac{3}{2}}{D} e^\theta y = \tfrac{1}{2} e^\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Solving these differential equations for the particular cases of $n=2$ and $n=3$, Mr. HARLEY arrived at the actual expression of the roots of the given algebraic equation for these cases. That all algebraic equations up to the fifth degree can be reduced to the above trinomial form, is well known.

A solution of (1) by means of definite triple integrals in the case of $n=4$ has been published by Mr. W. H. L. RUSSELL; and I am informed that a general solution of the equation by means of a definite single integral has been obtained by the same analyst.

While the subject seems to be more important with relation to differential than with reference to algebraic equations, the connexion into which the two subjects are brought must itself be considered as a very interesting fact. As respects the former of these subjects, it may be observed that it is a matter of quite fundamental importance to ascertain for what forms of the function ϕ (D), equations of the type

$$u + \phi(D) e^{n\theta} u = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

admit of finite solution. We possess theorems which enable us to deduce from each known integrable form an infinite number of others. Yet there is every reason to think

* Proceedings of the Literary and Philosophical Society of Manchester, No. 12, Session 1861–62.

that the number of really primary forms—of forms the knowledge of which, in combination with such known theorems, would enable us to solve all equations of the above type that are finitely solvable—is extremely small. It will, indeed, be a most remarkable conclusion, should it ultimately prove that the forms in question stand in absolute and exclusive connexion with the class of algebraic equations here considered.

The following paper is a contribution to the general theory under the aspect last mentioned. In endeavouring to solve Mr. HARLEY's equation by definite integrals, I was led to perceive its relation to a more general equation, and to make this the subject of investigation. The results will be presented in the following order:—

First, I shall show that if u stand for the mth power of any root of the algebraic equation

$$y^n-xy^{n-1}-1=0,$$

then u, considered as a function of x, will satisfy the differential equation

$$[\mathrm{D}]^n u+\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{n-1}\left(\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right)e^{n\theta}u=0,$$

in which $e^\theta=x$, $\mathrm{D}=\frac{d}{d\theta}$, and the notation

$$[a]^b=a(a-1)(a-2)\,..\,(a-b+1)$$

is adopted.

Secondly, I shall show that for particular values of m, the above equation admits of an immediate first integral, constituting a differential equation of the $n-1$th order, and that the results obtained by Mr. HARLEY are particular cases of this depressed equation, their difference of form arising from difference of determination of the arbitrary constant.

Thirdly, I shall solve the general differential equation by definite integrals.

Fourthly, I shall determine the arbitrary constants of the solution so as to express the mth power of that real root of the proposed algebraic equation which reduces to 1 when $x=0$.

The differential equation which forms the chief subject of these investigations certainly occupies an important place, if not one of exclusive importance, in the theory of that large class of differential equations of which the type is expressed in (3). At present, I am not aware of the existence of any differential equations of that particular type which admit of finite solution at all, otherwise than by an ultimate reduction to the form in question, or by a resolution into linear equations of the first order. It constitutes, in fact, a generalization of the form

$$u+\frac{a(\mathrm{D}-2)^2\pm n^2}{\mathrm{D}(\mathrm{D}-1)}e^{2\theta}u=0$$

given in my memoir "On a General Method in Analysis" (Philosophical Transactions for 1844, Part II.).

Formation of the Differential Equation.—General finite integral.

2. PROPOSITION.—*If* u *represent the* mth *power of any root of the algebraic equation*

$$y^n - xy^{n-1} - 1 = 0,$$

then u, *considered as a function of* x, *satisfies the linear differential equation*

$$[\mathrm{D}]^n u + \left[\frac{n-1}{n}\mathrm{D} + \frac{m}{n} - 1\right]^{n-1}\left(\frac{\mathrm{D}}{n} - \frac{m}{n} - 1\right)e^{n\theta}u = 0,$$

in which $e^\theta = x$, *and* $\mathrm{D} = \frac{d}{d\theta}$.

And the complete integral of the above differential equation will be

$$u = \mathrm{C}_1 y_1^m + \mathrm{C}_2 y_2^m \ldots + \mathrm{C}_n y_n^m,$$

$y_1, y_2, \ldots y_n$ *being the* n *roots of the given algebraic equation.*

Representing y^n by z, we may give to the proposed algebraic equation the form

$$z = b + xz^{\frac{n-1}{n}}, \quad \ldots \ldots \ldots \quad (1)$$

in which $b=1$. Hence by LAGRANGE's theorem

$$u = z^{\frac{m}{n}} = b^{\frac{m}{n}} + b^{\frac{n-1}{n}}\frac{d}{db}\left(b^{\frac{m}{n}}\right)x + \frac{d}{db}\left(b^{\frac{2(n-1)}{n}}\frac{d}{db}\,b^{\frac{m}{n}}\right)\frac{x^2}{1.2} + \text{\&c.}, \quad \ldots \quad (2)$$

the general term of the expansion being

$$\left(\frac{d}{db}\right)^{r-1}\left\{b^{\frac{r(n-1)}{n}}\frac{d}{db}\,b^{\frac{m}{n}}\right\}\frac{x^r}{1.2..r}, \quad \ldots \ldots \ldots \quad (3)$$

which, on effecting the operations indicated, becomes

$$\frac{m\left[\frac{m+r(n-1)}{n} - 1\right]^{r-1} b^{\frac{m-r}{n}}}{n[r]^r}x^r \quad \ldots \ldots \ldots \quad (4)$$

We see then that u is expanded in a series of the form

$$u_0 + u_1 x + u_2 x^2 + \text{\&c. } ad\ inf.,$$

in which, since $b=1$,

$$u_r = \frac{m\left[\frac{m+(n-1)r}{n} - 1\right]^{r-1} \times (1)^{\frac{m-r}{n}}}{n[r]^r}; \quad \ldots \ldots \ldots \quad (5)$$

and this expression will represent the first term as well as the succeeding coefficients of the Lagrangian development, provided that we interpret the form $[p]^0$ by 1, and $[p]^{-1}$ by $\frac{1}{p+1}$.

As $1^{\frac{1}{n}}$ admits of n distinct values, the above development may be made to represent the mth power of any one of the n roots of the given algebraic equation. In particular,

if we give to $1^{\frac{1}{n}}$ the particular value 1, we have

$$u_r = \frac{m\left[\frac{m+(n-1)r}{n}-1\right]^{r-1}}{n[r]^r},$$

and the expansion then represents the mth power of that particular root which, when $x=0$, reduces to 1. The law of the series upon which the formation of the differential equation depends is, as we shall perceive, independent of these determinations.

Changing r into $r-n$, we have

$$u_{r-n} = \frac{m\left[\frac{m+(n-1)r}{n}-n\right]^{r-n-1} \times 1^{\frac{m-r}{n}+1}}{n[r-n]^{r-n}},$$

whence the law of the series is seen to be

$$[r]^r u_r + \left[\frac{m+(n-1)r}{n}-1\right]^{n-1}\left(\frac{r}{n}-\frac{m}{n}-1\right)u_{n-r}=0, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and therefore, by what is shown in my memoir "On a General Method in Analysis," the differential equation defining u will be

$$[\mathrm{D}]^n u + \left[\frac{m+(n-1)\mathrm{D}}{n}-1\right]^{n-1}\left(\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right)e^{n\theta}u=0, \quad . \quad . \quad . \quad . \quad (\mathrm{I})$$

in which $e^{\theta}=x$ and $\mathrm{D}=\frac{d}{d\theta}$.

3. As u may here represent the mth power of *any* of the roots of the given equation, it is evident that the general integral of the above differential equation will be

$$u=\mathrm{C}_1 y_1^m + \mathrm{C}_2 y_2^m \ldots + \mathrm{C}_n y_n^m, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

exception arising, however, in the case in which for a particular value of m the n particular integrals y_1^m, y_2^m, .. y_n^m cease to be independent. In such cases the above value of u constitutes an integral, but not the general integral of the differential equation.

For instance, if $m=-1$, and if we reduce the given algebraic equation to the form

$$(y^{-1})^n + xy^{-1} - 1 = 0,$$

it is evident that, except when $n=2$, we shall have

$$y_1^{-1}+y_2^{-1} \ldots + y_n^{-1} = 0.$$

Here then

$$u=\mathrm{C}_1 y_1^{-1} + \mathrm{C}_2 y_2^{-1} \ldots + \mathrm{C}_n y_n^{-1}$$

may be reduced to the form

$$u=(\mathrm{C}_1-\mathrm{C}_n)y_1^{-1} + (\mathrm{C}_2-\mathrm{C}_n)y_2^{-1} \ldots + (\mathrm{C}_{n-1}-\mathrm{C}_n)y_{n-1}^{-1},$$

virtually involving but $n-1$ arbitrary constants.

Such cases of failure may, however, be treated by giving to the integral a form which for the particular value of m shall become indeterminate, and then seeking the limiting

value. In the above example we may write

$$u=C_1y_1^{m}..+C_{n-1}y_{n-1}^{m}+C_n\frac{y_1^{m}+y_2^{m}..+y_n^{m}}{m+1},$$

the last term of which becomes a vanishing fraction when $m=-1$. The true limiting form is seen to be

$$u=C_1y_1^{-1}..+C_{n-1}y_{n-1}^{-1}+C_n(y_1^{-1}\log y_1..+y_n^{-1}\log y_n). \quad . \quad . \quad . \quad . \quad (8)$$

This is the complete integral of (I.) when $m=-1$.

4. The theory of these failing cases may be viewed also in another aspect. When

$$u=C_1y_1^{m}+C_2y_2^{m}..+C_ny_n^{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

is an integral, but not the general integral of the differential equation (I), it must be the general integral of a differential equation involved in (I), but of a lower order. We may in fact conclude that such reduced differential equation will be deducible from the higher one by a process of integration. Let us apply this consideration to the foregoing example.

When $m=-1$, the equation (I) becomes

$$D(D-1)..(D-n+1)u+\frac{1}{n}\left[\frac{n-1}{n}D-\frac{1}{n}-1\right]^{n-1}(D-n+1)e^{n\theta}u=0.$$

Hence operating on both members with $(D-n+1)^{-1}$, we have

$$D(D-1)..(D-n+2)u+\frac{1}{n}\left[\frac{n-1}{n}D-\frac{1}{n}-1\right]^{n-1}e^{n\theta}u=Ce^{(n-1)\theta}.$$

It must then be possible to determine C so as to cause this differential equation to be satisfied by (9). First let us seek to determine C so as to cause the equation to admit of any of the particular integrals $y_1^{m}, y_2^{m}, .. y_n^{m}$. Substituting for u the Lagrangian expansion reduced by making $m=-1$, and giving to b any of the particular values included in the form $1^{\frac{1}{n}}$, we shall, on equating coefficients, find

$$C=\frac{-[n-3]^{n-2}}{n},$$

whence it appears that if n be greater than 2, $C=0$. Thus the reduced differential equation becomes

$$[D]^{n-1}u+\frac{1}{n}\left[\frac{n-1}{n}D-\frac{1}{n}-1\right]^{n-1}e^{n\theta}u=0; \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and this, when n is greater than 2, is satisfied by

$$u=C_1y_1^{-1}+C_2y_2^{-1}..+C_ny_n^{-1},$$

which in effect contains $n-1$ arbitrary constants, and so constitutes the complete integral of the differential equation.

If $n=2$, the differential equation becomes

$$Du+\tfrac{1}{2}(\tfrac{1}{2}D-\tfrac{3}{2})e^{2\theta}u=\frac{-1}{2}e^{\theta}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

which is satisfied by $u=y_1^{-1}$ and by $u=y_2^{-1}$, but, as is evident from its unhomogeneous form, not by $u=C_1y_1^{-1}+C_2y_2^{-1}$. In this case, in fact, the condition $y_1^{-1}+y_2^{-1}=0$ not being fulfilled, the primary differential equation (I) suffers no change in the form of its general solution.

Mr. HARLEY'S results are in effect transformations of (10) and (11). Since $u=y^{-1}$, it is seen that u will satisfy the algebraic equation

$$u^n+xu-1=0.$$

Transform this by assuming

$$x=-n(1-n)^{\frac{1-n}{n}}x'^{\frac{1-n}{n}},\quad u=(1-n)^{-\frac{1}{n}}x'^{-\frac{1}{n}}u',$$

and we have

$$u'^n-nu'+(n-1)x'=0,$$

which is Mr. HARLEY'S algebraic equation in form. Hence, if $x'=e^{\theta'}$ and $D'=\frac{d}{d\theta'}$, we shall have

$$e^{\theta}=-n(1-n)^{\frac{1-n}{n}}e^{\frac{1-n}{n}\theta'},\quad u=(1-n)^{-\frac{1}{n}}e^{-\frac{1}{n}\theta'}u',\quad D=\frac{n}{1-n}D'.$$

And (10) will become

$$\left[\frac{nD'}{1-n}\right]^{n-1}e^{-\frac{1}{n}\theta'}u'+\frac{1}{n}\left[-D'-\frac{1}{n}-1\right]^{n-1}(-n)^n(1-n)^{1-n}e^{\left(1-n-\frac{1}{n}\right)\theta'}u'=0.$$

Multiply by $e^{\left(n-1+\frac{1}{n}\right)\theta'}$, and we have

$$\left[\frac{n\left(D'-n+1-\frac{1}{n}\right)}{1-n}\right]^{n-1}e^{(n-1)\theta'}u'-[-D'+n-2]^{n-1}(-n)^{n-1}(1-n)^{1-n}u'=0.$$

Now

$$\left[\frac{n\left(D'-n+1-\frac{1}{n}\right)}{1-n}\right]^{n-1}=\left[\frac{n}{1-n}D'+n-\frac{1}{1-n}\right]^{n-1}=(-1)^{n-1}\left[\frac{n}{n-1}D'-\frac{2n-1}{n-1}\right]^{n-1}$$

and

$$[-D'+n-2]^{n-1}=(-1)^{n-1}[D']^{n-1}.$$

Hence

$$\left[\frac{n}{n-1}D'-\frac{2n-1}{n-1}\right]^{n-1}e^{(n-1)\theta'}u'-\left(\frac{n}{n-1}\right)^{n-1}[D']^{n-1}u'=0,$$

or

$$[D']^{n-1}u'-\left(\frac{n-1}{n}\right)^{n-1}\left[\frac{n}{n-1}D'-\frac{2n-1}{n-1}\right]^{n-1}e^{(n-1)\theta'}u'=0,$$

which is Mr. HARLEY'S equation (1), art. 1. When $n=2$, we obtain from (11), by the same transformations, Mr. HARLEY'S second equation (2), art. 1.

Not only for the particular value $m=-1$, but apparently for all integer values of m, the general differential equation (I) admits of one integration. It may be said that while the differential equation determining the form of the mth power of a root of the algebraic equation is in general of the nth order, this equation may, when m is an integer, be reduced to an equation of the $n-1$th order; not, however, like the higher equation,

unvarying in its type. I have thus verified some other particular forms obtained by Mr. HARLEY.

Solution of the Differential Equation by Definite Integrals.

5. On account of the difficulty of the investigation, I propose to employ two distinct methods leading to coincident results.

First Method.—Operating on both sides of the given differential equation (I) with $\{[\mathrm{D}]^n\}^{-1}$, we have

$$u+\frac{\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{n-1}\left(\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right)}{[\mathrm{D}]^n}e^{n\theta}u=\mathrm{C}_0+\mathrm{C}_1e^{\theta}\cdots+\mathrm{C}_{n-1}e^{(n-1)\theta}, \quad . \quad . \quad (1)$$

$\mathrm{C}_0, \mathrm{C}_1, \ldots \mathrm{C}_{n-1}$ being arbitrary constants. Let us represent

$$\frac{\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{n-1}\left(\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right)}{[\mathrm{D}]^n}e^{n\theta}\mathrm{U},$$

whatever the nature of the subject U, by $\varrho\mathrm{U}$, then the differential equation becomes

$$u+\varrho\, u=\mathrm{C}_0+\mathrm{C}_1e^{\theta}\cdots+\mathrm{C}_{n-1}e^{(n-1)\theta},$$

or

$$(1+\varrho)u=\mathrm{C}_0+\mathrm{C}_1e^{\theta}\cdots+\mathrm{C}_{n-1}e^{(n-1)\theta};$$

$$\therefore \quad u=(1+\varrho)^{-1}\{\mathrm{C}_0+\mathrm{C}_1e^{\theta}\cdots+\mathrm{C}_{n-1}e^{(n-1)\theta}\}=\Sigma_i\mathrm{C}_i(1+\varrho)^{-1}e^{i\theta},$$

the summation extending from $i=0$ to $i=n-1$.

Now

$$(1+\varrho)^{-1}e^{i\theta}=(1-\varrho+\varrho^2-\varrho^3\ldots)e^{i\theta}.$$

But if

$$\varphi(\mathrm{D})=\frac{\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{n-1}\left(\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right)}{[\mathrm{D}]^n},$$

we have

$$\begin{aligned}\varrho e^{i\theta}&=\varphi(\mathrm{D})e^{n\theta}e^{i\theta},\\ \varrho^2e^{i\theta}&=\varphi(\mathrm{D})e^{n\theta}\varphi(\mathrm{D})e^{(n+i)\theta}=\varphi(\mathrm{D})\varphi(\mathrm{D}-n)e^{(2n+i)\theta},\\ \varrho^pe^{i\theta}&=\varphi(\mathrm{D})\varphi(\mathrm{D}-n)\ldots\varphi\big(\mathrm{D}-(p-1)n\big)e^{(pn+i)\theta}.\end{aligned}$$

But from the form of $\varphi(\mathrm{D})$ it is easily seen that

$$\varphi(\mathrm{D})\varphi(\mathrm{D}-n)=\frac{\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{2(n-1)}\left[\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right]^2}{[\mathrm{D}]^{2n}},$$

and generally

$$\varphi(\mathrm{D})\varphi(\mathrm{D}-n)\ldots\varphi\big(\mathrm{D}-(p-1)n\big)=\frac{\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{p(n-1)}\left[\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right]^p}{[\mathrm{D}]^{pn}};$$

$$\therefore\ \varrho^pe^{i\theta}=\frac{\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{p(n-1)}\left[\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right]^p}{[\mathrm{D}]^{pn}}e^{(pn+i)\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now

$$[a]^b=a(a-1)\ldots(a-b+1)$$

$$=\frac{\Gamma(a+1)}{\Gamma(a-b+1)},$$

provided that $a+1$ and $a-b+1$ are positive. This law we can extend symbolically to expressions in which D appears, provided that, in the application of the symbolic forms thence arising, D shall admit of an interpretation which shall effectively make the subjects of the symbol Γ to be positive numerical magnitudes. Under this condition we have then

$$\varrho^p e^{i\theta}=\frac{\dfrac{\Gamma\left(\frac{n-1}{n}\mathrm{D}+\frac{m}{n}\right)}{\Gamma\left(\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-p(n-1)\right)}\times\dfrac{\Gamma\left(\frac{\mathrm{D}}{n}-\frac{m}{n}\right)}{\Gamma\left(\frac{\mathrm{D}}{n}-\frac{m}{n}-p\right)}}{\dfrac{\Gamma(\mathrm{D}+1)}{\Gamma(\mathrm{D}-pn+1)}}e^{(pn+i)\theta}$$

$$=\Phi(\mathrm{D})\Psi(\mathrm{D})e^{(pn+i)\theta},$$

where

$$\Phi(\mathrm{D})=\frac{\Gamma\left(\frac{n-1}{n}\mathrm{D}+\frac{m}{n}\right)\Gamma\left(\frac{\mathrm{D}}{n}-\frac{m}{n}\right)}{\Gamma(\mathrm{D}+1)},$$

$$\Psi(\mathrm{D})=\frac{\Gamma(\mathrm{D}-pn+1)}{\Gamma\left(\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-p(n-1)\right)\Gamma\left(\frac{\mathrm{D}}{n}-\frac{m}{n}-p\right)}.$$

Now

$$\Psi(\mathrm{D})e^{(pn+i)\theta}=\Psi(pn+i)e^{(pn+i)\theta}$$

$$=\frac{\Gamma(i+1)}{\Gamma\left(\frac{n-1}{n}i+\frac{m}{n}\right)\Gamma\left(\frac{i-m}{n}\right)}e^{(pn+i)\theta}.$$

We see then that the conditions

$$(n-1)i+m\overline{\gtrless}0,\quad i-m\overline{\gtrless}0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

must be satisfied. For $i=0$ these conditions are inconsistent, and the proposed employment of Γ therefore unlawful. For values of i greater than 0 the conditions will be found to amount to this, viz. that m must lie between the limits $-(n-1)$ and 1. We shall suppose m thus conditioned, and shall consider first the case in which $i>0$.

Here then we have, interpreting D by $pn+i$ in $\Psi(\mathrm{D})$, but leaving it unchanged in $\Phi(\mathrm{D})$,

$$\varrho^p e^{i\theta}=\frac{\Gamma\left(\frac{n-1}{n}\mathrm{D}+\frac{m}{n}\right)\Gamma\left(\frac{\mathrm{D}}{n}-\frac{m}{n}\right)}{\Gamma(\mathrm{D}+1)}\ \frac{\Gamma(i+1)}{\Gamma\left(\frac{n-1}{n}i+\frac{m}{n}\right)\Gamma\left(\frac{i-m}{n}\right)}e^{(pn+i)\theta}, \quad . \quad . \quad . \quad . \quad (4)$$

it being seen that if we similarly interpreted D in $\Phi(\mathrm{D})$ the conditions relative to Γ would be satisfied throughout.

Hence if we write

$$\frac{\Gamma(i+1)C_i}{\Gamma\left(\frac{n-1}{n}i+\frac{m}{n}\right)\Gamma\left(\frac{i-m}{n}\right)}=A_i,$$

we shall have

$$(1-\varrho+\varrho^2-\varrho^3+\&c.)C_ie^{i\theta}=\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D+1)}A_i(e^{i\theta}-e^{(n+i)\theta}+e^{(2n+i)\theta}-\&c.)$$

$$=\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D+1)}\frac{A_ie^{i\theta}}{1+e^{n}},$$

and therefore

$$\Sigma_iC_i(1+\varrho)^{-1}e^{i\theta}=\Sigma_i\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D+1)}\frac{A_ie^{i\theta}}{1+e^{n\theta}}, \quad \ldots \ldots \quad (5)$$

the summation extending from $i=1$ to $i=n-1$.

Consider next the case in which $i=0$. We have, when p is not less than 1,

$$\begin{aligned}\varrho^pC_0&=\varrho^{p-1}\varrho C_0\\&=\varrho^{p-1}\varphi(D)e^{n\theta}C_0\\&=C_0\varphi(n)\varrho^{p-1}e^{n\theta}.\end{aligned}$$

But changing in (4) p into $p-1$, and i into n,

$$\varrho^{p-1}e^{i\theta}=\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D+1)}\frac{\Gamma(n+1)}{\Gamma\left(n-1+\frac{m}{n}\right)\Gamma\left(\frac{n-m}{n}\right)}e^{pn\theta}.$$

Hence, if we write

$$C_0\varphi(n)\frac{\Gamma(n+1)}{\Gamma\left(n-1+\frac{m}{n}\right)\Gamma\left(\frac{n-m}{n}\right)}=-A_n, \quad \ldots \ldots \quad (5')$$

we have for all positive integral values of p,

$$\varrho^pC_0=-A_n\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D+1)}e^{pn\theta},$$

and therefore

$$(1-\varrho+\varrho^2-\varrho^3+\&c.)C_0=C_0+\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D+1)}(A_ne^{n\theta}-A_ne^{2n\theta}+A_ne^{3n\theta}-\&c.);$$

$$\therefore\quad (1+\varrho)^{-1}C_0=C_0+\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D+1)}\frac{A_ne^{n\theta}}{1+e^{n\theta}}.$$

Combining this with (5), we find

$$u=C_0+\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D+1)}\left\{\frac{A_1e^{\theta}+A_2e^{2\theta}\ldots+A_ne^{n\theta}}{1+e^{n\theta}}\right\}$$

$$=C_0+\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D)}D^{-1}\left\{\frac{A_1e^{\theta}+A_2e^{2\theta}\ldots+A_ne^{n\theta}}{1+e^{n\theta}}\right\}.$$

Now, resolving the rational fraction, we have

$$D^{-1}\frac{A_1e^{\theta}+A_2e^{2\theta}\ldots+A_ne^{n\theta}}{1+e^{n\theta}}=D^{-1}\left\{\frac{N_1e^{\theta}}{1-\alpha_1e^{\theta}}+\frac{N_2e^{\theta}}{1-\alpha_2e^{\theta}}\ldots+\frac{N_ne^{\theta}}{1-\alpha_ne^{\theta}}\right\}$$

$$=B_1\log(1-\alpha_1e^{\theta})+B_2\log(1-\alpha_2e^{\theta})\ldots+B_n\log(1-\alpha_ne^{\theta}),$$

$\alpha_1, \alpha_2, \ldots \alpha_n$ being the nth roots of -1, and $B_i=\frac{-N_i}{\alpha_i}$. Hence

$$u=C_0+\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D)}\{B_1\log(1-\alpha_1e^{\theta})\ldots+B_n\log(1-\alpha_ne^{\theta})\}. \quad . \quad . \quad (6)$$

In this expression $B_1, \ldots B_n$, being generated from the arbitrary constants $C_0, C_1, \ldots C_{n-1}$, may themselves be regarded as arbitrary constants. And this being done, C_0 will become a dependent constant, the form of which it will be necessary to determine.

First, however, let us endeavour to interpret by a definite integral the symbolic function of D.

We know that a and b being positive quantities,

$$\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}=\int_0^1 dt\ t^{a-1}(1-t)^{b-1}=\int_0^{\infty}\frac{dt\ t^{a-1}}{(1+t)^{a+b}}.$$

If we employ the second of these forms, we shall have

$$\frac{\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)}{\Gamma(D)}\phi(e^{\theta})=\int_0^{\infty}dt\ \frac{t^{\frac{n-1}{n}D+\frac{m}{n}-1}}{(1+t)^{D}}\phi(e^{\theta})$$

$$=\int_0^{\infty}dt\ t^{\frac{m}{n}-1}\left(\frac{t^{\frac{n-1}{n}}}{1+t}\right)^{D}\phi(e^{\theta})$$

$$=\int_0^{\infty}dt\ t^{\frac{m}{n}-1}\phi\left(\frac{t^{\frac{n-1}{n}}e^{\theta}}{1+t}\right)$$

by a known symbolical form of TAYLOR's theorem. Hence if

$$\frac{t^{\frac{n-1}{n}}}{1+t}=T,$$

we have

$$u=C_0+B_1\int_0^{\infty}dt\ t^{\frac{m}{n}-1}\log(1-\alpha_1Te^{\theta})\cdots+B_n\int_0^{\infty}dt\ t^{\frac{m}{n}-1}\log(1-\alpha_nTe^{\theta}). \quad . \quad . \quad (7)$$

6. In determining C_0 the following theorem will be of use, viz.:—

If r *be a positive integer, and* a *positive and less than* r, *then*

$$\Gamma(a)\Gamma(r-a)=\frac{[r-a-1]^{r-1}\pi}{\sin(a\pi)}. \qquad (8)$$

This may be proved as follows:—

Let i be the greatest integer in a, and let $a-i=a'$. Then

$$\Gamma(a)\Gamma(r-a)=[a-1]^i\Gamma(a')\times[r-a-1]^{r-i-1}\Gamma(1-a').$$

But a' being a positive proper fraction,

$$\Gamma(a')\Gamma(1-a')=\frac{\pi}{\sin(a'\pi)},$$

and

$$[a-1]^i=(a-1)(a-2)\ldots(a-i)$$
$$=(-1)^i(i-a)(i-a-1)\ldots(1-a),$$
$$[r-a-1]^{r-i-1}=(r-a-1)(r-a-2)\ldots(i-a+1);$$
$$\therefore \quad [r-a-1]^{r-i-1}[a-1]^i=(-1)^i(r-a-1)\ldots(i-a+1)\times(i-a)\ldots(1-a)$$
$$=(-1)^i[r-a-1]^{r-1}.$$

Hence

$$\Gamma(a)\Gamma(r-a)=(-1)^i[r-a-1]^{r-1}\frac{\pi}{\sin(a'\pi)}.$$

But

$$\sin(a'\pi)=\sin(a\pi-i\pi)=(-1)^i\sin(a\pi),$$
$$\therefore \quad \Gamma(a)\Gamma(r-a)=\frac{[r-a-1]^{r-1}\pi}{\sin(a\pi)},$$

as was to be proved.

Now in the instance before us we have by (5′)

$$C_0=-A_n\frac{\Gamma\left(n-1+\frac{m}{n}\right)\Gamma\left(\frac{n-m}{n}\right)}{\Gamma(n+1)\phi(n)},$$

where

$$\phi(n)=\frac{\left[n+\frac{m}{n}-2\right]^{n-1}\left(-\frac{m}{n}\right)}{[n]^n}.$$

Hence, since $\Gamma(n+1)=[n]^n$,

$$C_0=A_n\frac{\Gamma\left(n-1+\frac{m}{n}\right)\Gamma\left(1-\frac{m}{n}\right)}{\left[n+\frac{m}{n}-2\right]^{n-1}\times\frac{m}{n}};$$

wherefore $1-\frac{m}{n}$ being a positive quantity, and n a positive integer, we have, by the

above theorem,

$$\Gamma\left(n-1+\frac{m}{n}\right)\Gamma\left(1-\frac{m}{n}\right)=\frac{\left[n-2+\frac{m}{n}\right]^{n-1}\pi}{\sin\left(1-\frac{m}{n}\right)\pi}$$

$$=\frac{\left[n-2+\frac{m}{n}\right]^{n-1}\pi}{\sin\left(\frac{m}{n}\pi\right)}.$$

Accordingly

$$C_0=\frac{A_n\pi}{\frac{m}{n}\sin\frac{m\pi}{n}}.$$

But since

$$\frac{N_1e^\theta}{1-\alpha_1e^\theta}\cdot\cdot+\frac{N_ne^\theta}{1-\alpha_ne^\theta}=\frac{A_1e^\theta\cdot\cdot+A_ne^{n\theta}}{1+e^{n\theta}},$$

we have

$$A_n=(-1)^{n-1}\left(\frac{N_1}{\alpha_1}\cdot\cdot+\frac{N_n}{\alpha_n}\right)\alpha_1\alpha_2\ldots\alpha_n$$

$$=(-1)^n\ (B_1\cdot\cdot+B_n)\times(-1)^n$$

$$=\ B_1\cdot\cdot+B_n.$$

Therefore, finally,

$$C_0=\frac{B_1+B_2\cdot\cdot+B_n}{\frac{m}{n}\sin\frac{m\pi}{n}}.$$

Substituting in (7), and replacing e^θ by x, we have

$$u=\frac{(B_1+B_2\cdot\cdot+B_n)\pi}{\frac{m}{n}\sin\frac{m\pi}{n}}+B_1\int_0^\infty dt\ t^{\frac{m}{n}-1}\log(1-\alpha_1xT)\cdot\cdot+B_n\int_0^\infty dt\ t^{\frac{m}{n}-1}\log(1-\alpha_nxT), \quad . \quad (9)$$

wherein, it must be remembered, that α_1, α_2, .. α_n are the several nth roots of -1, and

$$T=\frac{t^{\frac{n-1}{n}}}{1+t}.$$

And this is the general integral of (I), B_1, B_2, .. B_n being the arbitrary constants of the solution.

Second Method.—7. For the *finite* solution of differential equations of the form

$$f_0(D)u+f_1(D)e^{n\theta}u=0,$$

it is usually convenient to reduce them to the form

$$u+\frac{f_1(D)}{f_0(D)}e^{n\theta}u=\{f_0(D)\}^{-1}0,$$

which falls under the general type

$$u+\phi(\mathrm{D})e^{n\theta}u=\mathrm{U}, \quad \ldots \ldots \ldots \quad (1)$$

U being a function of θ when the inverse operation $\{f_0(\mathrm{D})\}^{-1}0$ has been performed.

The theory of equations of the above type has been discussed fully in my memoir "On a General Method in Analysis." In particular it is there shown that the above equation can be converted into another of the same type,

$$v+\psi(\mathrm{D})e^{n\theta}v=\mathrm{V},$$

by assuming

$$u=\mathrm{P}_n\frac{\phi(\mathrm{D})}{\psi(\mathrm{D})}v, \quad \mathrm{V}=\left\{\mathrm{P}_n\frac{\phi(\mathrm{D})}{\psi(\mathrm{D})}\right\}^{-1}\mathrm{U}, \quad \ldots \ldots \ldots \quad (2)$$

where

$$\mathrm{P}_n\frac{\phi(\mathrm{D})}{\psi(\mathrm{D})}=\frac{\phi(\mathrm{D})\phi(\mathrm{D}-n)\phi(\mathrm{D}-2n)\ldots \textit{ad inf.}}{\psi(\mathrm{D})\psi(\mathrm{D}-n)\psi(\mathrm{D}-2n)\ldots \textit{ad inf.}}.$$

This theory I shall apply here, not to the ordinary finite solution, but to the solution by definite integrals of the differential equation (I). In doing this I shall give to U and V the particular values 0. We are justified in doing this by the canons relating to the arbitrary constants which are laid down in the memoir; but it will suffice here to direct attention to the fact that while the processes employed are strictly speaking particular, they lead to a solution involving the requisite number of arbitrary constants, and at the same time of the proper *form*, as manifested by the succession of the indices in its development.

Giving then to (I) the form

$$u+\frac{\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{n-1}\left(\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right)}{[\mathrm{D}]^n}e^{n\theta}u=0,$$

assume as the transformed equation

$$v+\frac{1}{[\mathrm{D}]^n}e^{n\theta}v=0.$$

Then by (2)

$$u=\mathrm{P}_n\left\{\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]\left(\frac{\mathrm{D}}{n}-\frac{m}{n}-1\right)\right\}v.$$

Now

$$\mathrm{P}_n\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{n-1}=\left(\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right)\left(\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-2\right)\ldots \textit{ad inf.},$$

since representing $\left[\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right]^{n-1}$ by $\phi(\mathrm{D})$, the first term in the factorial expression of $\phi(\mathrm{D}-n)$ will so follow the last term in that of $\phi(\mathrm{D})$ as to leave the law of factorial succession unbroken. Again, if $\mathrm{A}_ie^{i\theta}$ be any term in the development of v, we have, i being a positive integer,

$$\left(\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-1\right)\left(\frac{n-1}{n}\mathrm{D}+\frac{m}{n}-2\right)\ldots \mathrm{A}_ie^{i\theta}$$

$$=\mathrm{A}_i\left(\frac{n-1}{n}i+\frac{m}{n}-1\right)\left(\frac{n-1}{n}i+\frac{m}{n}-2\right)\ldots e^{i\theta}$$

$$=\mathrm{A}_i\mathrm{C}\Gamma\left(\frac{n-1}{n}i+\frac{m}{n}\right)e^{i\theta},$$

C being a constant, the value of which does not change with i. Hence we may write

$$P_n\left[\frac{n-1}{n}D+\frac{m}{n}-1\right]^{n-1}=C\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right),$$

and in like manner

$$P_n\left(\frac{D}{n}-\frac{m}{n}-1\right)=C'\Gamma\left(\frac{D}{n}-\frac{m}{n}\right).$$

The legitimacy of the introduction of Γ depends upon the condition

$$\frac{n-1}{n}i+\frac{m}{n}>0,\ \frac{i}{n}-\frac{m}{n}>0,$$

so that the value $i=0$ is inadmissible, as we have already assumed. Moreover m must lie between the limits $-(n-1)$ and 1.

Since $e^\theta=x$, the equation for v is equivalent to

$$\frac{d^nv}{dx^n}+v=0,$$

whence

$$v=c_1e^{\alpha_1x}+c_2e^{\alpha_2x}\ldots+c_ne^{\alpha_nx},$$

in which $\alpha_1, \alpha_2, \ldots \alpha_n$ are the nth roots of -1. This value of v can be expanded in ascending powers of x in the form

$$\begin{aligned}v&=v_0+v_1x+v_2x^2+\&c.\\&=v_0+v_1e^\theta+v_2e^{2\theta}+\&c.\end{aligned}$$

Hence $u-u_0$ representing that part of u which contains positive and integral powers of x, we shall have

$$u-u_0=CC'\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)(v-v_0).$$

Now
$$\begin{aligned}v-v_0&=C_1(e^{\alpha_1x}-1)+C_2(e^{\alpha_2x}-1)\ldots+C_n(e^{\alpha_nx}-1)\\&=\Sigma C_i(e^{\alpha_ix}-1),\end{aligned}$$

the summation extending from $i=1$ to $i=n$. Hence, merging CC' in the arbitrary constants $C_1, \ldots C_n$, we have

$$u=u_0+\Sigma C_i\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)(e^{\alpha_ix}-1), \quad \ldots\ldots\ldots (3)$$

in which $x=e^\theta$. This expression we now propose to interpret by definite integrals.

Now
$$e^{\alpha_ix}-1=\int_0^x\alpha_ie^{\alpha_ih}dh.$$

Substituting and merging α_i in the arbitrary constant C_i, we have

$$\begin{aligned}u&=u_0+\Sigma C_i\Gamma\left(\frac{n-1}{n}D+\frac{m}{n}\right)\Gamma\left(\frac{D}{n}-\frac{m}{n}\right)\int_0^xe^{\alpha_ih}dh\\&=u_0+\Sigma C_i\int_0^\infty dse^{-s}s^{\frac{n-1}{n}D+\frac{m}{n}-1}\int_0^\infty dte^{-t}t^{\frac{D}{n}-\frac{m}{n}-1}\int_0^xe^{\alpha_i}\,dh\end{aligned}$$

on interpreting the Γ functions in the usual manner. We may therefore write

$$u=u_0+\Sigma C_i\int_0^\infty\int_0^\infty dsdte^{-(s+t)}s^{\frac{m}{n}-1}t^{-\frac{m}{n}-1}\int_0^{xs^{\frac{n-1}{n}}t^{\frac{1}{n}}}e^{\alpha_i h}\,dh,$$

since by the symbolical form of TAYLOR's theorem

$$s^{\frac{n-1}{n}\mathrm{D}}t^{\frac{\mathrm{D}}{n}}\phi(x)=\left(s^{\frac{n-1}{n}}t^{\frac{1}{n}}\right)^{\mathrm{D}}\phi(x)=\phi\left(xs^{\frac{n-1}{n}}t^{\frac{1}{n}}\right).$$

Let us now transform the double integral relative to s and t by assuming

$$s=vt,$$

and making v and t the new system of variables. We shall have

$$dsdt=tdvdt,$$

while the limits of v and t will be 0 and ∞. Hence

$$u=u_0+\Sigma C_i\int_0^\infty\int_0^\infty dvdte^{-(1+v)t}v^{\frac{m}{n}-1}t^{-1}\int_0^{xv^{\frac{n-1}{n}}t}e^{\alpha_i h}\,dh.$$

Again, transform the double integral relative to t and h, by assuming $h=ty$. We shall have $dh=tdy$, and the limits of y will be 0 and $xv^{\frac{n-1}{n}}$. Whence

$$u=u_0+\Sigma C_i\int_0^\infty\int_0^\infty\int_0^{xv^{\frac{n-1}{n}}}dvdtdye^{-(1+v-\alpha_i y)t}v^{\frac{m}{n}-1}.$$

Integrating with respect to t, we have

$$u=u_0+\Sigma C_i\int_0^\infty dv\int_0^{xv^{\frac{n-1}{n}}}dy\,\frac{v^{\frac{m}{n}-1}}{1+v-\alpha_i y}.$$

Now integrating with respect to y, and merging $\frac{-1}{\alpha_i}$ in the arbitrary constants,

$$\left.\begin{aligned}u&=u_0+\Sigma C_i\int_0^\infty dv\,v^{\frac{m}{n}-1}\left\{\log\left(1+v-\alpha_i xv^{\frac{n-1}{n}}\right)-\log(1+v)\right\}\\&=u_0+\Sigma C_i\int_0^\infty dv\,v^{\frac{m}{n}-1}\log\left(1-\frac{\alpha_i xv^{\frac{n-1}{n}}}{1+v}\right)\end{aligned}\right\}\quad\ldots\quad(4)$$

It remains to determine u_0.

Developing the function under the sign of integration in ascending powers of x, and effecting the integration for each term separately, we find, for the coefficient of x^n, the expression

$$u_n=\Sigma C_i\frac{\Gamma\left(\frac{m}{n}+n-1\right)\Gamma\left(1-\frac{m}{n}\right)}{n\Gamma(n)};$$

but from the law of the series as expressed in (6), art. 2,

$$u_n=-\frac{\left[\frac{m}{n}+n-2\right]^{n-1}\times\left(-\frac{m}{n}\right)}{[n]^n}u_0.$$

Equating these values,

$$u_0=\Sigma C_i\frac{\Gamma\left(\frac{m}{n}+n-1\right)\Gamma\left(1-\frac{m}{n}\right)}{\frac{m}{n}\left[\frac{m}{n}+n-2\right]^{n-1}}$$

$$=\Sigma C_i\frac{\pi}{\frac{m}{n}\sin\frac{m\pi}{n}}$$

by the reductions of art. 6.

Hence, finally,

$$u=\Sigma\frac{C_i\pi}{\frac{m}{n}\sin\frac{m\pi}{n}}+\Sigma C_i\int_0^\infty dv\, v^{\frac{m}{n}-1}\log\left(1-\alpha_i\frac{xv^{\frac{n-1}{n}}}{1+v}\right), \quad \text{(II)}$$

which agrees with the previous result.

Determination of the Constants.

8. I propose here to determine the constants of the general integral (II), so as to obtain an expression for the mth power of that particular (real) root of the equation

$$y^n-xy^{n-1}-1=0$$

which becomes unity when $x=0$.

We have

$$u=\Sigma C_i\frac{\pi}{\frac{m}{n}\sin\frac{m\pi}{n}}+\Sigma C_i\int_0^\infty dv\, v^{\frac{m}{n}-1}\log(1-\alpha_i xV), \quad (1)$$

where $V=\frac{v^{\frac{n-1}{n}}}{1+v}$, and α_i represents in succession the different nth roots of -1.

The coefficient of x^r in the expansion of this value of u in ascending powers of x will be found to be

$$-\Sigma C_i\alpha_i^r\frac{\Gamma\left(\frac{m+(n-1)r}{n}\right)\Gamma\left(\frac{r-m}{n}\right)}{r\Gamma(r)},$$

and its coefficient in the expansion of y^m by LAGRANGE's theorem is, for the particular root in question,

$$\frac{m\left[\frac{m+(n-1)r}{n}-1\right]^{r-1}}{n[r]^r},$$

equating which we have

$$\Sigma C_i\alpha_i^r=-\frac{m\left[\frac{m+(n-1)r}{n}-1\right]^{r-1}}{n\Gamma\left(\frac{m+(n-1)r}{n}\right)\Gamma\left(\frac{r-m}{n}\right)}.$$

But by the theorem of art. 6,

$$\Gamma\left(\frac{m+(n-1)r}{n}\right)\Gamma\left(\frac{r-m}{n}\right)=\Gamma\left(\frac{r-m}{n}\right)\Gamma\left(r-\frac{r-m}{n}\right)$$

$$=\left[r-\frac{r-m}{n}-1\right]^{n-1}\frac{\pi}{\sin\left(\frac{r-m}{n}\pi\right)}$$

$$=\frac{\left[\frac{m+(n-1)r}{n}-1\right]^{r-1}\pi}{\sin\left(\frac{m-r}{n}\pi\right)}.$$

Hence

$$\Sigma_i C_i \alpha_i^r = -\frac{m\sin\left(\frac{r-m}{n}\pi\right)}{n\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Giving, in this equation, to r any particular system of n values, we shall obtain a system of n linear equations for the determination of the n constants C_1, C_2, .. C_n. We shall form this system by giving to r the values 1, 2, .. n.

Now α_j representing any *particular* root selected from the series α_1, α_2, .. α_n, multiply the above typical equation by α_j^{n-r}, and then, giving to r the successive values 1, 2 .. n, form the sum of the equations thus arising. The result may be expressed in the form

$$\Sigma_i C_i \Sigma_r \alpha_i^r \alpha_j^{n-r} = -\frac{m}{n\pi}\Sigma_r \sin\left(\frac{r-m}{n}\pi\right)\alpha_j^{n-r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the summations with respect to i and r being both from 1 to n inclusive.

But

$$\Sigma_r \alpha_i^r \alpha_j^{n-r} = \alpha_i \alpha_j^{n-1} + \alpha_i^2 \alpha_j^{n-2} .. + \alpha_i^n$$

$$= \alpha_i(\alpha_j^{n-1} + \alpha_i \alpha_j^{n-2} .. + \alpha_i^{n-1})$$

$$= \alpha_i \frac{\alpha_j^n - \alpha_i^n}{\alpha_j - \alpha_i}.$$

Now when α_i is not equal to α_j, this expression vanishes, since $\alpha_i^n = \alpha_j^n = -1$. When, however, $\alpha_i = \alpha_j$, the fraction $\frac{\alpha_j^n - \alpha_i^n}{\alpha_j - \alpha_i}$ becomes indeterminate, and its true limiting value is seen to be $n\alpha_j^n = -n$. Hence (3) becomes

$$-nC_j = -\frac{m}{n\pi}\Sigma_r \sin\left(\frac{r-m}{n}\pi\right)\alpha_j^{n-r},$$

$$\therefore \quad C_j = \frac{m}{n^2\pi}\Sigma_r \sin\left(\frac{r-m}{n}\pi\right)\alpha_j^{n-r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We have thus solved the linear system of equations. We have still to reduce this solution by effecting the summation in the second member.

Now to α_j we may give the form $e^{\frac{2j+1}{n}\pi\sqrt{-1}}$, which will represent all the nth roots of -1 in succession if we give to j the series of values 1, 2, .. n. Hence substituting for α_j

the above value, and giving to $\sin\left(\frac{m-r}{n}\pi\right)$ its exponential form, we have

$$\Sigma_r \sin\left(\frac{m-r}{n}\pi\right)\alpha_j^{-r}=\Sigma_r\frac{e^{\frac{m-r-(2j+1)r}{n}\pi\sqrt{-1}}-e^{\frac{-(m-r)-(2j+1)r}{n}\pi\sqrt{-1}}}{2\sqrt{-1}}$$

$$=\frac{e^{\frac{m\pi}{n}\sqrt{-1}}\,\Sigma_r e^{\frac{-2(j+1)r\pi}{n}\sqrt{-1}}-e^{\frac{-m\pi}{n}\sqrt{-1}}\,\Sigma_r e^{\frac{-2jr\pi}{n}\sqrt{-1}}}{2\sqrt{-1}}.$$

Now in general

$$\Sigma_r e^{kr\pi\sqrt{-1}}=e^{k\pi\sqrt{-1}}+e^{2k\pi\sqrt{-1}}\ldots+e^{nk\pi\sqrt{-1}}$$

$$=\frac{e^{(n+1)k\pi\sqrt{-1}}-e^{k\pi\sqrt{-1}}}{e^{k\pi\sqrt{-1}}-1}$$

$$=e^{\frac{k(n+1)\pi}{2}\sqrt{-1}}\times\frac{e^{\frac{kn\pi}{2}\sqrt{-1}}-e^{\frac{-kn\pi}{2}\sqrt{-1}}}{e^{\frac{k\pi}{2}\sqrt{-1}}-e^{\frac{-k\pi}{2}\sqrt{-1}}}$$

$$=e^{\frac{k(n+1)}{2}\pi\sqrt{-1}}\times\frac{\sin\frac{kn\pi}{2}}{\sin\frac{k\pi}{2}}.$$

Putting therefore $$k=-\frac{2(j+1)}{n},$$

we have $$\Sigma_r e^{\frac{-2(j+1)r\pi}{n}\sqrt{-1}}=e^{\frac{-(j+1)(n+1)}{n}\pi\sqrt{-1}}\,\frac{\sin(j+1)\pi}{\sin\frac{(j+1)\pi}{n}},$$

and putting $$k=e\frac{-2j}{n},$$

$$\Sigma_r e^{\frac{-2jr\pi}{n}\sqrt{-1}}=e^{\frac{-j(n+1)}{n}\pi\sqrt{-1}}\,\frac{\sin j\pi}{\sin\frac{j\pi}{n}}.$$

Hence $$\Sigma_r \sin\left(\frac{m-r}{n}\pi\right)\alpha_j^{-r}=\frac{1}{2\sqrt{-1}}\left\{\begin{aligned}&e^{\frac{m-(j+1)(n+1)}{n}\pi\sqrt{-1}}\,\frac{\sin(j+1)\pi}{\sin\frac{(j+1)\pi}{n}}\\&-e^{\frac{-m-j(n+1)}{n}\pi\sqrt{-1}}\,\frac{\sin j\pi}{\sin\frac{j\pi}{n}}\end{aligned}\right\}$$

Now $$\frac{\sin(j+1)\pi}{\sin\frac{(j+1)\pi}{n}}=0$$

for all values of j taken from the series 1, 2, .. n except the value $n-1$, for which the expression becomes indeterminate in form, and has for its true value

$$\frac{\pi\cos n\pi}{\frac{\pi}{n}\cos\frac{n\pi}{n}}=\frac{n\cos n\pi}{\cos\pi}=\pm n,$$

as n is odd or even.

So too
$$\frac{\sin j\pi}{\sin\frac{j\pi}{n}}=0$$
for all values of j taken from the series 1, 2, .. n except the value n, for which its true value is
$$\frac{n\cos n\pi}{\cos\pi}=\pm n$$
as n is odd or even.

Hence when j stands for any of the integers 1, 2, .. $n-2$, we have
$$\Sigma_r \sin\left(\frac{m-r}{n}\pi\right)\alpha_j^{-r}=0.$$
When $j=n-1$, we have
$$\Sigma_r \sin\left(\frac{m-r}{n}\pi\right)\alpha_j^{-r}=\pm\frac{n}{2\sqrt{-1}}e^{\frac{m-n(n+1)}{n}\pi\sqrt{-1}},$$
the upper or lower sign being taken according as n is odd or even. To the second member we may give the form
$$\pm\frac{n}{2\sqrt{-1}}e^{\frac{m\pi}{n}\sqrt{-1}}\left(\cos(n+1)\pi-\sqrt{-1}\sin(n+1)\pi\right)=\frac{n}{2\sqrt{-1}}e^{\frac{m\pi}{n}\sqrt{-1}},$$
since $\sin(n+1)\pi=0$, $\cos(n+1)\pi=\pm1$, as n is odd or even.

Thus when $j=n-1$, we have
$$\Sigma_r \sin\left(\frac{m-r}{n}\pi\right)\alpha_j^{-r}=\frac{n}{2\sqrt{-1}}e^{\frac{m\pi}{n}\sqrt{-1}}.$$
In the same way when $j=n$, we find
$$\Sigma_r \sin\left(\frac{m-r}{n}\pi\right)\alpha_j^{-r}=-\frac{n}{2\sqrt{-1}}e^{\frac{-m\pi}{n}\sqrt{-1}}.$$

It results therefore that, according as j is less than $n-1$, equal to $n-1$, or equal to n, we shall have
$$C_j=0,\ \text{or}\ \frac{m}{n\pi}\frac{e^{\frac{m}{n}\pi\sqrt{-1}}}{2\sqrt{-1}},\ \text{or}\ \frac{-m}{n\pi}\frac{e^{\frac{-m}{n}\pi\sqrt{-1}}}{2\sqrt{-1}}.$$
In the general integral (II), art. 7, we shall therefore have
$$\Sigma C_i=\frac{m}{n\pi}\left(\frac{e^{\frac{m\pi}{n}\sqrt{-1}}-e^{-\frac{m\pi}{n}\sqrt{-1}}}{2\sqrt{-1}}\right)=\frac{m}{n\pi}\sin\frac{m\pi}{n},$$
$$u=1+\frac{m}{n\pi}\left\{\frac{e^{\frac{m\pi}{n}\sqrt{-1}}}{2\sqrt{-1}}\int_0^\infty dv\,v^{\frac{m}{n}-1}\log(1-\alpha_{n-1}x\mathrm{V})-\frac{e^{\frac{-m\pi}{n}\sqrt{-1}}}{2\sqrt{-1}}\int_0^\infty dv\,v^{\frac{m}{n}-1}\log(1-\alpha_n x\mathrm{V})\right\},\quad(5)$$
where $\mathrm{V}=\frac{v^{\frac{n-1}{n}}}{1+v}$.

Now $$\alpha_{n-1}=e^{(2(n-1)+1)\pi\sqrt{-1}}=e^{(2n-1)\pi\sqrt{-1}}=e^{-\pi\sqrt{-1}},$$

$$\alpha_n\ =e^{(2n+1)\pi\sqrt{-1}}\ =e^{\pi\sqrt{-1}};$$

therefore, finally,

$$u=1+\frac{m}{2n\pi\sqrt{-1}}\left\{e^{\frac{m\pi}{n}\sqrt{-1}}\int_0^\infty dv\, v^{\frac{m}{n}-1}\log\left(1-e^{\frac{-\pi}{n}\sqrt{-1}}x\mathrm{V}\right)-e^{\frac{-m\pi}{n}\sqrt{-1}}\int_0^\infty dv\, v^{\frac{m}{n}-1}\log\left(1-e^{\frac{\pi}{n}\sqrt{-1}}x\mathrm{V}\right)\right\}. \quad (6$$

It is seen, from the form of this expression, that it represents a *real* value.

If we substitute v for $v^{\frac{1}{n}}$, a change which does not affect the limits, there results

$$u=y^m=1+\frac{m}{2\pi\sqrt{-1}}\left\{e^{\frac{m\pi}{n}\sqrt{-1}}\int_0^\infty dv\, v^{m-1}\log\left(1-e^{\frac{-\pi}{n}\sqrt{-1}}x\mathrm{V}\right)-e^{-\frac{m\pi}{n}\sqrt{-1}}\int_0^\infty dv\, v^{m-1}\log\left(1-e^{\frac{\pi}{n}\sqrt{-1}}x\mathrm{V}\right)\right\}$$

in which $\mathrm{V}=\frac{v^{n-1}}{1+v^n}$. This expression we shall now reduce to an equivalent *real* form.

Reduction of the expression for y^m.

9. We shall somewhat simplify the general expression above found for y^m by integrating by parts. The integrated portion will be found to vanish at both limits.

Representing $\frac{d\mathrm{V}}{dv}$ by V', we have

$$\int mv^{m-1}\log\left(1-e^{\frac{\pm\pi}{n}\sqrt{-1}}x\mathrm{V}\right)dv=v^m\log\left(1-e^{\frac{\pm\pi}{n}\sqrt{-1}}x\mathrm{V}\right)+xe^{\frac{\pm\pi}{n}\sqrt{-1}}\int\frac{v^m\mathrm{V}'dv}{1-e^{\frac{\pm\pi}{n}\sqrt{-1}}x\mathrm{V}}.$$

Now, expanding the logarithm in the integrated portion, and putting for V its value $\frac{v^{n-1}}{1+v^n}$, we see that that portion will consist of a series of terms of the form

$$\frac{\mathrm{A}v^{m+(n-1)r}}{(1+v^n)^r},$$

r being for each such term a positive integer.

All these terms vanish when $v=0$, since, by the conditions to which m is subject, $m+(n-1)r$ is positive.

Again, they vanish when v is made infinite, since in this case

$$\frac{\mathrm{A}v^{m+(n-1)r}}{(1+v^n)^r}=\mathrm{A}v^{m-r},$$

and, by the conditions relative to m, the index $m-r$ is negative.

We have, then, on applying the above reduction to the terms of the general value of y^m,

$$y^m=1+\frac{1}{2\pi\sqrt{-1}}\left\{e^{\frac{(m-1)\pi}{n}\sqrt{-1}}\int_0^\infty\frac{xv^m\mathrm{V}'dv}{1-x\mathrm{V}e^{-\frac{\pi}{n}\sqrt{-1}}}-e^{\frac{-(m-1)\pi}{n}\sqrt{-1}}\int_0^\infty\frac{xv^m\mathrm{V}'dv}{1-x\mathrm{V}e^{\frac{\pi}{n}\sqrt{-1}}}\right\}.$$

Now substitute for the imaginary exponentials their trigonometrical value, and there results

$$y^m=1+\frac{x}{\pi}\int_0^\infty\frac{\left(\sin\left(\frac{m-1}{n}\pi\right)-x\mathrm{V}\sin\frac{m\pi}{n}\right)v^m\mathrm{V}'dv}{1-2x\mathrm{V}\cos\frac{\pi}{n}+x^2\mathrm{V}^2}.$$

As x enters this expression only in combination with V, it is suggested to us to represent xV by V. If we do this the final theorem will be

THEOREM. *If* y^m *represent the* m*th power of that real root of the equation*

$$y^n-xy^{n-1}-1=0$$

which reduces to 1 *when* $x=0$, *then, supposing* m *to be between the limits* 1 *and* $-n+1$, *the value of* y^m *will be*

$$y^m=1+\frac{1}{\pi}\int_0^\infty\frac{\left(\sin\left(\frac{m-1}{n}\pi\right)-\mathrm{V}\sin\frac{m\pi}{n}\right)v^m\frac{d\mathrm{V}}{dv}dv}{1-2\mathrm{V}\cos\frac{\pi}{n}+\mathrm{V}^2}, \quad \ldots\ldots\ldots \text{(IV)}$$

in which

$$\mathrm{V}=\frac{xv^{n-1}}{1+v^n}.$$

10. Hence too we have the value of a remarkable definite integral, viz.

$$\int_0^\infty\frac{\left(\sin\frac{m-1}{n}\pi-\mathrm{V}\sin\frac{m\pi}{n}\right)\frac{d\mathrm{V}}{dv}v^m dv}{1-2\mathrm{V}\cos\frac{\pi}{n}+\mathrm{V}^2}=\pi(y^m-1) \quad \ldots\ldots \text{(V)}$$

under the above conditions and with the above interpretations.

It may be desirable to verify this result.

Since
$$\mathrm{V}=\frac{xv^{n-1}}{1+v^n},$$

we shall have
$$\frac{d\mathrm{V}}{dv}=\frac{(n-1)\mathrm{V}}{v}-\frac{n\mathrm{V}^2}{x},$$

so that the definite integral is resolvable into

$$(n-1)\int_0^\infty\frac{v^{m-1}\mathrm{V}\left(\sin\frac{(m-1)\pi}{n}-\mathrm{V}\sin\frac{m\pi}{n}\right)\mathrm{V}'dv}{1-2\mathrm{V}\cos\frac{\pi}{n}+\mathrm{V}^2}$$

$$-\frac{n}{x}\int_0^\infty\frac{v^m\mathrm{V}^2\left(\sin\frac{(m-1)\pi}{n}-\mathrm{V}\sin\frac{m\pi}{n}\right)dv}{1-2\mathrm{V}\cos\frac{\pi}{n}+\mathrm{V}^2}.$$

Now it may be shown that

$$\frac{\sin\left(\frac{m-1}{n}\pi\right)-V\sin\frac{m\pi}{n}}{1-2V\cos\frac{\pi}{n}+V^2}=\Sigma_r\sin\left(\frac{m-r-1}{n}\pi\right)V^r,$$

the summation with respect to r extending from $r=0$ to $r=\infty$. Hence the first member of (V) may be developed in the form

$$(n-1)\Sigma_r\int_0^\infty v^{m-1}\sin\left(\frac{m-r-1}{n}\pi\right)V^{r+1}dv$$

$$\frac{-n}{x}\Sigma_r\int_0^\infty v^m\sin\left(\frac{m-r-1}{n}\pi\right)V^{r+2}\,dv.$$

Now
$$\int_0^\infty v^{m-1}V^{r+1}dv=x^{r+1}\int_0^\infty\frac{v^{m+(r+1)(n-1)}dv}{(1+v^n)^{r+1}}$$

$$=\frac{\Gamma\left(\frac{m+(r+1)(n-1)}{n}\right)\Gamma\left(\frac{r-m+1}{n}\right)}{n\Gamma(r+1)}x^{r+1},$$

and
$$\int_0^\infty v^mV^{r+1}dv=x^{r+2}\frac{\Gamma\left(\frac{m+1+(r+2)(n-1)}{n}\right)\Gamma\left(\frac{r-m+1}{n}\right)}{n\Gamma(r+2)}x^{r+2}$$

$$=\frac{m+(r+1)(n-1)}{n(r+1)}\;\frac{\Gamma\left(\frac{m+(r+1)(n-1)}{n}\right)\Gamma\left(\frac{r-m+1}{n}\right)}{n\Gamma(r+1)}x^{r+2}.$$

Hence the total coefficient of x^{r+1} in (V) is

$$\sin\frac{(m-r-1)\pi}{n}\;\frac{\Gamma\left(\frac{m+(r+1)(n-1)}{n}\right)\Gamma\left(\frac{r-m+1}{n}\right)}{n\Gamma(r+1)}\left\{n-1-n\times\frac{m+(r+1)(n-1)}{n(r+1)}\right\}$$

$$=\frac{\sin\frac{(m-r-1)\pi}{n}\Gamma\left(\frac{m+(r+1)(n-1)}{n}\right)\Gamma\left(\frac{r-m+1}{n}\right)}{n\Gamma(r+1)}\times\frac{-m}{r+1},$$

and therefore that of x^r is

$$\frac{\sin\left(\frac{m-r}{n}\pi\right)\Gamma\left(\frac{m+r(n-1)}{n}\right)\Gamma\left(\frac{r-m}{n}\right)}{n\Gamma(r)}\times\frac{-m}{r}.$$

Now
$$\Gamma\left(\frac{m+r(n-1)}{n}\right)\Gamma\left(\frac{r-m}{n}\right)=\Gamma\left(\frac{r-m}{n}\right)\Gamma\left(r-\frac{r-m}{n}\right)$$

$$=\left[r-1-\frac{r-m}{n}\right]^{r-1}\frac{\pi}{\sin\left(\frac{r-m}{n}\pi\right)}.$$

Therefore the coefficient of x^r is

$$\frac{m\left[r-1-\frac{r-m}{n}\right]^{r-1}\pi}{n[r]^r};$$

and this is, by art. 2, equal to πu_r in the expansion of y^m in ascending powers of x. Hence, the lowest value of r in the expansion of the definite integral being unity, we see that the value of that integral will be expressed by $\pi(y^m-1)$, as was to be shown.

It will be observed that the function under the sign of definite integration does not become infinite within the limits. Ordinary methods of approximation might therefore be applied. I apprehend, however, that it is not in this direction that the value of such results is to be sought.

INDEX

TO THE

PHILOSOPHICAL TRANSACTIONS

FOR THE YEAR 1864.

A.

B.

C.

D.

J.

L.

M.

N.

P.

R.

S.

T.

V.

W.

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2

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Fig. 1.


Fig. 2.


Fig. 3.


Fig. 4.


Fig. 5.


Fig. 6.


Fig. 7.


Fig. 8.

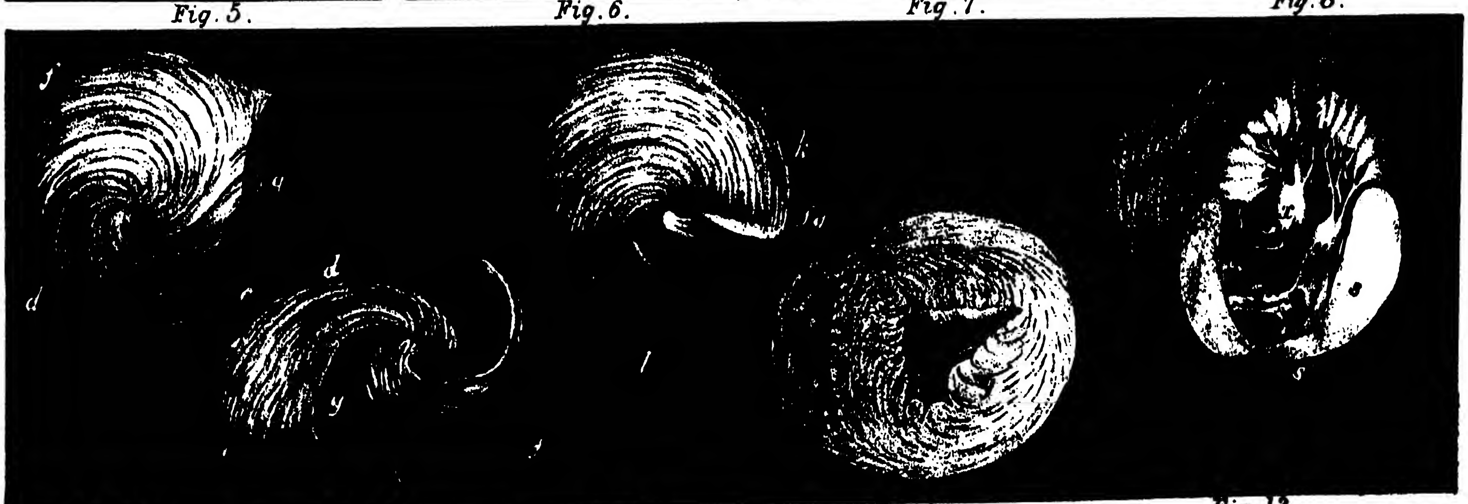
Fig. 9. Fig. 10. Fig. 11. Fig. 12. Fig. 13.


Fig. 14.

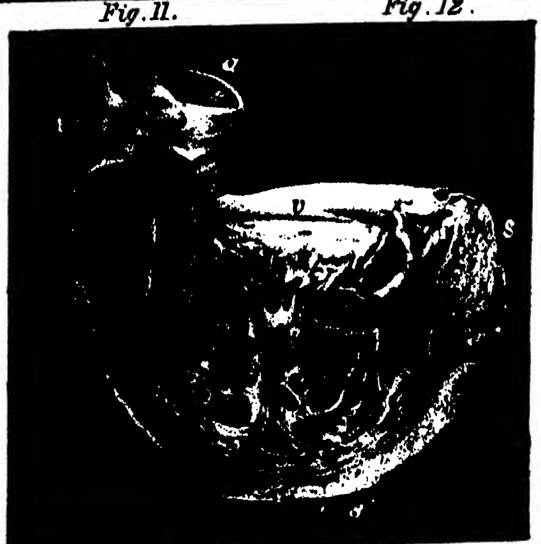
Fig. 15.


Fig. 16. Fig. 17.

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Fig. 18

Fig. 19.

Fig. 20

Fig. 21.

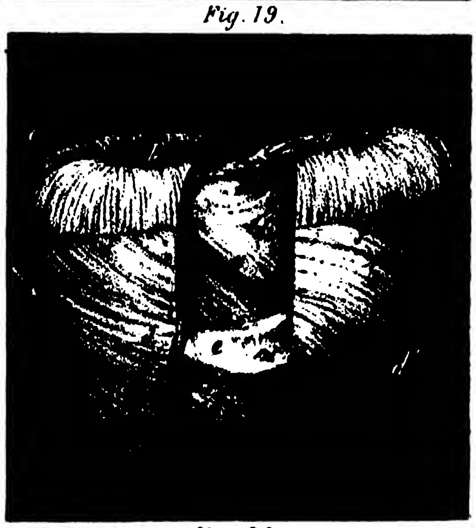
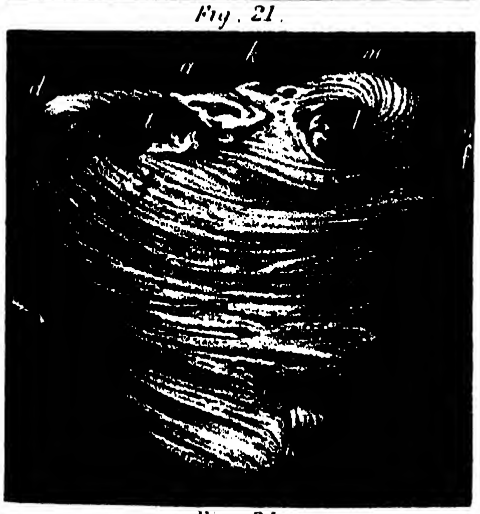


Fig. 22.

Fig. 23.



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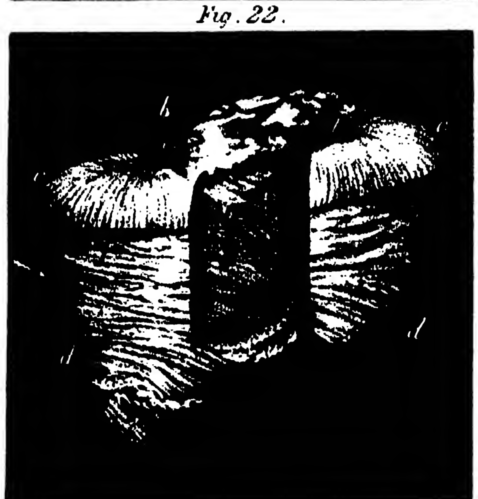

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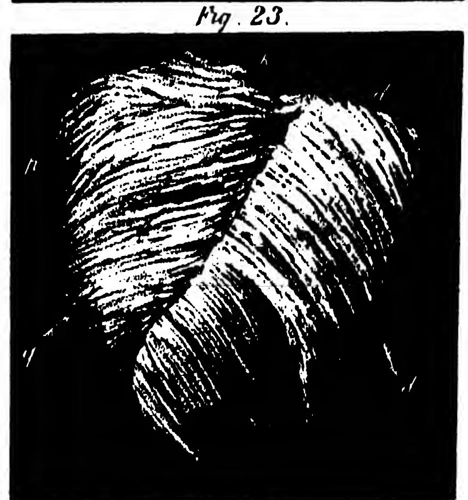

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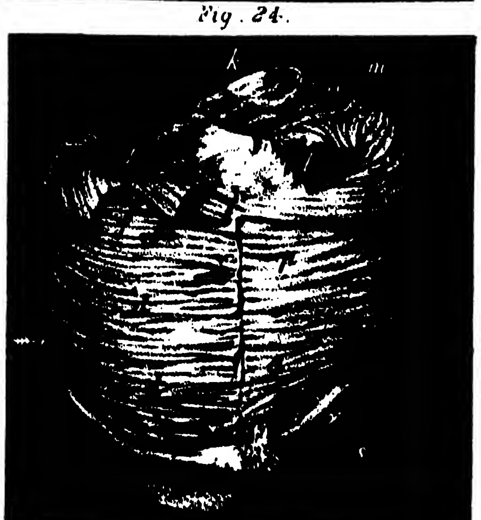

Fig. 27.

Fig. 28.

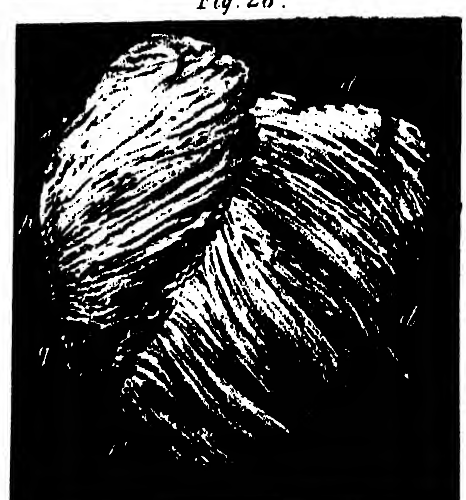

Fig. 29

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Fig. 30.


Fig. 31.


Fig. 32.


Fig. 33.


Fig. 34.


Fig. 35.


Fig. 36.


Fig. 37.


Fig. 38.


Fig. 39.


Fig. 40.


Fig. 41.


Fig. 42.


Fig. 43.


Fig. 44.

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Fig. 45.

Fig. 46.

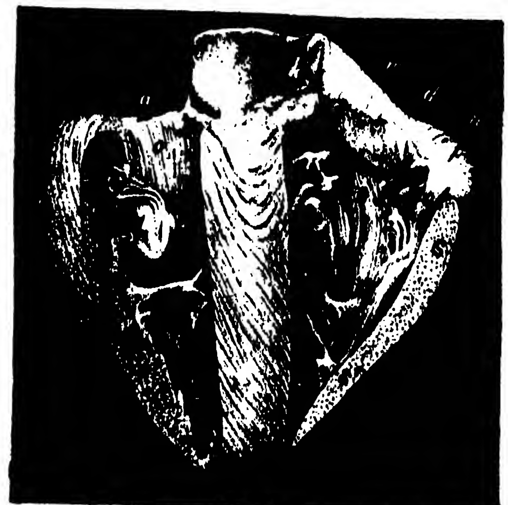

Fig. 47.

Fig. 48.

Fig. 49.

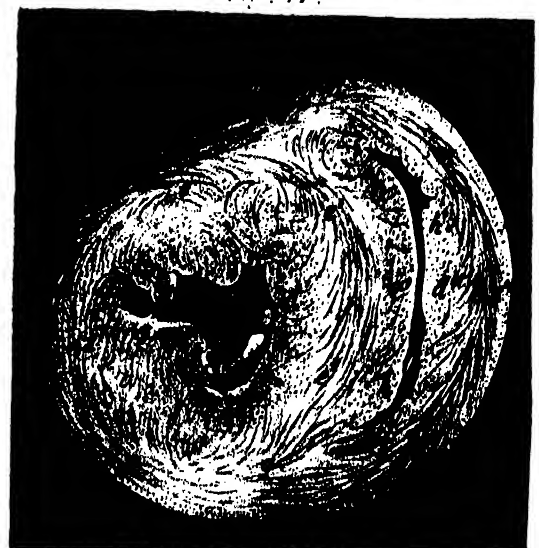

Fig. 50.

Fig. 51.

Fig. 52.

Fig. 53.

Fig. 54.

Diagram. 1.

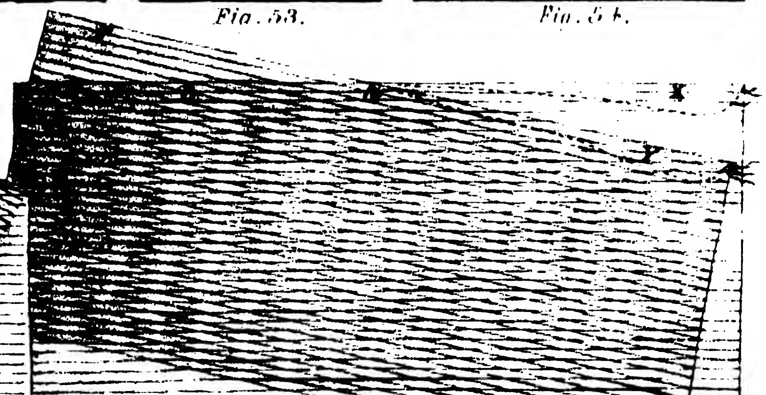

Diagram. 2.

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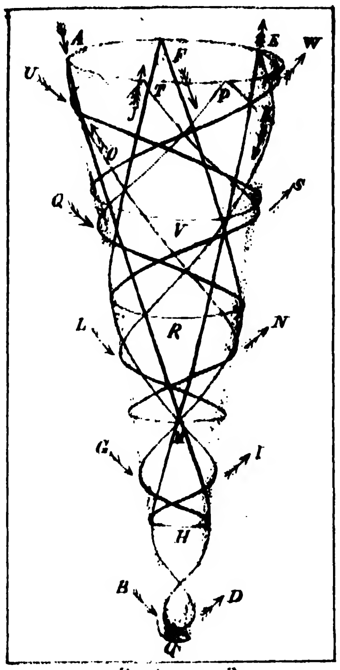
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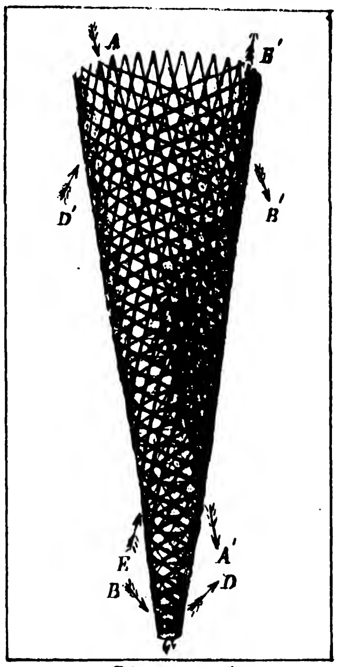
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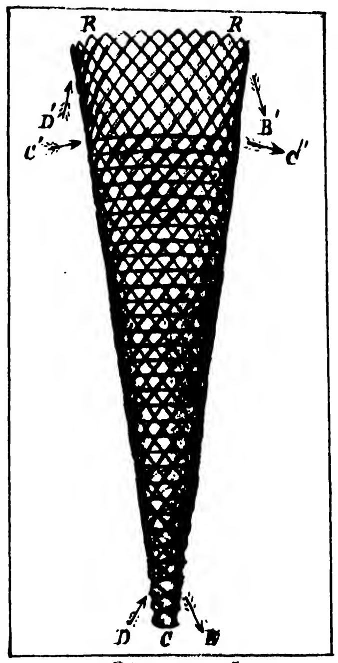
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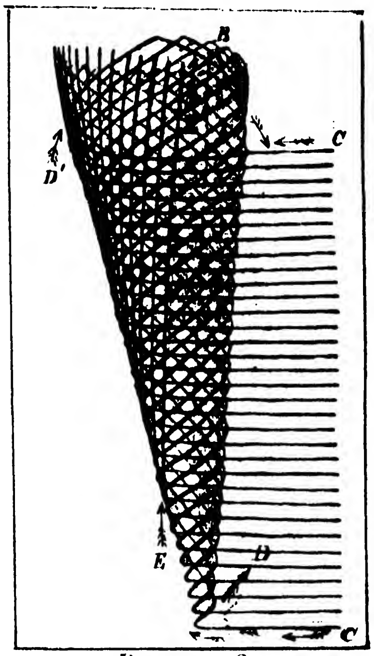
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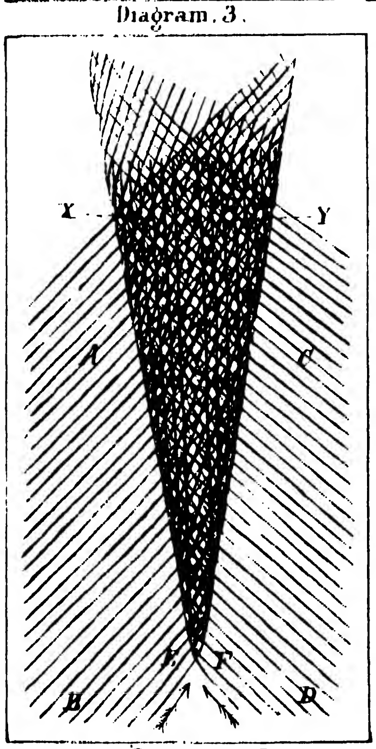
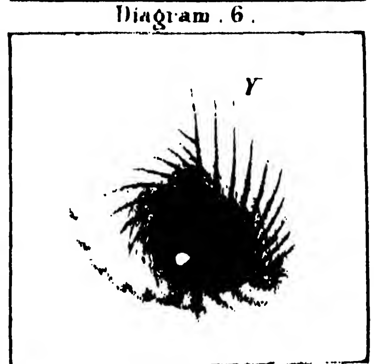


Diagram. 7.


Diagram. 8.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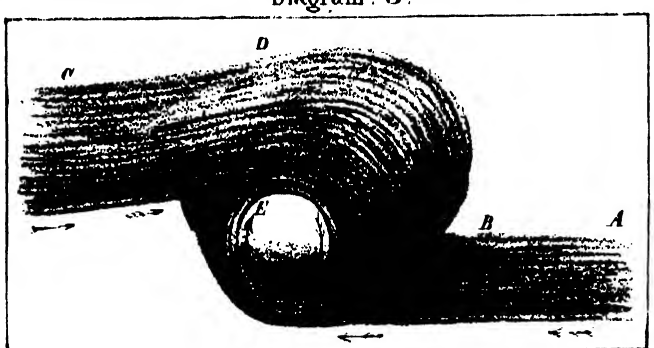


Diagram. 10.


Diagram. 11.

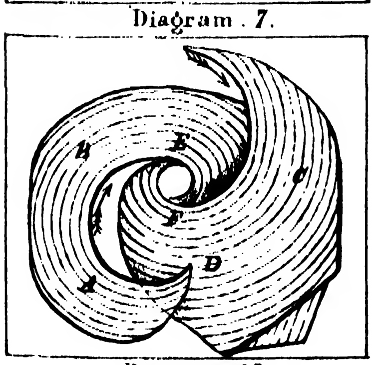


Diagram. 12

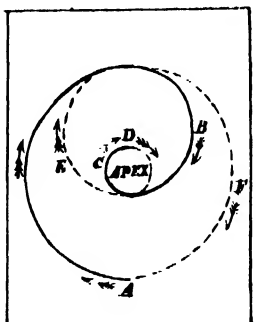


Diagram 13.

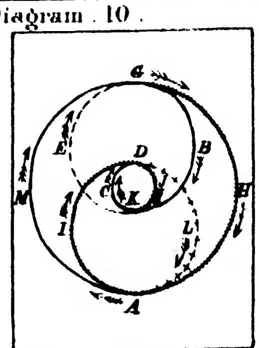
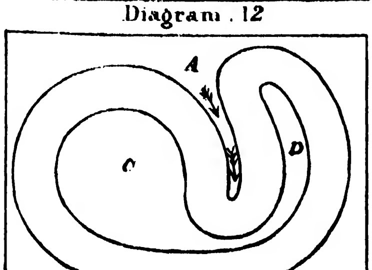


Diagram. 14.


Fig. 55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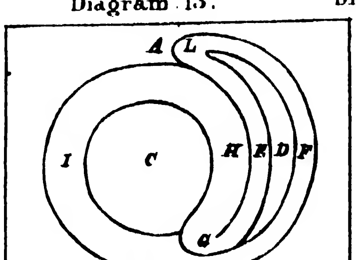
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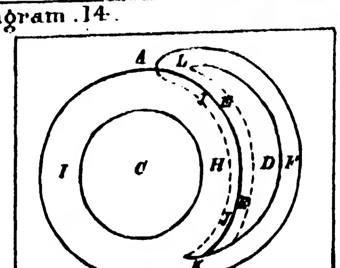
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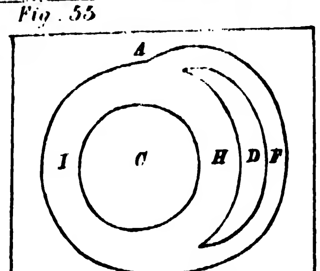
Diagram. 18.

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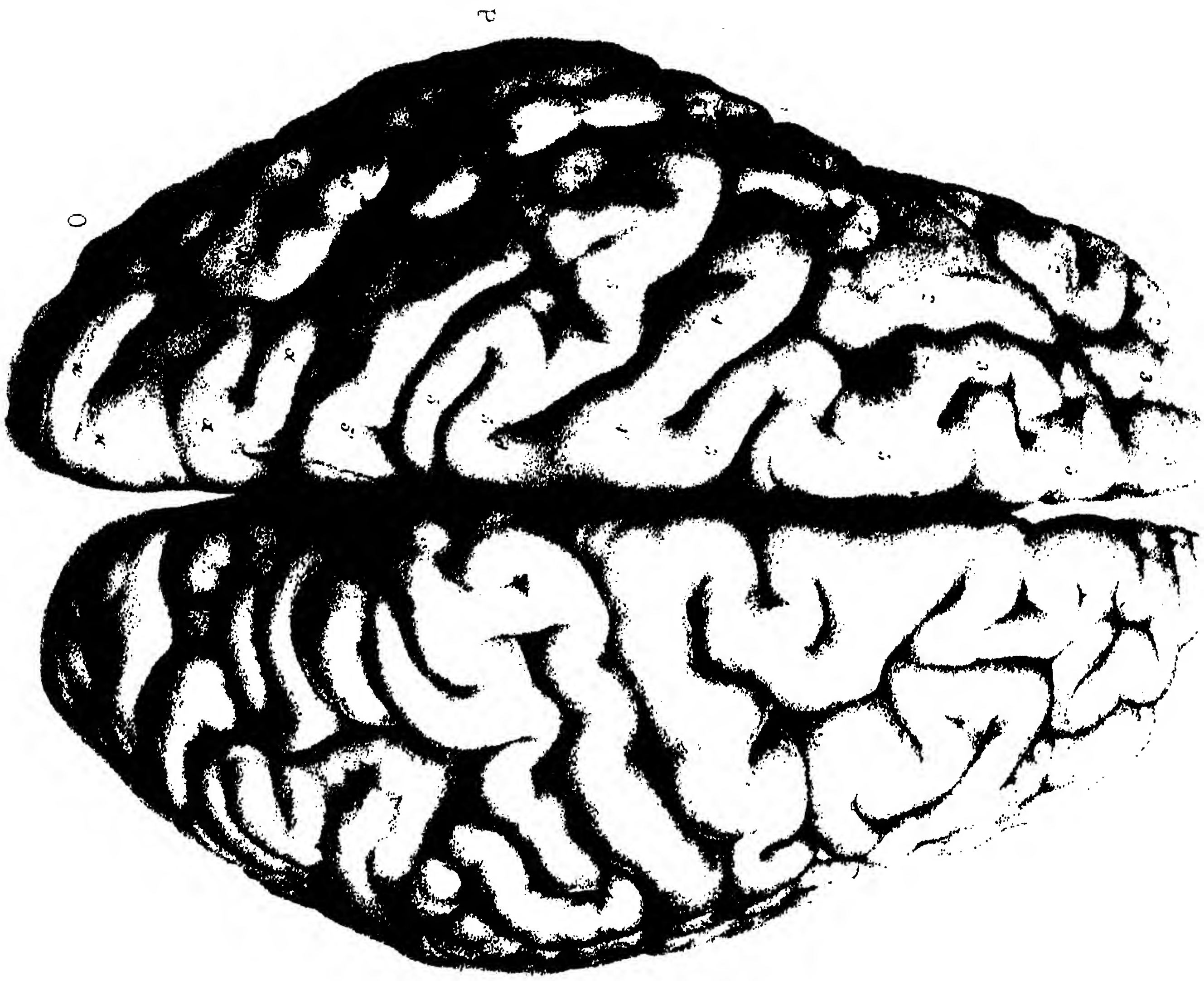
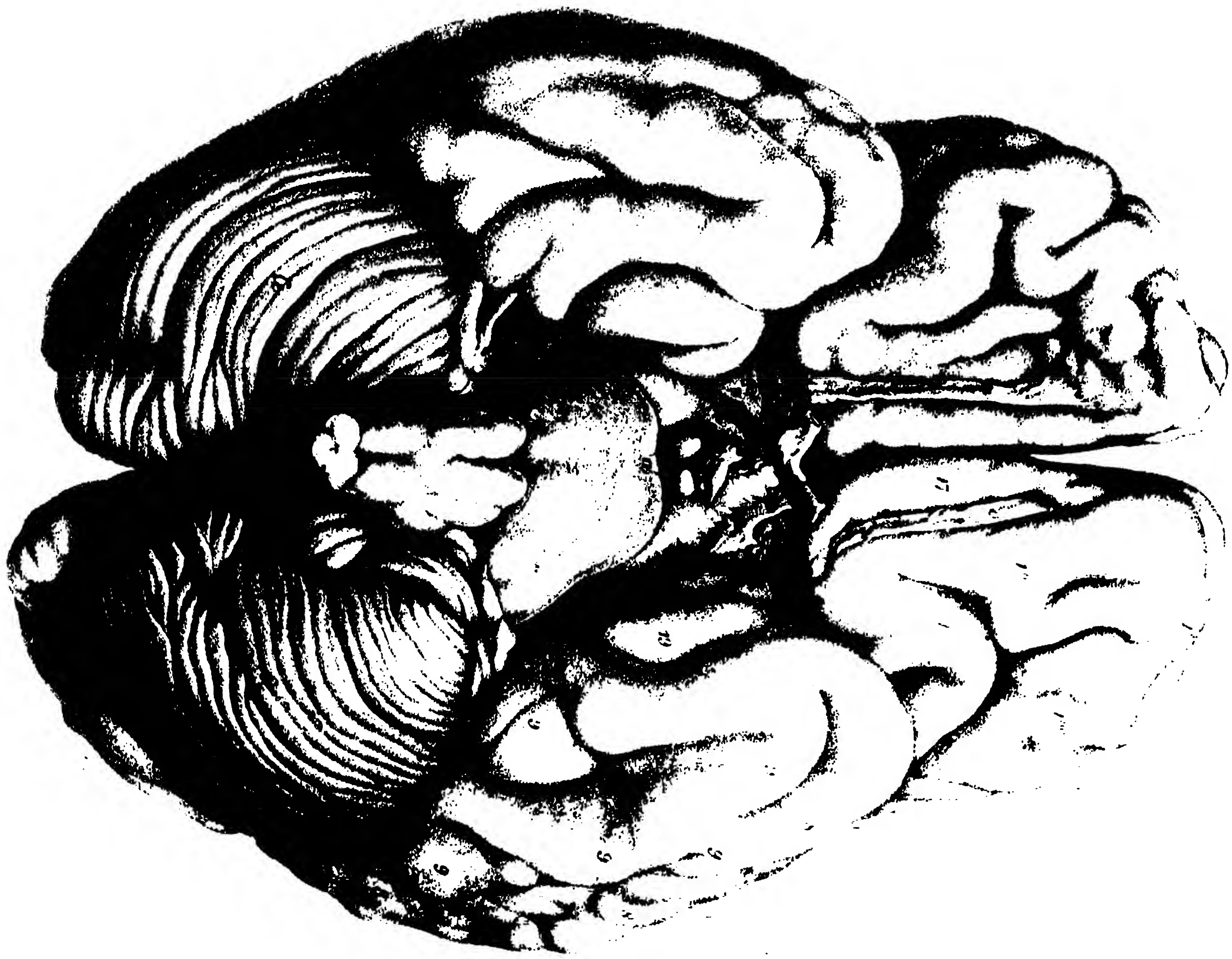
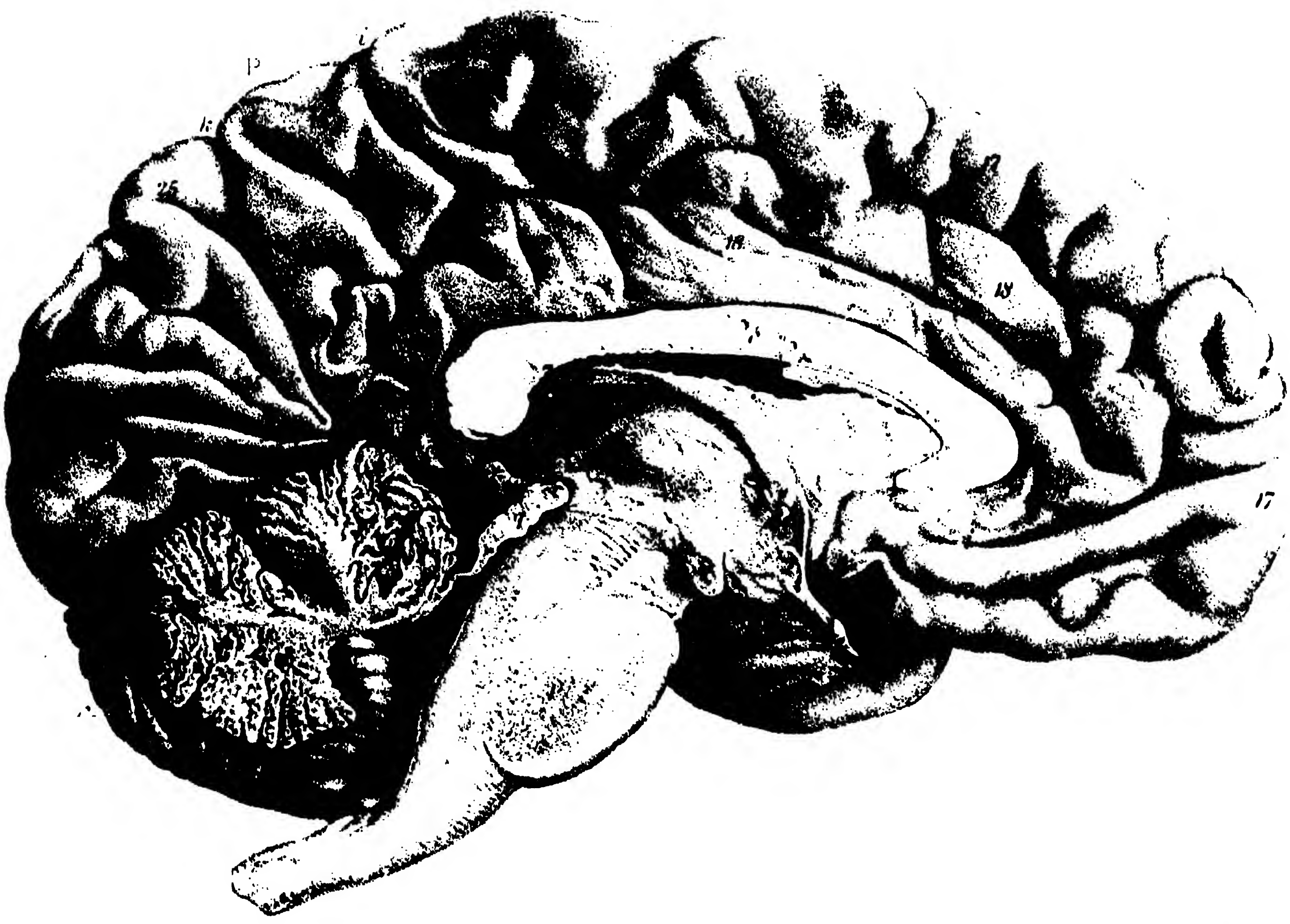






M. Williams, lith.

Fig. 5.

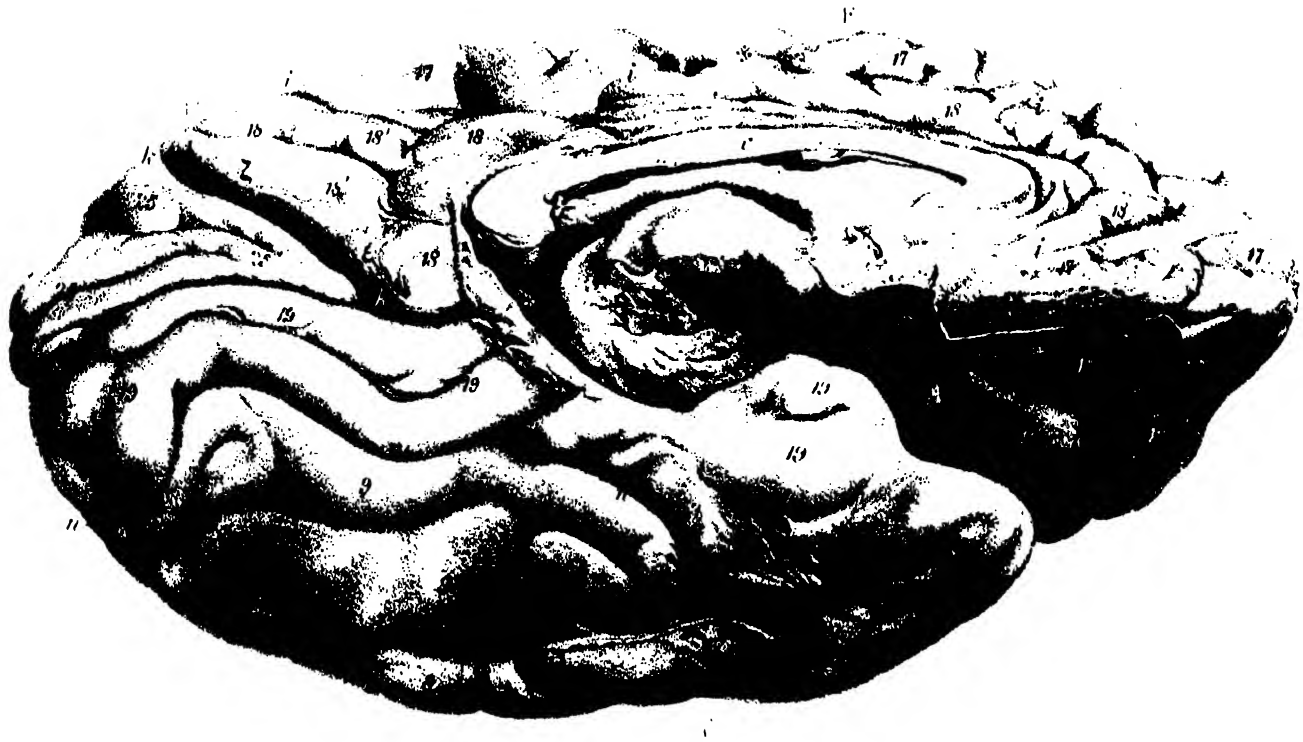
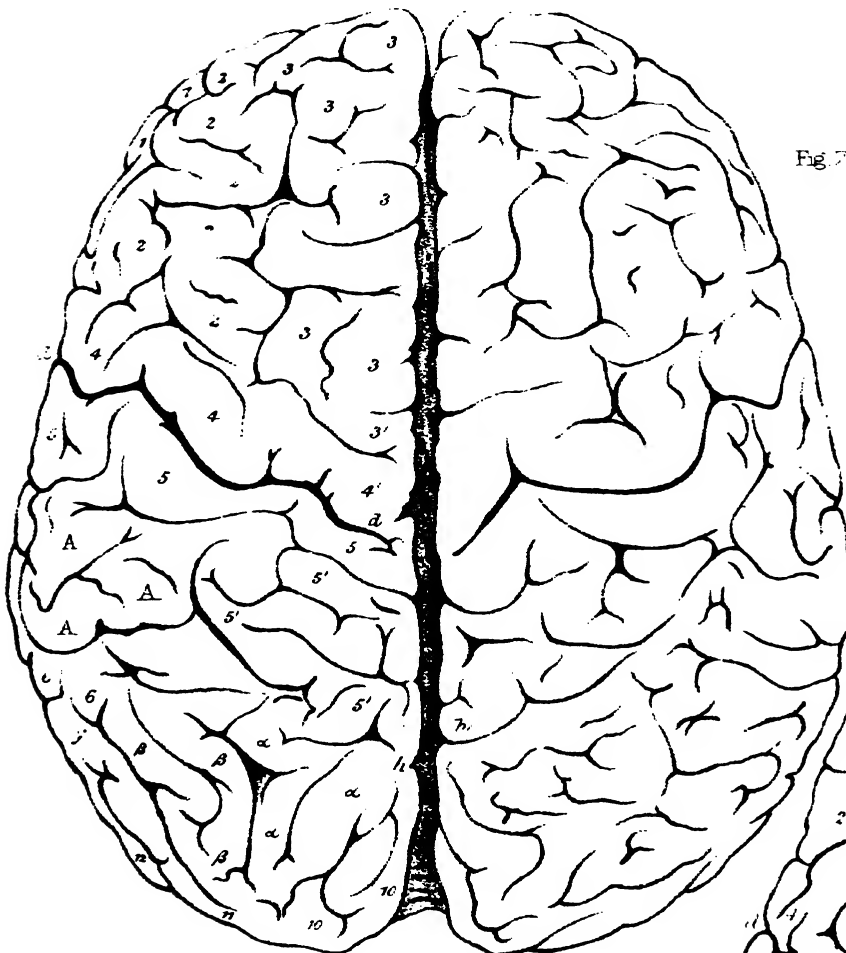


Fig. 6.

Edwin M. Williams, lith.

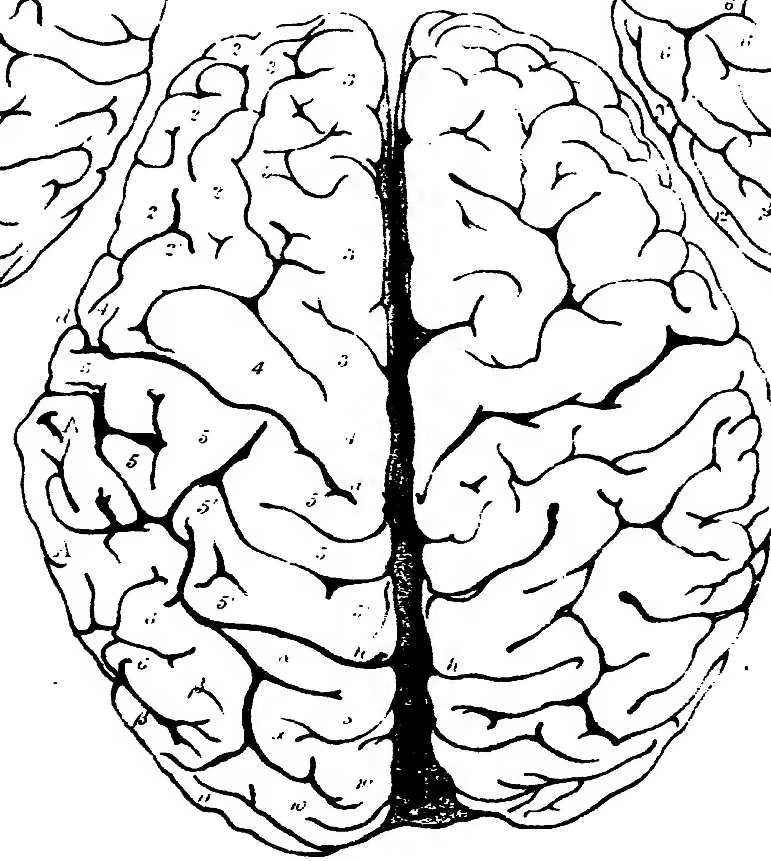
H. Watkins phot.

Fig. 7



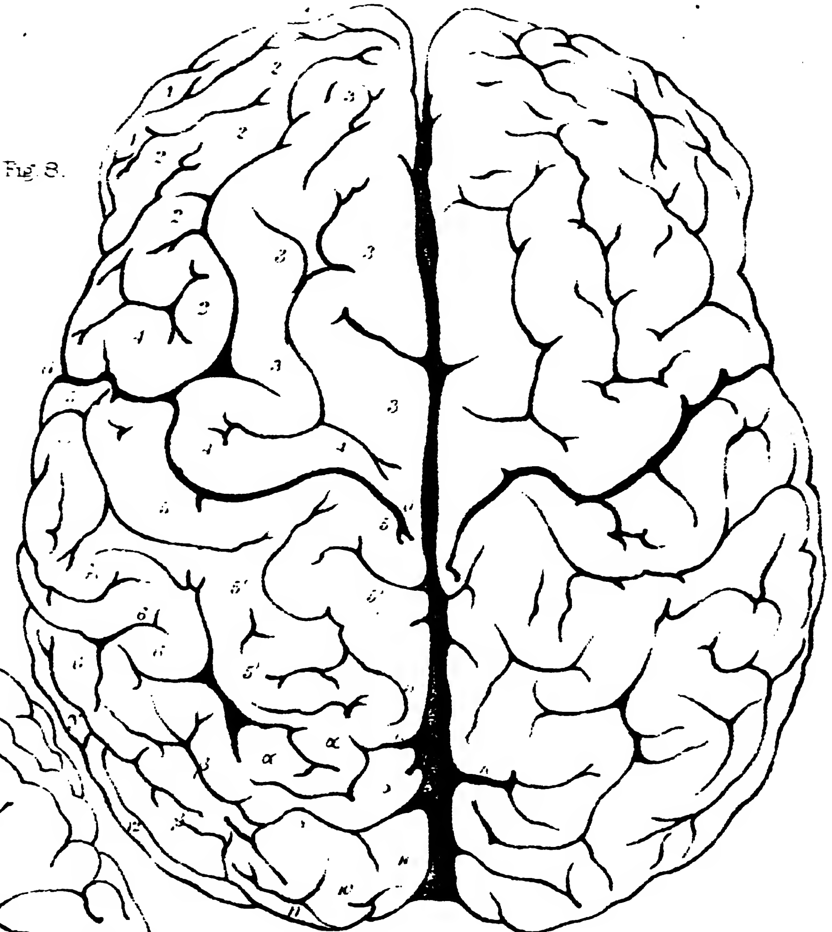
European
(recent.)

Fig. 9.

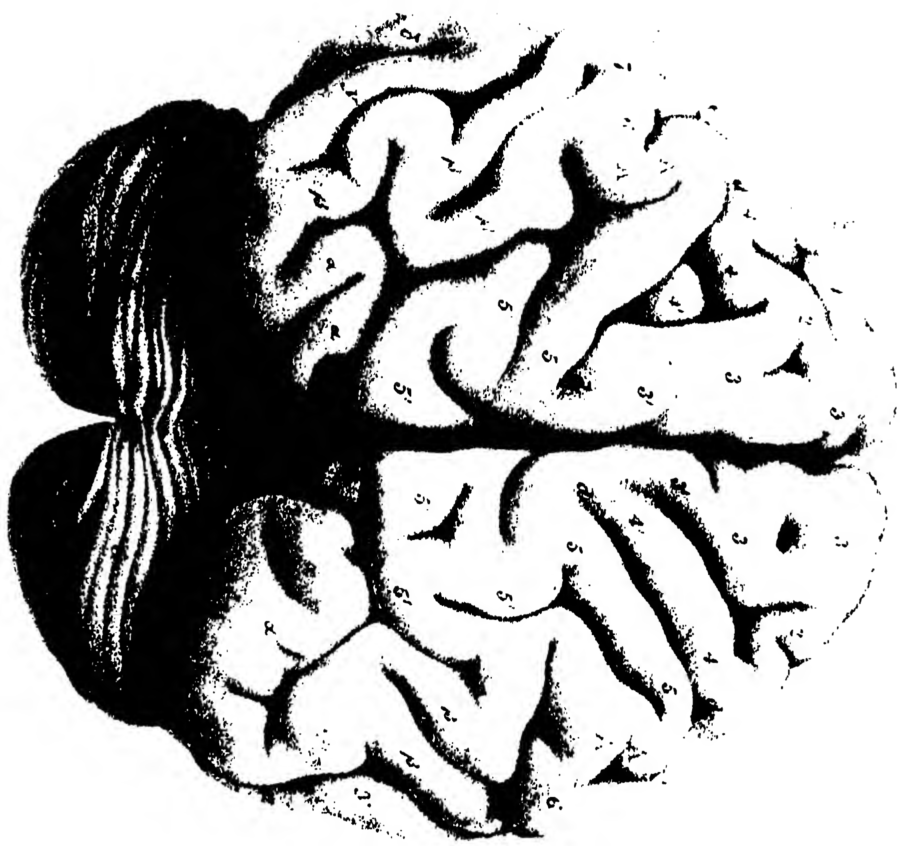


Bushwoman.
(preserved.)

Fig. 8.



Hottentot Venus.
(restored.)



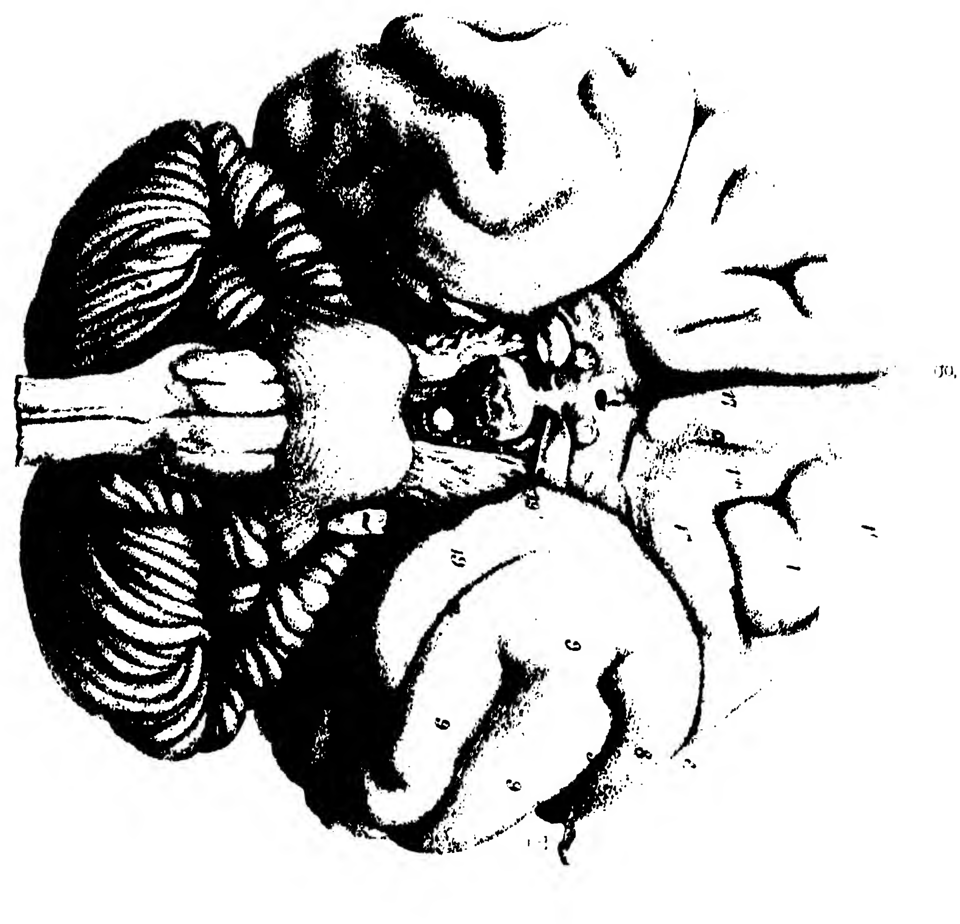


Fig. 16.

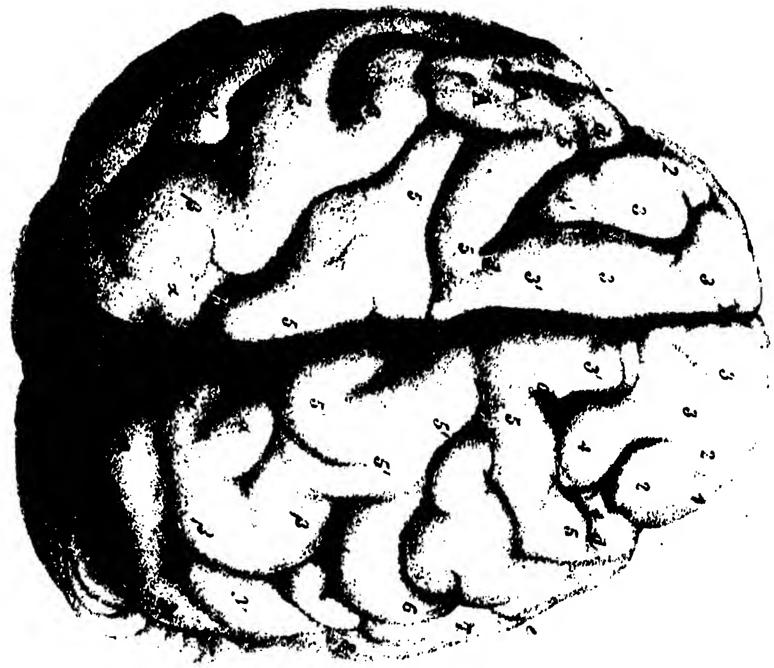


Fig. 14.

Fig. 17.

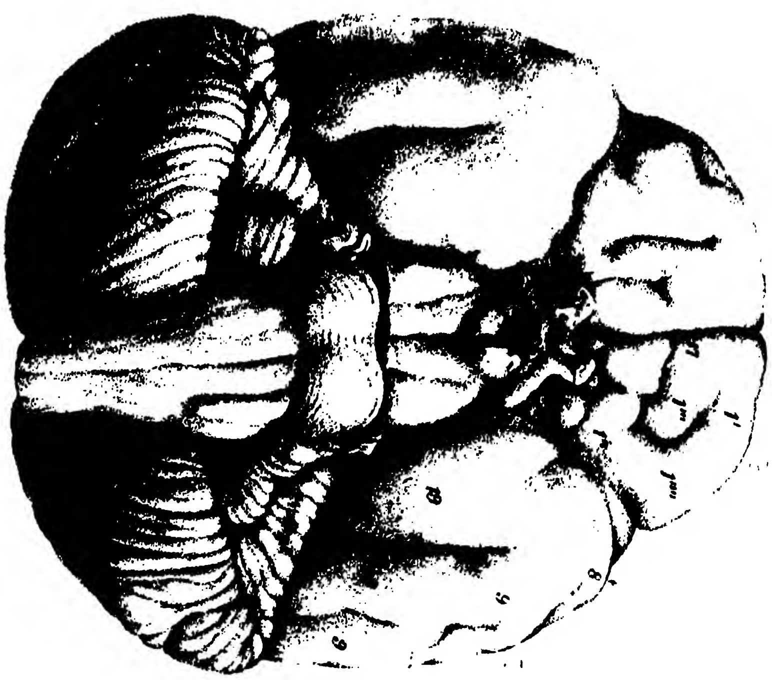
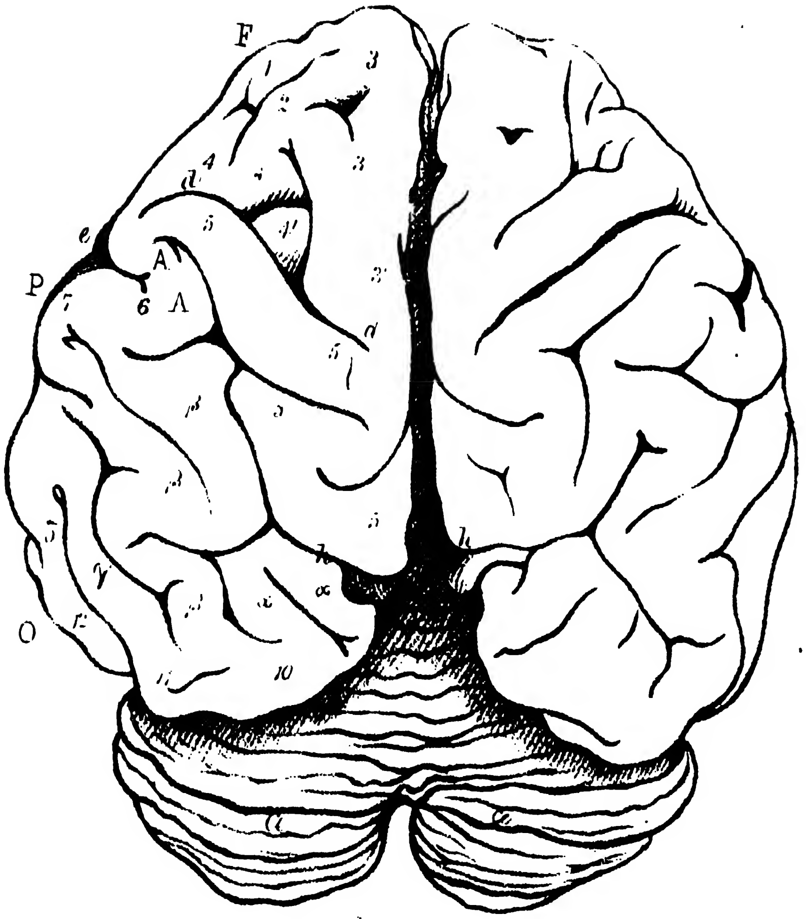


Fig. 15.

H. Watkins, phot.

Edwin M. Williams, lith.

Fig. 18



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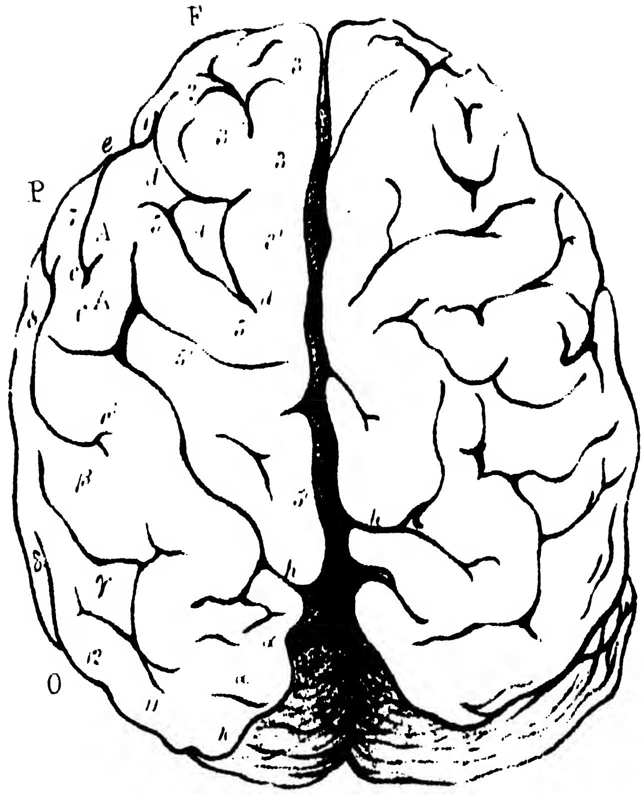


Fig. 21

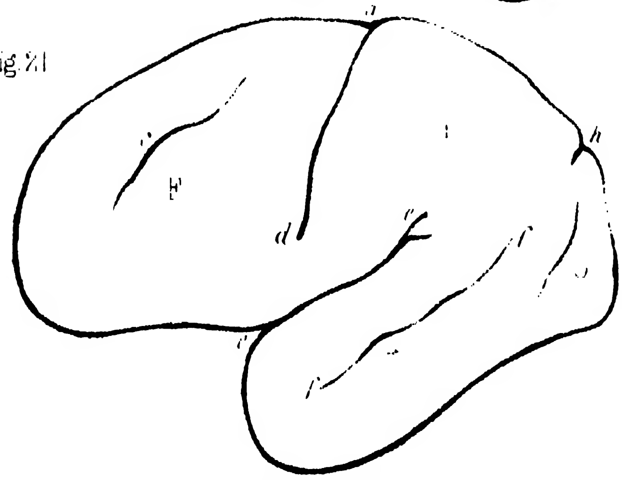


Fig. 20

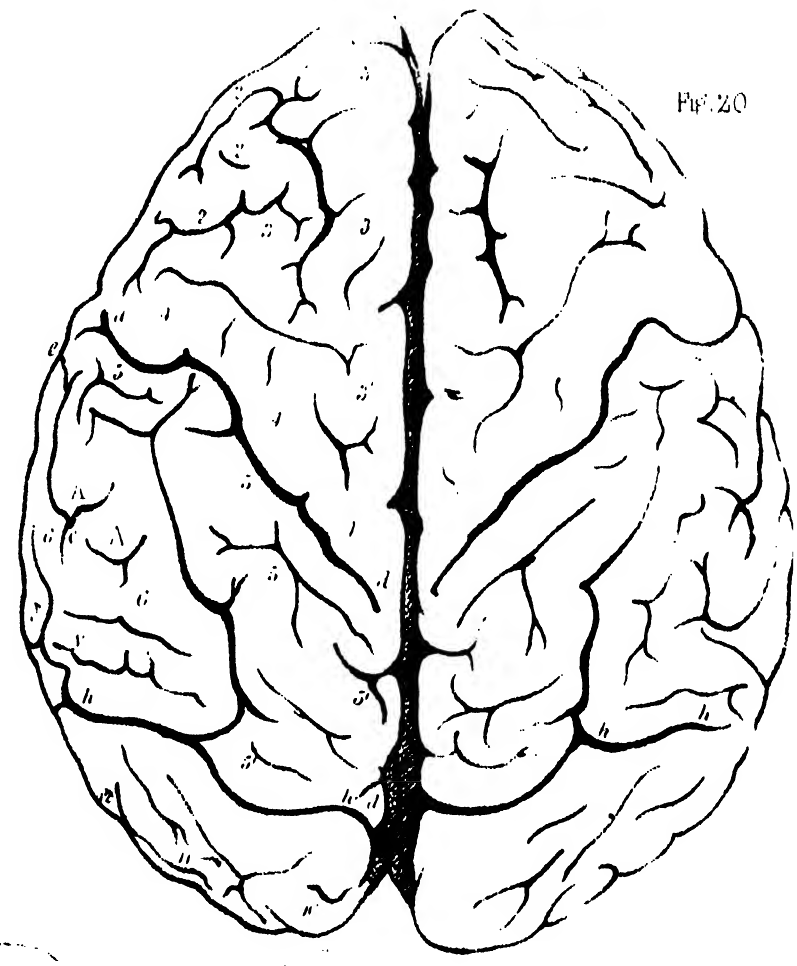


Fig. 22

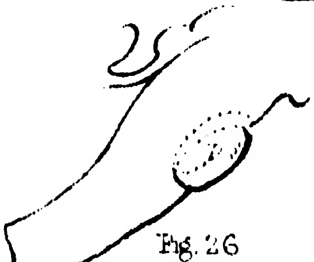
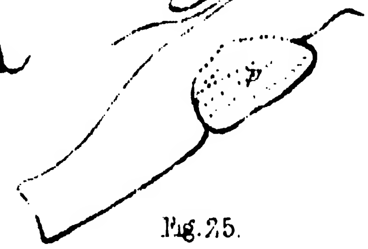


Fig. 26

Fig. 23.

Fig. 24.



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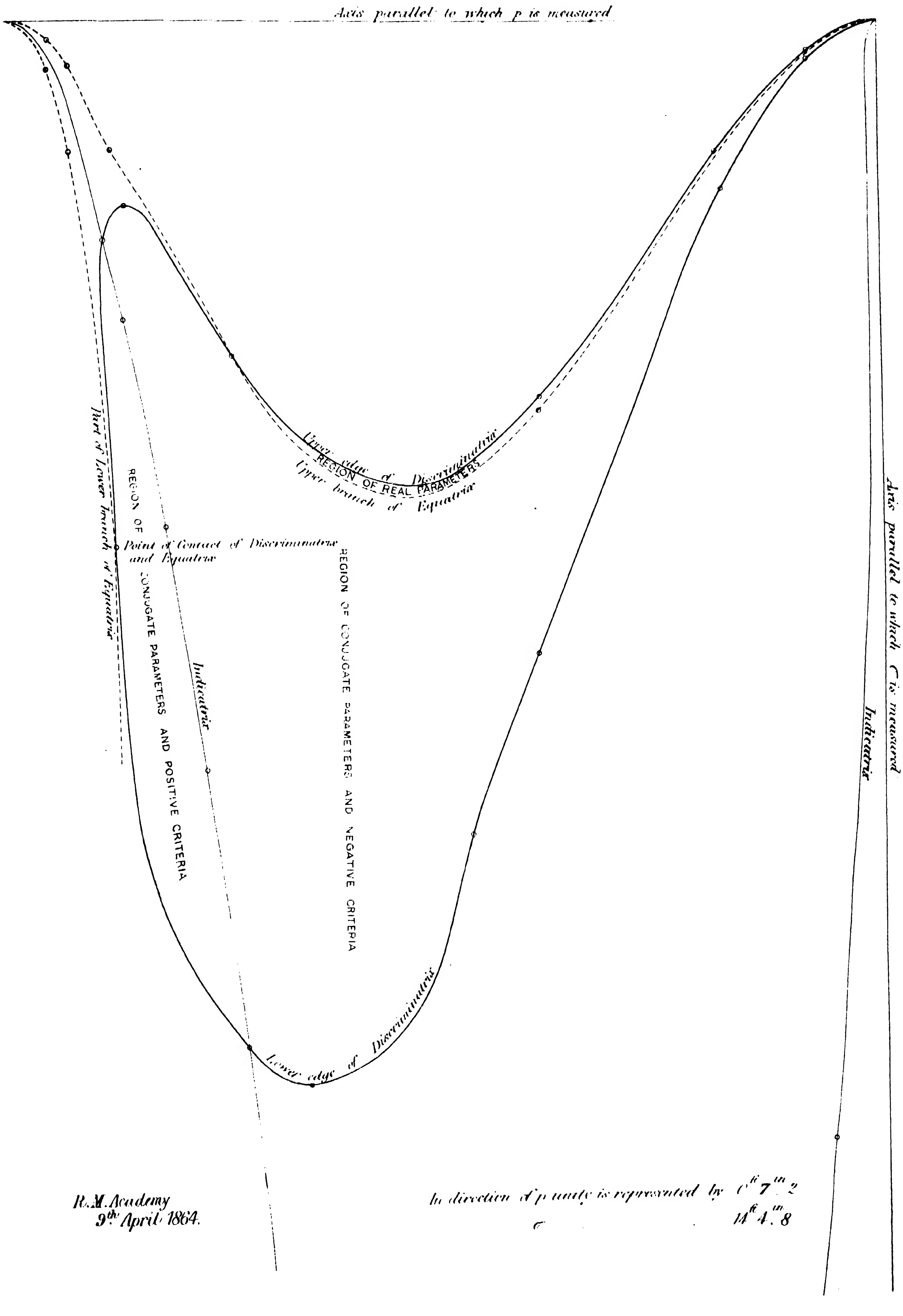
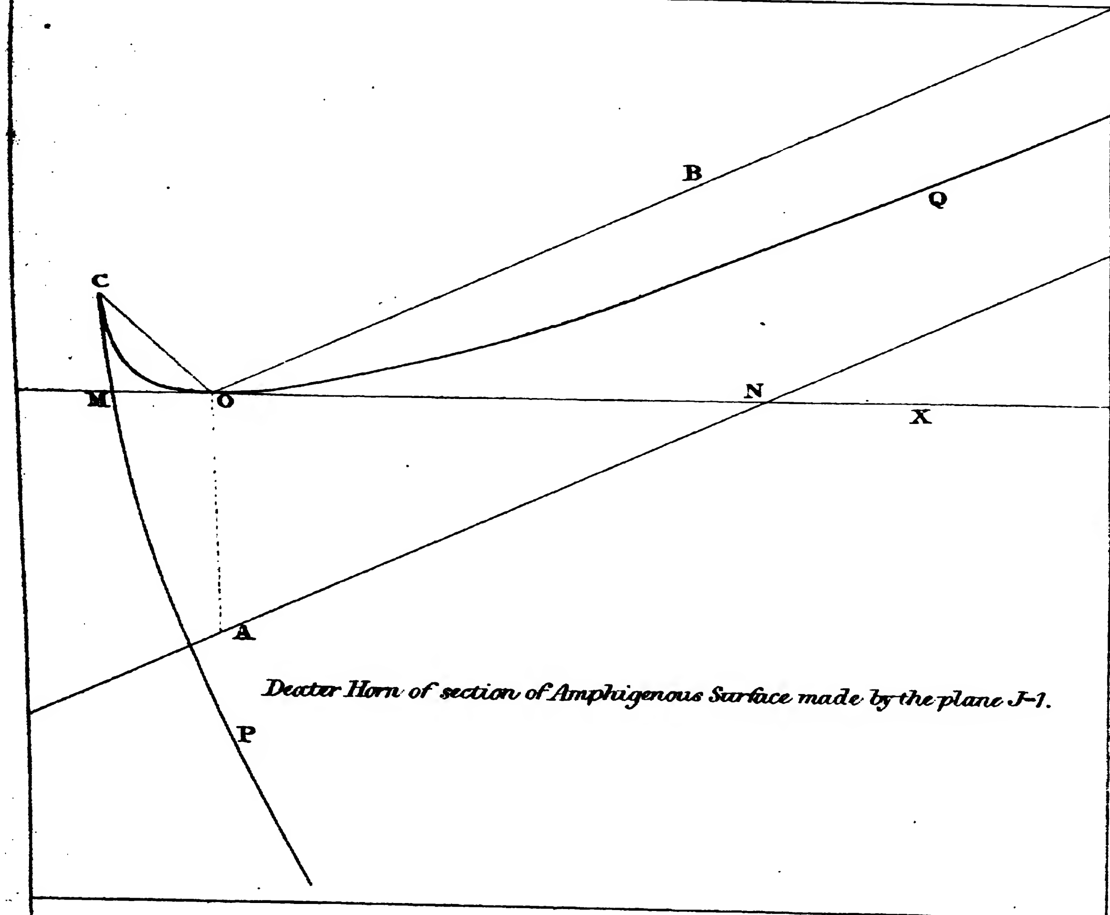
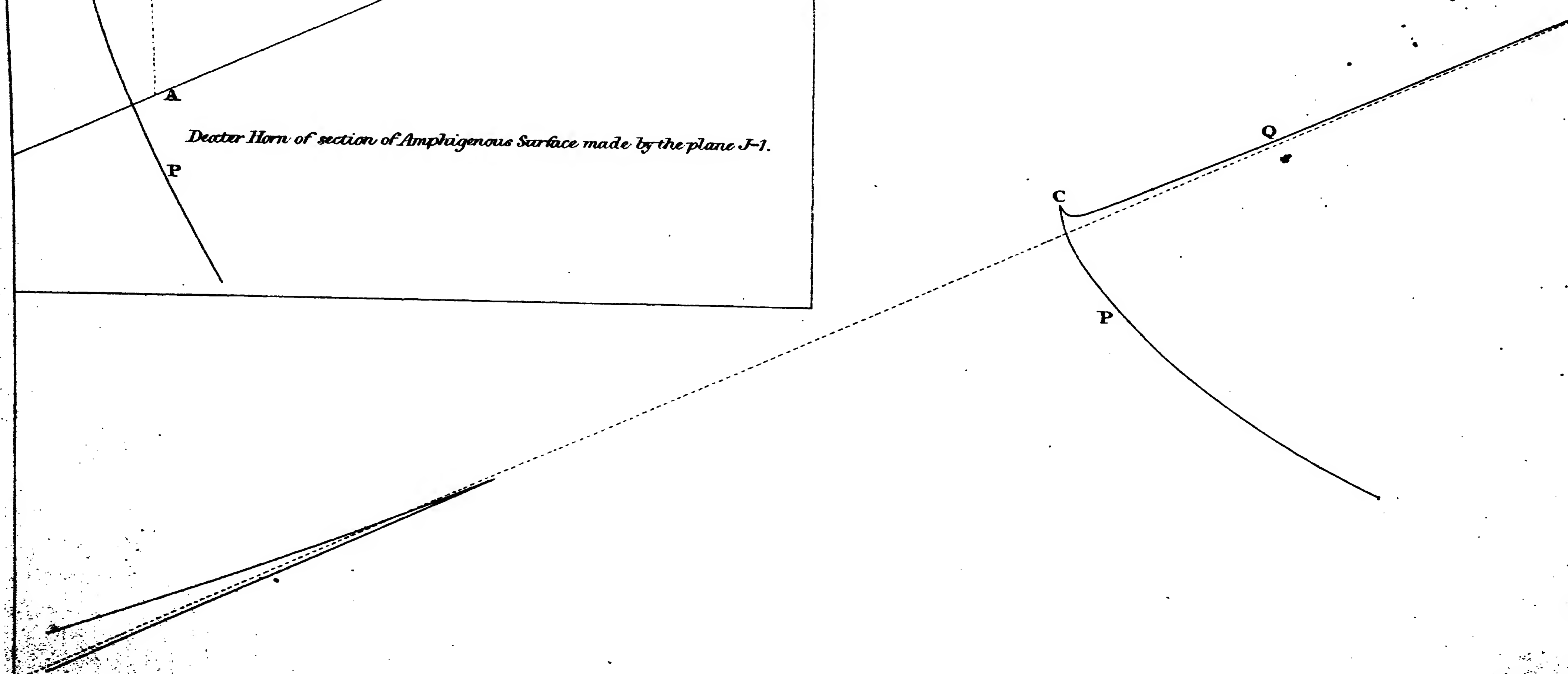


R. M. Academy
9th April 1864.

In direction of p unity is represented by 0ft. 7in. 2
C 14ft. 4in. 8



Dexter Horn of section of Amphigenous Surface made by the plane J–1.



Complete Section of the Amphigenous Surface made by the plane J—1.